Effects of squeezing on quantum nonlocality of superpositions of coherent states

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We analyze effects of squeezing upon superpositions of coherent states (SCSs) and entangled coherent states for Bell-inequality tests. We find that external squeezing can always increase the degrees of Bell violations, if the squeezing direction is properly chosen, for the case of photon parity measurements. On the other hand, when photon on/off measurements are used, the squeezing operation can enhance the degree of Bell violations only for moderate values of amplitudes and squeezing. We point out that a significant improvement is required over currently available squeezed SCSs in order to directly demonstrate a Bell-inequality violation in a real experiment.

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I. INTRODUCTION

Einstein, Podolsky, and Rosen (EPR) questioned completeness of quantum mechanics based on the idea of local realism [1]. Bell suggested a profound and useful inequality imposed by local hidden variable theories, which reflects EPR's idea [2]. A couple of refined versions of Bell's inequality followed the original one [3,4], and numerous experimental demonstrations have also been performed [5,6]. In these studies, quantum states of light have played a crucial role. Indeed, all Bell-inequality tests in which the spacelike separation between two local parties is satisfied have been performed using photons. In the meantime, it is worth noting that a loophole-free Bell-inequality test is yet to be performed. The major obstacle in typical photon-based experiments, where two local parties are separate enough, is probably the detection loophole [7]. Very recently, a Bell-inequality test free from the detection loophole was performed using remote atomic qubits [8], however, it did not satisfy the spacelike separation required for a loophole-free Bell test.

Recently, various types of continuous-variable states have been studied in order to suggest proposals for loophole-free Bell-inequality tests [9]. As non-Gaussian continuous-variable states have rich structures in the phase space, it is important to explore possibility of efficient Bell-inequality tests using those states. Among non-Gaussian continuous-variable states, superpositions of two coherent states (SCSs) [10,11] in free-traveling optical fields have been found a very useful tool for fundamental tests of quantum theory [12–17] as well as for quantum information applications [18–23]. In particular, they are useful for Bell-inequality tests using various measurements such as photon on/off detection, photon number detection, and homodyne detection [12–15]. Once single-mode SCSs are generated, a 50:50 beam splitter can be used to generate entangled coherent states (ECSs) [24] with which one can perform Bell-inequality tests [12,13,15–17].

Recently, "squeezed" SCSs were generated and detected [25–27], where the size of the states ($\alpha=\sqrt{2.6}$) was reasonably large for fundamental tests of quantum theory and implementations of quantum information processing [28]. Squeezed SCSs can be more robust against decoherence than unsqueezed ones [29] while they have similar nonclassical properties and usefulness in quantum information applications [30–32]. Remarkably, it has been clearly pointed out that the squeezed SCSs recently generated can be used for proof-of-principle experiments such as quantum teleportation and single qubit gates without any modifications [32]. This strongly motivates us to study effects of squeezing on SCSs and ECSs for various purposes.

In this paper, we study effects of squeezing on SCSs and ECSs for the purpose of Bell-inequality tests using photon parity measurements and on/off measurements. We show that the squeezing operation can increase the degrees of Bell violations when photon parity measurements are used, while it depends on the values of amplitudes and squeezing for the case of photon on/off measurements. We also point out that fidelity of the generated SCS should be improved up to at least 92% with respect to ideal state in order to demonstrate direct Bell violations in real experiments.

This paper is organized as follows. In Sec. II, two different approaches to entangle and squeeze SCSs are briefly presented. One is to pass a squeezed SCS through a beam splitter to generate an entangled state, and the other is to apply the two-mode squeezing operation on an ECS. We then analyze, in Sec. III, the effects of the single-mode and two-mode squeezing for Bell-inequality tests. In Sec. IV, we apply our theoretical evaluation to experimentally feasible squeezed SCSs considering experimental imperfections. A summary is given in Sec. V with final remarks on prospects for experimental tests of Bell inequalities using SCSs.

II. ENTANGLING AND SQUEEZING SUPERPOSITIONS OF COHERENT STATES

We present two particular types of SCSs, namely, even and odd SCSs, as

$$|\mathrm{SCS}_{\pm}(\gamma)\rangle = \mathcal{N}_{\pm}(\gamma)(|\gamma\rangle \pm |-\gamma\rangle), \quad (1)$$

where $\mathcal{N}_{\pm}$ are normalization factors, $|\gamma\rangle$ is a coherent state of amplitude γ, and γ is assumed to be real for simplicity without loss of generality. The SCS with the plus (minus) sign

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between the coherent states in Eq. (1) is called an even (odd) SCS because it contains an even (odd) number of photons regardless of the value of γ. The size of a SCS may be defined by the magnitude of the amplitude γ. The ECSs at modes a and b are defined as

$$|\Phi_\pm\rangle = \mathcal{N}_\pm(|\gamma\rangle_a|\gamma\rangle_b \pm |-\gamma\rangle_a|-\gamma\rangle_b),$$

$$|\Psi_\pm\rangle = \mathcal{N}_\pm(|\gamma\rangle_a|-\gamma\rangle_b \pm |-\gamma\rangle_a|\gamma\rangle_b), \quad (2)$$

which can be generated by splitting $|\mathrm{SCS}_\pm(\sqrt{2}\gamma)\rangle$ at a 50:50 beam splitter with an appropriate phase. We refer to the normalization factor $\mathcal{N}_\pm$ as $\mathcal{N}_\pm(\sqrt{2}\gamma)$ hereafter. Note that $|\Phi_-\rangle$ and $|\Psi_-\rangle$ are maximally entangled (i.e., each of them contains 1 ebit), which in general show stronger Bell violations than $|\Phi_+\rangle$ and $|\Psi_+\rangle$ [13].

