Interplay between chirality and electron-electron interactions in graphene systems

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Outline

1. Introduction
2. Electronic structure of graphene multilayers
3. Exciton condensation in graphene double layers
4. Pseudospin magnetism in graphene bilayers
5. Conclusion
Introduction

- What is graphene?

- Why is it important?

Interplay between chirality and electron-electron interactions in graphene systems
1. Introduction

1) Graphene

- Two-dimensional honeycomb lattice of carbon atoms.

- 2D Dirac-like equation with linear dispersion near $K/K'$.

- New electron-electron interaction physics? Example: magnetism, superconductivity
1. Introduction

2) Extraordinary properties of graphene

- Chiral
- Dirac-like equation
  \[ \sigma_{xy} = \pm \frac{(4e^2/h)(1/2 + n)}{1} \]
- Quantum Hall effect
  - Wallace, Phys. Rev. (1947)
  - Novoselov et al., Nature (2005)

- \( \mu > 10^4 \text{cm}^2/\text{Vs} \)
- High mobility

- \( E_g \sim W^{-1} \)
- Device application
  - Han et al., PRL (2007)

Interplay between chirality and electron-electron interactions in graphene systems
Electronic structure of graphene multilayers

⇒ Described by a set of chiral systems
2. Electronic structure of graphene multilayers

1) Electronic structure of monolayer graphene

- Effective theory around $K$

$$H_{B}^{mono} = v_{F} \begin{pmatrix} 0 & pe^{-i\phi_{p}} \\ pe^{i\phi_{p}} & 0 \end{pmatrix}$$

$$E(p) = \pm v_{F} p$$

$v_{F}$ = in-plane velocity
$\phi_{k}$ = $\tan^{-1}(k_{y} / k_{x})$
$p = \hbar k$

Interplay between chirality and electron-electron interactions in graphene systems
2. Electronic structure of graphene multilayers

2) Electronic structure of bilayer graphene

- **4-band model around** $K$

$$H_{4\text{band}}^{bi} = \begin{pmatrix}
0 & v_F p e^{-i\phi} & 0 & 0 \\
v_F p e^{i\phi} & 0 & t_\perp & 0 \\
0 & t_\perp & 0 & v_F p e^{-i\phi} \\
0 & 0 & v_F p e^{i\phi} & 0
\end{pmatrix}$$

Interplay between chirality and electron-electron interactions in graphene systems
2. Electronic structure of graphene multilayers

2) Electronic structure of bilayer graphene

- Effective theory around $K$

\[ H_{bi}^B = -\frac{1}{2m} \begin{pmatrix} 0 & p^2 e^{-i2\phi_p} \\ p^2 e^{i2\phi_p} & 0 \end{pmatrix} \]

\[ E(p) = \pm \frac{p^2}{2m} \]

- $m = t_\perp / 2v_F$

What is the effective theory for arbitrarily stacked multilayers?
3) Chirality and Pseudospin

- Effective theory of monolayer graphene

\[
H_{B}^{\text{mono}} = v_F \begin{pmatrix}
0 & p e^{-i\phi_p} \\
p e^{i\phi_p} & 0
\end{pmatrix} = v_F p \mathbf{\sigma} \cdot \mathbf{n}_1(\phi_p)
\]

\[
\mathbf{n}_1(\phi_p) = (\cos \phi_p, \sin \phi_p)
\]

\[
\phi_p = \tan^{-1}(p_y/p_x)
\]

- Energy spectrum

\[
E_1(p) = \pm v_F p
\]
3) Chirality and Pseudospin

- Effective theory of bilayer graphene

\[
H_B^{bi} = \frac{1}{2m} \begin{pmatrix}
\alpha_b & \beta_t \\
0 & p^2 e^{-i2\phi_p}
\end{pmatrix} = -\frac{p^2}{2m} \sigma \cdot n_2(\phi_p)
\]

\[
n_2(\phi_p) = (\cos 2\phi_p, \sin 2\phi_p)
\]

- Energy spectrum

\[
E_2(p) = \pm \frac{p^2}{2m}
\]

\[
\phi_p = \tan^{-1}(p_y/p_x)
\]
2. Electronic structure of graphene multilayers

3) Chirality and Pseudospin

- Effective theory of $J$-chiral system

\[
H_J = \varepsilon_0 \begin{pmatrix}
0 & p^J e^{-iJ\phi_p} \\
p^J e^{iJ\phi_p} & 0
\end{pmatrix} = \varepsilon_0 p^J \mathbf{\sigma} \cdot \mathbf{n}_J(\phi_p)
\]

\[
\mathbf{n}_J(\phi_p) = (\cos J\phi_p, \sin J\phi_p)
\]

\[
\phi_p = \tan^{-1}(p_y/p_x)
\]

