Interplay between chirality and electron-electron interactions in graphene systems

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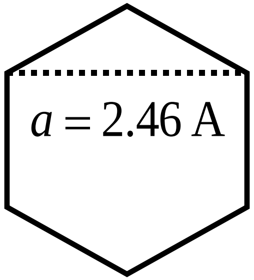
Outline

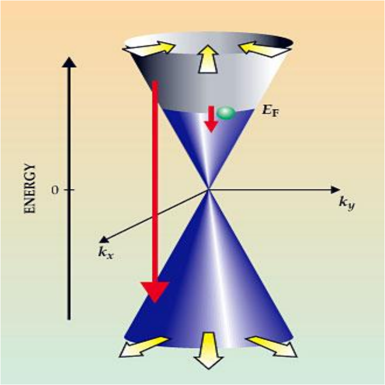
1. Introduction
2. Electronic structure of graphene multilayers
3. Exciton condensation in graphene double layers
4. Pseudospin magnetism in graphene bilayers
5. Conclusion

Introduction

- What is graphene?



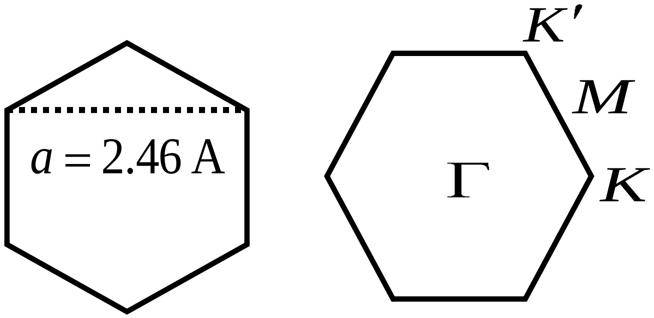
- Why is it important?



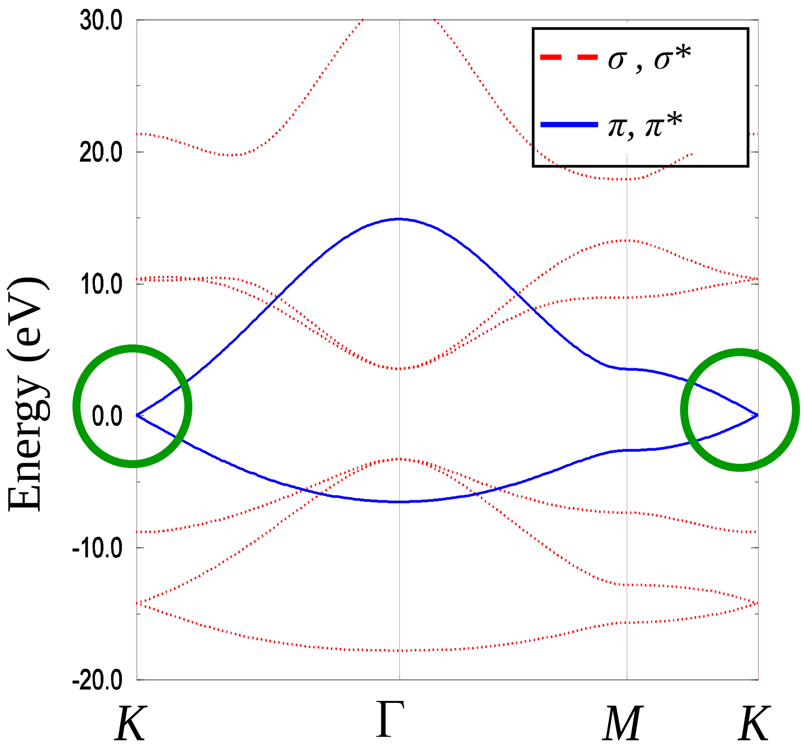
1. Introduction

1) Graphene

- Two-dimensional honeycomb lattice of carbon atoms.



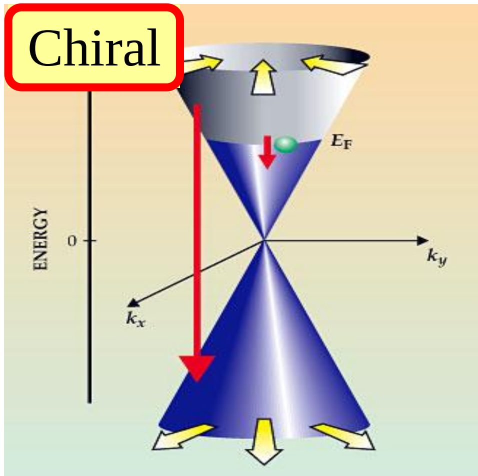
- 2D Dirac-like equation with linear dispersion near K/K'.



- New electron-electron interaction physics? Example: magnetism, superconductivity

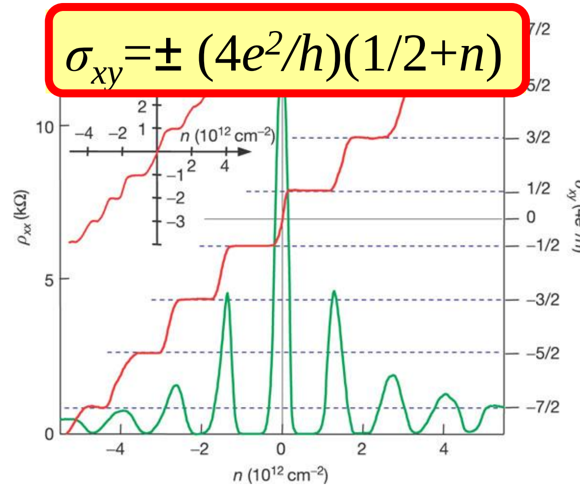
1. Introduction

2) Extraordinary properties of graphene



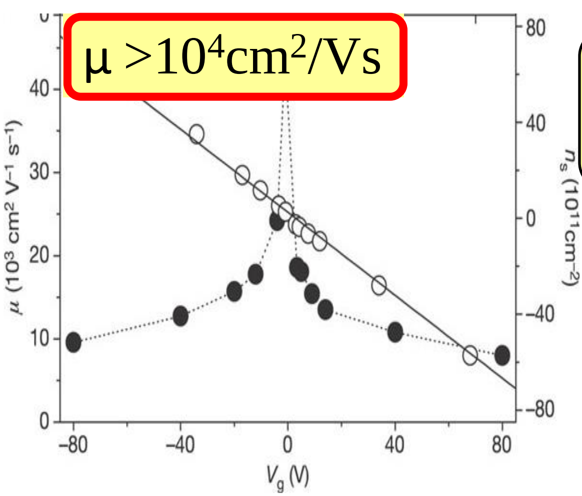
Dirac-like equation

Wallace, Phys. Rev. (1947)



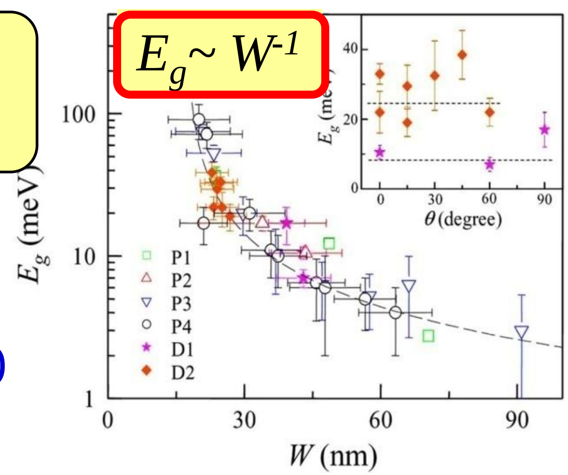
Quantum Hall effect

Novoselov et al. Nature (2005)



High mobility

Zhang et al. Nature (2005)

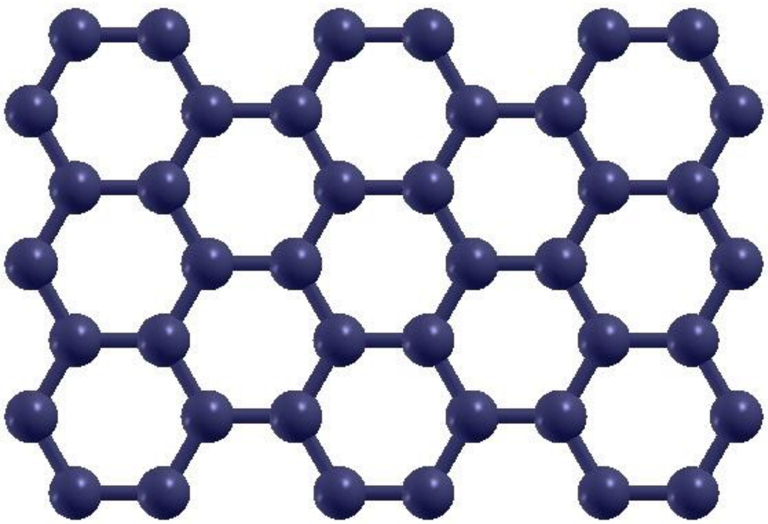


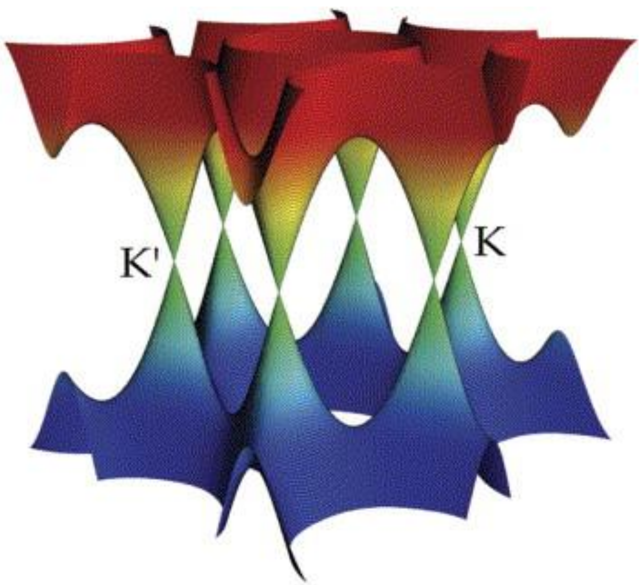
Device application

Han et al. PRL (2007)

Interplay between chirality and electron-electron interactions in graphene systems

Electronic structure of graphene multilayers





$\Rightarrow$ Described by a set of chiral systems

2. Electronic structure of graphene multilayers

1) Electronic structure of monolayer graphene

- Effective theory around K

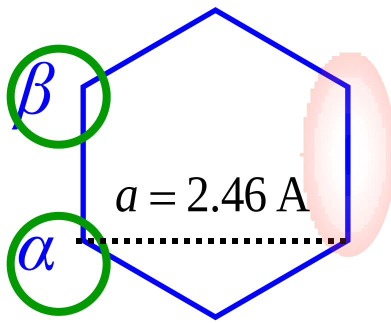
$$H_B^{mono} = v_F \begin{array}{c} \begin{array}{cc} \quad\alpha\quad & \quad\quad\beta \end{array} \\ \begin{pmatrix} 0 & pe^{-i\phi_{\mathbf{p}}} \\ pe^{i\phi_{\mathbf{p}}} & 0 \end{pmatrix} \end{array}$$

$$E(\mathbf{p}) = \pm v_F \, p$$

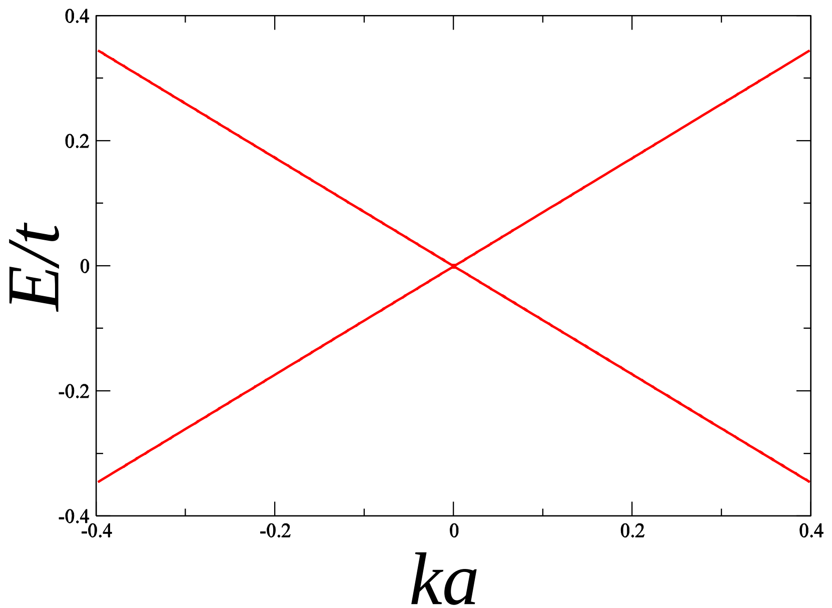
$v_F =$ in-plane velocity

$\phi_{\mathbf{k}} = \tan^{-1}(k_y / k_x)$

$\mathbf{p} = \hbar\mathbf{k}$



$$t = \frac{2}{\sqrt{3}} \frac{\hbar v_F}{a}$$

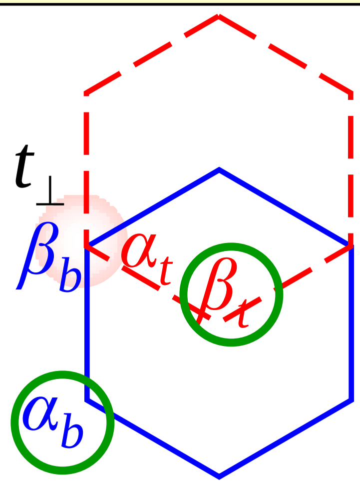


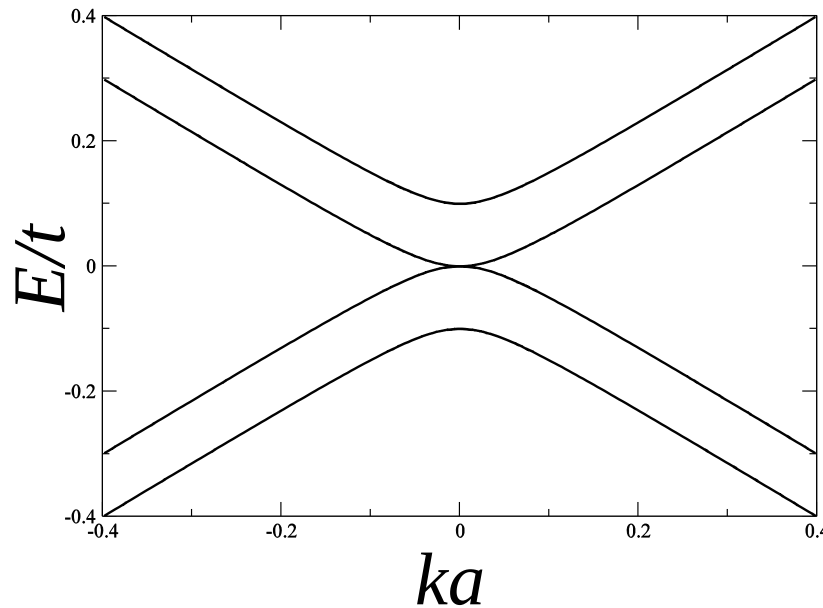
2. Electronic structure of graphene multilayers

