Title

2008 APS March Meeting

Chirality Sum Rule in Graphene Multilayers

Hongki Min and A. H. MacDonald

Department of Physics, The University of Texas at Austin

Phys. Rev. B **77**, 155416 (2008)

Outline

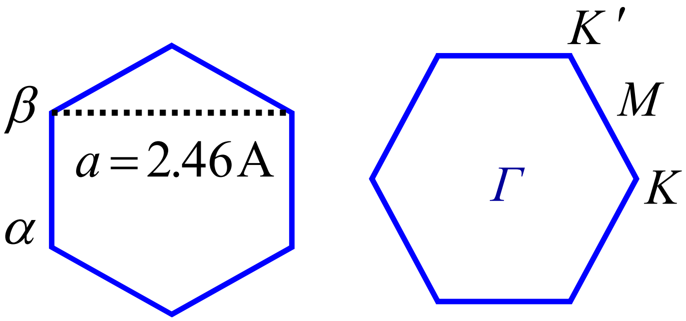
The low energy electronic structure of arbitrarily stacked graphene multilayers are described by a set of chiral 2D electron systems.

1. Monolayer Graphene
2. Multilayer Graphene
3. Chirality Sum Rule
4. Applications: Pseudospin Magnetism

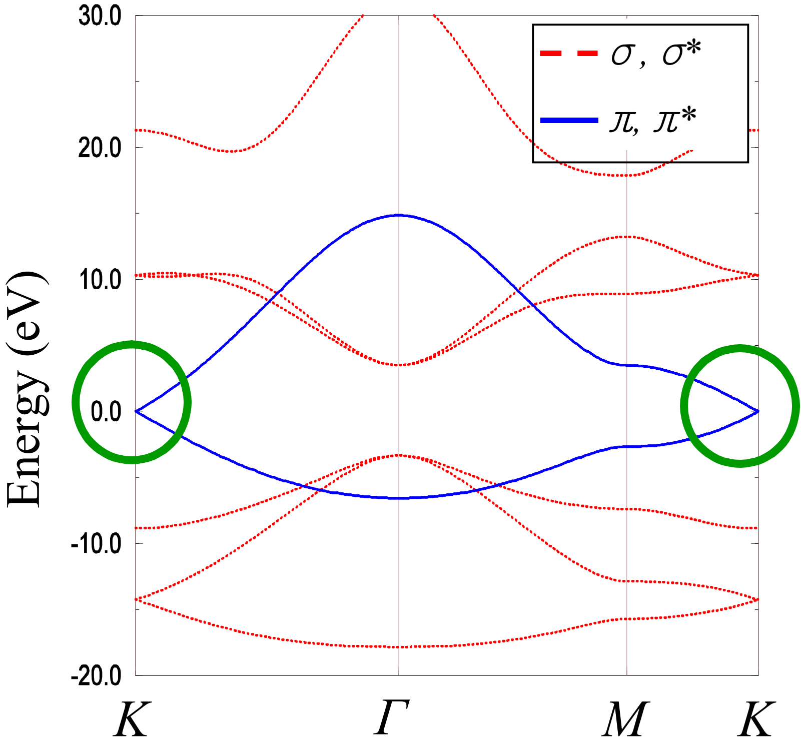
1. Monolayer Graphene

1) Graphene

- Graphene is a two-dimensional honeycomb lattice of carbon atoms.



- At low energies near *K*/*K′*, energy bands are described by a 2D Dirac-like equation with linear dispersion.



1. Monolayer Graphene

2) Effective theory

- At low energies near *K*, graphene is described by

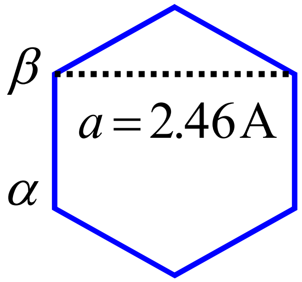
$$H = v_F \begin{array}{cc} \alpha & \beta \\ \left(\begin{array}{cc} 0 & pe^{-i\phi_{\mathbf{p}}} \\ pe^{i\phi_{\mathbf{p}}} & 0 \end{array}\right) \end{array} = v_F\, p\, \boldsymbol{\sigma}\cdot\mathbf{n}$$

