Probing the Universe with Large-Scale Structure

General Relativistic Effects and Gauge-Invariant Formalism

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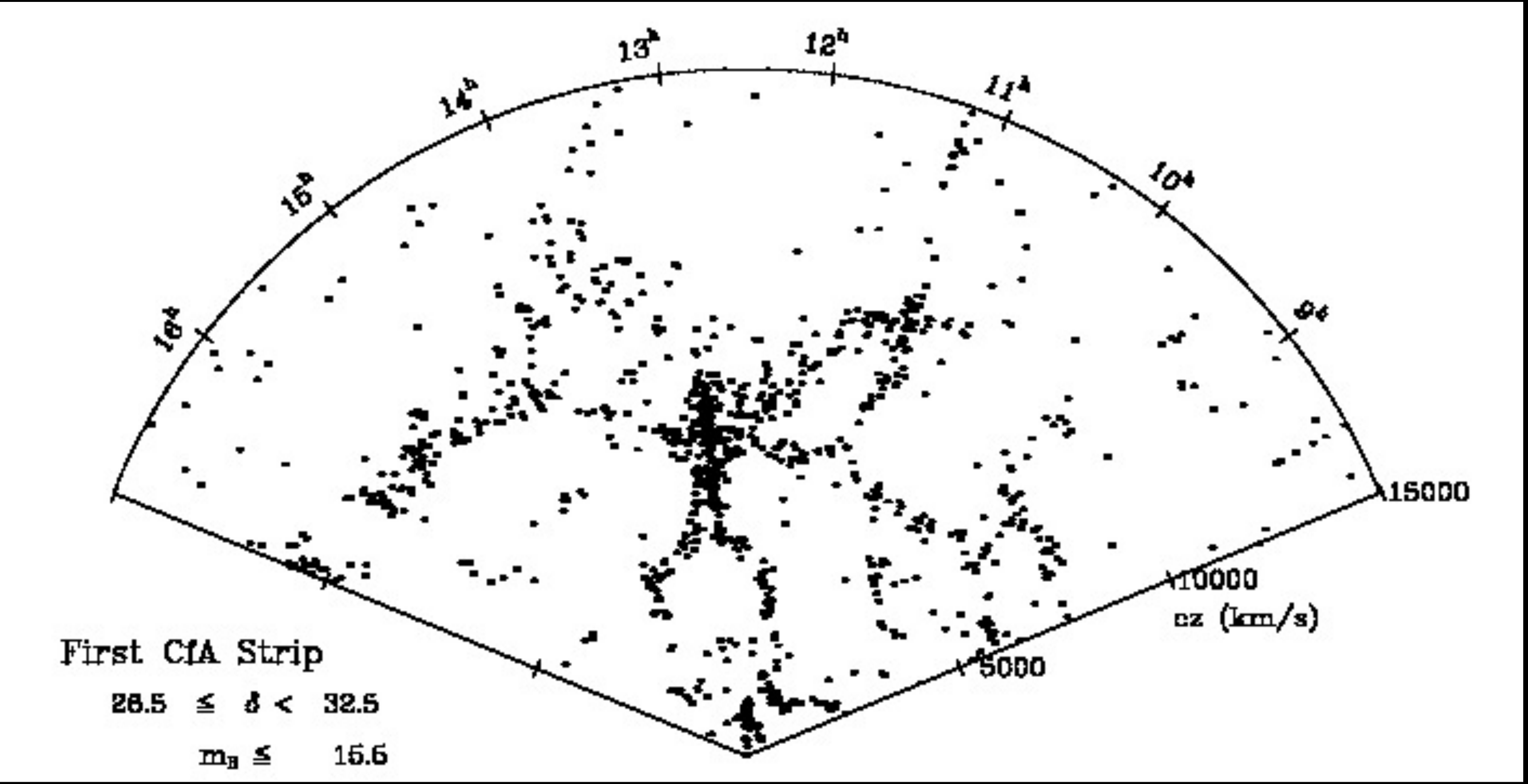
Universität Zürich[UZH]

I. PRECISION COSMOLOGY:

Past, Today, and Future

Galaxy Surveys

- The CfA galaxy redshift survey in 1977 ~ 1985
  - 18,000 galaxies, $z\sim 0.05$ *CfA great wall*