For Bell-inequality tests, we shall use two types of entangled states, i.e., entangled squeezed SCSs (ESSs) and squeezed ECSs (SECSs). The former can be obtained by beam-splitting after single-mode-squeezing SCSs, and the latter by two-mode-squeezing after beam-splitting a SCSs as shown in Fig. 1. The squeezed SCSs (SSCS) and ESSs can be represented as

$$|\mathrm{SSCS}_\pm(\gamma)\rangle = S(s)|\mathrm{SCS}_\pm(\gamma)\rangle, \quad (3)$$

$$|\psi_\pm\rangle = B_{ab}|\mathrm{SSCS}_\pm(\sqrt{2}\gamma)\rangle_a|0\rangle_b, \quad (4)$$

where $S_a(s)=\exp[\frac{s}{2}(a^2-a^{\dagger 2})]$ is the single-mode squeezing operator, $B_{ab}=\exp[\frac{\pi}{4}(a^\dagger b-a^\dagger b)]$ the 50:50 beam splitter operator, and a and $a^\dagger$ (b and $b^\dagger$) the bosonic annihilation and creation operators for mode a (mode b). The ESSs become the same as $|\Psi_\pm\rangle$ for the case of $s=0$. The SECSs are

$$|\Phi^s_\pm\rangle = S_{ab}(s)|\Phi_\pm\rangle,$$

$$|\Psi^s_\pm\rangle = S_{ab}(s)|\Psi_\pm\rangle, \quad (5)$$

where $S_{ab}(s)=\exp[s(ab-a^\dagger b^\dagger)]$ is the two-mode squeezing operator. We assume that the squeezing parameter s is real for both $S_a(s)$ and $S_{ab}(s)$. The corresponding state is then squeezed along the real axis in the phase space for $s>0$ while it is squeezed along the imaginary axis for $s<0$.

III. VIOLATIONS OF BELL'S INEQUALITY WITH PHOTON PARITY AND ON/OFF MEASUREMENT SCHEMES

A. Bell-CHSH inequality with the Wigner functions

Banaszek and Wódkiewicz (BW) studied Bell's inequality in the phase space, in terms of the Wigner (Q) function based upon photon number parity (on/off) measurements and the displacement operation [33]. The Wigner function approach is based upon the Bell's inequality version of Clauser, Horne, Shimony, and Holt (CHSH), while the Q function is upon the version of Clauser and Horne (CH) [33]. The displaced parity operator used for the Bell-CHSH inequality is

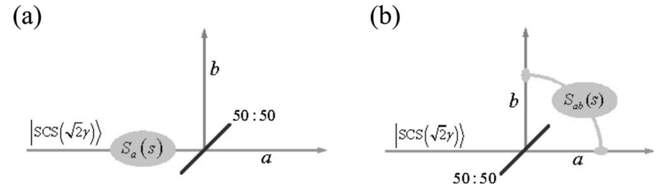

FIG. 1. Entangling and squeezing procedure for (a) an ESS and (b) a SECS. The ESS is obtained by single-mode-squeezing $|\mathrm{SCS}_\pm(\sqrt{2}\gamma)\rangle$ and feeding it into a 50:50 beam splitter, whereas the SECS by feeding $|\mathrm{SCS}_\pm(\sqrt{2}\gamma)\rangle$ into a 50:50 beam splitter and two-mode-squeezing it.

$$\mathcal{P}(\alpha) = \Pi^{\mathrm{even}}(\alpha) - \Pi^{\mathrm{odd}}(\alpha)$$
$$= D(\alpha)\sum_{n=0}^{\infty}(|2n\rangle\langle 2n| - |2n+1\rangle\langle 2n+1|)D^\dagger(\alpha), \quad (6)$$

where $D(\alpha)=\exp(\alpha a^\dagger-\alpha^* a)$ is the displacement operator, and the Bell operator is

$$\mathcal{B}_{\mathrm{CHSH}} = \mathcal{P}_a(\alpha)\otimes\mathcal{P}_b(\beta) + \mathcal{P}_a(\alpha')\otimes\mathcal{P}_b(\beta) + \mathcal{P}_a(\alpha)\otimes\mathcal{P}_b(\beta') - \mathcal{P}_a(\alpha')\otimes\mathcal{P}_b(\beta'). \quad (7)$$

The Wigner functions for state ρ may be obtained by taking the average of the parity operator $\mathcal{P}(\alpha)$ as [33,35]

$$W(\alpha) = \frac{2}{\pi}\mathrm{Tr}[\rho\mathcal{P}(\alpha)] \quad (8)$$

and for two-mode state ρ_{ab} as

$$W(\alpha,\beta) = \left(\frac{2}{\pi}\right)^2\mathrm{Tr}[\rho_{ab}\mathcal{P}_a(\alpha)\otimes\mathcal{P}_b(\alpha)]. \quad (9)$$

Thus the Bell-CHSH inequality can be represented by the Wigner function as

$$|B_{\mathrm{CHSH}}| = \left(\frac{\pi}{2}\right)^2|W(\alpha,\beta)+W(\alpha',\beta)+W(\alpha,\beta')-W(\alpha',\beta')| \le 2, \quad (10)$$

where $W(\alpha,\beta)$ is the two-mode Wigner function and we refer to $B_{\mathrm{CHSH}}=\langle\mathcal{B}_{\mathrm{CHSH}}\rangle$ as the Bell-CHSH function. This inequality can be violated with appropriate measurement operators and entangled states, and its maximum value, $2\sqrt{2}$, is known as Cirel'son's bound [34].

