Testing quantum nonlocality by generalized quasiprobability functions

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We derive a Bell inequality based on a generalized quasiprobability function, which is parametrized by one nonpositive real value. Two types of known Bell inequalities formulated in terms of the Wigner and the Q functions are included as limiting cases. We investigate violations of our Bell inequalities for single-photon entangled states and two-mode squeezed vacuum states when varying the detector efficiency. We show that the Bell inequality for the Q function allows the lowest detection efficiency for violations of local realism.

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I. INTRODUCTION

Ever since the famous arguments of Einstein-Podolsky-Rosen (EPR) [1], quantum nonlocality has been a central issue for understanding the conceptual foundations of quantum mechanics. Quantum nonlocality can be demonstrated by the violation of Bell inequalities (BIs) [2] which are obeyed by local-realistic (LR) theories. Realizations of BI tests are thus of great importance in testing the validity of quantum theories against LR theories. In addition, BI tests play a practical role in the detection of entanglement, which is one of the main resources for quantum information processing. Bell inequality tests for two-dimensional systems have already been realized [3], while BI tests in higher-dimensional and continuous-variable systems remain an active area of research [4,5].

Phase-space representations are a convenient tool for the analysis of continuous-variable states as they provide insights into the boundaries between quantum and classical physics. Any quantum state $\hat{\rho}$ can be fully characterized by the quasiprobability function defined in phase space [6]. In contrast to the probability functions in classical phase-space, the quasiprobability function is not always positive. For example, the Wigner function of the single-photon state has negative values in certain regions of phase space [7]. Since the negativity of the quasiprobability function inevitably reflects a nonclassical feature of quantum states, the relation between negativity of quasiprobabilities and quantum nonlocality has been investigated [8,9]. Bell argued [8] that the original EPR state will not exhibit nonlocality since its Wigner function is positive everywhere and hence serves as a classical probability distribution for hidden variables. On the other hand, Banaszek and Wódkiewicz (BW) showed how to demonstrate quantum nonlocality using the Q and the Wigner functions [9]. They suggested two distinct types of BIs: one of which is formulated via the Q function and referred to in this paper as the BW-Q inequality while the other is formulated using the Wigner function and is referred to as the BW-W inequality. Remarkably, the BW-W inequality was shown to be violated by the EPR state [9]. This indicates that there is no direct relation between the negativity of the Wigner function and the nonlocality.

Quasiprobability functions can be parametrized by one real parameter s as [6,10]

$$W(\alpha;s)=\frac{2}{\pi(1-s)}\mathrm{Tr}[\hat{\rho}\hat{\Pi}(\alpha;s)], \tag{1}$$

where $\hat{\Pi}(\alpha;s)=\Sigma_{n=0}^{\infty}[(s+1)/(s-1)]^n|\alpha,n\rangle\langle\alpha,n|$ and $|\alpha,n\rangle$ is the number state displaced by the complex variable α in phase space. It is produced by applying the Glauber displacement operator $\hat{D}(\alpha)$ to the number state $|n\rangle$. We call $W(\alpha;s)$ the s-parameterized quasiprobability function, which becomes the P, the Wigner, and the Q functions when setting $s=1,0,-1$ [10], respectively. For nonpositive s, the function $W(\alpha;s)$ can be written as a convolution of the Wigner function and a Gaussian weight

$$W(\alpha;s)=\frac{2}{\pi|s|}\int d^2\beta\, W(\beta)\exp\left(-\frac{2|\alpha-\beta|^2}{|s|}\right). \tag{2}$$

This can be identified with a smoothed Wigner function affected by noise, which is modeled by Gaussian smoothing [11–13]. Therefore decreasing s reduces the negativity of the Wigner function and is thus often considered to be a loss of quantumness. For example, the Q function ($s=-1$), which is positive everywhere in phase space, can be identified with the Wigner function smoothed over the area of measurement uncertainty.