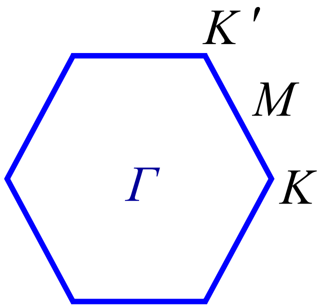
$$\boldsymbol{\sigma}_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \boldsymbol{\sigma}_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$$

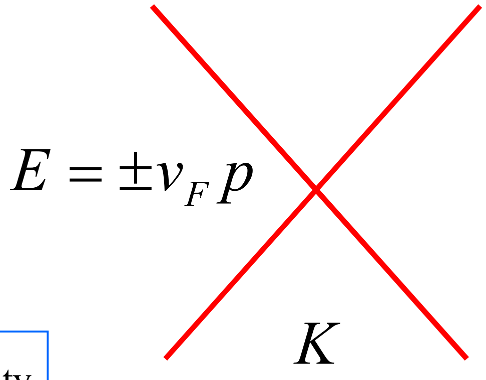
$$\mathbf{n}(\phi_{\mathbf{p}}) = (\cos\phi_{\mathbf{p}}, \sin\phi_{\mathbf{p}})$$

$v_F =$ in-plane velocity

$\phi_{\mathbf{p}} = \tan^{-1}(p_y / p_x)$







$$E = \pm v_F\, p$$

- **σ** refers to sublattice *α* and *β* instead of spin ↑ and ↓.

⇒ pseudospin

1. Monolayer Graphene

3) Chirality

- Chirality is defined by a projection of spin (**σ**) on the direction of motion (**n**).

$$H_J = \varepsilon_0 \begin{pmatrix} 0 & p^J e^{-iJ\phi_{\mathbf{p}}} \\ p^J e^{iJ\phi_{\mathbf{p}}} & 0 \end{pmatrix} = \varepsilon_0 p^J \boldsymbol{\sigma} \cdot \mathbf{n}_J(\phi_{\mathbf{p}})$$

$$\mathbf{n}_J(\phi_{\mathbf{p}}) = (\cos J\phi_{\mathbf{p}}, \sin J\phi_{\mathbf{p}})$$

$$\phi_{\mathbf{p}} = \tan^{-1}(p_y / p_x)$$

- Energy spectrum is given by

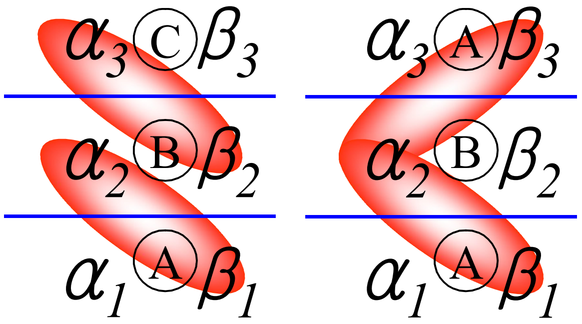
$$E_J(\mathbf{p}) = \pm\varepsilon_0 p^J$$

⇒ Monolayer graphene is described by *J*=1 chiral 2D electron gas.