Using Eqs. (1) and (8), the Wigner functions for the even and odd SCSs can be calculated as

$$W^{\mathrm{SCS}}_\pm(\alpha) = \mathcal{N}^2_\pm[W_{\sqrt{2}\gamma}(\alpha) + W_{-\sqrt{2}\gamma}(\alpha) \pm 2X_{\sqrt{2}\gamma}(\alpha)], \quad (11)$$

where $W_\gamma(\alpha)=2e^{-2|\alpha-\gamma|^2}/\pi$ is the Wigner function of coherent state $|\gamma\rangle$ and $X_\gamma(\alpha)=2e^{-2|\alpha|^2}\cos[4\,\mathrm{Im}(\alpha^*\gamma)]/\pi$. The Wigner functions of the ESSs can be obtained using Eqs. (4) and (9), and they can also be expressed as

$$W_{\psi_\pm}(\alpha,\beta) = W^{\mathrm{SCS}}_\pm\left(\frac{\alpha^s-\beta^s}{\sqrt{2}}\right)W_0\left(\frac{\alpha+\beta}{\sqrt{2}}\right), \quad (12)$$

where $W_0(\alpha)$ is the Wigner function of the vacuum and the superscript s is used to indicate

$$\alpha^s = \alpha\cosh s + \alpha^*\sinh s = e^s\,\mathrm{Re}\,\alpha + ie^{-s}\,\mathrm{Im}\,\alpha. \quad (13)$$

for an arbitrary complex number α. The two-mode Wigner

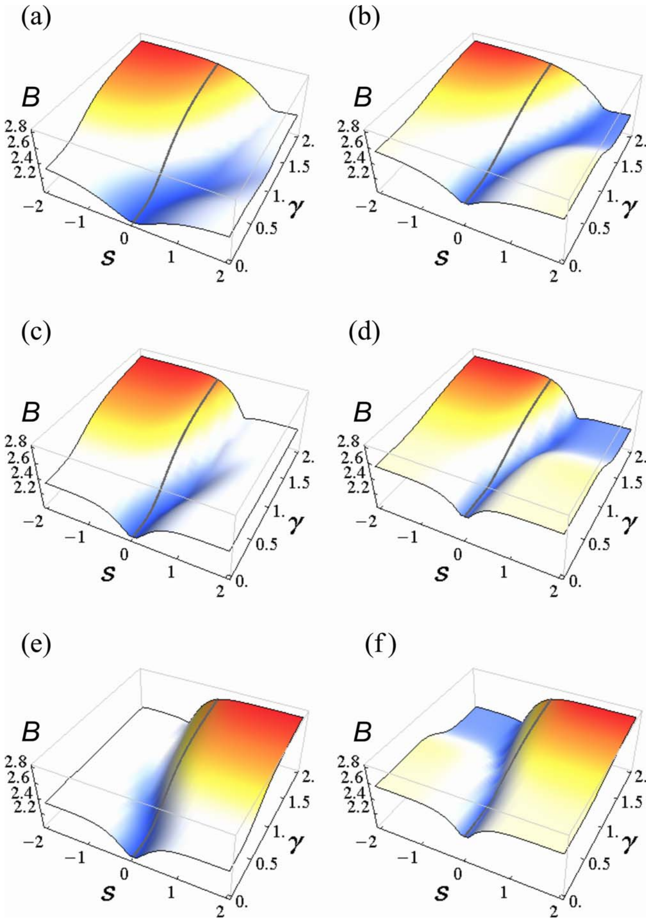

FIG. 2. (Color online) Optimized Bell-CHSH function $B=|B_{\mathrm{CHSH}}|_{\max}$ for (a) ψ_+ (b) ψ_- (c) Φ_+^s (d) Φ_-^s (e) Ψ_+^s (f) Ψ_-^s for parity measurements. The split line in each graph indicates no squeezing ($s=0$). Note that the plots for $\psi_\pm$ are similar to the ones for $\Phi_\pm^s$, and that B's of $\Psi_\pm^s$ are rather similar and symmetric to the ones $\Phi_\pm^s$ with respect to $s=0$ line. One can observe that for small γ squeezing in any direction can enhance Bell violations, whereas for large γ squeezing in specific direction only can enhance them. In any case, squeezing causes Bell violations to increase monotonically from the nonsqueezed values and converge to specific ones.

