Effects of disorder on magnetic vortex dynamics

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Effects of disorder and internal dynamics on vortex wall propagation
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Effects of disorder on magnetic vortex gyration
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Outline

I. Effects of disorder on vortex wall propagation
   • Field-induced propagation
   • Current-induced propagation
   • Enhanced damping and internal dynamics

II. Effects of disorder on vortex gyration
   • Gyration in a disordered film
   • Gyration in a single pinning potential

Extrinsic disorder and excitation of internal domain wall structures enhance the effective damping.
Motivation

Domain wall structure
Field induced domain wall motion

Magnetization dynamics

• Landau-Lifshits-Gilbert (LLG) equation

\[ \frac{\partial}{\partial t} \mathbf{M} = -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{M} \mathbf{M} \times \frac{\partial}{\partial t} \mathbf{M} \]

\( \mathbf{M} = \) magnetization  \( \gamma = \) gyromagnetic ratio  \( \mathbf{H}_{\text{eff}} = \) Effective magnetic field  \( \alpha = \) Gilbert damping

Collective coordinate approach

• Transverse wall and vortex wall
Collective coordinate approach

Two-coordinate model for vortex core position

• **Equations of motion** $(X, Y)$

\[
\alpha \mathbf{DV} = \mathbf{F} + \mathbf{V} \times \mathbf{G}
\]

- $\mathbf{V} =$ time derivative of $(X, Y)$
- $\mathbf{D} =$ viscosity tensor
- $\mathbf{F} =$ generalized force
- $\mathbf{G} =$ gyroscopic tensor
- $\alpha =$ Gilbert damping

\[
m \ddot{\mathbf{v}} + \Gamma \mathbf{v} = q(\mathbf{E} + \mathbf{v} \times \mathbf{B})
\]

- $D_{ij} = \int d^3x \frac{\partial \mathbf{m}}{\partial X_i} \cdot \frac{\partial \mathbf{m}}{\partial X_j}$
- $F_i = \gamma \int d^3x H_{\text{eff}} \cdot \frac{\partial \mathbf{m}}{\partial X_i} = -\frac{\gamma}{\mu_0 M} \frac{\partial E}{\partial X_i}$
- $G_{ij} = \int d^3x \mathbf{m} \cdot \left( \frac{\partial \mathbf{m}}{\partial X_i} \times \frac{\partial \mathbf{m}}{\partial X_j} \right)$

Thiele, PRL 30, 230 (1973);
Schryer and Walker, JAP 45, 5406 (1974);
Thiaville et al, EPL 69, 990 (2005);
Li and Zhang, PRB 70, 024417 (2004);
He et al, PRB 73, 184408 (2006);
Tretiakov et al, PRL 100, 127204 (2008);
Clarke et al, PRB 78, 134412 (2008);
.....

2D massless charged particle moving through a medium with a viscosity tensor $\alpha \mathbf{D}$ in the presence of an in-plane electric field $\mathbf{F}$ and a perpendicular magnetic field $\mathbf{G}$.
Does this reduced model, which ignores the extrinsic disorder and internal structure of the domain wall, provide a good interpretation of experiments?
Simulation

- LLG equation solver
  - Objected Oriented MicroMagnetic Framework
- Permalloy (Ni\textsubscript{80}Fe\textsubscript{20}) thin film
  - 200 nm width, 20 nm thickness, 5 nm cell size
  - 10000 nm ~ 15000 nm length
- Disorder model
  - Correlated variation in $M_s$ to model thickness fluctuations
  \[
  D = \frac{\sqrt{\langle M(r) - M_s \rangle^2}}{M_s}
  \]
- Removing finite size effects
  - Compensating fields at ends
  - Absorbing boundary conditions at ends
$D = \sqrt{\Delta M^2(r)/M_s}$

Enhanced propagation by disorder

Pinned by disorder

$\mu_0 H = 3 \text{ mT}$

Velocity estimation by ensemble average

\[ \Delta = 0.025 \]

\[ X = 0 \times (\text{nm}) \]

\[ \Delta = 0.075 \]

\[ X = 0 \times (\text{nm}) \]

\[ \mu_0 H = 3 \text{ mT} \]

\[ V_{DW} = 34.2 \text{ m/s} \]

\[ V_{DW} = 53.3 \text{ m/s} \]

\[ V_{DW} = 63.5 \text{ m/s} \]

\[ V_{DW} = 77.7 \text{ m/s} \]