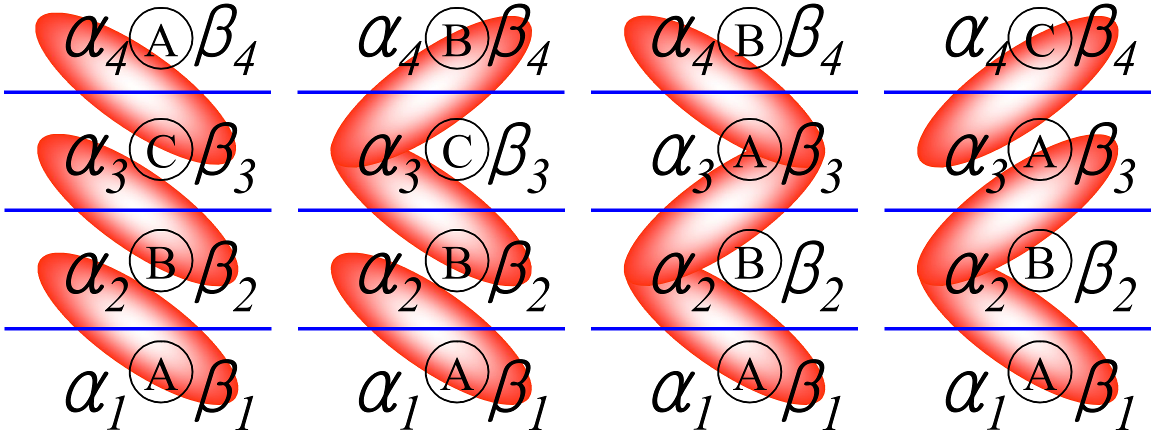
2. Multilayer Graphene

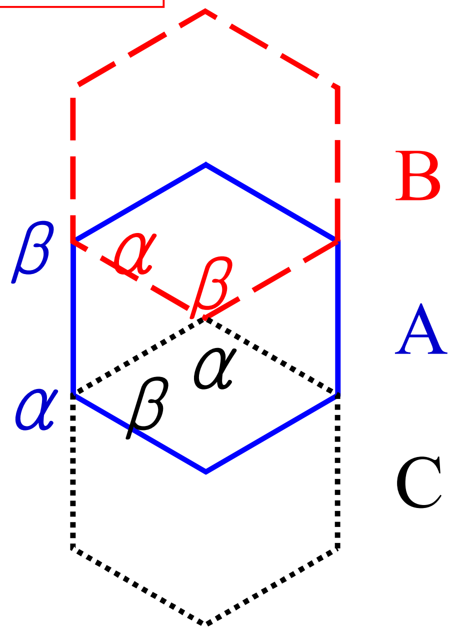
1) Stacking diagrams

- Layers are linked by vertical interlayer hopping



Ordered stacks:
Guinea et al., PRB **73**, 245426 (2006);
Koshino et al., PRB **76**, 085425 (2007);
Mañes et al., PRB **75**, 155424 (2007);
Nakamura et al., PRB **77**, 045429 (2008).





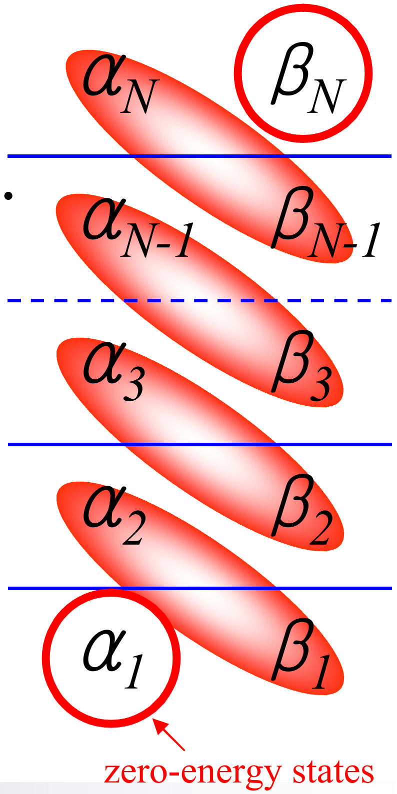
2. Multilayer Graphene

2) Effective theory

Min et al. PRB **77**, 155416 (2008)

- Identify zero-energy states from the diagram.
- Obtain the effective theory using degenerate state perturbation theory.
- Example: ABC stacked *N*-layer

Isolated α_1 and β_N are zero-energy states.



$$H_N^{ABC} = \varepsilon_0 \begin{array}{c} \begin{array}{cc} \alpha_1 & \beta_N \end{array} \\ \begin{pmatrix} 0 & p^N e^{-iN\phi_{\mathbf{p}}} \\ p^N e^{iN\phi_{\mathbf{p}}} & 0 \end{pmatrix} \end{array}$$