2) Electronic structure of bilayer graphene

- 4-band model around K

$$\begin{array}{cc} & \begin{matrix} \alpha_b & \quad\beta_b & \quad\alpha_t & \quad\beta_t \end{matrix} \\ H^{bi}_{4band} = & \begin{pmatrix} 0 & v_F p e^{-i\phi_p} & 0 & 0 \\ v_F p e^{i\phi_p} & 0 & t_\perp & 0 \\ 0 & t_\perp & 0 & v_F p e^{-i\phi_p} \\ 0 & 0 & v_F p e^{i\phi_p} & 0 \end{pmatrix} \end{array}$$





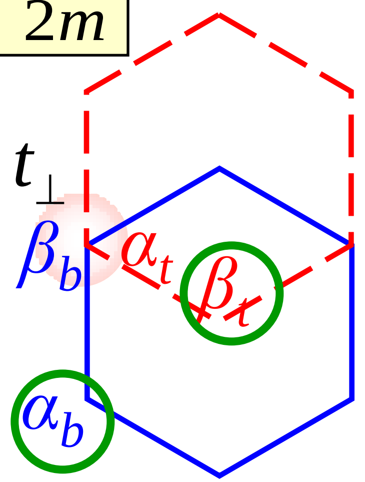
2) Electronic structure of bilayer graphene

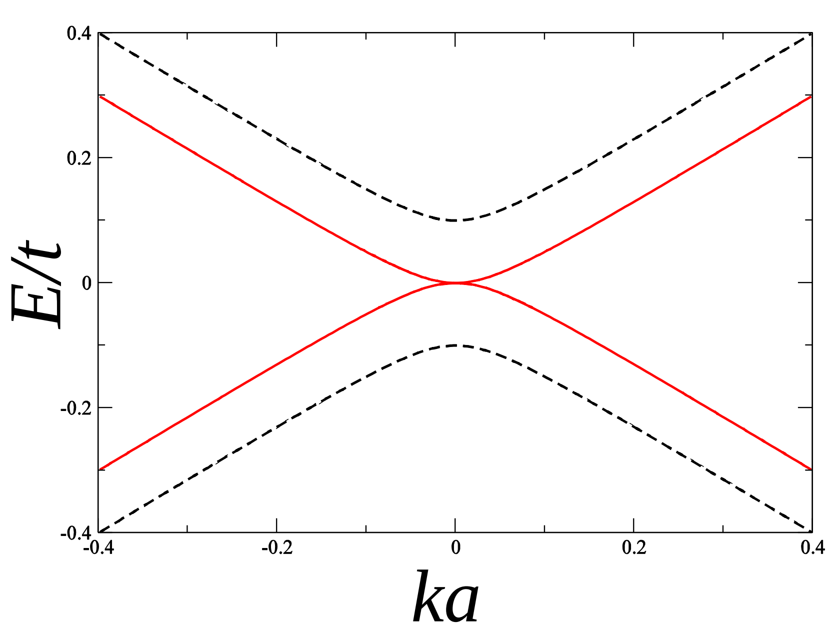
- Effective theory around K

$$
H_B^{bi} = -\frac{1}{2m}\begin{array}{c} \begin{array}{cc} \alpha_b & \beta_t \end{array} \\ \begin{pmatrix} 0 & p^2 e^{-i2\phi_{\mathbf{p}}} \\ p^2 e^{i2\phi_{\mathbf{p}}} & 0 \end{pmatrix} \end{array}
$$

$$
E(\mathbf{p}) = \pm \frac{p^2}{2m}
$$

$$
m = t_\perp / 2v_F^2
$$





What is the effective theory for arbitrarily stacked multilayers?

3) Chirality and Pseudospin

- Effective theory of monolayer graphene

$$H_B^{mono} = v_F \begin{array}{cc} \alpha & \beta \\ \end{array}$$

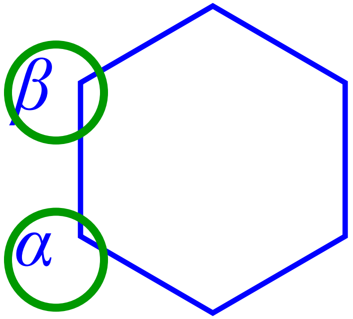
$$H_B^{mono} = v_F \begin{pmatrix} 0 & pe^{-i\phi_{\mathbf{p}}} \\ pe^{i\phi_{\mathbf{p}}} & 0 \end{pmatrix} = v_F\, p\boldsymbol{\sigma}\cdot\mathbf{n}_1(\phi_{\mathbf{p}})$$

$$\mathbf{n}_1(\phi_{\mathbf{p}}) = (\cos\phi_{\mathbf{p}}, \sin\phi_{\mathbf{p}})$$

$$\phi_{\mathbf{p}} = \tan^{-1}(p_y / p_x)$$

- Energy spectrum

$$E_1(\mathbf{p}) = \pm v_F\, p$$



3) Chirality and Pseudospin

- Effective theory of bilayer graphene

$$\begin{array}{ccc} & \alpha_b & \beta_t \end{array}$$

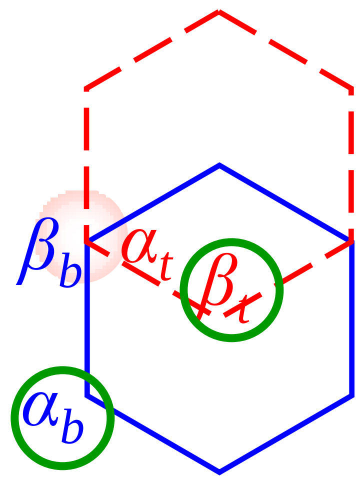
$$\boxed{H_B^{bi} = -\frac{1}{2m}\begin{pmatrix} 0 & p^2 e^{-i2\phi_\mathbf{p}} \\ p^2 e^{i2\phi_\mathbf{p}} & 0 \end{pmatrix} = -\frac{p^2}{2m}\boldsymbol{\sigma}\cdot\mathbf{n}_2(\phi_\mathbf{p})}$$

$$\boxed{\mathbf{n}_2(\phi_\mathbf{p}) = (\cos 2\phi_\mathbf{p}, \sin 2\phi_\mathbf{p})}$$

$$\boxed{\phi_\mathbf{p} = \tan^{-1}(p_y / p_x)}$$

- Energy spectrum

$$\boxed{E_2(\mathbf{p}) = \pm\frac{p^2}{2m}}$$



3) Chirality and Pseudospin

- Effective theory of *J*-chiral system

"↑" "↓" $\Leftarrow$ Pseudospin

$$\boxed{H_J = \varepsilon_0 \begin{pmatrix} 0 & p^J e^{-iJ\phi_{\mathbf{p}}} \\ p^J e^{iJ\phi_{\mathbf{p}}} & 0 \end{pmatrix} = \varepsilon_0 p^J \boldsymbol{\sigma} \cdot \mathbf{n}_J(\phi_{\mathbf{p}})}$$

$$\boxed{\mathbf{n}_J(\phi_{\mathbf{p}}) = (\cos J\phi_{\mathbf{p}}, \sin J\phi_{\mathbf{p}})}$$

$$\boxed{\phi_{\mathbf{p}} = \tan^{-1}(p_y / p_x)}$$

- Energy spectrum

$$\boxed{E_J(\mathbf{p}) = \pm \varepsilon_0 p^J}$$

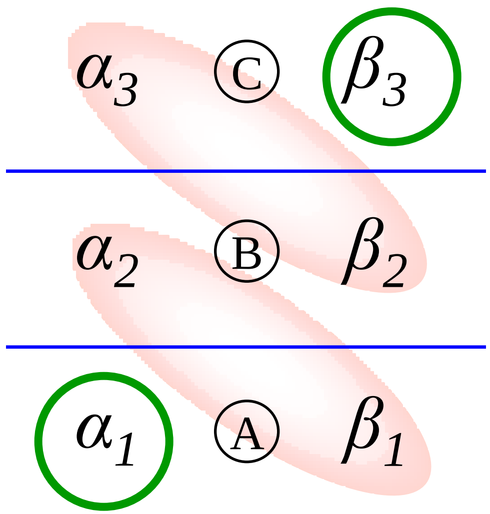
Monolayer graphene : *J*=1
Bilayer graphene : *J*=2

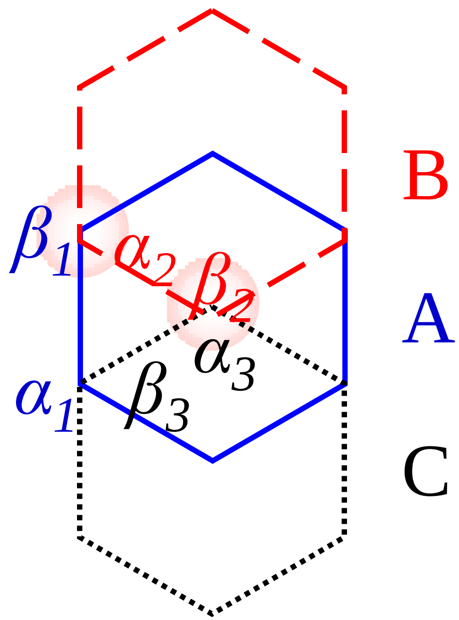
2. Electronic structure of graphene multilayers

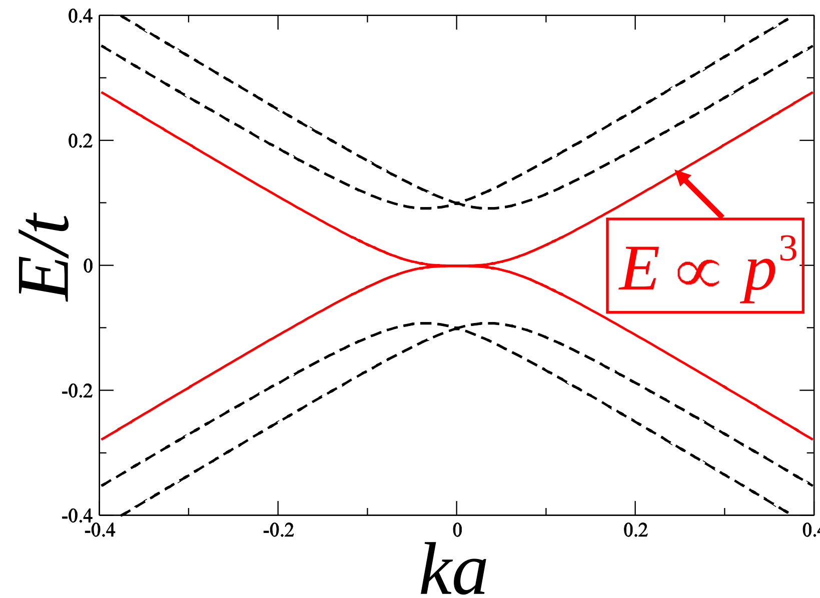
4) Multilayer stacking

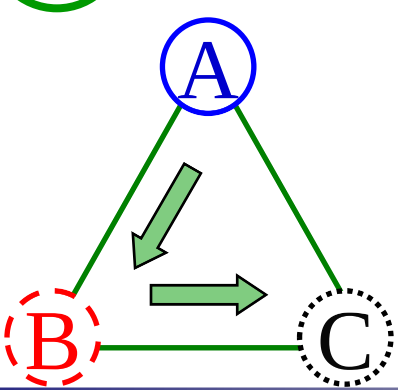
Min and MacDonald, PRB **77**, 155416 (2008)