No disorder

Pinning by disorder

\[ \mu_0 H = 3 \text{ mT} \]

\[ D = 0.025 \]

\[ D = 0.05 \]

\[ D = 0.075 \]

\[ D = 0.025 \]

\[ D = 0.05 \]

\[ D = 0.075 \]

\[ D = 0.025 \]

\[ D = 0.05 \]

\[ D = 0.075 \]
Disorder can enhance/suppress velocity

Domain wall velocity as a function of field

\[ V_{DW}(\text{m/s}) \]

\[ \mu_0 H \text{ (mT)} \]

\[ D=0 \]

\[ D=0.025 \]

\[ D=0.05 \]
Ideal model with different damping constants

Domain wall velocity as a function of fields

Results can be understood in terms of an effective damping that increases with the disorder.
Current induced domain wall motion

Magnetization dynamics

- Landau-Lifshits-Gilbert (LLG) equation

\[
\begin{align*}
\left( \frac{\partial}{\partial t} + \mathbf{v}_s \cdot \nabla \right) \mathbf{M} &= -\gamma \mathbf{M} \times \mathbf{H}_{\text{eff}} + \frac{\alpha}{\mathbf{M}} \mathbf{M} \times \left( \frac{\partial}{\partial t} + \frac{\beta}{\alpha} \mathbf{v}_s \cdot \nabla \right) \mathbf{M} \\
\end{align*}
\]

\(\mathbf{M}\) = magnetization  \(\mathbf{H}_{\text{eff}}\) = Effective magnetic field  \(\gamma\) = gyromagnetic ratio

\(\alpha\) = Gilbert damping  \(\beta\) = spin-transfer torque parameter  \(\mathbf{v}_s\) = spin velocity

Collective coordinate approach: Vortex wall

- Equations of motion  \((X, Y)\)

\[
\begin{align*}
\Gamma \left( \mathbf{V} - \frac{\beta}{\alpha} \mathbf{v}_s \right) &= \mathbf{F} + (\mathbf{V} - \mathbf{v}_s) \times \mathbf{G} \\
\end{align*}
\]

\(\mathbf{V}\) = domain wall velocity  \(\mathbf{D}\) = viscosity tensor  \(\mathbf{F}\) = generalized force  \(\mathbf{G}\) = gyroscopic tensor
Changing $\alpha$ in high current regime: $v_s = 2.5v_c$ ($\beta = 0$)

Ideal model with different $\beta/\alpha$:

$\Gamma \left( V - \frac{\beta}{\alpha} v_s \right) = F + (V - v_s) \times G$

$X/\lambda$ vs $Y/\lambda$

Trajectories depend on $\beta/\alpha$.
\[ D = \sqrt{\Delta M^2(r)/M_s} \]

Deviated trajectory by disorder

\[ \beta/\alpha = 1 \]

\[ J = 2 \times 10^{13} \text{ A/m}^2 \]
Disorder enhances the effective damping

Disorder vs velocity

Damping vs velocity

\[ J = 2 \times 10^{13} \text{ A/m}^2 > J_c, \beta = 0 \]
Disorder enhances the effective damping

Domain wall velocity as a function of disorder

\[ D = \frac{\sqrt{\Delta M^2(r)}}{M_s} \]

Extracted effective damping constant could vary sample by sample
Energy dissipation

Dissipation through spin wave and vortex core

\begin{align*}
\frac{dE}{dtdx} &= \alpha \left( \frac{fJ}{\text{ns} \cdot \text{nm}} \right) \\
\alpha &= 0.01 \\
\mu_0 H &= 3 \text{ mT}
\end{align*}

\[
\frac{dE}{dt} \propto -\alpha \left( \frac{\partial M}{\partial t} \right)^2
\]
Energy dissipation

Dissipation through spin wave and vortex core

\( \alpha = 0.01 \)
\( \mu_0 H = 3 \text{ mT} \)

\[ \frac{dE}{dt dx} \left( \frac{\text{fJ}}{\text{ns} \cdot \text{nm}} \right) \]