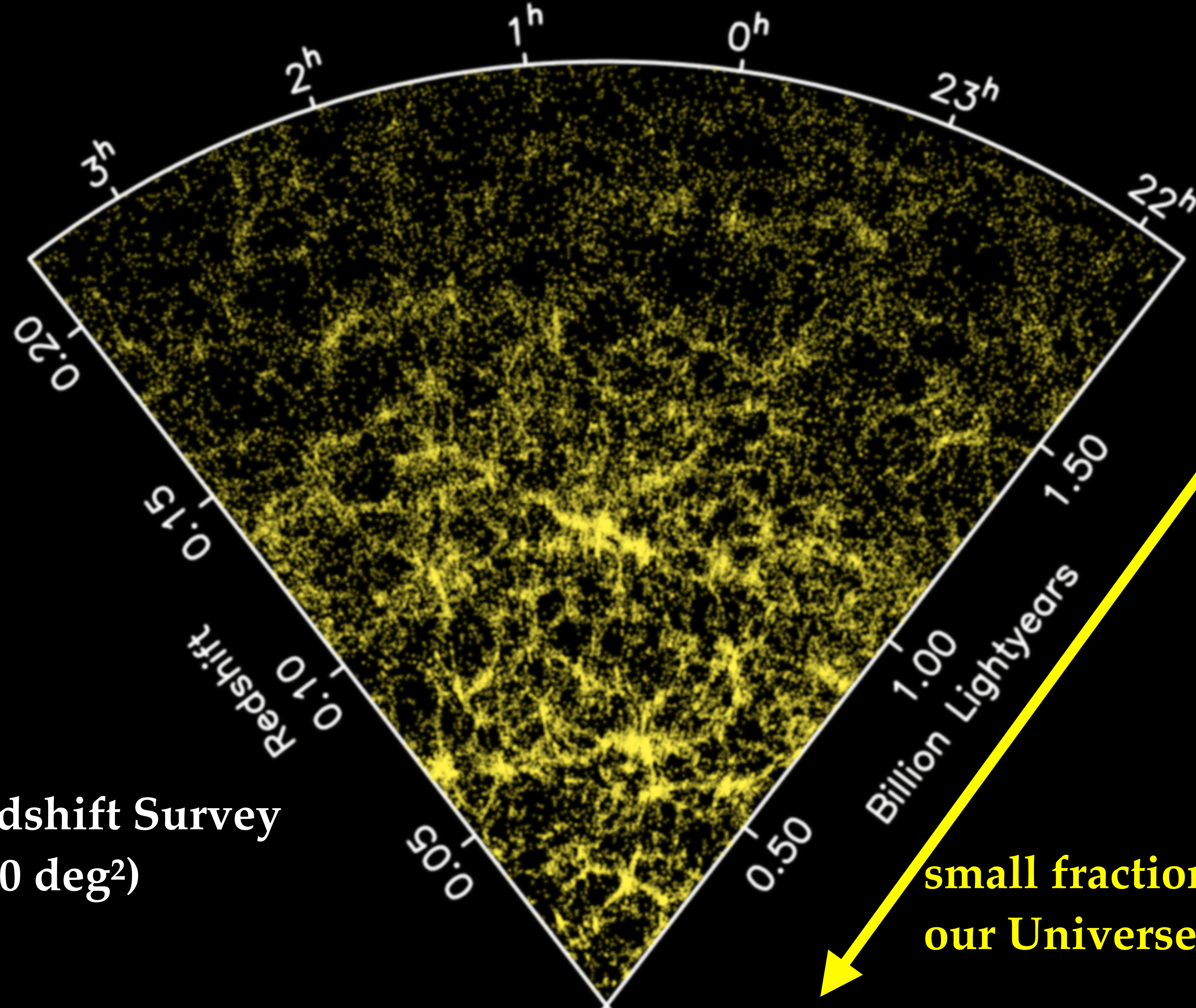


Huchra, Davis, Latham, Tonry, ApJS, 1983

de Lapparent, Geller, Huchra, ApJ, 1991

Galaxy Surveys

redshift ~ 0.2



2dF Galaxy Redshift Survey
1997~2002 (1500 deg²)

small fraction of
our Universe

Galaxy Surveys

- **Sloan Digital Sky Survey**
  - **2000 ~ 2008**
  - **1/4 sky (10,000 deg^2)**
  - **1 million galaxies**
  - **120,000 quasars**
  - **~ 24 magnitude**
  - ***z ~ 0.5***