- Example: ABC trilayer







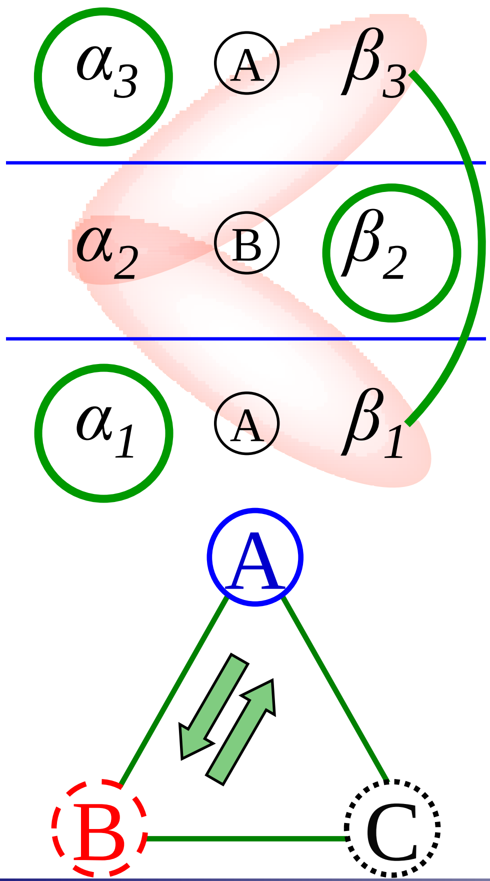


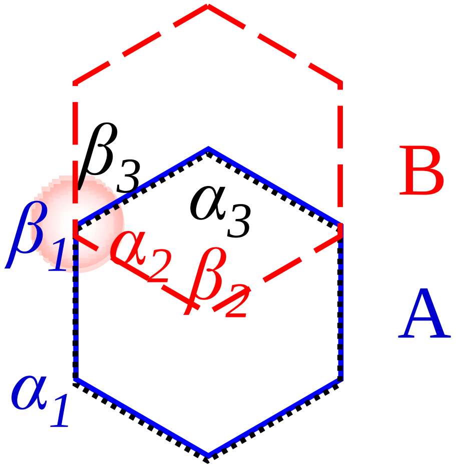
***J*=3** chiral 2D electron gas

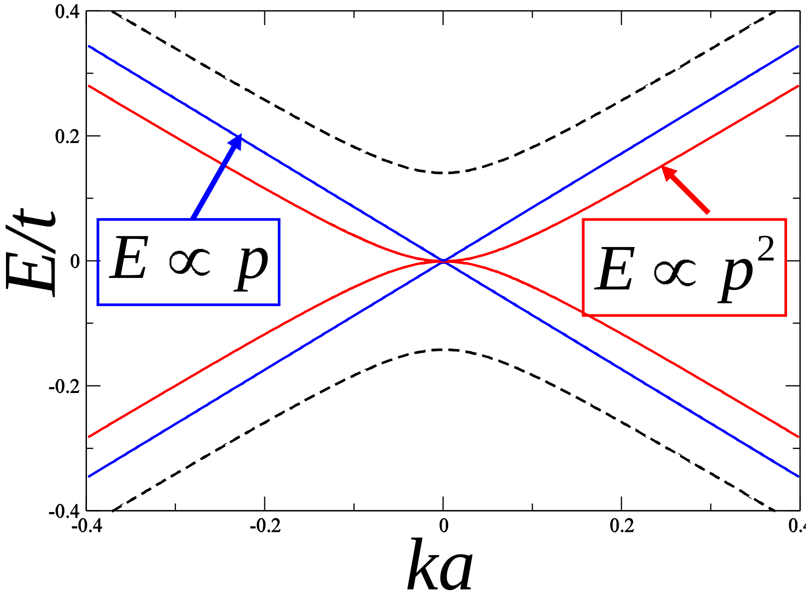
4) Multilayer stacking

Min and MacDonald, PRB **77**, 155416 (2008)

- Example: ABA trilayer







***J*=1, *J*=2** chiral 2D electron gas

5) Effective theory (ABC)

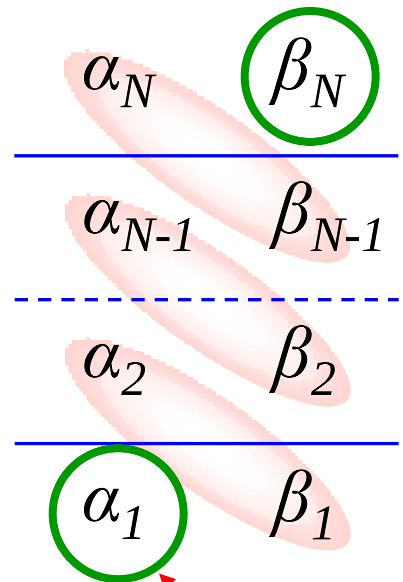
Min and MacDonald, PRB **77**, 155416 (2008)

- Identify zero-energy states from the stacking diagram.

$\Rightarrow$ Ex: isolated α_1 and β_N are zero-energy states.

- Obtain the effective theory using degenerate state perturbation theory.

Ex: ABC stacked N-layer



$$\boxed{H = H_\perp + H_{||}}$$

r, r' : zero-energy states

$$\boxed{\langle \Psi_r | H | \Psi_{r'} \rangle = \langle \Psi_r | H_{||} \text{[redacted]} \hat{Q}(-H_\perp^{-1})\hat{Q}H_{||} \_^{n-1} | \Psi_{r'} \rangle}$$

$$\Rightarrow \text{Ex: } \boxed{\langle \alpha_1 | H | \beta_N \rangle = -t_\perp (v_F p e^{-i\phi_{\mathbf{p}}} / t_\perp)^N}$$

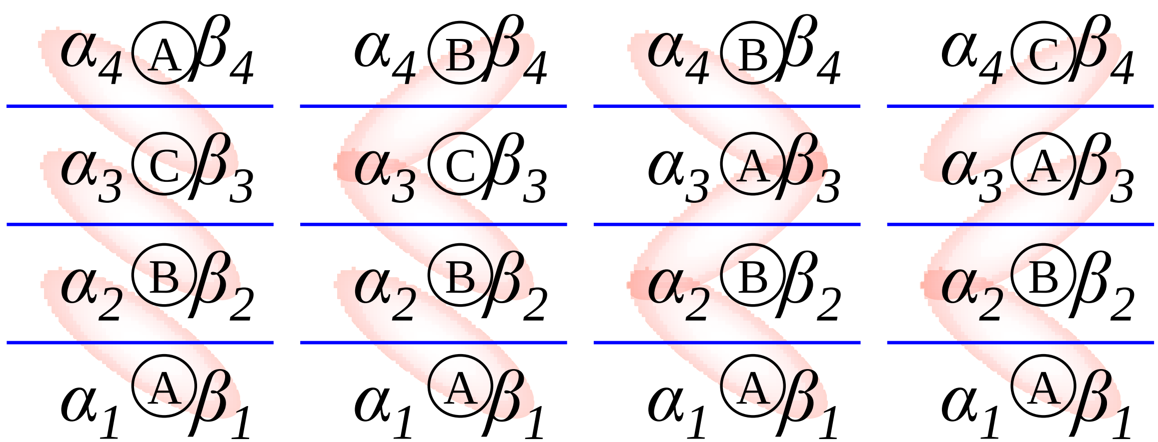
N-chiral system

2. Electronic structure of graphene multilayers

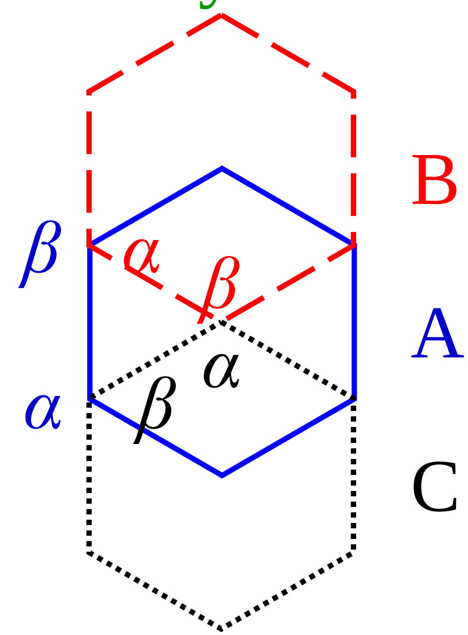
6) Chiral decomposition

Min and MacDonald, PRB **77**, 155416 (2008)

- The system is decomposed to different chiral systems.



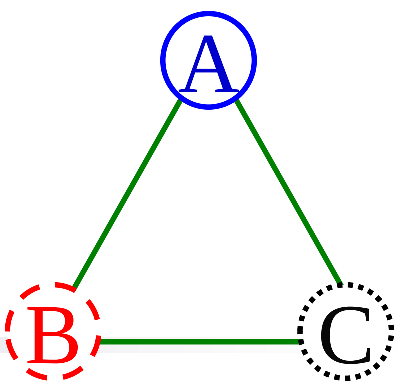
***J*=4** ***J*=3, 1** ***J*=2, 2** ***J*=1, 3**



$$\Rightarrow \boxed{H_N^{eff} = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{N_D}}}$$

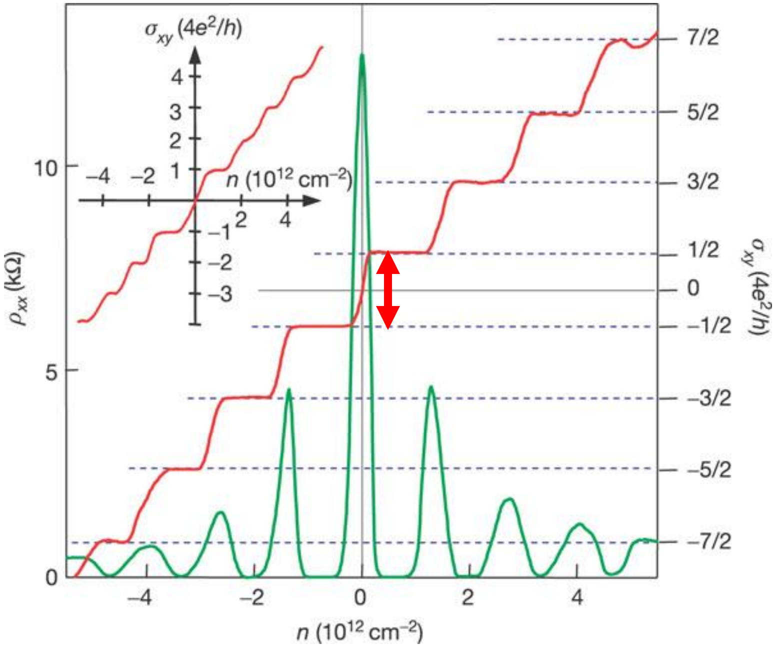
$$\boxed{\sum_{i=1}^{N_D} J_i = N}$$

Chirality sum rule



7) New quantum Hall effects

- QHE in monolayer graphene

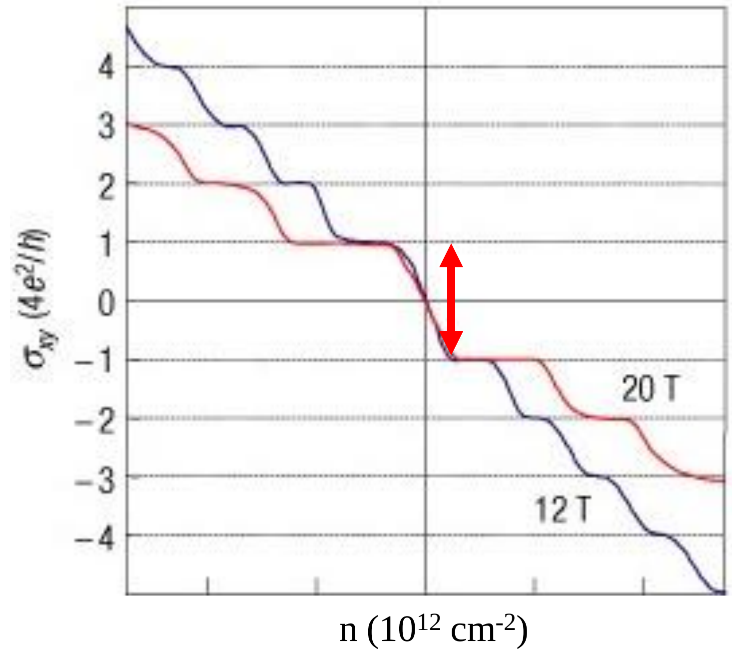


Novoselov et al, Nature **438**, 197 (2005)

$$\sigma_{xy} = \pm \frac{4e^2}{h}\left(\frac{1}{2} + n\right)$$

n=0,1,2,3,…

- QHE in bilayer graphene



Novoselov et al, Nat. Phys. **2**, 177 (2006)

$$\sigma_{xy} = \pm \frac{4e^2}{h}\left(\quad + n\right)$$

n=0,1,2,3,…

Interplay between chirality and electron-electron interactions in graphene systems

7) New quantum Hall effects

Min and MacDonald, PRB **77**, 155416 (2008)