functions for the ECSs are calculated in the same manner as [13]

$$W_{\Phi_\pm}(\alpha,\beta)=\mathcal{N}_\pm^2[W_\gamma(\alpha)W_\gamma(\beta) + W_{-\gamma}(\alpha)W_{-\gamma}(\beta) \pm 2X_\gamma(\alpha)X_\gamma(\beta) \mp 2Y_\gamma(\alpha)Y_\gamma(\beta)],$$

$$W_{\Psi_\pm}(\alpha,\beta)=\mathcal{N}_\pm^2[W_\gamma(\alpha)W_{-\gamma}(\beta) + W_{-\gamma}(\alpha)W_\gamma(\beta) \pm 2X_\gamma(\alpha)X_\gamma(\beta) \pm 2Y_\gamma(\alpha)Y_\gamma(\beta)], \quad (14)$$

where $Y_\gamma(\alpha)=2e^{-2|\alpha|^2}\sin[4\,\mathrm{Im}(\alpha^*\gamma)]/\pi$. The Wigner functions for SECSs are then

$$W_{\Phi_\pm^s}(\alpha,\beta)=W_{\Phi_\pm}(\tilde{\alpha}^s,\tilde{\beta}^s),$$

$$W_{\Psi_\pm^s}(\alpha,\beta)=W_{\Psi_\pm}(\tilde{\alpha}^s,\tilde{\beta}^s), \quad (15)$$

where

$$\tilde{\alpha}^s=\alpha\cosh s+\beta^*\sinh s, \quad \tilde{\beta}^s=\beta\cosh s+\alpha^*\sinh s. \quad (16)$$

Note that when $s=0$, $W_{\Psi_\pm^s}(\alpha,\beta)=W_{\psi_\pm}(\alpha,\beta)$ and $W_{\Phi_\pm^s}(\alpha,\beta)=W_{\psi_\pm}(-\beta,\alpha)$.

It is known that the Bell violation for an ECS approaches Cirel'son's bound [34] when the amplitude γ becomes large [13]. Figure 2 shows that a couple of characteristic properties in common when squeezing is applied to the states being considered. The squeezing operation increases the degree of the Bell violation up to some extent for small γ, but has a tendency of degrading it for squeezing in the specific direction for larger γ. For example, the squeezing in both the real and imaginary directions in the phase space, enhances violation for ψ_+ (ψ_-) until γ reaches around 0.5 (1.0). On the other hand, for larger values of γ squeezing in the real direction (i.e., $s>0$) decreases the degree of violation while squeezing in the imaginary direction (i.e., $s<0$) increases the violation.

In fact, for larger γ, squeezing along the real axis makes the interference fringes less sharp and this could be related to the decrease in the Bell violations. In the case of $\Phi_\pm^s$ with large γ, since $\tilde{\alpha}^s(\tilde{\beta}^s)\to\alpha'(-\alpha'^*)$ as $s\to-\infty$ where $\alpha'=\frac{1}{2}e^{-s}(\alpha-\beta^*)$, and hence $W_{\Phi_\pm^s}(\alpha,\beta)\to W_{\Phi_\pm}(\alpha',-\alpha'^*)$, which is the very condition when maximum violations occur for $W_{\Phi_\pm}(\alpha,\beta)$. But as $s\to\infty$, the interference part $X_\gamma(\tilde{\alpha}^s)X_\gamma(\tilde{\beta}^s)-Y_\gamma(\tilde{\alpha}^s)Y_\gamma(\tilde{\beta}^s)$ in the Wigner function fades out, which may play a crucial role in degrading the Bell violations. The case of $\Psi_\pm^s$ can be explained in a similar way. Therefore, in the case of photon parity measurements, squeezing in a well-chosen quadrature direction can enhance Bell violations of tested states, though its contribution gets slighter as the amplitudes of the states grow larger.

B. Bell-CH inequality with the Q functions

The operator used for tests of the Bell-CH inequality is

$$\mathcal{B}_{\mathrm{CH}}=\mathcal{Q}_a(\alpha)\otimes\mathcal{Q}_b(\beta)+\mathcal{Q}_a(\alpha')\otimes\mathcal{Q}_b(\beta)+\mathcal{Q}_a(\alpha)\otimes\mathcal{Q}_b(\beta') - \mathcal{Q}_a(\alpha')\otimes\mathcal{Q}_b(\beta')-\mathcal{Q}_a(\alpha)\otimes\mathcal{I}_b-\mathcal{I}_a\otimes\mathcal{Q}_b(\beta), \quad (17)$$

where

$$\mathcal{Q}(\alpha)=D(\alpha)|0\rangle\langle 0|D^\dagger(\alpha) \quad (18)$$

is a displaced "no photon" operator and $\mathcal{I}$ is the identity operator. Subsequently, the Bell-CH function $B_{\mathrm{CH}}=\langle\mathcal{B}_{\mathrm{CH}}\rangle$ is given in terms of Q representation as

$$B_{\mathrm{CH}}=\pi^2[Q_{ab}(\alpha,\beta)+Q_{ab}(\alpha',\beta)+Q_{ab}(\alpha,\beta')-Q_{ab}(\alpha',\beta')] - \pi[Q_a(\alpha)+Q_b(\beta)], \quad (19)$$

where $Q_a(\alpha)$ and $Q_b(\beta)$ are marginal Q functions in the corresponding modes. As implied above, the Q functions of single-mode state ρ and two-mode state ρ_{ab} can be obtained using the operator $\mathcal{Q}(\alpha)$ as $(1/\pi)\mathrm{Tr}[\rho\mathcal{Q}(\alpha)]$ and $(1/\pi)^2\mathrm{Tr}[\rho_{ab}\mathcal{Q}_a(\alpha)\otimes\mathcal{Q}_b(\beta)]$, respectively [35]. The Q functions for the SSCS are then given as

$$Q_\pm^{\mathrm{SSCS}}(\alpha)=\mathcal{N}_\pm^2[Q_{\sqrt{2}\gamma}^+(\alpha)+Q_{\sqrt{2}\gamma}^-(\alpha)\pm 2Q_{\sqrt{2}\gamma}^{\mathrm{X}}(\alpha)], \quad (20)$$