The purpose of this paper is to propose a method for testing quantum nonlocality using the s-parametrized quasiprobability function. We will first formulate a generalized BI in terms of the s-parametrized quasiprobability function in Sec. II. This will lead us to a *s -parameterized Bell inequality*, which includes the BW-Q and the BW-W inequalities as limiting cases. We will then present a measurement scheme to test BIs using imperfect detectors in Sec. III. The measured Bell expectation value can be written as a function of the parameter s and the overall detector efficiency η. In Sec. IV violations of BIs will be demonstrated for single-photon entangled states and in Sec. V for two-mode squeezed vacuum states (TMSSs). We find the range of s and η which allows observing nonlocal properties of these two

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types of states. We will show that the test involving the Q function permits the lowest detector efficiency for observing violations of local realism. We also find that the degree of violation is irrespective of the negativity of the quasiprobability function. Finally, in Sec. VI, we discuss the characteristics and applications of the s-parametrized BI.

II. GENERALIZED BELL INEQUALITIES OF QUASIPROBABILITY FUNCTIONS

We begin by formulating a generalized BI in terms of quasiprobability functions. Suppose that two spatially separated parties, Alice and Bob, independently choose one of two observables, denoted by $\hat{A}_1,\hat{A}_2$ and $\hat{B}_1,\hat{B}_2$, respectively. No restriction is placed on the number of possible measurement outcomes (which may be infinite). We assume that the measurement operators of the local observables $\hat{A}_1,\ \hat{A}_2,\ \hat{B}_1,\ \hat{B}_2$ can be written as

$$\hat{A}_a=\hat{O}(\alpha_a;s),\quad \hat{B}_a=\hat{O}(\beta_b;s),\quad \text{for } a,b=1,2$$

using a Hermitian operator

$$\hat{O}(\alpha;s)=\begin{cases}(1-s)\hat{\Pi}(\alpha;s)+s\mathbb{1} & \text{if } -1<s\leq 0\\ 2\hat{\Pi}(\alpha;s)-\mathbb{1} & \text{if } s\leq -1\end{cases} \tag{3}$$

parametrized by a real nonpositive number s and an arbitrary complex variable α. Here, $\mathbb{1}$ is the identity operator. The possible measurement outcomes of $\hat{O}(\alpha;s)$ are given by its eigenvalues,

$$\lambda_n=\begin{cases}(1-s)\left(\dfrac{s+1}{s-1}\right)^n+s & \text{if } -1<s\leq 0\\ 2\left(\dfrac{s+1}{s-1}\right)^n-1 & \text{if } s\leq -1,\end{cases} \tag{4}$$

and their eigenvectors are the displaced number states. The maximum and the minimum measurement outcomes of $\hat{O}(\alpha;s)$ for any nonpositive s are $\lambda_{\max}=1$ and $\lambda_{\min}=-1$, respectively. For $s=0$, we have $\hat{O}(\alpha;0)=\hat{\Pi}(\alpha;0)=\Sigma_{n=0}^{\infty}(-1)^n|\alpha,n\rangle\langle\alpha,n|$, the displaced parity operator, while for $s=-1$ we find that $\hat{O}(\alpha;-1)=2|\alpha\rangle\langle\alpha|-\mathbb{1}$ projects onto the coherent states.

A Bell operator can be constructed using the measurement operators $\hat{A}_a,\hat{B}_b$ by way of a construction similar to the Clauser-Horne-Shimony-Holt (CHSH) combination

$$\hat{\mathcal{B}}=\hat{C}_{1,1}+\hat{C}_{1,2}+\hat{C}_{2,1}-\hat{C}_{2,2}, \tag{5}$$

where $\hat{C}_{a,b}=\hat{A}_a\otimes\hat{B}_b$ is the correlation operator. Since the expectation values of the local observables are bounded by $|\langle\hat{A}_a\rangle|<1$ and $|\langle\hat{B}_b\rangle|\leq 1$ for any nonpositive s, the expectation value of the Bell operator defined in Eq. (5) is bounded by $|\langle\hat{\mathcal{B}}\rangle|\equiv|\mathcal{B}|\leq 2$ in LR theories. Note that the expectation value of $\hat{\Pi}(\alpha;s)$ for a given density operator $\hat{\rho}$ is proportional to the s-parameterized quasiprobability function [6,10]