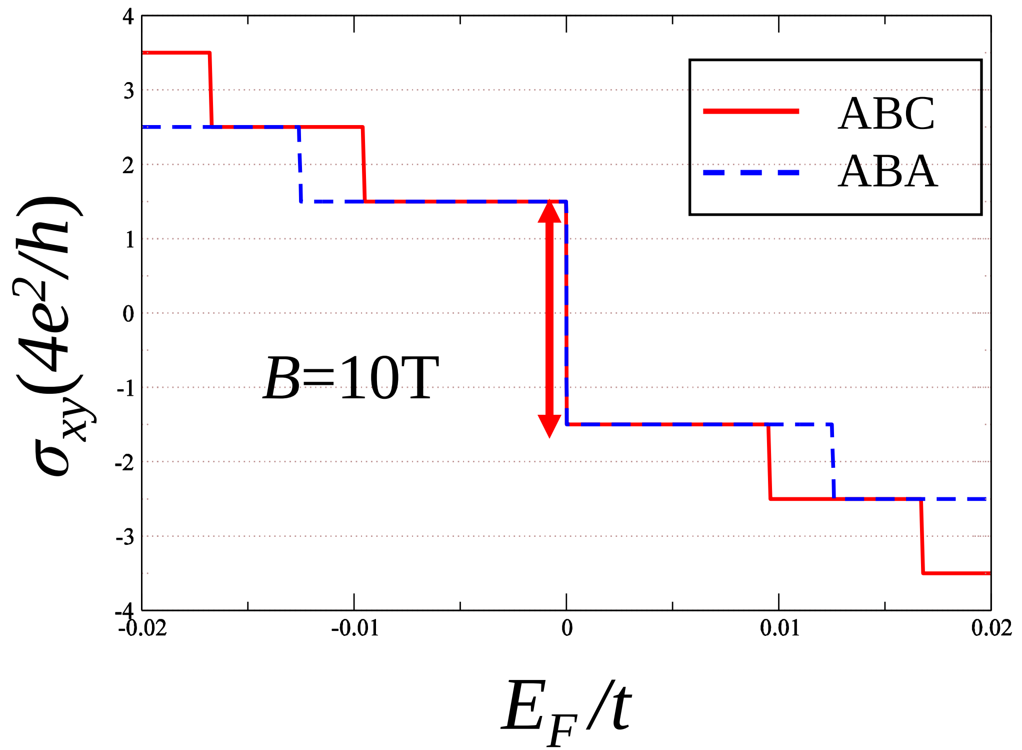
- QHE in multilayer graphene

$$\sigma_{xy}^{J} = \pm \frac{4e^2}{h}\left(\frac{J}{2} + n\right)$$

$$\sum_{i=1}^{N_D} J_i = N$$

n=0,1,2,3,…

$$\Rightarrow \sigma_{xy} = \pm \frac{4e^2}{h}\left(\boxed{\frac{N}{2}} + n\right)$$



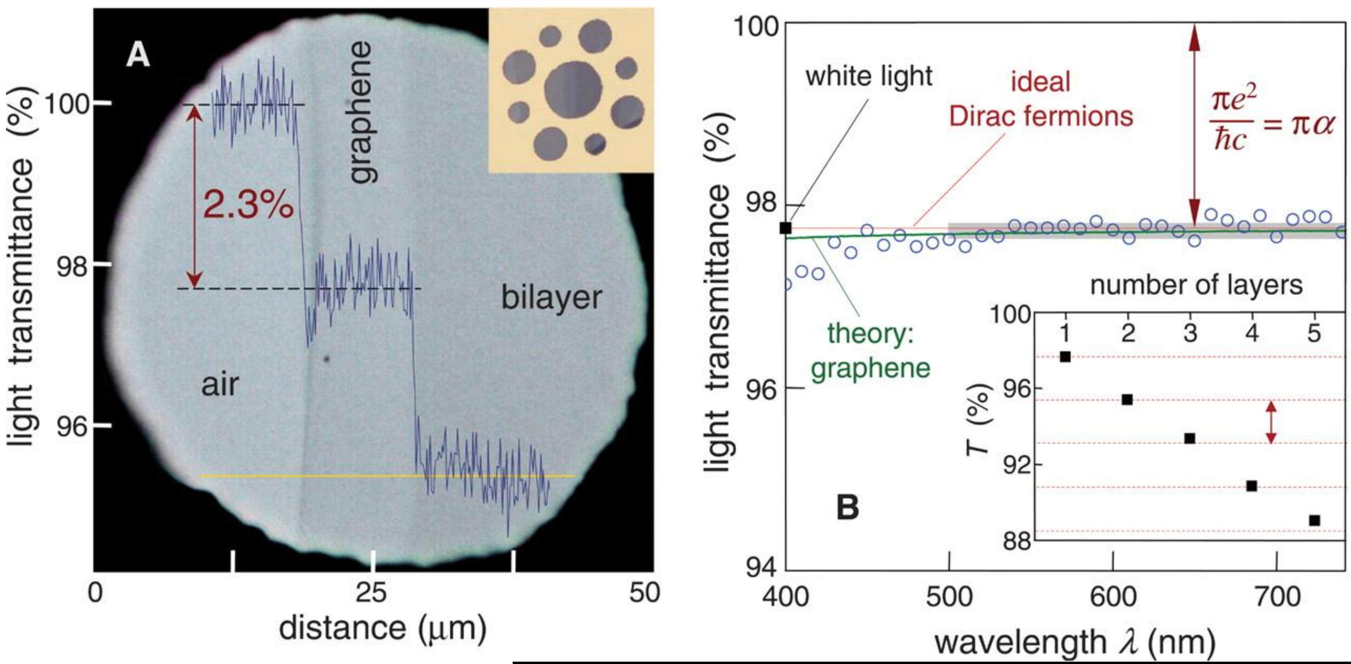
- Example: Trilayer

$\Rightarrow$ N=3 quantum Hall conductivity

E_F = Fermi energy
t = intralayer hopping

8) Optical conductivity and transmittance

Nair et al, Science **320**, 1308 (2008)



$$\sigma_{uni} = \frac{\pi}{2}\frac{e^2}{h}$$

$$T = \left(1 + \frac{2\pi}{c} N\sigma_{uni}\right)^{-2} \approx 1 - N\pi\alpha$$

Optical conductivity is given by σ_{uni} per layer over a broad range of λ.

8) Optical conductivity and transmittance

Min and MacDonald, PRL **103**, 067402 (2009)

- Monolayer graphene (J=1)

$$\sigma_{uni} = \frac{\pi}{2}\frac{e^2}{h}$$

- J-chiral system

$$\sigma_R = J\sigma_{uni}$$

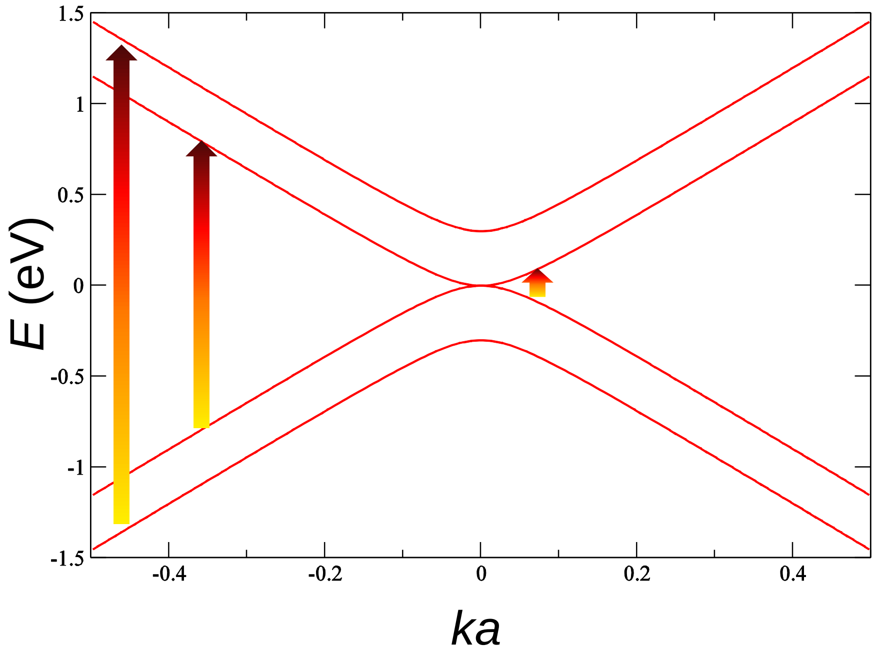
- Chiral decomposition and sum rule for arbitrarily stacked N-layer graphene

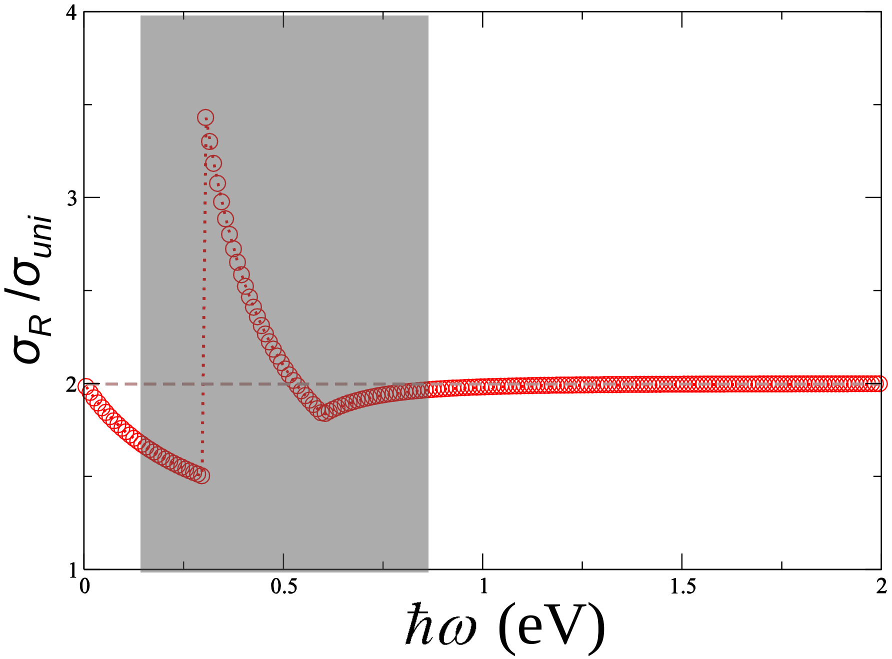
$$H_N^{eff} = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{N_D}} \qquad \sum_{i=1}^{N_D} J_i = N \quad \Rightarrow \quad \sigma_R \approx N\sigma_{uni}$$

At low frequencies, chiral decomposition and sum rum give $N\sigma_{uni}$.

8) Optical conductivity and transmittance

- Bilayer: Low and high frequency limit



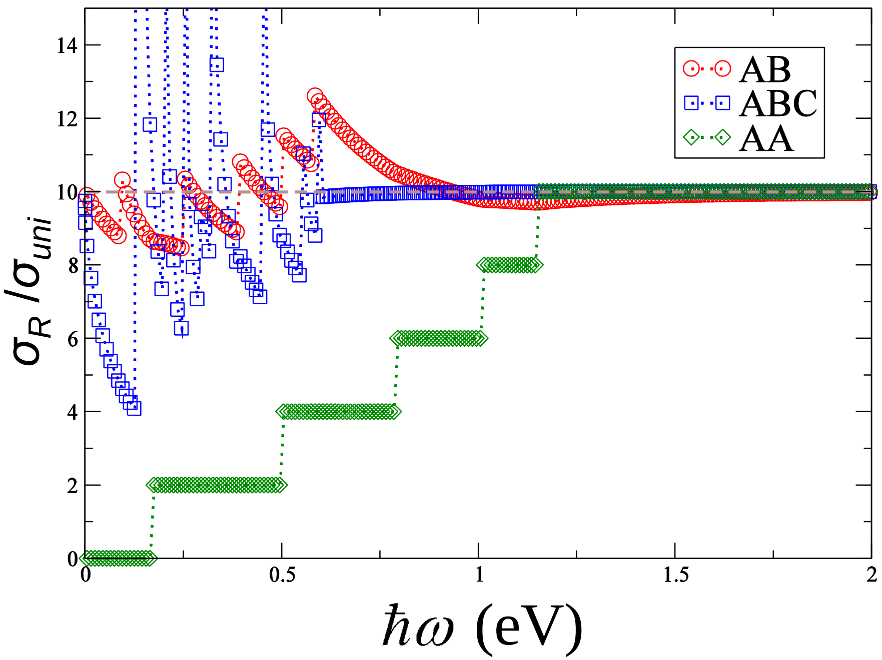


At high frequencies, N=2 decoupled stack gives $N\sigma_{uni}$.
At low frequencies, N=2 chiral system gives $N\sigma_{uni}$.

8) Optical conductivity and transmittance

Min and MacDonald, PRL **103**, 067402 (2009)

- 10 layer graphene stacks



300 nm ↔ 4.13 eV

Optical conductivity measurements provide a useful way to identify the number of layers and stacking sequences.

Summary of Part 2

- Arbitrarily stacked multilayer graphene is described by a set of chiral systems at low energies.