- Energy spectrum

\[
E_J(p) = \pm \varepsilon_0 p^J
\]

Monolayer graphene : $J=1$

Bilayer graphene : $J=2$

Interplay between chirality and electron-electron interactions in graphene systems
2. Electronic structure of graphene multilayers

4) Multilayer stacking

- Example: ABC trilayer

Min and MacDonald, PRB 77, 155416 (2008)

Interplay between chirality and electron-electron interactions in graphene systems
4) Multilayer stacking

- Example: ABA trilayer

$J=1, J=2$ chiral 2D electron gas
5) Effective theory (ABC)  Min and MacDonald, PRB 77, 155416 (2008)

- Identify zero-energy states from the stacking diagram.
  ⇒ Ex: isolated $\alpha_1$ and $\beta_N$ are zero-energy states.

- Obtain the effective theory using degenerate state perturbation theory.

$$H = H_\perp + H_\parallel$$

$$\langle \Psi_r | H | \Psi_{r'} \rangle = \langle \Psi_r | H_\parallel \hat{Q}(-H_\perp^{-1})\hat{Q}H_\parallel^{-1} | \Psi_{r'} \rangle$$

⇒ Ex: $\langle \alpha_1 | H | \beta_N \rangle = -t_\perp (v_F p e^{-i\phi_F} / t_\perp)^N$
6) Chiral decomposition

- The system is decomposed to different chiral systems.

\[
\begin{align*}
\alpha_4 \beta_4 & \quad \alpha_4 \beta_4 & \quad \alpha_4 \beta_4 & \quad \alpha_4 \beta_4 \\
\alpha_3 \beta_3 & \quad \alpha_3 \beta_3 & \quad \alpha_3 \beta_3 & \quad \alpha_3 \beta_3 \\
\alpha_2 \beta_2 & \quad \alpha_2 \beta_2 & \quad \alpha_2 \beta_2 & \quad \alpha_2 \beta_2 \\
\alpha_1 \beta_1 & \quad \alpha_1 \beta_1 & \quad \alpha_1 \beta_1 & \quad \alpha_1 \beta_1 \\
\hline
J=4 & \quad J=3, 1 & \quad J=2, 2 & \quad J=1, 3
\end{align*}
\]

\[H_{\text{eff}}^N = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{ND}}\]

\[\sum_{i=1}^{N_D} J_i = N\]

Chirality sum rule
2. Electronic structure of graphene multilayers

7) New quantum Hall effects

- QHE in monolayer graphene

\[ \sigma_{xy} = \pm \frac{4e^2}{h} \left( \frac{1}{2} + n \right) \]

\[ n = 0, 1, 2, 3, \ldots \]


- QHE in bilayer graphene

\[ \sigma_{xy} = \pm \frac{4e^2}{h} \left( \frac{1}{2} + n \right) \]

\[ n = 0, 1, 2, 3, \ldots \]

7) New quantum Hall effects

• QHE in multilayer graphene

\[
\sigma_{xy}^J = \pm \frac{4e^2}{h} \left( \frac{J}{2} + n \right)
\]

\[
\sum_{i=1}^{N_D} J_i = N
\]

\[n=0,1,2,3,\ldots\]

\[
\Rightarrow \sigma_{xy} = \pm \frac{4e^2}{h} \left( \frac{N}{2} + n \right)
\]

• Example: Trilayer

⇒ \(N=3\) quantum Hall conductivity

\(B=10T\)

\(\sigma_{xy}(4e^2/h)\)

\(E_F/t\)

\(E_F = \text{Fermi energy} \quad t = \text{intralayer hopping}\)
2. Electronic structure of graphene multilayers

8) Optical conductivity and transmittance

Optical conductivity is given by $\sigma_{uni}$ per layer over a broad range of $\lambda$.

$$\sigma_{uni} = \frac{\pi}{2} \frac{e^2}{h}$$

$$T = \left(1 + \frac{2\pi}{c} N\sigma_{uni} \right)^{-2} \approx 1 - N\pi\alpha$$

2. Electronic structure of graphene multilayers

8) Optical conductivity and transmittance

- Monolayer graphene ($J=1$)
  \[ \sigma_{uni} = \frac{\pi e^2}{2h} \]

- $J$-chiral system
  \[ \sigma_R = J \sigma_{uni} \]

- Chiral decomposition and sum rule for arbitrarily stacked $N$-layer graphene
  \[ H_{N}^{\text{eff}} = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{ND}} \]
  \[ \sum_{i=1}^{N_D} J_i = N \implies \sigma_R \approx N \sigma_{uni} \]

At low frequencies, chiral decomposition and sum rum give $N\sigma_{uni}$. 
8) Optical conductivity and transmittance

- Bilayer: Low and high frequency limit

At high frequencies, $N=2$ decoupled stack gives $N\sigma_{uni}$.
At low frequencies, $N=2$ chiral system gives $N\sigma_{uni}$.