Increased rate of energy dissipation from vortex core motion indicates excitation of the internal degrees of freedom in domain wall structure.
Summary of Part I

- Disorder can affect domain wall dynamics significantly.
- Excitation of internal degrees of freedom by disorder gives enhanced effective damping.
- Caution is needed when using idealized models to interpret domain wall motion measurements.
Vortex in a magnetic disk

Diameter=400 nm, thickness=10 nm, cell=2.5 nm

Study the enhanced damping ratio in vortex gyration
Displacement of vortex core by static fields

$\mu_0 H_y = 10 \text{ mT}$

No magnetic field

$H_x, H_y, H_p$

$t = 200 \text{ ps}$
Gyration frequency map from Kerr microscopy

Compton and Crowell, PRL 97, 137202 (2007);
Compton, Chen and Crowell, PRB 81, 144412 (2010)

Gyration with a frequency that is a characteristic of a pining site
\[ D = 0.05, \mu_0 H_p = 0.1 \text{ mT}, \, dt_p = 0.2 \text{ ns} \]

\[ D = \sqrt{\Delta M^2(r)/M_s} \]

Gyration frequency map from simulation

Disorder image

Gyration frequency map

Gyration with a frequency that is a characteristic of a pining site
Gyration with a frequency that is a characteristic of a pining site.
Time evolution of gyration radius

$D = 0.05, \mu_0 H_p = 20 \, \text{mT}, \, dt_p = 1 \, \text{ns}$

Transition from free to trapped regimes

Correlation length $\approx 10 \, \text{nm}$

No disorder
Vortex gyration in a single pinning potential

Single pinning potential with a radius of 10 nm

\[ \delta = \frac{\Delta M_c}{M_s} \]

\[ H_p = 20 \text{ mT} \]

\[ t = 1 \text{ ns} \]

400 nm
Collective coordinate approach

Two-coordinate model for vortex core position

- Equations of motion 

\[ \alpha \mathbf{D} \mathbf{V} = \mathbf{F} + \mathbf{V} \times \mathbf{G} \]

\[ R = R_0 \exp \left( -\frac{t}{\tau} \right), \quad \phi = 2\pi ft \]

\[ \Rightarrow \quad 2\pi f_{\tau} C \alpha = 1 \]

\[ C = \frac{D_{\phi\phi}}{G + \alpha D_{R\phi}} \]

Geometrical factor

\[ \Rightarrow \quad \frac{f_0 \tau_0}{f \tau} = \frac{\alpha_{\text{eff}} C}{\alpha_0 C_0} \]

\[ \text{By comparing free and trapped regimes, the effective damping can be determined.} \]
Enhanced decay rate by a pinning potential

Evolution of gyration radius

\[ \delta = \Delta M_c / M_s \]

![Graph showing the evolution of gyration radius over time with different values of \( \delta \).](image)

- **Trapped at** \( r=10 \text{ nm} \)
- **Measured from the peak position in the spectrum**
- **Measured from the line width in the spectrum**

**Inset:**

\[ \frac{f_0 \tau_0}{f \tau} = \frac{\alpha_{\text{eff}} C}{\alpha_0 C_0} \]

Transition from free to trapped regimes
Deformation by a pinning potential

Evolution of deformation factor

\[ \delta = \frac{\Delta M_c}{M_s} \]

Deformation of vortex should be taken into account

Deformation of vortex structure

Deformation of a vortex structure

\[ C = \frac{D_{\phi\phi}}{G + \alpha D_{R\phi}} \]

\[ \frac{f_0 \tau_0}{f \tau} = \frac{\alpha}{\alpha_0} \frac{C}{C_0} \]
Enhanced effective damping

Enhancement ratio as a function of depth

Enhanced damping mostly comes from the change in geometry of the vortex.

\[ \frac{\alpha_{\text{eff}}}{\alpha_0} = \frac{f_0\tau_0}{f\tau} \frac{C_0}{C} \]

\[ \delta = \frac{\Delta M_c}{M_s} \]
As the frequency increases, the vortex gets more excited increasing the effective damping.
Summary of Part II

- The enhanced damping rate could be measured by the gyration experiments up to the deformation factor.

- The larger the gyration frequency, the bigger the effective damping.

- The effective damping is enhanced by disorder and internal excitations of domain wall structure.

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