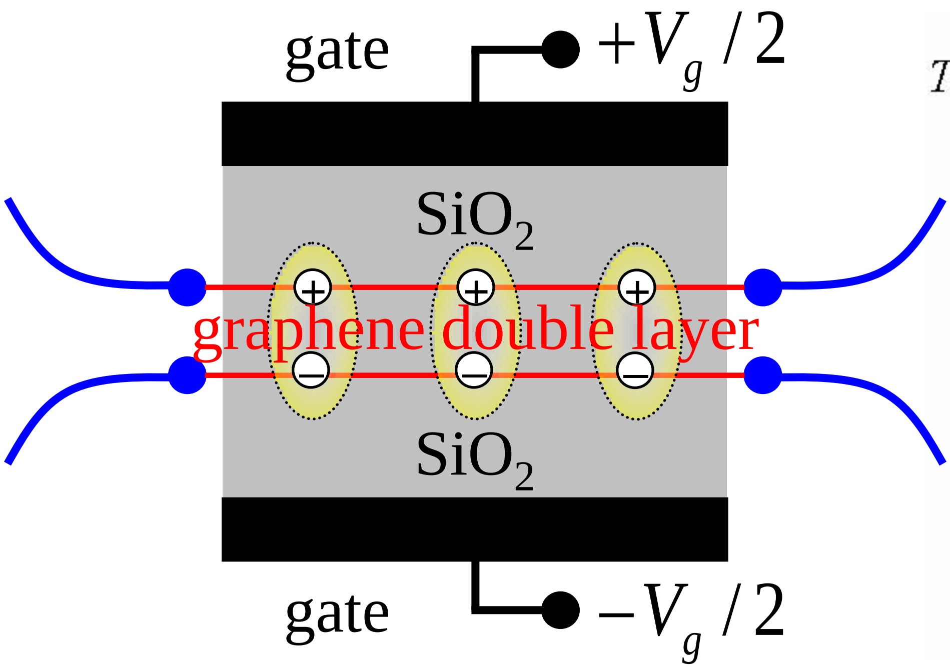
$$\boxed{H_N^{eff} = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{N_D}}}$$

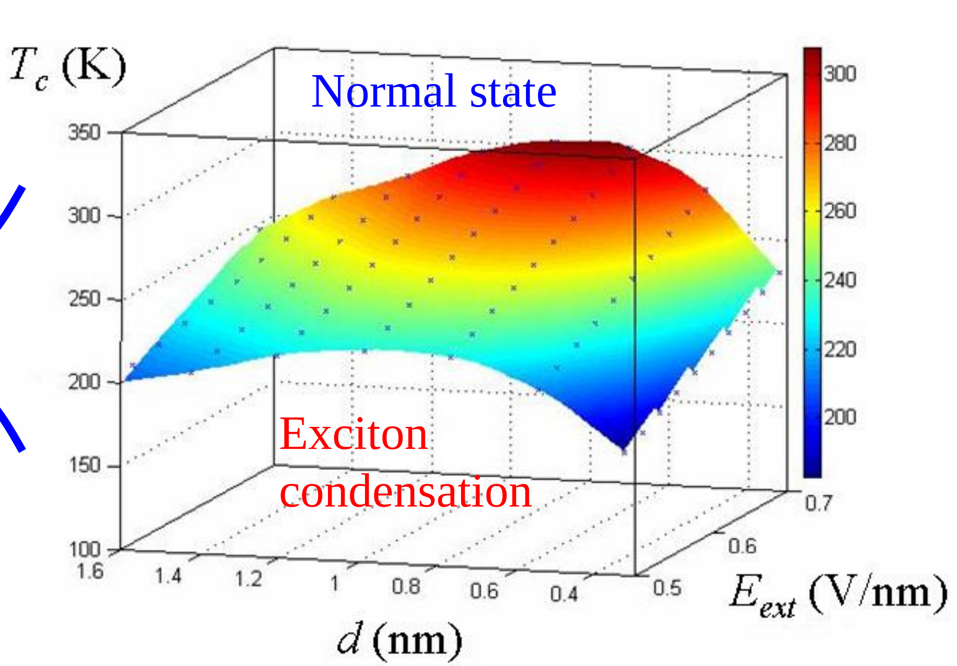
- Chirality sum is always the number of layers.

$$\boxed{\sum_{i=1}^{N_D} J_i = N}$$

What is the effects of electron-electron interactions?

Exciton condensation in graphene double layers



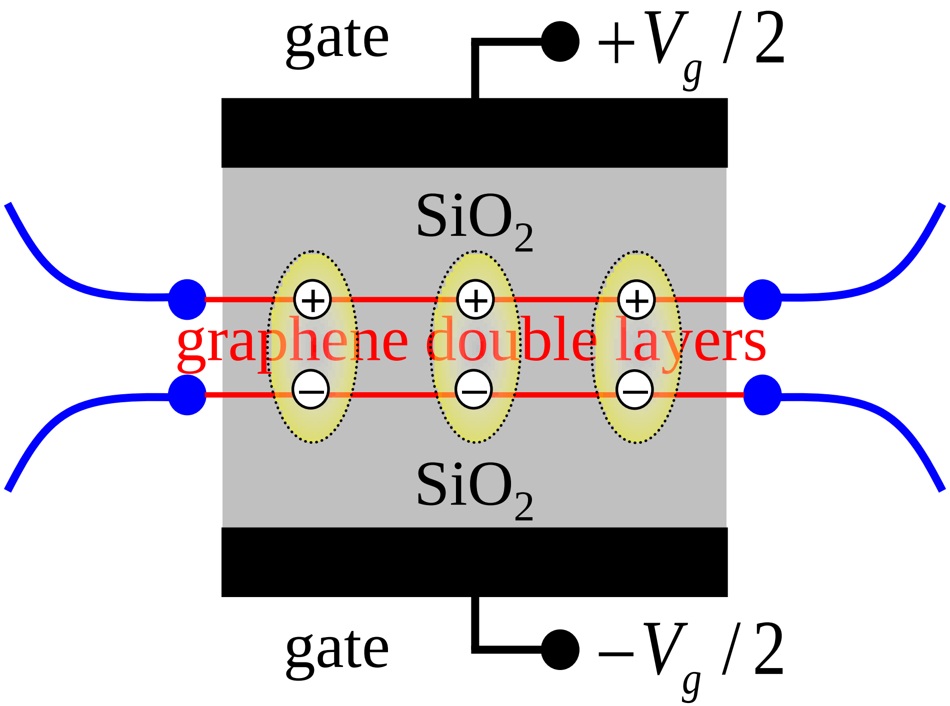


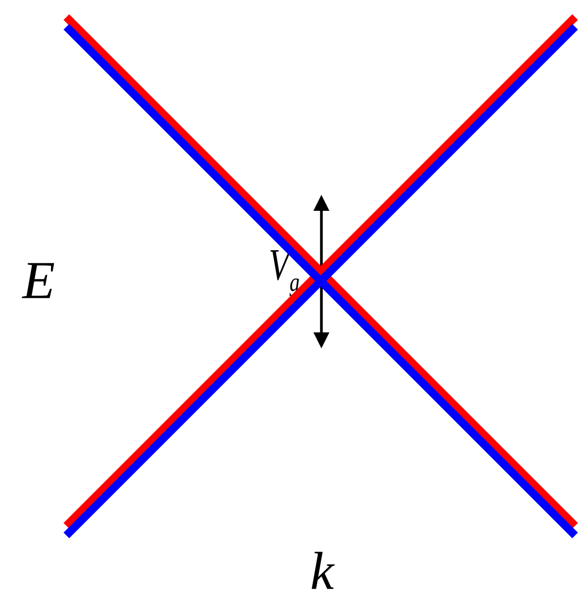
$\Rightarrow$ Room-temperature superfluidity

3.Exciton condensation in graphene double layers

1) System

- Two single-layer graphene sheets separated by SiO_2 dielectric barrier in the no tunneling limit.



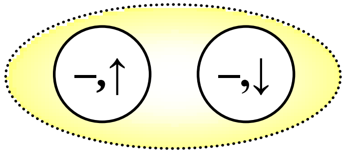


$\Rightarrow$ High-temperature exciton condensation

2) Pair condensation

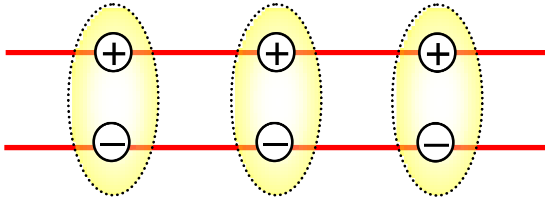
- Cooper pairs

$$\boxed{\left\langle \hat{\psi}^{+}_{\uparrow}(\mathbf{r})\hat{\psi}^{+}_{\downarrow}(\mathbf{r})\right\rangle \neq 0}$$



- Bilayer excitons

$$\boxed{\left\langle \hat{\psi}^{+}_{b}(\mathbf{r})\hat{\psi}_{t}(\mathbf{r})\right\rangle \neq 0}$$

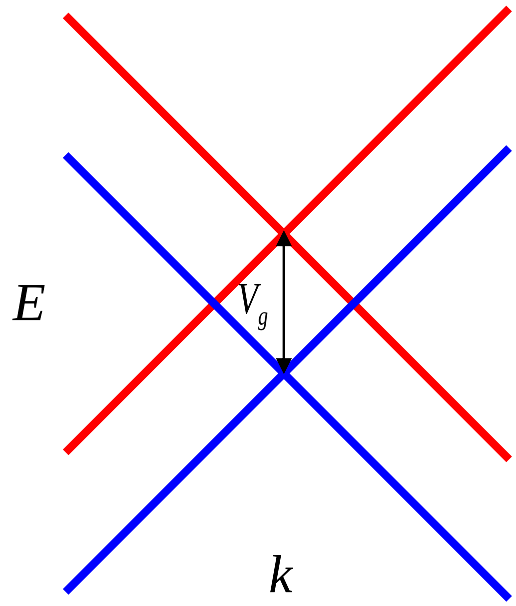


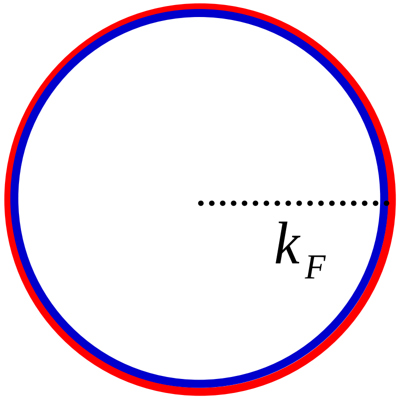
$\Rightarrow$ Exciton condensation is spontaneous interlayer coherence.

3.Exciton condensation in graphene double layers

3) Why graphene?

- Gapless semiconductor
- Perfect particle-hole symmetry
- Atomically thin 2D system

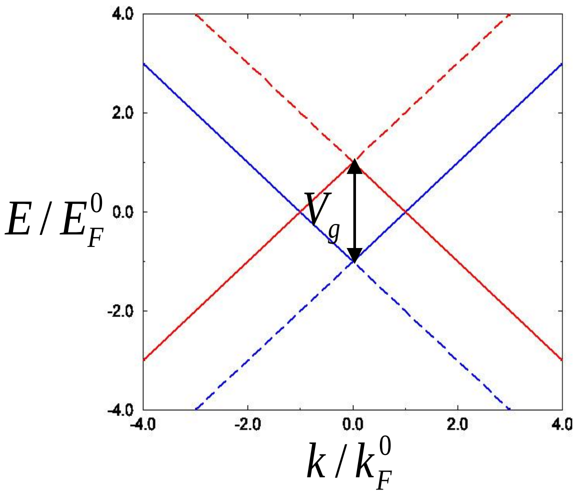


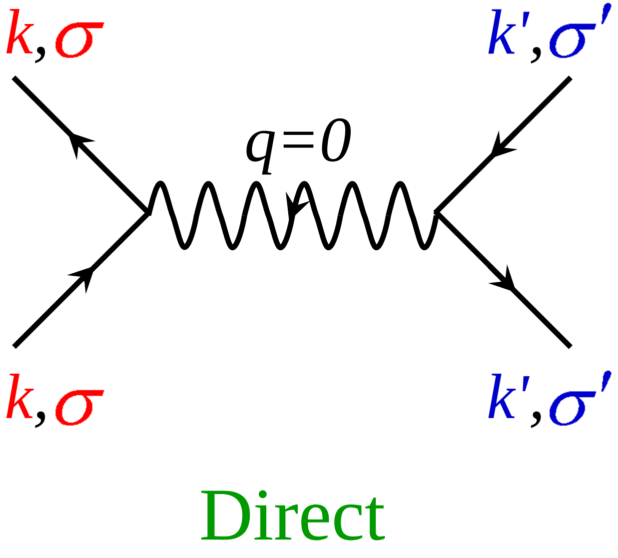


3.Exciton condensation in graphene double layers

4) Numerical calculation

- A self-consistent mean-field theory neglecting remote bands





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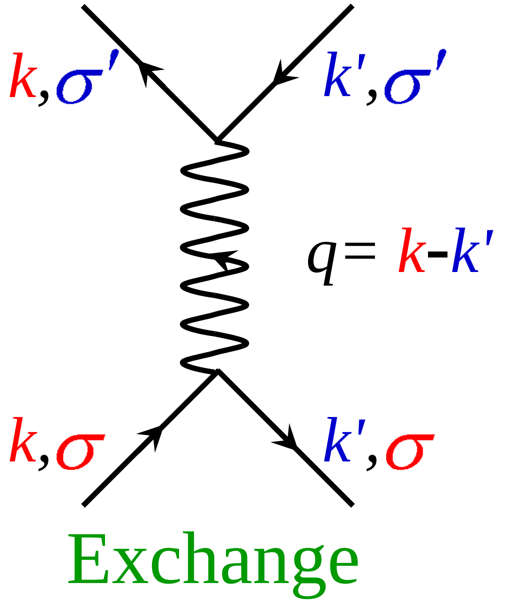
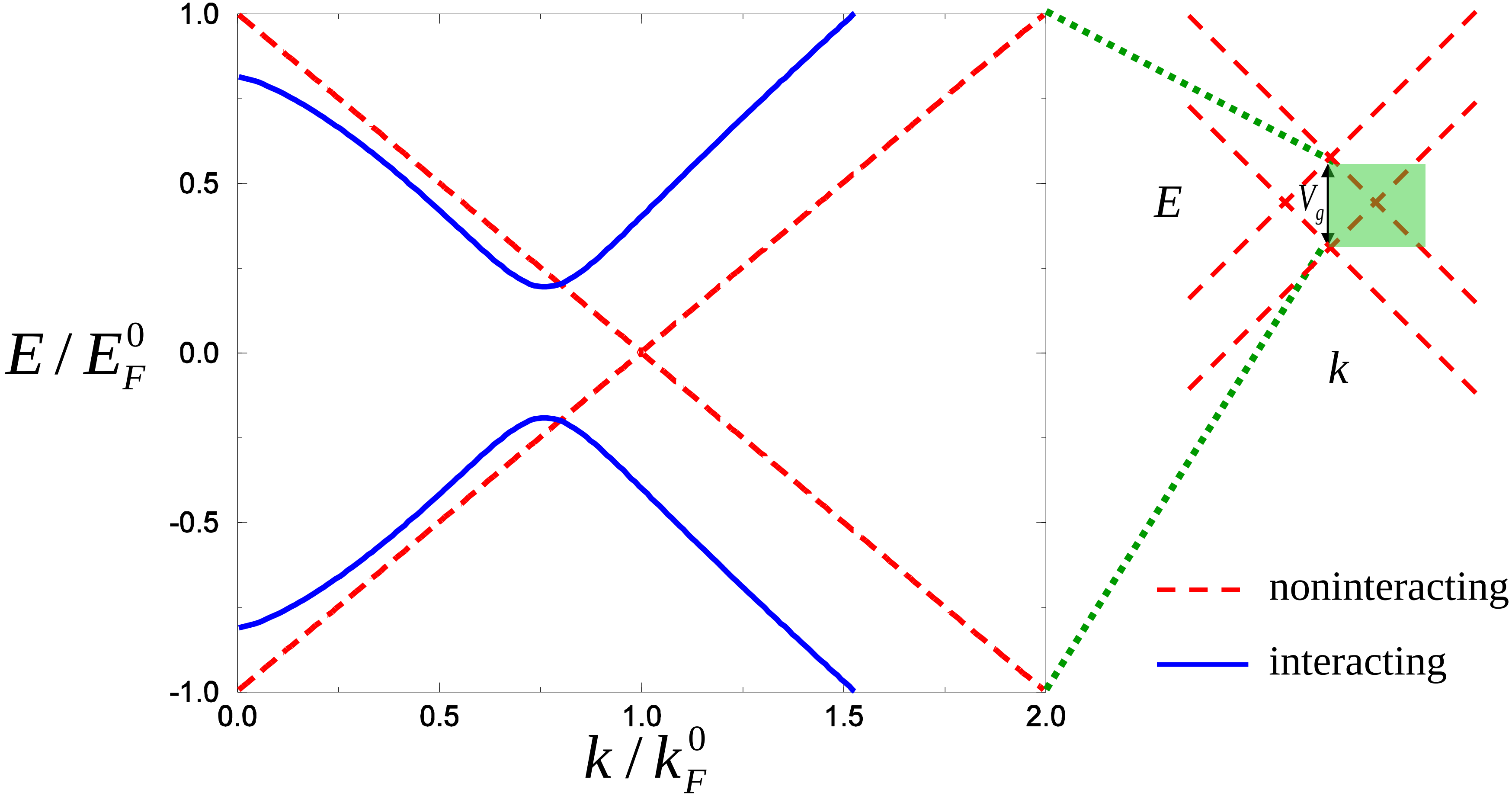