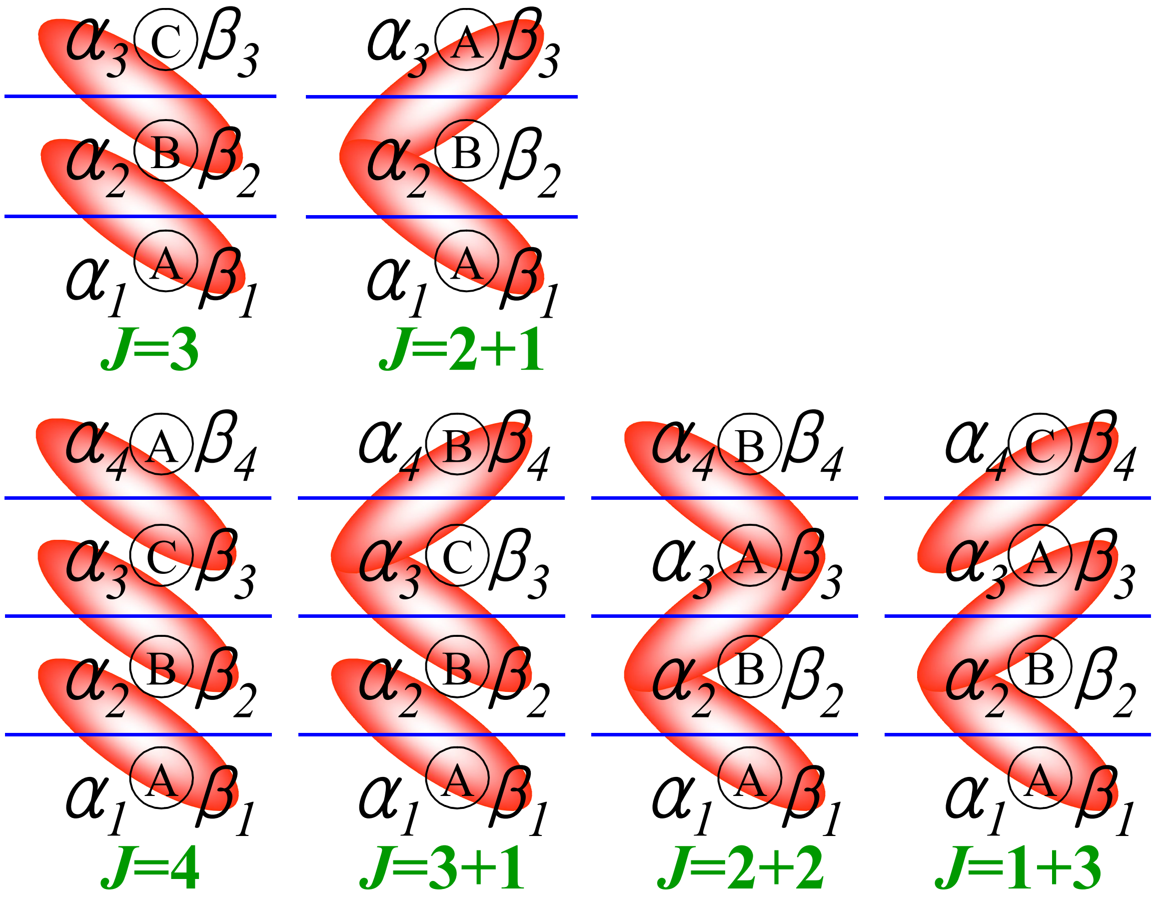
$\Rightarrow$ ABC stacked *N*-layer graphene is described by *N*-chiral 2D electron gas.

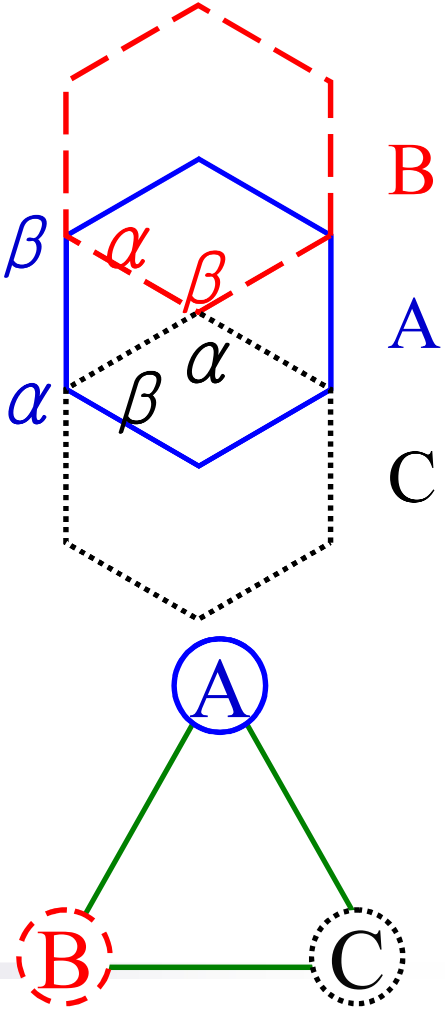
2. Multilayer Graphene

3) Example: N=3 and N=4 layers

Min et al. PRB **77**, 155416 (2008)

- The system is decomposed to different chiral systems.