$$\begin{aligned}W(\alpha;s)&=\frac{2}{\pi(1-s)}\mathrm{Tr}[\hat{\rho}\hat{\Pi}(\alpha;s)]\\&=\frac{2}{\pi(1-s)}\sum_{n=0}^{\infty}\left(\frac{s+1}{s-1}\right)^n\langle\alpha,n|\hat{\rho}|\alpha,n\rangle,\end{aligned} \tag{6}$$

from which both the Wigner and the Q functions can be recovered by setting $s=0$ and $s=-1$, respectively. We do not consider the case $s>0$ when the eigenvalues of $\hat{\Pi}(\alpha;s)$ are not bounded. We thus obtain the following generalized BI:

$$\begin{aligned}|\mathcal{B}|_{\{-1<s\leq 0\}}=\Bigg|&\frac{\pi^2(1-s)^4}{4}[W(\alpha_1,\beta_1;s)+W(\alpha_1,\beta_2;s)\\&+W(\alpha_2,\beta_1;s)-W(\alpha_2,\beta_2;s)]\\&+\pi s(1-s)^2[W(\alpha_1;s)+W(\beta_1;s)]+2s^2\Bigg|\leq 2,\end{aligned}$$

$$\begin{aligned}|\mathcal{B}|_{\{s\leq -1\}}=|&\pi^2(1-s)^2[W(\alpha_1,\beta_1;s)+W(\alpha_1,\beta_2;s)\\&+W(\alpha_2,\beta_1;s)-W(\alpha_2,\beta_2;s)]-2\pi(1-s)\\&\times[W(\alpha_1;s)+W(\beta_1;s)]+2|\leq 2,\end{aligned} \tag{7}$$

where $W(\alpha,\beta;s)=[4/\pi^2(1-s)^2]\mathrm{Tr}[\hat{\rho}\hat{\Pi}(\alpha;s)\otimes\hat{\Pi}(\beta;s)]$ is the two-mode s-parametrized quasiprobability functions and $W(\alpha;s)$ and $W(\beta;s)$ are its marginal distributions. We call Eq. (7) the s-parametrized Bell inequality for quasiprobability functions. This BI is equivalent to the BW-W inequality when $s=0$, which has the form of the standard CHSH inequality [14], and the BW-Q inequality when $s=-1$ in the form of the BI proposed by Clauser and Horn [15]. In these cases the corresponding generalized quasiprobability function reduces to the Wigner function $W(\alpha,\beta)=W(\alpha,\beta;0)$ and the Q function $Q(\alpha,\beta)=W(\alpha,\beta;-1)$, respectively [9].

III. TESTING QUANTUM NONLOCALITY

In this section we present a scheme to test quantum nonlocality using the s-parametrized BIs. For a valid quantum nonlocality test, the measured quantities should satisfy the LR conditions, which are assumed when deriving BIs. Thus, here, we employ the direct measurement scheme of quasiprobability functions using photon number detectors proposed in [12].

A pair of entangled states generated from a source of correlated photons is distributed between Alice and Bob, each of whom makes a local measurement by way of an unbalanced homodyne detection (see Fig. 1). Each local measurement is carried out using a photon number detector with quantum efficiency η_d preceded by a beam splitter with transmissivity T. Coherent fields $|\xi\rangle$ and $|\delta\rangle$ enter through the other input ports of each beam splitter. For high transmissivity $T\to 1$ and strong coherent fields $\xi,\delta\to\infty$, the beam splitters of Alice and Bob can be described by the displacement operators $\hat{D}(\alpha)$ and $\hat{D}(\beta)$, respectively, where $\alpha=\xi\sqrt{(1-T)/T}$ and $\beta=\delta\sqrt{(1-T)/T}$ [12]. In measurements $[(s+1)/(s-1)]^{\hat{n}}$ with $\hat{n}=\Sigma_n n|n\rangle\langle n|$, the photon number operator is performed on the outgoing modes using perfect photon number detectors. Then

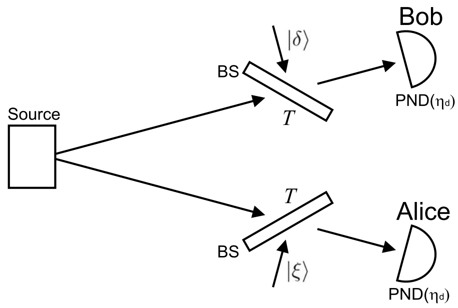

FIG. 1. The optical setup for the BI test. Each local measurement is carried out after mixing the incoming field with a coherent state (denoted by $|\xi\rangle$ for Alice and $|\delta\rangle$ for Bob) in a beam splitter (BS) of high transmissivity T. The photon number detectors (PNDs) have efficiency η_d.

the expectation value directly yields the value of the s-parametrized quasiprobability function at the point in phase space specified by the complex variables α and β. For example, the Wigner function can be obtained by the parity measurements $(-1)^{\hat{n}}(s=0)$ and the Q function by on-off (i.e., photon presence or absence) measurements $(s=-1)$.