5) Energy band structure

Min et al. PRB **78**, 121401(R) (2008)

- Cooper instability



3.Exciton condensation in graphene double layers

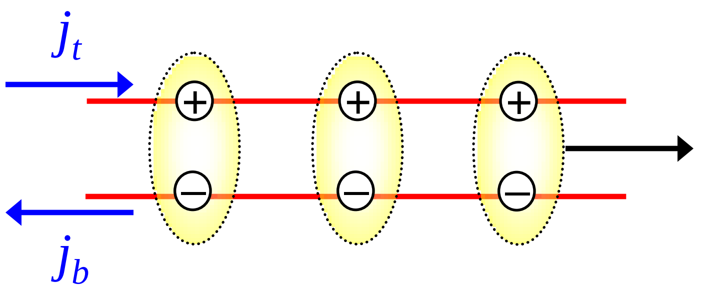
6) Kosterlitz-Thouless (KT) transition

- In 2D, superfluidity is destroyed by phase fluctuations.

$$\boxed{k_B T_{KT} = \frac{\pi}{2}\rho_s(k_B T_{KT})}$$

7) Exciton superfluidity

- Counter-flow current



superfluid density

$$\boxed{j_Q = \frac{e}{\hbar}\rho_s Q}$$

exciton momentum

$$\boxed{j_Q \rightarrow \rho_s \rightarrow T_{KT}}$$

8) Phase diagram

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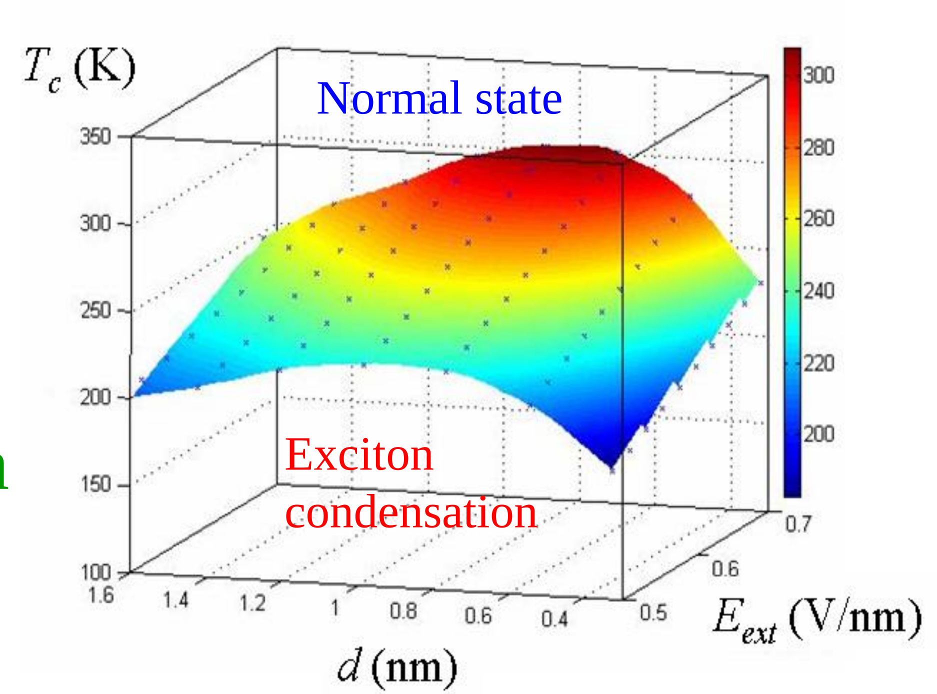
- $T_c \uparrow$ as $E_{ext} \uparrow$
- Optimal layer separation
- $E_{ext} \sim 0.7$ V/nm, $d \sim 1$ nm

$\Rightarrow T_c \sim 300$ K



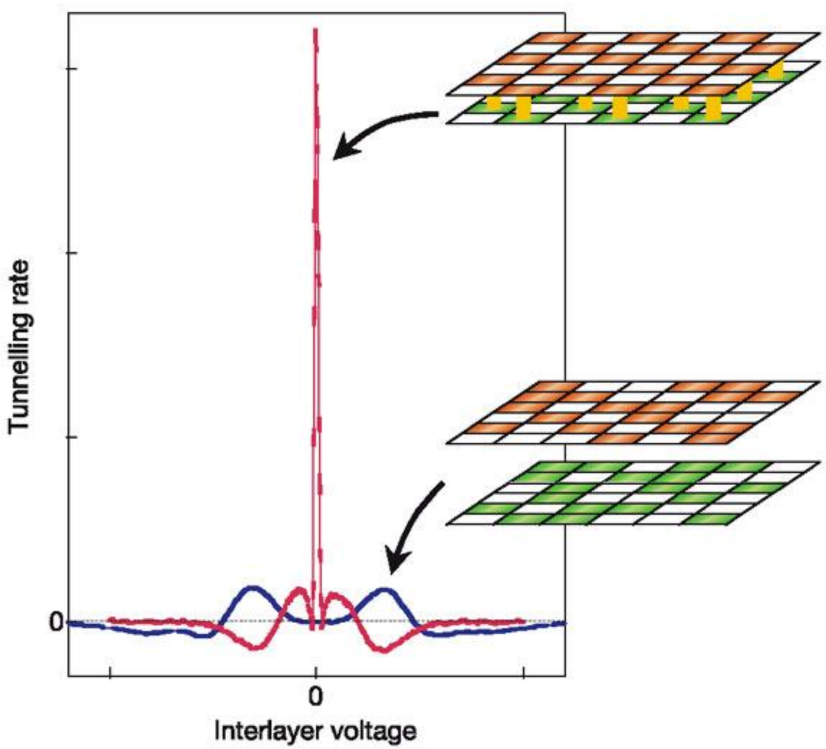
- Comparison with BCS superconductivity

Cooper pair : limited by $\omega_D \Rightarrow T_c \sim 10$ K

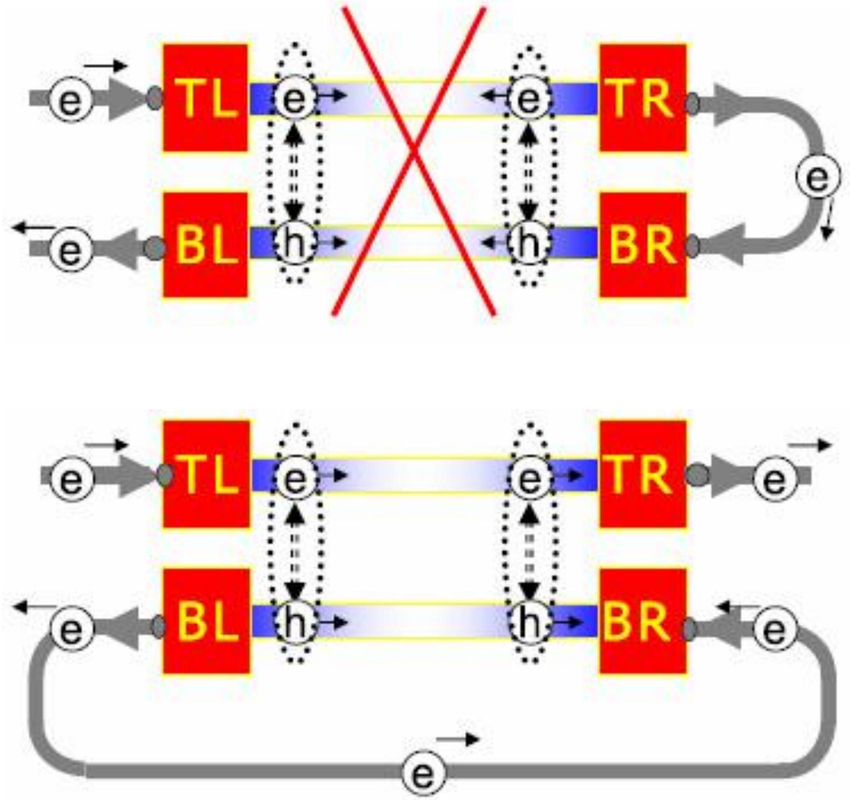
Bilayer exciton : limited by $v_F/d \Rightarrow T_c \sim 300$ K

9) Experimental search

Tunneling



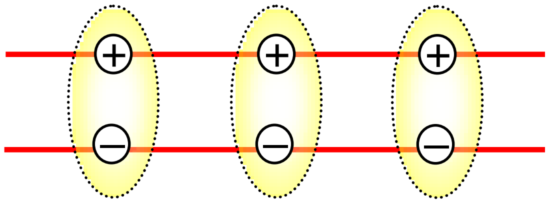
Transport



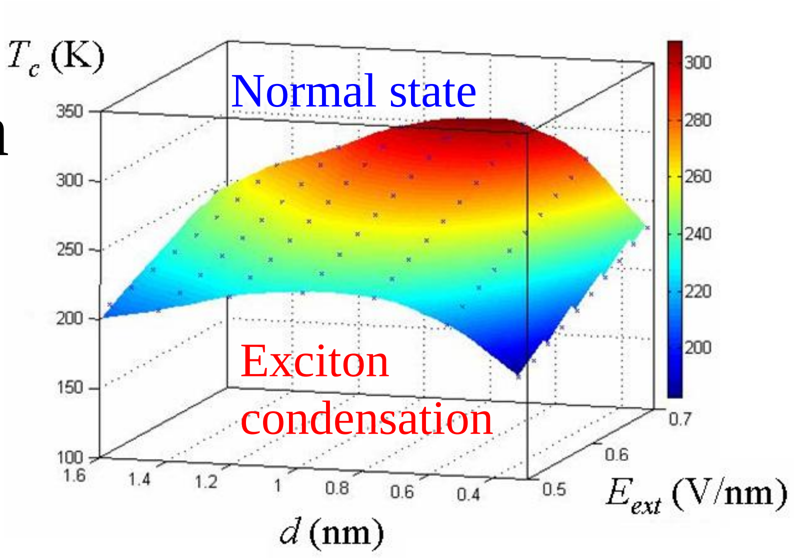
Eisenstein and MacDonald, Nature **432**, 691 (2004)   Su and MacDonald, Nat. Phys. **4**, 799 (2008)

Summary of Part 3

- Bilayer exciton condensation is spontaneous interlayer coherence.

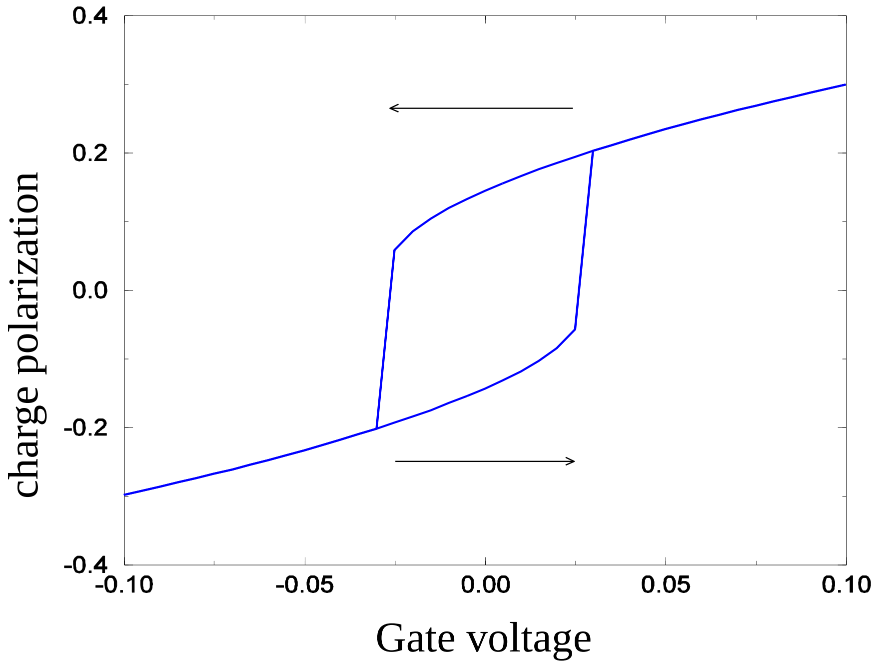


- Bilayer exciton condensation can occur in decoupled graphene double layers at high temperatures.