2. Multilayer Graphene

4) Energy band structure

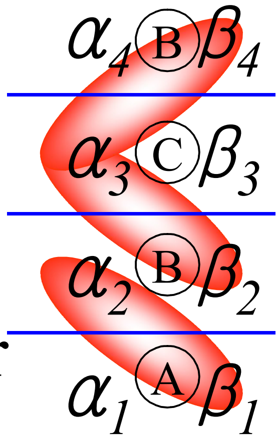
Min et al. PRB **77**, 155416 (2008)

- Energy spectrum for *J*-chiral system

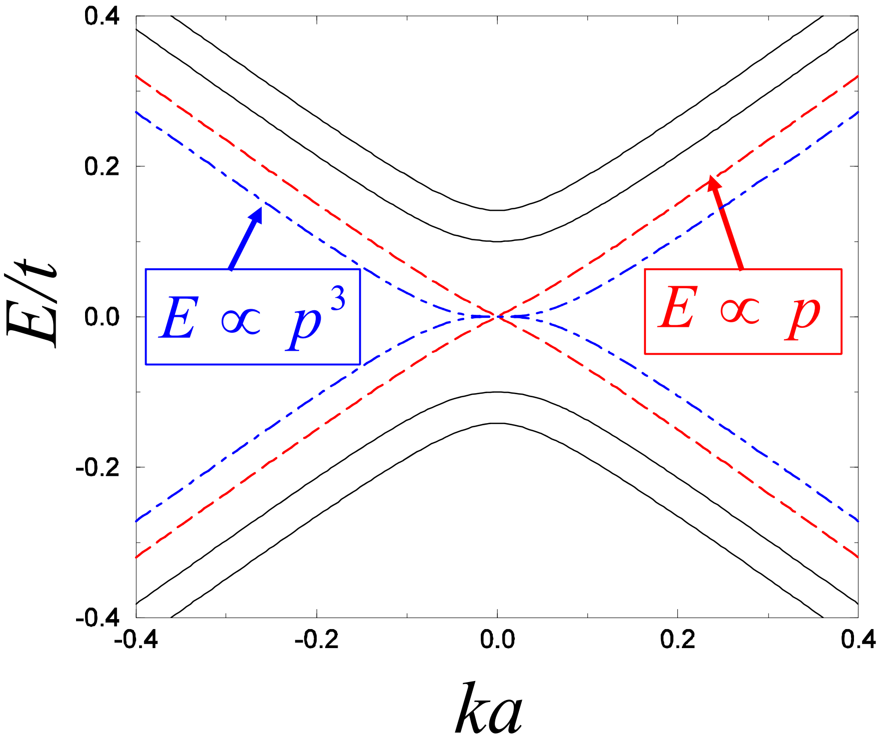
$$\boxed{E_J(\mathbf{p}) \propto p^J}$$

- Example: ABCB tetralayer

$$\boxed{E_{ABCB}(\mathbf{p}) \propto p, p^3}$$



***J*=3+1**



⇒ ABCB tetralayer is described by a combination of *J*=3 and *J*=1 chiral 2D systems

3. Chirality Sum Rule

1) Chirality sum rule

Min et al. PRB **77**, 155416 (2008)

- *J*-chiral system is described by

$$\boxed{H_J = \varepsilon_0 p^J \boldsymbol{\sigma} \cdot \mathbf{n}_J(\phi_{\mathbf{p}})}$$

$$\boxed{\mathbf{n}_J(\phi_{\mathbf{p}}) = (\cos J\phi_{\mathbf{p}}, \sin J\phi_{\mathbf{p}})}$$

- Arbitrarily stacked graphene multilayers are described by a set of chiral pseudospin doublets.

$$\boxed{H_N^{eff} = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{N_D}}}$$

N_D= Number of doublets

- Although the number of doublets in an *N*-layer system depends on the stacking sequence, the pseudospin chirality sum is always *N*.