Let us now consider the effects of the detector efficiencies η. If the true photon number distribution is given by $P(n)$, then the measured distribution can be written as a function of the overall detection efficiency $\eta=\eta_d T$ as $P_\eta(m)=\sum_{n=m}^{\infty}P(n)\binom{n}{m}(1-\eta)^{n-m}\eta^m$ [16]. For $\alpha=0$ the measured quasiprobability function is

$$\begin{aligned}W_\eta(0;s) &= \frac{2}{\pi(1-s)}\sum_{m=0}^{\infty}\left(\frac{s+1}{s-1}\right)^m P_\eta(m)\\ &= \frac{2}{\pi(1-s)}\sum_{n=0}^{\infty}\left(1-\eta+\eta\frac{s+1}{s-1}\right)^n P(n)\\ &= \frac{W\left(0;-\frac{1-s-\eta}{\eta}\right)}{\eta}\\ &\equiv \frac{W(0;s')}{\eta}.\end{aligned} \tag{8}$$

The s-parametrized quasiprobability function measured by a detector with efficiency η can therefore be identified with the quasiprobability function with parameter $s'=-(1-s-\eta)/\eta$. Other sources of noise (e.g., dark counts and mode mismatch) could be included into this approach but are neglected here for simplicity.

Finally, the expectation value of observable (3) is given as

$$\langle\hat{O}(\alpha;s)\rangle_\eta = \begin{cases}\dfrac{\pi(1-s)^2}{2\eta}W(\alpha;s')+s & \text{if } -1<s\le 0\\[2ex] \dfrac{\pi(1-s)}{\eta}W(\alpha;s')-1 & \text{if } s\le -1,\end{cases} \tag{9}$$

where $\langle\cdot\rangle_\eta$ represents the expectation value obtained by measurement with efficiency η. Note that Eq. (9) is the statistical average of directly measured data without postselection. The expectation value of the Bell operator (5) written as a function of s and η is given by

$$\begin{aligned}\langle\hat{\mathcal{B}}_{\{-1<s\le 0\}}\rangle_\eta &= \frac{\pi^2(1-s)^4}{4\eta^2}\Bigg[W\left(\alpha_1,\beta_1;-\frac{1-s-\eta}{\eta}\right)\\ &\quad + W\left(\alpha_1,\beta_2;-\frac{1-s-\eta}{\eta}\right)\\ &\quad + W\left(\alpha_2,\beta_1;-\frac{1-s-\eta}{\eta}\right)\\ &\quad - W\left(\alpha_2,\beta_2;-\frac{1-s-\eta}{\eta}\right)\Bigg]\\ &\quad + \frac{\pi s(1-s)^2}{\eta}\Bigg[W\left(\alpha_1;-\frac{1-s-\eta}{\eta}\right)\\ &\quad + W\left(\beta_1;-\frac{1-s-\eta}{\eta}\right)\Bigg]+2s^2,\end{aligned}$$

$$\begin{aligned}\langle\hat{\mathcal{B}}_{\{s\le -1\}}\rangle_\eta &= \frac{\pi^2(1-s)^2}{\eta^2}\Bigg[W\left(\alpha_1,\beta_1;-\frac{1-s-\eta}{\eta}\right)\\ &\quad + W\left(\alpha_1,\beta_2;-\frac{1-s-\eta}{\eta}\right)+W\Bigg(\alpha_2,\beta_1;\\ &\quad -\frac{1-s-\eta}{\eta}\Bigg)-W\left(\alpha_2,\beta_2;-\frac{1-s-\eta}{\eta}\right)\Bigg]\\ &\quad -\frac{2\pi(1-s)}{\eta}\Bigg[W\left(\alpha_1;-\frac{1-s-\eta}{\eta}\right)\\ &\quad + W\left(\beta_1;-\frac{1-s-\eta}{\eta}\right)\Bigg]+2.\end{aligned} \tag{10}$$

Note that the Bell expectation values in Eq. (10) for $s=0$ and $s=-1$ give the same results as tests of the BW-W and the BW-Q inequalities, respectively.