Interplay between chirality and electron-electron interactions in graphene systems
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2. Electronic structure of graphene multilayers

8) Optical conductivity and transmittance
   • 10 layer graphene stacks

Optical conductivity measurements provide a useful way to identify the number of layers and stacking sequences.

Min and MacDonald, PRL 103, 067402 (2009)

Optical conductivity measurements provide a useful way to identify the number of layers and stacking sequences.

300 nm ↔ 4.13 eV
Summary of Part 2

• Arbitrarily stacked multilayer graphene is described by a set of chiral systems at low energies.

\[ H_{N}^{\text{eff}} = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{ND}} \]

• Chirality sum is always the number of layers.

\[ \sum_{i=1}^{N_D} J_i = N \]

What is the effects of electron-electron interactions?
Exciton condensation in graphene double layers

$\Rightarrow$ Room-temperature superfluidity

Interplay between chirality and electron-electron interactions in graphene systems
3. Exciton condensation in graphene double layers

1) System

- Two single-layer graphene sheets separated by SiO$_2$ dielectric barrier in the no tunneling limit.

\[ +V_g \]  
\[ -V_g \]  

\[ \Rightarrow \text{High-temperature exciton condensation} \]
Exciton condensation in graphene double layers

2) Pair condensation

- Cooper pairs

\[ \langle \hat{\psi}_{\uparrow}^{+}(\mathbf{r})\hat{\psi}_{\downarrow}^{+}(\mathbf{r}) \rangle \neq 0 \]

- Bilayer excitons

\[ \langle \hat{\psi}_{b}^{+}(\mathbf{r})\hat{\psi}_{t}(\mathbf{r}) \rangle \neq 0 \]

⇒ Exciton condensation is spontaneous interlayer coherence.
3. Exciton condensation in graphene double layers

3) Why graphene?

- Gapless semiconductor
- Perfect particle-hole symmetry
- Atomically thin 2D system
3. Exciton condensation in graphene double layers

4) Numerical calculation

- A self-consistent mean-field theory neglecting remote bands

Interplay between chirality and electron-electron interactions in graphene systems
3. Exciton condensation in graphene double layers

5) Energy band structure

- Cooper instability

\[ E / E_F^0 \]

\[ k / k_F^0 \]

Min et al. PRB 78, 121401(R) (2008)

Interplay between chirality and electron-electron interactions in graphene systems
6) Kosterlitz-Thouless (KT) transition

- In 2D, superfluidity is destroyed by phase fluctuations.

\[ k_B T_{KT} = \frac{\pi}{2} \rho_s (k_B T_{KT}) \]

7) Exciton superfluidity

- Counter-flow current

\[ j_Q = \frac{e}{\hbar} \rho_s Q \]

\[ j_Q \rightarrow \rho_s \rightarrow T_{KT} \]**
3. Exciton condensation in graphene double layers

8) Phase diagram

- \( T_c \uparrow \) as \( E_{ext} \uparrow \)
- Optimal layer separation
- \( E_{ext} \sim 0.7 \, \text{V/nm}, \, d \sim 1 \, \text{nm} \)

\[ \Rightarrow T_c \sim 300 \, \text{K} \]

- Comparison with BCS superconductivity
  
  Cooper pair : limited by \( \omega_D \) \( \Rightarrow T_c \sim 10 \, \text{K} \)
  
  Bilayer exciton : limited by \( \nu_F/d \) \( \Rightarrow T_c \sim 300 \, \text{K} \)
3. Exciton condensation in graphene double layers

9) Experimental search

Tunneling

Transport


Interplay between chirality and electron-electron interactions in graphene systems
3. Exciton condensation in graphene double layers

Summary of Part 3

• Bilayer exciton condensation is spontaneous interlayer coherence.

• Bilayer exciton condensation can occur in decoupled graphene double layers at high temperatures.