subsequently for ESSs as

$$Q_{\psi_\pm}(\alpha,\beta) = Q_\pm^{\mathrm{SSCS}}\left(\frac{\alpha-\beta}{\sqrt{2}}\right) Q_0\left(\frac{\alpha+\beta}{\sqrt{2}}\right), \qquad (21)$$

where

$$Q_\gamma^\pm(\alpha) = \cos\,\theta Q_{\pm\gamma_{-s}}(\alpha_s), \qquad (22)$$

$$Q_\gamma^{\mathrm{X}}(\alpha) = \cos\,\theta Q_0(\alpha_s) e^{-|\gamma_{-s}|^2}\cos[2\,\mathrm{Im}(\alpha_s^*\gamma_s)], \qquad (23)$$

where $\alpha_s = \alpha\cos(\theta/2) + \alpha^*\sin(\theta/2)$, $\gamma_{-s} = \gamma\cos(\theta/2) - \gamma^*\sin(\theta/2)$, $\theta/2 = \tan^{-1}(\tanh\frac{s}{2})$, $Q_\gamma(\alpha) = e^{-|\alpha-\gamma|^2}/\pi$, and $Q_0(\alpha) = e^{-|\alpha|^2}/\pi$. Note that as $s\to\infty(-\infty)$, $\alpha_s \to \sqrt{2}\,\mathrm{Re}[\alpha](i\sqrt{2}\,\mathrm{Im}[\alpha])$.

In the meantime, the Q functions for SECSs are

$$Q_{\Phi_\pm^s}(\alpha,\beta) = \mathcal{N}_\pm^2[Q_{++}(\alpha,\beta) + Q_{--}(\alpha,\beta) \pm 2Q_+^{\mathrm{XY}}(\alpha,\beta)], \quad (24)$$

$$Q_{\Psi_\pm^s}(\alpha,\beta) = \mathcal{N}_\pm^2[Q_{+-}(\alpha,\beta) + Q_{-+}(\alpha,\beta) \pm 2Q_-^{\mathrm{XY}}(\alpha,\beta)], \quad (25)$$

where

$$Q_{\pm\pm}(\alpha,\beta) = \cos^2\,\theta Q_{\pm\gamma_{\mp s}}(\tilde{\alpha}_s) Q_{\pm\gamma_{\mp s}}(\tilde{\beta}_s),$$

$$Q_{\pm\mp}(\alpha,\beta) = \cos^2\,\theta Q_{\pm\gamma_{\mp s}}(\tilde{\alpha}_s) Q_{\mp\gamma_{\pm s}}(\tilde{\beta}_s), \qquad (26)$$

$$Q_\pm^{\mathrm{XY}}(\alpha,\beta) = \cos^2\,\theta Q_0(\tilde{\alpha}_s) Q_0(\tilde{\beta}_s) e^{-2|\gamma_{\mp s}|^2} \times \cos[2\,\mathrm{Im}(\alpha_s^*\gamma_s \pm \beta_s^*\gamma_s)], \qquad (27)$$

with $\tilde{\alpha}_s = \alpha\cos(\theta/2) + \beta^*\sin(\theta/2)$ and $\tilde{\beta}_s = \beta\cos(\theta/2) + \alpha^*\sin(\theta/2)$. Since the Bell-CH inequality is equivalent to the Bell-CHSH inequality for the case of bipartite systems and dichotomic measurements, the above Bell-CH function can be replaced with the Bell-CHSH function, which shall be further clarified in the following subsection.

C. Bell-CHSH inequality with on/off measurements

One can test the Bell-CHSH inequality by the following displaced "on/off" measurement operator

$$\mathcal{O}(\alpha) = \Pi^{\mathrm{on}}(\alpha) - \Pi^{\mathrm{off}}(\alpha) = D^\dagger(\alpha)\left(\sum_{n=1}^{\infty}|n\rangle\langle n| - |0\rangle\langle 0|\right)D(\alpha), \qquad (28)$$

which assigns +1 or −1 to each measured result depending on whether (any) photons are detected or not at a detector such as an avalanche photodiode. Then the Bell-CHSH inequality can be represented in the same way as done in Eq. (10) just with $\mathcal{P}$ replaced with $\mathcal{O}$, so that the Bell-CHSH function becomes

$$B_{\mathrm{CHSH}} = A(\alpha,\beta) + A(\alpha',\beta) + A(\alpha,\beta') - A(\alpha',\beta') \qquad (29)$$

with

$$A(\alpha,\beta) = 1 - 2\pi Q_a(-\alpha) - 2\pi Q_b(-\beta) + 4\pi^2 Q_{ab}(-\alpha,-\beta), \qquad (30)$$

where the Q functions are the ones obtained in the previous subsection. It is worth noting that this Bell-CHSH function is related to the previous Bell-CH function as