IV. VIOLATION BY SINGLE-PHOTON ENTANGLED STATES

We investigate violations of the s-parametrized BI (7) for the single-photon entangled state [17]

$$|\Psi\rangle = \frac{1}{\sqrt{2}}(|0,1\rangle+|1,0\rangle), \tag{11}$$

where $|n,m\rangle$ is the state with n photons in Alice's mode and m photons in Bob's mode. This state is created by a single photon incident on a 50:50 beam splitter. Its two-mode s-parametrized quasiprobability function is given by

$$\begin{aligned}W_\Psi(\alpha,\beta;s) &= \frac{4}{\pi^2(1-s)^2}\left(-\frac{1+s}{1-s}+\frac{2}{(1-s)^2}|\alpha+\beta|^2\right)\\ &\quad\times\exp\left[-\frac{2(|\alpha|^2+|\beta|^2)}{1-s}\right],\end{aligned} \tag{12}$$

and its marginal single-mode distribution is

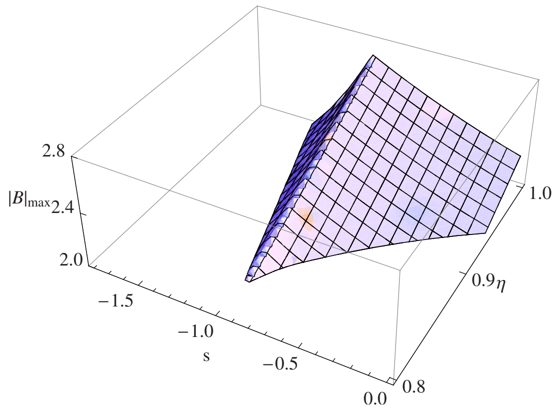

FIG. 2. (Color online) Maximum Bell expectation value $|\mathcal{B}|=|\langle\hat{\mathcal{B}}\rangle|$ for the single-photon entangled state. Only the range of parameters s and detector efficiencies η with $|\mathcal{B}|>2$ are shown.

$$W_{\Psi}(\alpha;s)=(1/\pi)(2-2\eta+4\eta^2|\alpha|^2)\exp[-2\eta|\alpha|^2]. \quad (13)$$

Note that for $0\geq s>-1$ Eq. (12) has negative values in certain regions of phase space but for $s=-1$ it becomes the Q function $W_{\Psi}(\alpha,\beta;-1)\geq 0$.

The maximum expectation values $|\mathcal{B}|_{\max}=|\langle\hat{\mathcal{B}}\rangle|_{\max}$ are obtained for properly chosen α_1, α_2, β_1, and β_2. Figure 2 shows the range of parameters s and detector efficiencies η for which the BI is violated, $|\mathcal{B}|_{\max}>2$. Interestingly, the degree of violation is not directly related to the negativity of the quasiprobability functions. The test of the BI using the Q function ($s=-1$) yields strong violations and is most robust to detector inefficiencies. This is because the observable (3) becomes dichotomized at $s=-1$ corresponding to detection of none vs some photons. For a given s, the amount of violation decreases with decreasing η. The minimum value of η indicates the required detector efficiency for a successful nonlocality test [18]. For example, the minimum bound is about 83% for the Q function ($s=-1$). We also find the minimum parameter s which allows demonstrating quantum nonlocality for a given detector efficiency. For example, for a perfect detector ($\eta=1$), the corresponding BI is violated when $s\gtrsim-1.43$.

V. VIOLATION BY TWO-MODE SQUEEZED STATES

We consider the TMSSs, i.e., a continuous-variable entangled state written as

$$|\mathrm{TMSS}\rangle=\mathrm{sech}\, r\sum_{n=0}^{\infty}\tanh^n r|n,n\rangle, \quad (14)$$

where $r>0$ is the squeezing parameter. It can be realized, for instance, by nondegenerate optical parametric amplifiers [19]. In the infinite squeezing limit $r\to\infty$, the TMSS becomes the normalized EPR state, which is the maximally entangled state associated with position and momentum [9].