What happens in coupled graphene bilayers?
Pseudospin magnetism in graphene bilayers

⇒ Spontaneous charge polarization

Interplay between chirality and electron-electron interactions in graphene systems
4. Pseudospin magnetism in graphene bilayers

1) Pseudospin $\Rightarrow$ Two-valued quantum degrees of freedom

- Bilayer graphene ($J=2$ chiral system)

  $\uparrow = \alpha_b$  $\downarrow = \beta_t$

\[
H_{bi}^b = -\frac{1}{2m} \begin{pmatrix} 0 & p^2 e^{-i2\phi_p} \\ p^2 e^{i2\phi_p} & 0 \end{pmatrix} = -\frac{p^2}{2m} \sigma \cdot \mathbf{n}_2(\phi_p)
\]

$\mathbf{n}_2(\phi_p) = (\cos 2\phi_p, \sin 2\phi_p)$

$\phi_p = \tan^{-1}(p_y/p_x)$

- Ferromagnetism means spontaneous spin polarization.

- Is there spontaneous charge polarization in the presence of electron-electron interactions?

  $\Rightarrow$ Pseudospin magnetism
4. Pseudospin magnetism in graphene bilayers

2) Numerical calculation

- A self-consistent mean-field theory

\[
H_{MF} = -B(k) \cdot \sigma
\]

\( B(k) \) : effective magnetic field
\( \sigma \) : pseudospin

\[
\begin{align*}
&k, \sigma \\
&\quad \rightarrow \\
&\quad q = 0 \\
&\quad \leftarrow \\
&k', \sigma'
\end{align*}
\]

Direct

\[
\begin{align*}
&k, \sigma' \\
&\quad \rightarrow \\
&\quad \rightarrow \\
&\quad \leftarrow \\
&\quad \leftarrow \\
&k', \sigma'
\end{align*}
\]

Exchange

\[
\begin{align*}
&k, \sigma' \\
&\quad \rightarrow \\
&\quad \rightarrow \\
&\quad \leftarrow \\
&\quad \leftarrow \\
&k', \sigma' \quad q = k-k'
\end{align*}
\]
4. Pseudospin magnetism in graphene bilayers

3) Example: Bilayer graphene ($J=2$) 

Min et al. PRB 77, 041407(R) (2008)

$B_{\parallel}(k) = (B_x, B_y)$

$H_{MF} = -B(k) \cdot \sigma$

$B(k) = (B_\perp \cos J\phi_k, B_\perp \sin J\phi_k, B_z)$

$\Rightarrow$ Spontaneous charge transfer between two layers

Interplay between chirality and electron-electron interactions in graphene systems
Pseudospin magnetism is stable for stronger interaction strength, for smaller doping, for larger chirality.

Min et al. PRB 77, 041407(R) (2008)

\[ J : \text{chirality} \]
\[ \alpha : \text{interaction strength} \]
\[ f : \text{doping} \]
4. Pseudospin magnetism in graphene bilayers

5) Chiral decomposition revisited

• Chiral decomposition in the electronic structure

\[ H_N^{\text{eff}} = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{ND}} \]

\[ \sum_{i=1}^{N_D} J_i = N \]

⇒ ABC stacked multilayers are excellent candidates.
Summary of Part 3

• Pseudospin is any two-valued quantum degrees of freedom.

• A chiral system is described by pseudospin language.

• Pseudospin analogue of ferromagnetism in chiral systems.

⇒ Pseudospin magnetism
Conclusion

New electronic device scheme?

Interplay between chirality and electron-electron interactions in graphene systems
5. Conclusion

1) Search for new ordered states in graphene systems

Dirac-like chiral wavefunction + Electron-electron interaction

⇒ New ordered states in graphene systems

Exciton condensation in decoupled graphene double layers
Pseudospin magnetism in coupled graphene bilayers

Interplay between chirality and electron-electron interactions in graphene systems
Interplay between chirality and electron-electron interactions in graphene systems

5. Conclusion

2) Collective field effect transistor (FET) vision

• Giant magnetoresistance (GMR)

⇒ Collective behavior of many electrons
5. Conclusion

3) New electronic device scheme

Example: Pseudospin magnetism

- Collective behavior of many electrons
- Can be switched with gate voltages using much less power.
- Can exhibit a pseudospin version of GMR and spin-transfer torque.

Pseudospintronics!
1. Electronic structure of graphene multilayers
Chiral decomposition in the electronic structure of graphene multilayers

Origin of universal optical conductivity and optical stacking sequence identification in multilayer graphene
Hongki Min, A. H. MacDonald

2. Exciton condensation in graphene bilayers
Room-temperature superfluidity in graphene bilayers
Hongki Min, Rafi Bistritzer, Jung-Jung Su, A. H. MacDonald

3. Pseudospin magnetism in graphene bilayers
Pseudospin magnetism in graphene
Hongki Min, Giovanni Borghi, Marco Polini, A. H. MacDonald