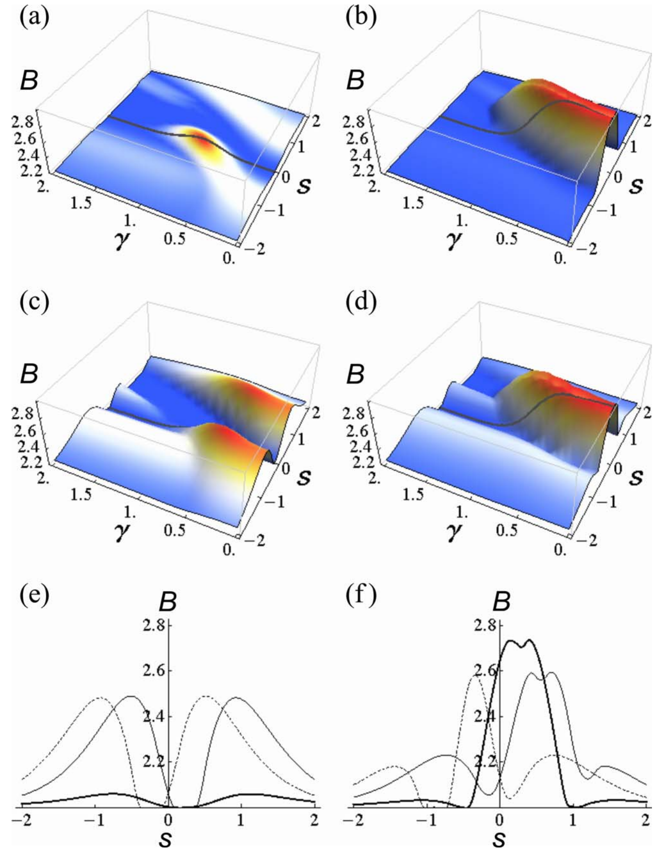

FIG. 3. (Color online) Optimized Bell-CHSH function $B = |B_{\mathrm{CHSH}}|_{\max}$ for (a) ψ_+ (b) ψ_- (c) Φ_+^s (d) Φ_-^s for on/off measurements. The lowest two plots are for (e) ψ_+ (thick), Φ_+^s (thin), and Ψ_+^s (dashed), respectively, with γ=0.5 and for (f) ψ_- (thick), Φ_-^s (thin), and Ψ_-^s (dashed) with γ=1.0. The plots of $\Psi_\pm^s$ not presented here are similar to those of $\Phi_\pm^s$ provided the sign of s is altered as in the parity measurement case. Note that for small γ, squeezing + states increases B up to some extent (e), and that for large γ, squeezing − states in specific direction only contributes to maximal values of B_{CHSH}'s (f).

$$B_{\mathrm{CHSH}}(\alpha,\beta) = 4B_{\mathrm{CH}}(-\alpha,-\beta) + 2. \qquad (31)$$

A Bell-inequality test with photon on/off measurements is obviously more feasible than that of photon number parity measurements. However, if the average photon number of the state under consideration is too large, Bell violations cannot be observed using photon on/off measurements because the possibility of getting a "off" result approaches zero [13]. Because of this, Bell violations for ESSs and SECSs shown in Fig. 3 show different behaviors compared to the cases of photon parity measurements. In the case of "+" states $(\Phi_+,\Psi_+(\psi_+))$, quadrature squeezing in any direction increases Bell violations only for small γ. Meanwhile, in the case of "−" states $[\Phi_-,\Psi_-(\psi_-)]$, squeezing in specific direction increases the violations only for $\gamma \gtrsim 1$, whereas it is not any desirable for violations for small γ. In any case, large squeezing in any quadrature direction causes Bell violations to eventually vanish. This is different from the cases for the parity measurements where large values of squeezing cause the Bell functions to converge to certain values (smaller or larger than the ones in the cases of no squeezing).

IV. ESTIMATION OF BELL VIOLATIONS WITH REALISTIC STATES

We are also interested in whether a recently generated SSCS [25], which can be immediately used to generate an ESS, may be used for tests of Bell's inequality. The size of the generated SSCS, an "even" one, was as large as $\gamma=\sqrt{2.6}$ and the squeezing degree was 3.5 dB along the real axis in the phase space. The ideal state that can be generated with a two-photon number state using the scheme described in Ref. [25] is [36]

$$|\phi_2\rangle=\sqrt{2/3}|2\rangle+\sqrt{1/3}|0\rangle. \tag{32}$$

This state is a very good approximation of an ideal SSCS

$$|\mathrm{SSCS}_+\rangle=S(s_0)|\mathrm{SCS}_+(\alpha_0)\rangle, \tag{33}$$

where $s_0=0.4$, $\alpha_0=\sqrt{2.6}$, and the fidelity between the two states is as high as $|\langle\phi_2|\mathrm{SSCS}_+\rangle|^2\approx 99\%$. If state (33) is injected into a 50:50 beam splitter, it becomes $|\psi_+\rangle$ with $\gamma=\sqrt{2.6/2}$. When this ideal two-mode state is used to obtain the Bell function B_{CHSH}, its optimized value is 2.419 (2.033) with photon number parity (on/off) measurements. In the meantime, state $|\phi_2\rangle$ shows a Bell-inequality violation as large as $B_{\mathrm{CHSH}}=2.401(2.006)$ using parity (on/off) measurements. In order to analyze the case of the actually generated (mixed) state ρ_{exp}, which is degraded by experimental imperfections such as non-unit efficiencies, noises, and errors related to measuring devices, we use the following Wigner function in Ref. [25],