For a nonpositive s the quasiprobability function of the TMSS is given by

$$W_{\mathrm{TMSS}}(\alpha,\beta;s)=\frac{4}{\pi^2R(s)}\exp\biggl(-\frac{2}{R(s)}\{S(s)(|\alpha|^2+|\beta|^2)-\sinh 2r(\alpha\beta+\alpha^*\beta^*)\}\biggr), \quad (15)$$

and its marginal single-mode distribution is

$$W_{\mathrm{TMSS}}(\alpha;s)=\frac{2}{\pi S(s)}\exp\left(-\frac{2|\alpha|^2}{S(s)}\right), \quad (16)$$

where $R(s)=s^2-2s\cosh 2r+1$ and $S(s)=\cosh 2r-s$. Note that these are positive everywhere in phase space. In Fig. 3(a) violations of the s-parametrized BI are shown for TMSSs. The test using the Q function ($s=-1$) is most robust with respect to detector inefficiencies. The amount of violation shows different tendencies depending on the squeezing parameter r. In the case of low squeezing rates, i.e., when the amplitudes of small-n number states are dominant, the violation is maximal if we choose the Q function ($s=-1$) as

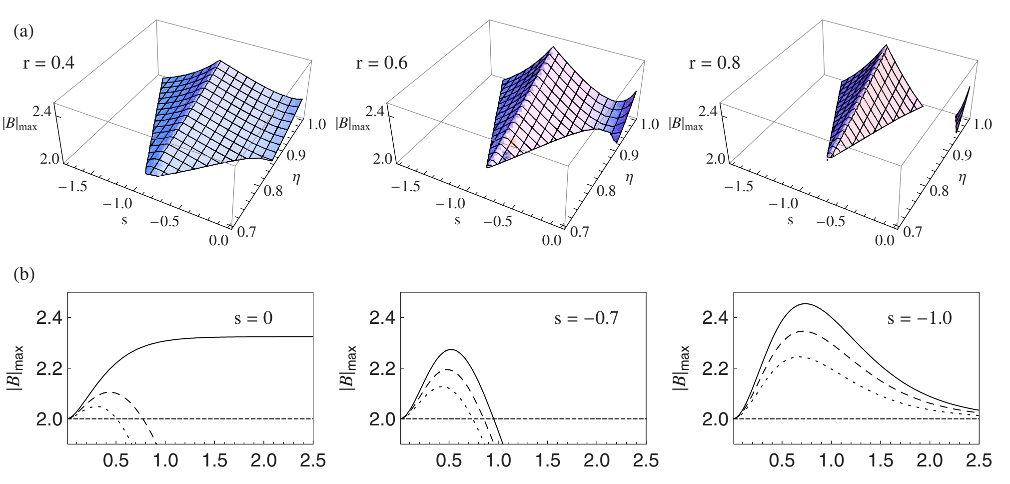

FIG. 3. (Color online) Demonstration of quantum nonlocality for TMSSs. (a) Maximum Bell values are shown for different squeezing r in the range of s and η where the BI is violated. (b) Violation of the BI as a function of the squeezing r for different s and $\eta=1$ (solid line), $\eta=0.95$ (dashed line), and $\eta=0.9$ (dotted line).

shown in Fig. 3(b). This implies that the dominant contribution to the violation comes from correlations between the vacuum and the photons present. For larger squeezing rates $r \gtrsim 1.2$, the violation reaches a maximal value $\mathcal{B} \approx 2.32$ when we test the Wigner function ($s=0$) [20]. This indicates that the parity measurements are effective for verifying higher-order number correlations. However, the parity measurements require very high detector efficiency as shown in Fig. 3(b). The range of s within which one can demonstrate nonlocality becomes narrower around $s=0$ and $s=-1$ with increasing squeezing rate r. This is because observable (3) is dichotomized at $s=0$ and $s=-1$.