$$\boxed{\sum_{i=1}^{N_D} J_i = N}$$

3. Chirality Sum Rule

2) Quantum Hall effect

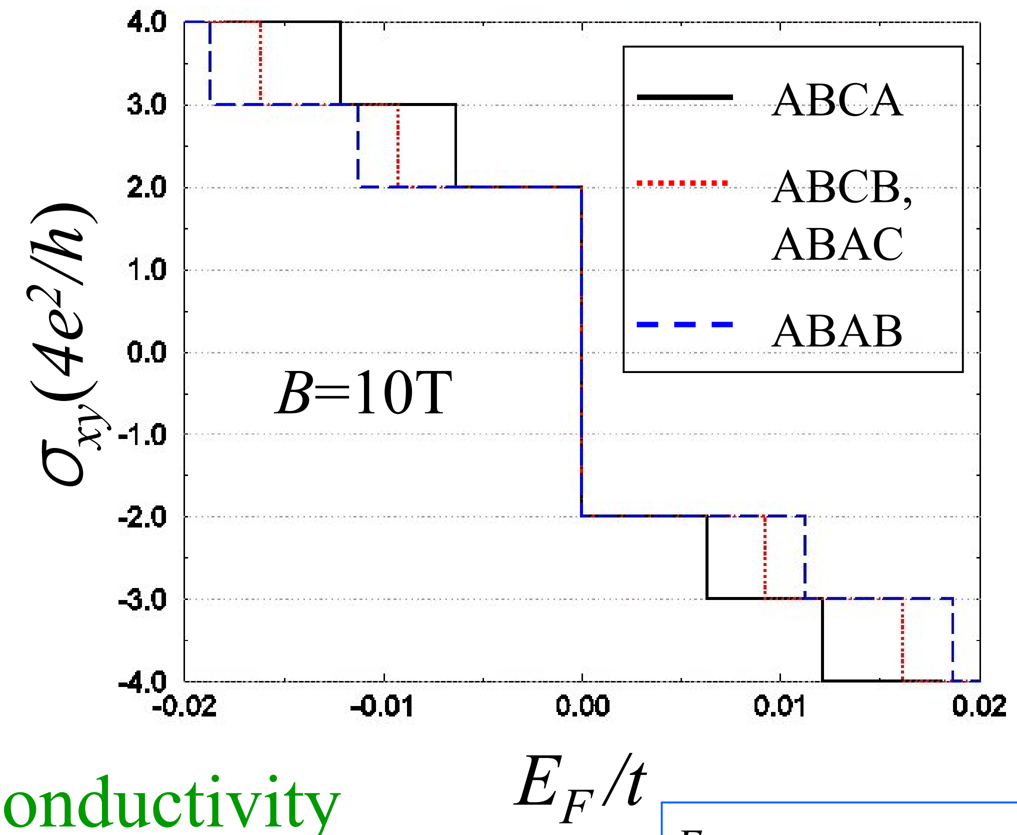
Min et al. PRB **77**, 155416 (2008)

- Chirality sum rule implies a new quantum Hall conductivity.

$$\boxed{\sum_{i=1}^{N_D} J_i = N}$$

$$\Rightarrow \boxed{\sigma_{xy} = \pm \frac{4e^2}{h}\left(\boxed{\frac{N}{2}} + n\right)}$$