$$W_{\mathrm{exp}}(x,p)=\frac{\exp(-x^2/\alpha-p^2/\beta)}{\pi\sqrt{\alpha\beta}\left[\left(1-\frac{\delta\alpha(1-\nu)^2}{2(\alpha-\beta)}\right)^2+\frac{1}{2}\left(\frac{\delta\alpha(1-\nu)^2}{2(\alpha-\beta)}\right)^2\right]}\left\{\frac{\delta^2}{2}\left[\frac{x^2}{\alpha}+\frac{\alpha\nu^2p^2}{\beta^2}\right]^2+2\delta\left[1-\delta\left(1+\frac{(\alpha\nu-\beta)^2}{2\beta(\alpha-\beta)}\right)\right]\left[\frac{x^2}{\alpha}+\frac{\alpha\nu^2p^2}{\beta^2}\right]\right.$$
$$\left.+\delta^2\frac{(\alpha\nu-\beta)^2}{2\beta(\alpha-\beta)}\left[\frac{x^2}{\alpha}-\frac{\alpha\nu^2p^2}{\beta^2}\right]+\left[1-\delta\left(1+\frac{(\alpha\nu-\beta)^2}{2\beta(\alpha-\beta)}\right)\right]^2+\frac{\delta^2}{2}\left[\frac{(\alpha\nu-\beta)^2}{2\beta(\alpha-\beta)}\right]^2\right\}, \tag{34}$$

where $x=\sqrt{2}\,\mathrm{Re}[\alpha]$, $p=\sqrt{2}\,\mathrm{Im}[\alpha]$ in our case, and the four parameters α, β, ν, and δ are defined by gain and various imperfection parameters [37]. However, since the Bell function depends very sensitively on such imperfection parameters, we assume perfect measuring devices with no errors, in which case

$$\alpha\rightarrow g,\quad \beta\rightarrow\alpha-(g-1)^2/g,\quad \nu\rightarrow 1/g,\quad \delta\rightarrow 1, \tag{35}$$

where g is an optical parametric amplifier gain describing the two-photon number state, and then the fidelity $F=\langle\phi_2|\rho_{\mathrm{exp}}|\phi_2\rangle$ depends only on g. Note also that for testing the on/off measurement case, we can transform the above Wigner function into the Q function simply by just replacing the parameters α, β, δ by $\alpha+1$, $\beta+1$, $\frac{\alpha}{\alpha+1}\delta$. As can be seen in Fig. 4, in order for ρ_{exp} to show Bell violations, the fidelity should be improved up to around 92% in the case of parity measurements. However, we note again that the violations are possible only when all the experimental imperfections nearly vanish, which is extremely demanding. When on/off measurements are used, the fidelity should be even more improved up to at least 99% to show a Bell violation.

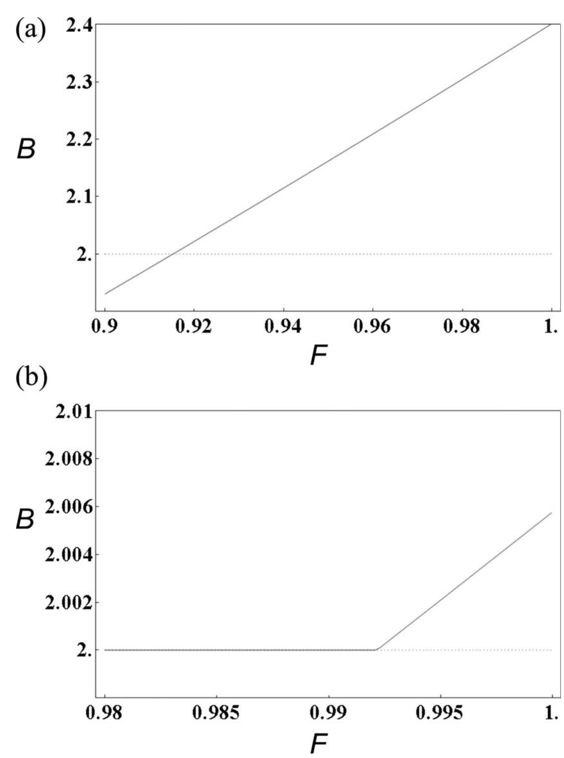

FIG. 4. Optimized Bell-CHSH function $B=|B_{\mathrm{CHSH}}|_{\mathrm{max}}$ vs fidelity F of ρ_{exp} with respect to $|\phi_2\rangle$ for the case of (a) photon parity measurements and (b) photon on/off measurements. Dotted line in each plot indicates the local realistic bound for Bell-CHSH inequality. Bell violation for the case of parity measurements can be observed when the fidelity approaches 92% while that for on/off measurements case cannot be observed until the fidelity goes over 99%.

V. REMARKS

We have studied how squeezing influences the degree of Bell-inequality violations of several different beam-split-entangled SCSs. It has been found that squeezing can always increase Bell violations, given the squeezing direction is properly chosen, for the case of photon parity measurements. On the other hand, in the case of the photon on/off measurements, squeezing can enhance Bell violations only for well-chosen values of amplitudes and squeezing. Therefore, it should be noted that for certain measurement schemes, the squeezing action is not always helpful in enhancing Bell violations of entangled states of light.

In order to demonstrate a Bell violation in a real experiment, a significant improvement is required over the currently available SSCS. For example, the fidelity of the generated state should be improved up to 92% even when all the other conditions including the efficiency of photon parity measurements are ideal. There are ongoing efforts to effectively generate high-fidelity SSCSs using currently available experimental resources [38]. It would be a more realistic target to perform homodyne tomography to reconstruct a generated SSCS and "indirectly" show a Bell violation using Eq. (7).

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[1] A. Einstein, B. Podolsky, and N. Rosen, Phys. Rev. **47**, 777 (1935).