What happens in coupled graphene bilayers?

Pseudospin magnetism in graphene bilayers



$\Rightarrow$ Spontaneous charge polarization

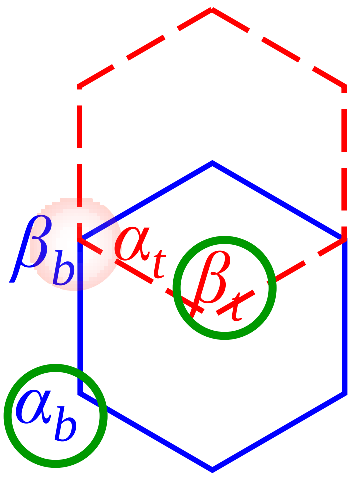
4. Pseudospin magnetism in graphene bilayers

1) Pseudospin $\Rightarrow$ Two-valued quantum degrees of freedom

- Bilayer graphene (J=2 chiral system)

“↑” = α_b “↓” = β_t

$$H_B^{bi} = -\frac{1}{2m}\begin{pmatrix} 0 & p^2 e^{-i2\phi_{\mathbf{p}}} \\ p^2 e^{i2\phi_{\mathbf{p}}} & 0 \end{pmatrix} = -\frac{p^2}{2m}\boldsymbol{\sigma}\cdot\mathbf{n}_2(\phi_{\mathbf{p}})$$



$$\mathbf{n}_2(\phi_{\mathbf{p}}) = (\cos 2\phi_{\mathbf{p}}, \sin 2\phi_{\mathbf{p}})$$

$$\phi_{\mathbf{p}} = \tan^{-1}(p_y / p_x)$$

- Ferromagnetism means spontaneous spin polarization.

- Is there spontaneous charge polarization in the the presence of electron-electron interactions?

$\Rightarrow$ Pseudospin magnetism

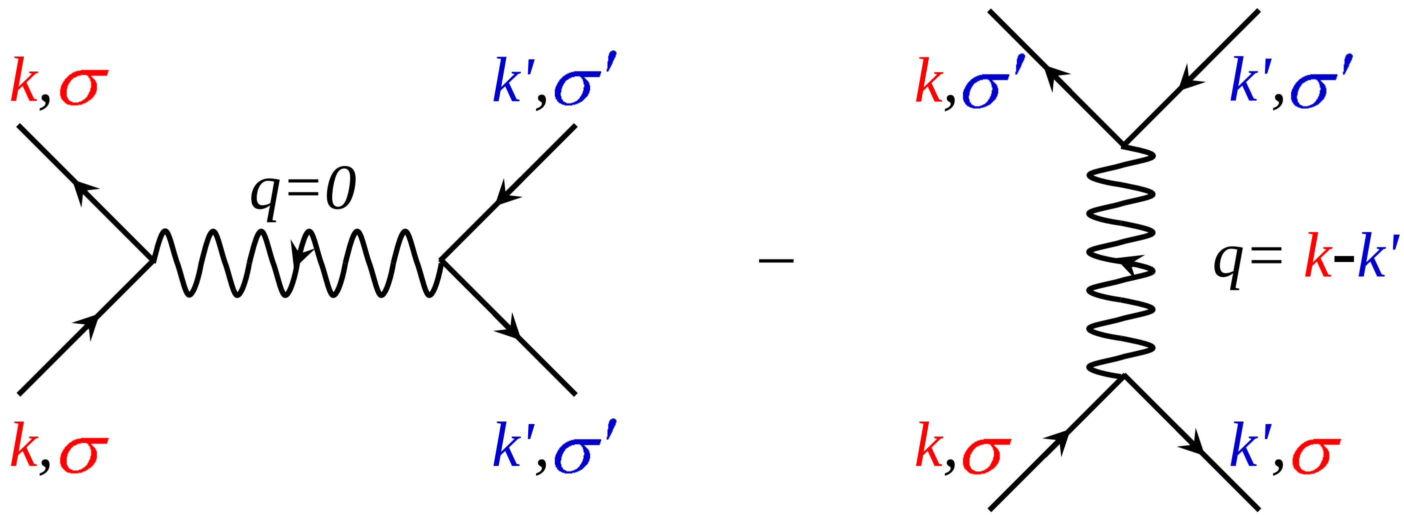
4. Pseudospin magnetism in graphene bilayers

2) Numerical calculation

- A self-consistent mean-field theory

$$\boxed{H_{MF} = -\mathbf{B}(\mathbf{k})\cdot\boldsymbol{\sigma}}$$

$\mathbf{B}(\mathbf{k})$: effective magnetic field
$\boldsymbol{\sigma}$: pseudospin



Direct　　　　　　　　Exchange

3) Example: Bilayer graphene (J=2)

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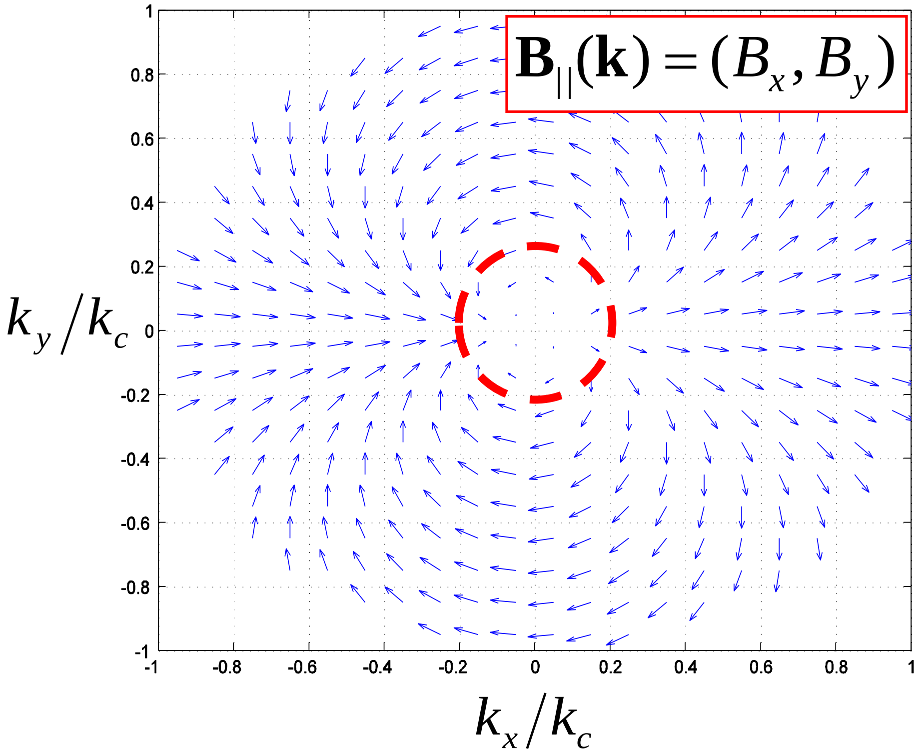

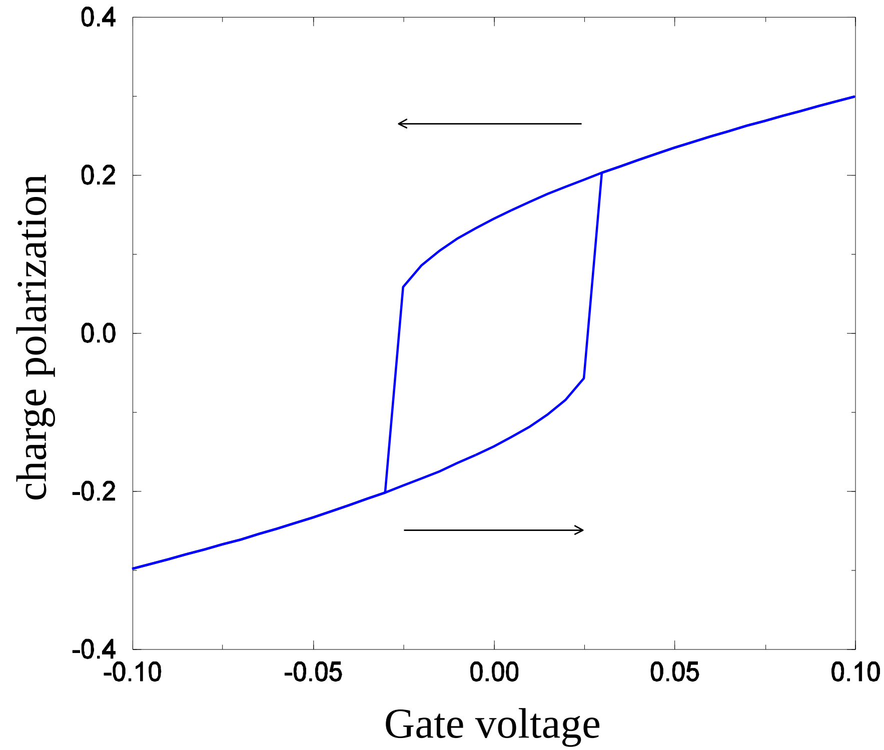

$$H_{MF} = -\mathbf{B}(\mathbf{k})\cdot\boldsymbol{\sigma}$$

$$\mathbf{B}(\mathbf{k}) = (B_\perp \cos J\phi_{\mathbf{k}}, B_\perp \sin J\phi_{\mathbf{k}}, B_z)$$

$\Rightarrow$ Spontaneous charge transfer between two layers

4) Phase diagram

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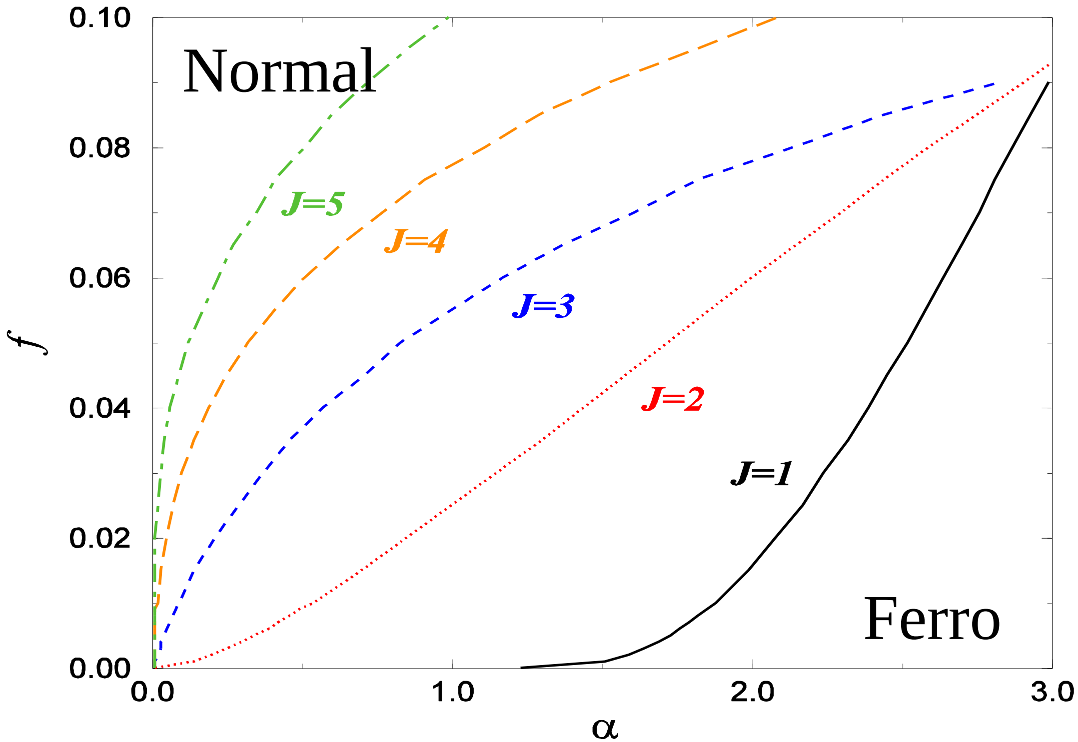


J : chirality

α : interaction strength

f : doping

- Pseudospin magnetism is stable for stronger interaction strength, for smaller doping, for larger chirality.