n=0,1,2,3,…



E_F = Fermi energy
t = intralayer hopping

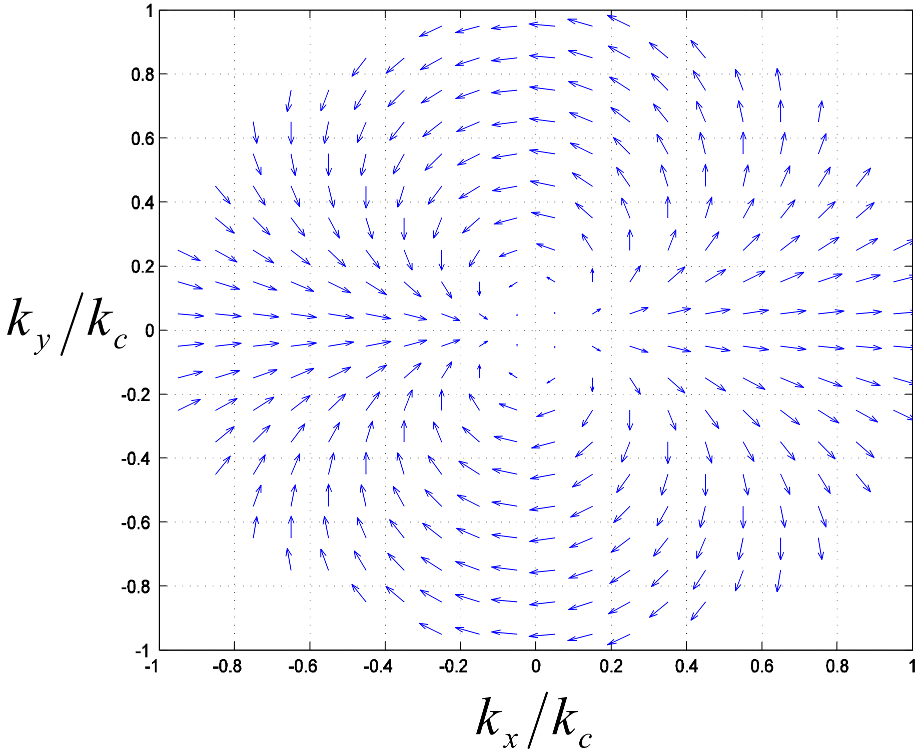
- Example: Tetralayer

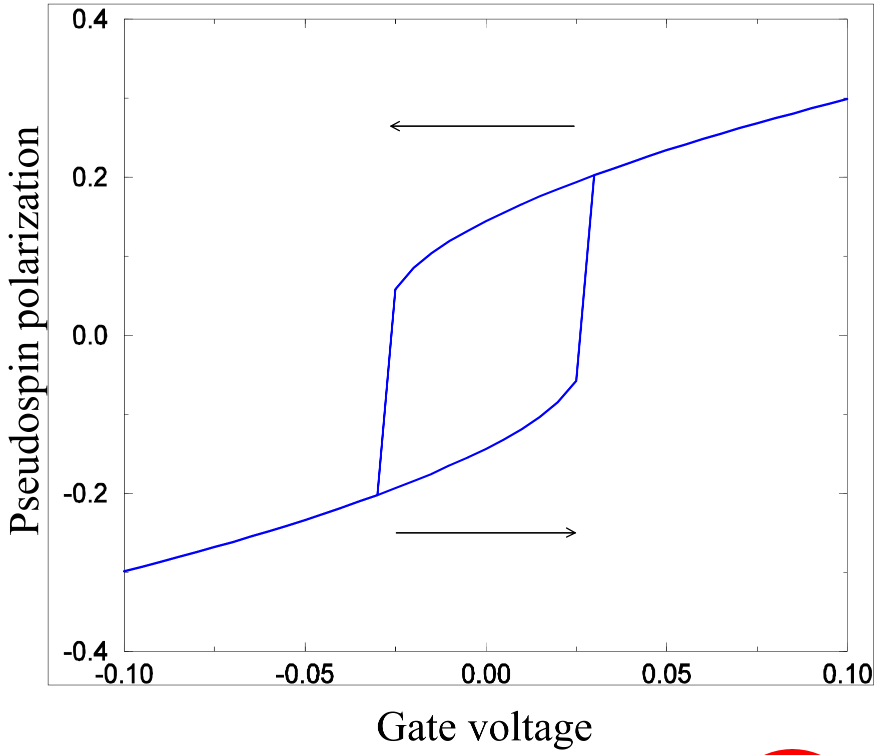
$\Rightarrow$ N=4 quantum Hall conductivity

4. Applications: Pseudospin Magnetism

1) Example: Bilayer graphene (*J*=2)

Min et al. PRB **77**, 041407(R) (2008)





$$\boxed{H_J = \varepsilon_0 p^J \boldsymbol{\sigma} \cdot \mathbf{n}_J(\phi_\mathbf{p})}$$