[2] J. S. Bell, Physics (Long Island City, N.Y.) **1**, 195 (1964).

[3] J. F. Clauser, M. A. Horne, A. Shimony, and R. A. Holt, Phys. Rev. Lett. **23**, 880 (1969).

[4] J. F. Clauser and M. A. Horne, Phys. Rev. D **10**, 526 (1974).

[5] S. J. Freedman and J. F. Clauser, Phys. Rev. Lett. **28**, 938 (1972).

[6] A. Aspect, Ph. Grangier, and G. Roger, Phys. Rev. Lett. **47**, 460 (1981).

[7] A. Garg and N. D. Mermin, Phys. Rev. D **35**, 3831 (1987).

[8] D. N. Matsukevich, P. Maunz, D. L. Moehring, S. Olmschenk, and C. Monroe, Phys. Rev. Lett. **100**, 150404 (2008).

[9] J. Wenger, M. Hafezi, F. Grosshans, R. Tualle-Brouri, and Ph. Grangier, Phys. Rev. A **67**, 012105 (2003); H. Nha and H. J. Carmichael, Phys. Rev. Lett. **93**, 020401 (2004); R. García-Patrón, J. Fiurášek, N. J. Cerf, J. Wenger, R. Tualle-Brouri, and Ph. Grangier, *ibid.* **93**, 130409 (2004); R. García-Patrón, J. Fiurášek, and N. J. Cerf, Phys. Rev. A **71**, 022105 (2005); M. Stobińska, P. Horodecki, R. Chhajlany, and R. Horodecki, e-print arXiv:0909.1545.

[10] B. Yurke and D. Stoler, Phys. Rev. Lett. **57**, 13 (1986).

[11] W. Schleich, M. Pernigo, and F. L. Kien, Phys. Rev. A **44**, 2172 (1991).

[12] D. Wilson, H. Jeong, and M. S. Kim, J. Mod. Opt. **49**, 851 (2002).

[13] H. Jeong, W. Son, M. S. Kim, D. Ahn, and Č. Brukner, Phys. Rev. A **67**, 012106 (2003).

[14] H. Jeong and Nguyen Ba An, Phys. Rev. A **74**, 022104 (2006).

[15] M. Stobińska, H. Jeong, and T. C. Ralph, Phys. Rev. A **75**, 052105 (2007).

[16] H. Jeong, Phys. Rev. A **78**, 042101 (2008).

[17] H. Jeong, M. Paternostro, and T. C. Ralph, Phys. Rev. Lett. **102**, 060403 (2009).

[18] S. J. van Enk and O. Hirota, Phys. Rev. A **64**, 022313 (2001).

[19] H. Jeong, M. S. Kim, and J. Lee, Phys. Rev. A **64**, 052308 (2001).

[20] H. Jeong and M. S. Kim, Quantum Inf. Comput. **2**, 208 (2002).

[21] H. Jeong and M. S. Kim, Phys. Rev. A **65**, 042305 (2002).

[22] T. C. Ralph, A. Gilchrist, G. J. Milburn, W. J. Munro, and S. Glancy, Phys. Rev. A **68**, 042319 (2003).

[23] W. J. Munro, K. Nemoto, G. J. Milburn, and S. L. Braunstein, Phys. Rev. A **66**, 023819 (2002).

[24] B. C. Sanders, Phys. Rev. A **45**, 6811 (1992).

[25] A. Ourjoumtsev, H. Jeong, R. Tualle-Brouri, and Ph. Grangier, Nature (London) **448**, 784 (2007).

[26] M. Sasaki, M. Takeoka, and H. Takahashi, Phys. Rev. A **77**, 063840 (2008); M. Takeoka, H. Takahashi, and M. Sasaki, *ibid.* **77**, 062315 (2008).

[27] H. Takahashi, K. Wakui, S. Suzuki, M. Takeoka, K. Hayasaka, A. Furusawa, and M. Sasaki, Phys. Rev. Lett. **101**, 233605 (2008).

[28] A. P. Lund, T. C. Ralph, and H. L. Haselgrove, Phys. Rev. Lett. **100**, 030503 (2008).

[29] A. Serafini, S. De Siena, F. Illuminati, and M. G. A. Paris, J. Opt. B: Quantum Semiclassical Opt. **6**, S591 (2004).

[30] A.-S. F. Obada and G. M. A. Al-Kader, Int. J. Mod. Phys. B **13**, 2299 (1999).

[31] F. Dell'Anno, S. De Siena, L. Albano Farias, and F. Illuminati, Eur. Phys. J. Spec. Top. **160**, 115 (2008).

[32] P. Marek and M. S. Kim, Phys. Rev. A **78**, 022309 (2008).

[33] K. Banaszek and K. Wódkiewicz, Phys. Rev. A **58**, 4345 (1998); Phys. Rev. Lett. **82**, 2009 (1999).

[34] B. S. Cirel'son, Lett. Math. Phys. **4**, 93 (1980).

[35] S. M. Barnett and P. M. Radmore, *Methods in Theoretical Quantum Optics* (Clarendon Press, Oxford, 1997).

[36] $|\phi_2\rangle = \sqrt{2/3}|2\rangle - \sqrt{1/3}|0\rangle$ in the original paper should be corrected to be the one we give here.

[37] Some minor typos in Ref. [25] have been corrected in the Wigner function (34).

[38] P. Marek, H. Jeong, and M. S. Kim, Phys. Rev. A **78**, 063811 (2008).