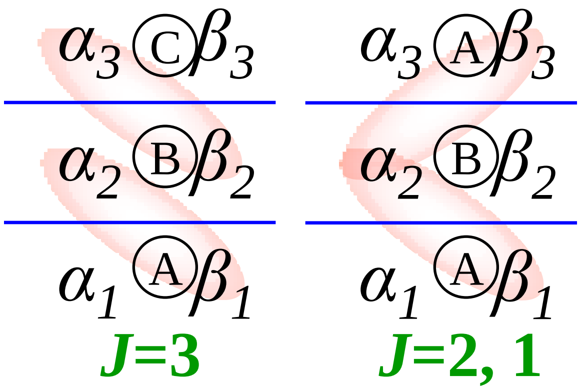
5) Chiral decomposition revisited

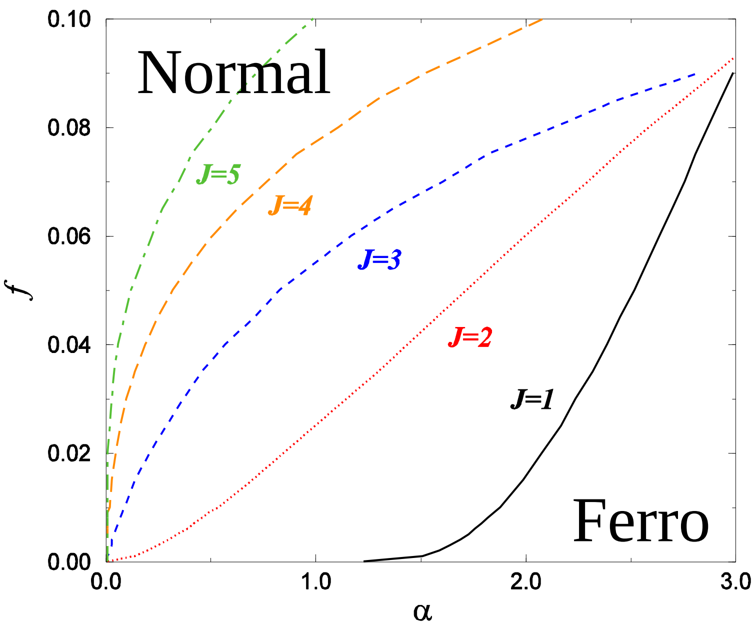
- Chiral decomposition in the electronic structure

$$\boxed{H_N^{eff} = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{N_D}}}$$

$$\boxed{\sum_{i=1}^{N_D} J_i = N}$$

$\Rightarrow$ ABC stacked multilayers are excellent candidates.

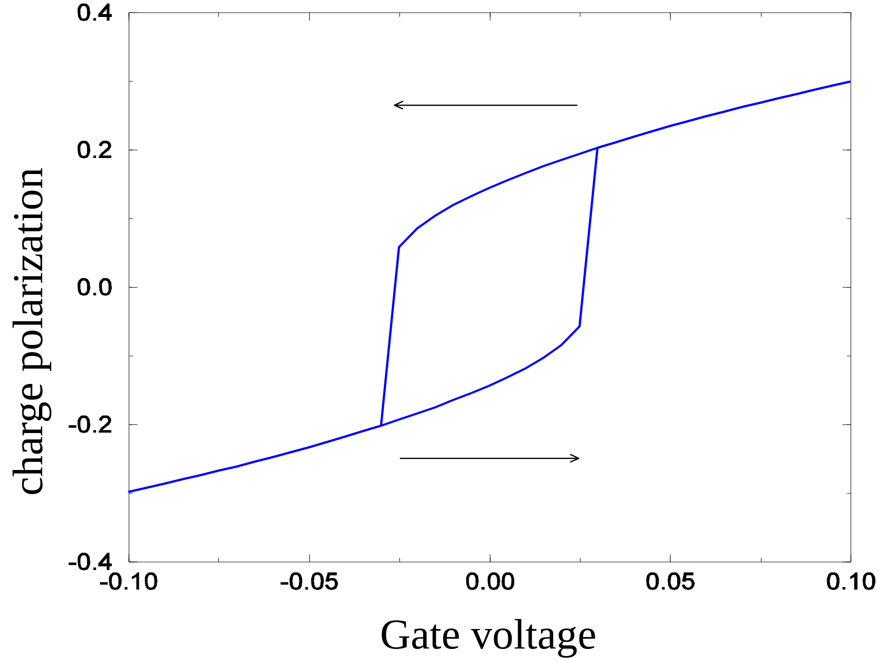




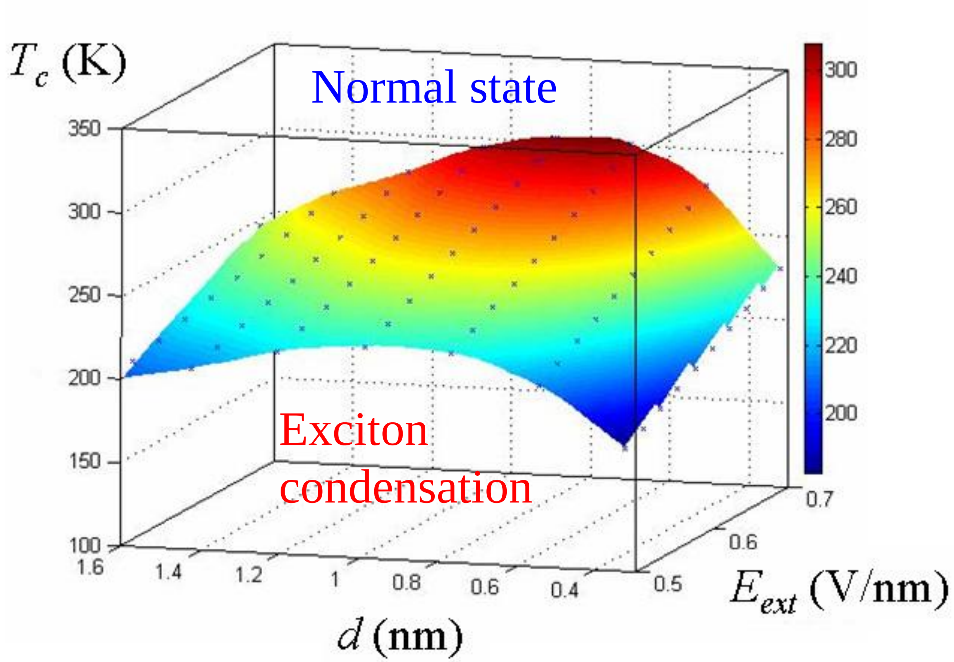
Summary of Part 3

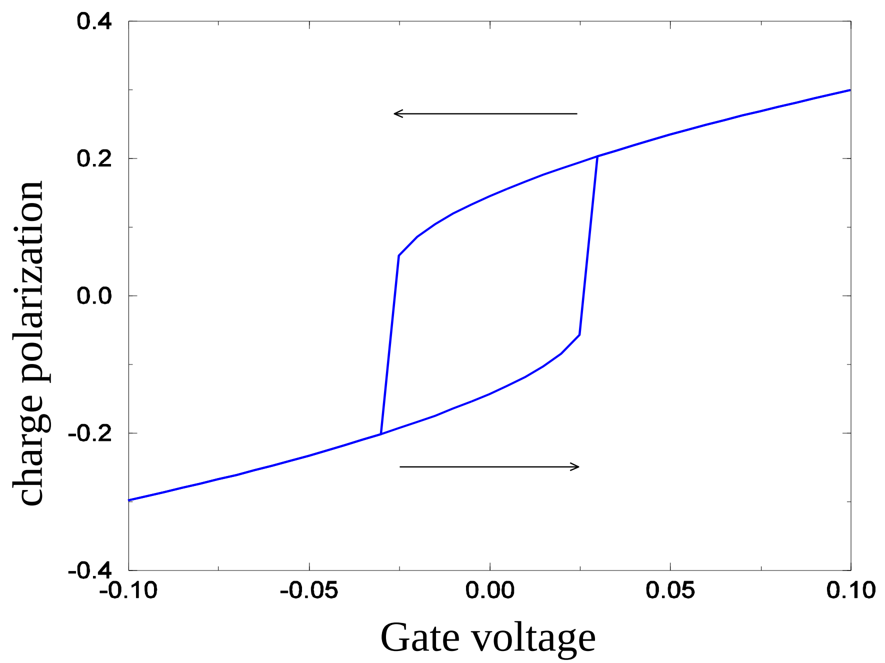
- Psuedospin is any two-valued quantum degrees of freedom.
- A chiral system is described by pseudospin language.
- Pseudospin analogue of ferromagnetism in chiral systems.

$\Rightarrow$ Pseudospin magnetism



Conclusion





$\Rightarrow$ New electronic device scheme?

5. Conclusion

1) Search for new ordered states in graphene systems

Dirac-like chiral wavefunction

+

Electron-electron interaction

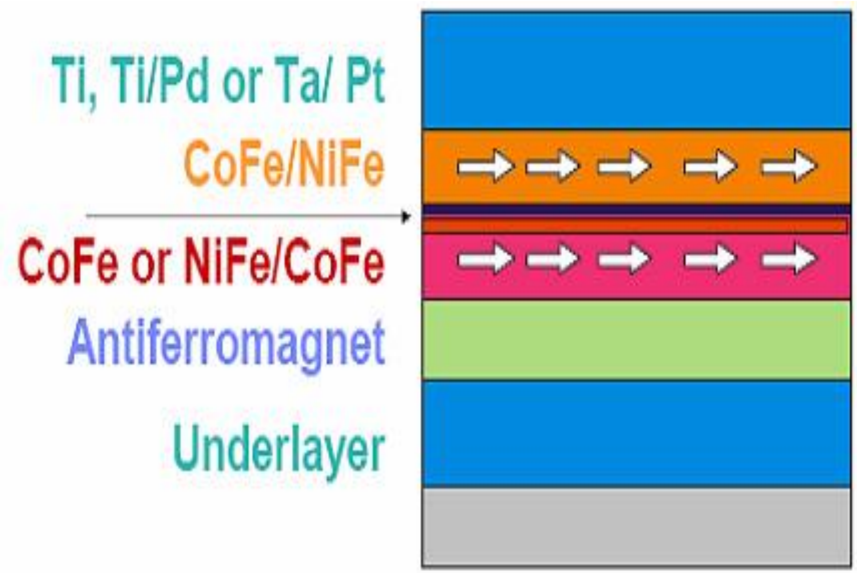
$\Rightarrow$ New ordered states in graphene systems

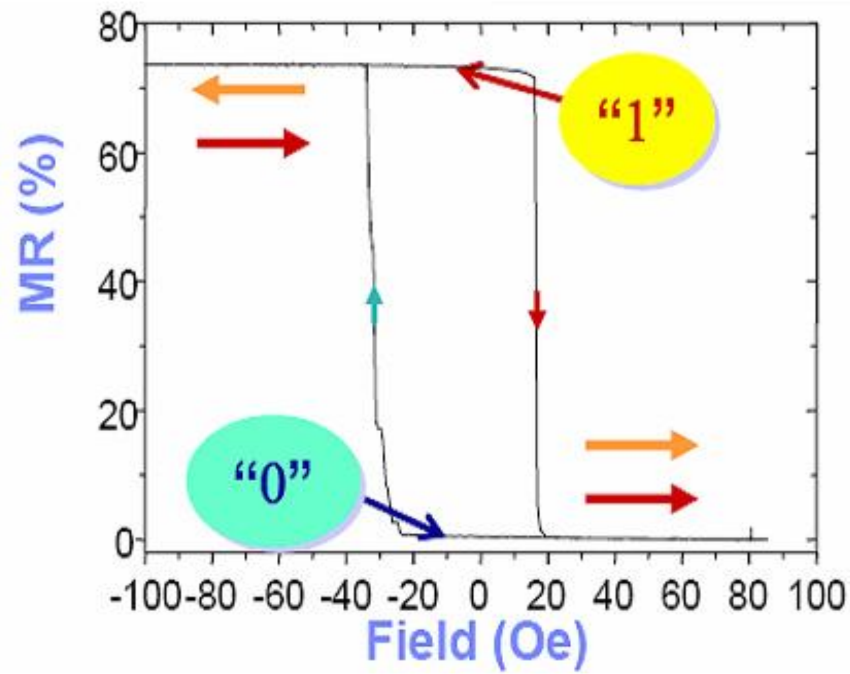
Exciton condensation in decoupled graphene double layers

Pseudospin magnetism in coupled graphene bilayers

2) Collective field effect transistor (FET) vision

- Giant magnetoresistance (GMR)





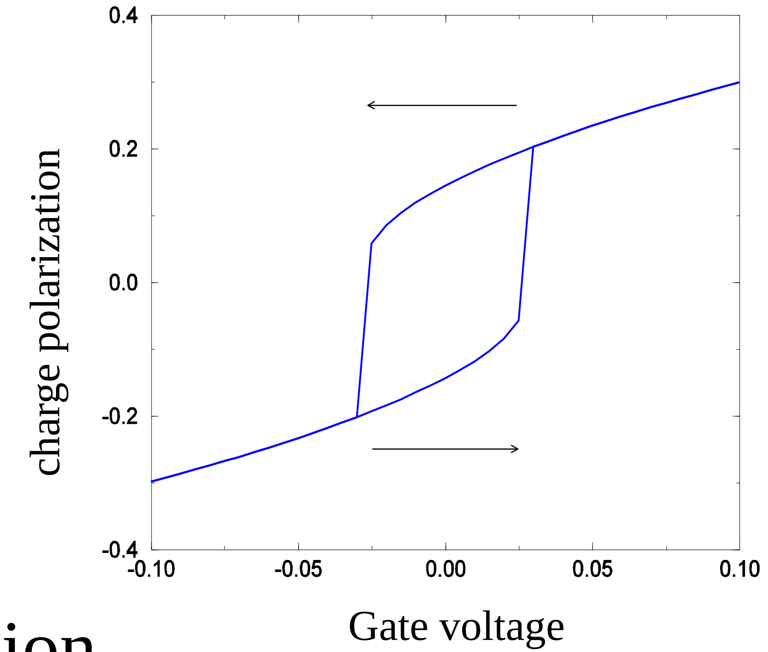
$\Rightarrow$ Collective behavior of many electrons

5. Conclusion

3) New electronic device scheme

Example: Pseudospin magnetism

- Collective behavior of many electrons
- Can be switched with gate voltages using much less power.
- Can exhibit a pseudospin version of GMR and spin-transfer torque.



Pseudospintronics!

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2. Exciton condensation in graphene bilayers

Room-temperature superfluidity in graphene bilayers

Phys. Rev. B **78**, 121401 (R) (2008)

Hongki Min, Rafi Bistritzer, Jung-Jung Su, A. H. MacDonald

3. Pseudospin magnetism in graphene bilayers

Pseudospin magnetism in graphene

Phys. Rev. B **77**, 041407 (R) (2008)

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