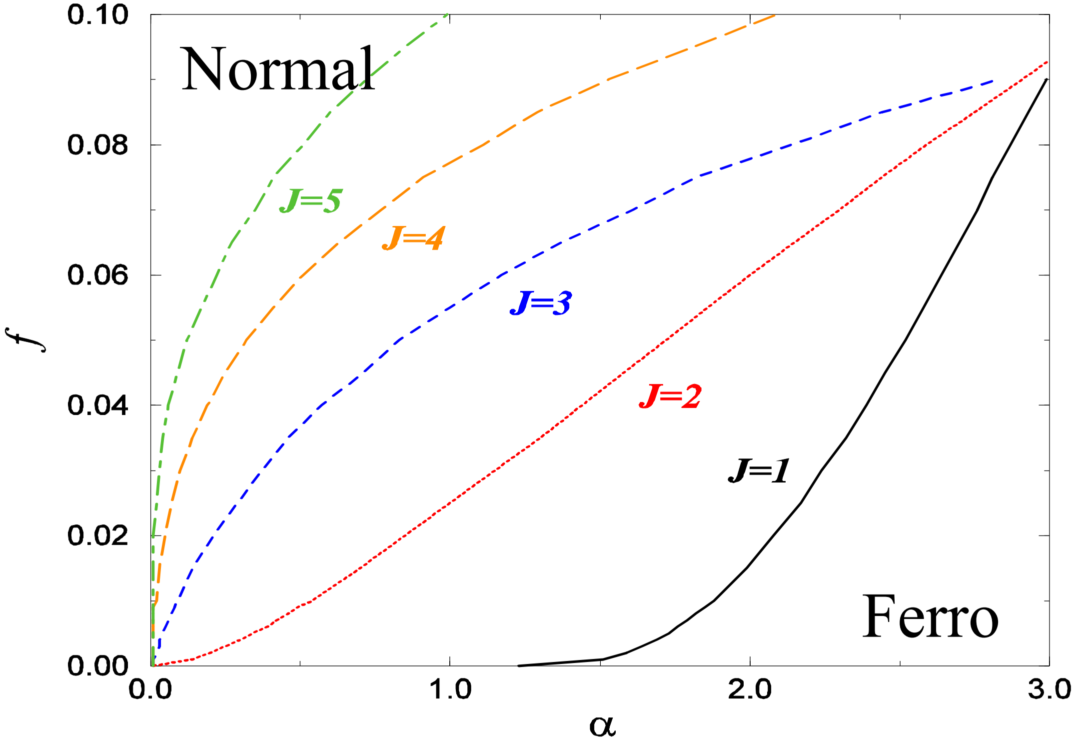
$$\boxed{\mathbf{n}_J(\phi_\mathbf{p}) = (n_\perp \cos J\phi_\mathbf{p}, n_\perp \sin J\phi_\mathbf{p}, n_z)}$$

$\Rightarrow$ The charge density spontaneously shifts to one of the two layers.

4. Applications: Pseudospin Magnetism

2) Phase diagram

Min et al. PRB **77**, 041407(R) (2008)



J : chirality

α : interaction strength

f : doping

- Pseudospin magnetism is stable for stronger interaction strength, for smaller doping, for larger chirality.
- ABC stacked multilayers are excellent candidates.

Summary

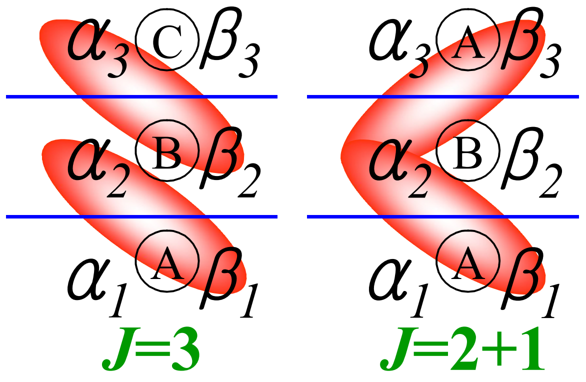
1) Chirality sum rule

- Arbitrarily stacked graphene multilayers are described by a set of chiral systems.

$$H_N^{eff} = H_{J_1} \otimes H_{J_2} \otimes \cdots \otimes H_{J_{N_D}}$$

$$\sum_{i=1}^{N_D} J_i = N$$

Min et al. PRB **77**, 155416 (2008)



2) Quantum Hall effect

- Chirality sum rule implies a new quantum Hall conductivity.

$$\sigma_{xy} = \pm \frac{4e^2}{h}\left(\boxed{\frac{N}{2}} + n\right)$$

n=0,1,2,3,…