VI. DISCUSSION AND CONCLUSIONS

We demonstrated that quantum nonlocality has no direct relation to the negativity of s-parametrized quasiprobability functions. In fact the Q function ($s=-1$) which never becomes negative can still be used to verify nonlocal properties as we showed in Fig. 2 and yields strong violations of the corresponding BI. This implies that the quantum properties of nonlocality and negativity of the quasiprobability functions should be considered distinct features of quantum mechanics. Furthermore, we showed that the Q function test allows the lowest detector efficiency for demonstrating quantum nonlocality. For example, it requires only $\eta \approx 83\%$ for a single-photon entangled state and $\eta \approx 75\%$ for TMSSs with $r=0.4$ to detect nonlocality. This indicates that two-mode correlations between vacuum and many photons can be more robust to detector inefficiencies than correlations between the vacuum and a single photon.

The parameter s determines the characteristics of the detected nonlocal correlations. For example, if we choose $s=-1$ the violation of the BI exhibits only correlations between vacuum and photons. In order to test higher-order photon number correlations, we need to increase s to zero, so that the factor $[(s+1)/(s-1)]^n$ multiplied to the photon number probability increases in Eq. (6). Although parity measurements ($s=0$) allow us to detect higher-order correlations effectively, they also require very high detector efficiencies as shown in Fig. 3. If we properly choose a certain parameter $-1 < s < 0$, e.g., $s=-0.7$, we can detect higher-order correlations with a lower detector efficiency than that required for testing the BI using the Wigner function. However, we note that the violation of the BI with $s=-0.7$ disappears with increasing squeezing rate as shown in Fig. 3(b); this restricts the possible applications to schemes using light that contains only a few photons.

Let us finally discuss whether we can regard decoherence effects as changes to s. Interactions with the environment and the detection noise tend to smoothen quasiprobability functions. For example, when solving the Fokker-Planck equation for the evolution of the Wigner function of a system interacting with a thermal environment, one obtains [21]

$$W(\alpha,\tau) = \frac{1}{t(\tau)^2}\int d^2\beta\, W^{\mathrm{th}}(\beta) W\left(\frac{\alpha - r(\tau)\beta}{t(\tau)}, \tau=0\right). \tag{17}$$

Here the parameters $r(\tau)=\sqrt{1-e^{-\gamma\tau}}$ and $t(\tau)=\sqrt{e^{-\gamma\tau}}$ are given in terms of the energy decay rate γ, and

$$W^{\mathrm{th}}(\beta) = \frac{2}{\pi(1+2\bar{n})}\exp\left(-\frac{2|\beta|^2}{1+2\bar{n}}\right) \tag{18}$$

is the Wigner function for the thermal state of average thermal photon number $\bar{n}$. The effect of the thermal environment is then identified with temporal changes of the parameter

$$s(\tau) \sim -\frac{r(\tau)^2}{t(\tau)^2}(1+2\bar{n}) = (1-e^{\gamma\tau})(1+2\bar{n}). \tag{19}$$

Therefore, one might be tempted to consider an environment in a thermal state as giving rise to a temporal change in s in Eq. (3). However, this idea is not applicable to tests of quantum nonlocality. The s-parametrized BI is derived for observables (3) which contain s as a deterministic value of LR theories. Thus the local-realistic bound is no longer valid when dynamical observables are considered (even though they give the same statistical average). However, this idea might be useful for witnessing entanglement [22].

In summary, we have formulated a BI in terms of the generalized quasiprobability function. This BI is parametrized by a nonpositive value s and includes previously proposed BIs such as the BW-W ($s=0$) and the BW-Q ($s=-1$) inequalities [9]. We employed a direct measurement scheme for quasiprobability functions [12] to test quantum nonlocality. The violation of BIs was demonstrated for two types of entangled states: single-photon entangled and two-mode squeezed vacuum states. We found the range of s and η which allow the observation of quantum nonlocal properties. We discussed the types of correlations and their robustness to detection inefficiencies for different values of s. We also demonstrated that the negativity of the quasiprobability function is not directly related to the violation of BIs. The realization of s-parametrized BI tests is expected along with the progress of photon detection technologies [23] in the near future. Our investigations can readily be extended to other types of states such as photon subtracted Gaussian states [24,25] or optical Schrödinger cat states [26].

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