Credit: SDSS

Galaxy Surveys

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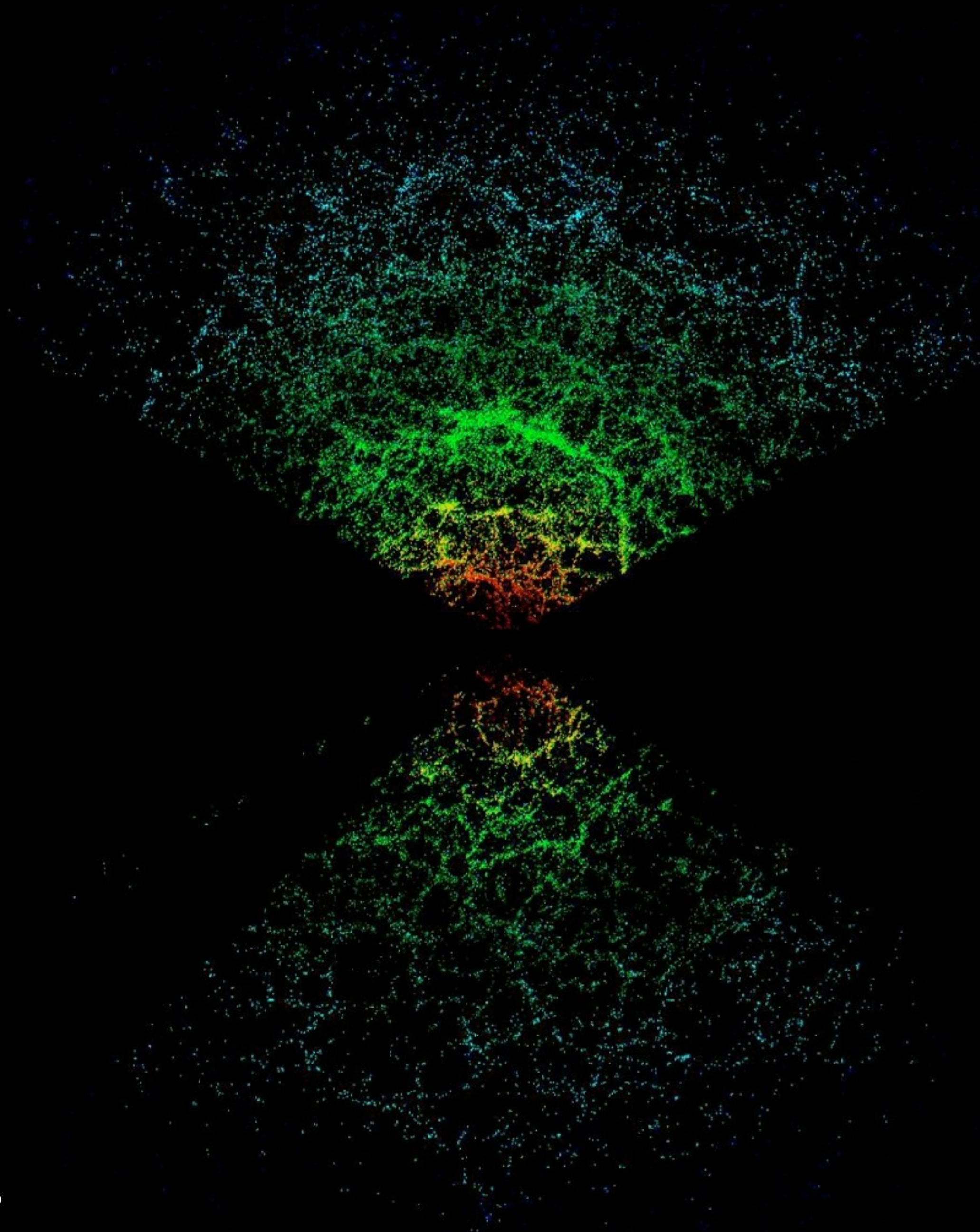
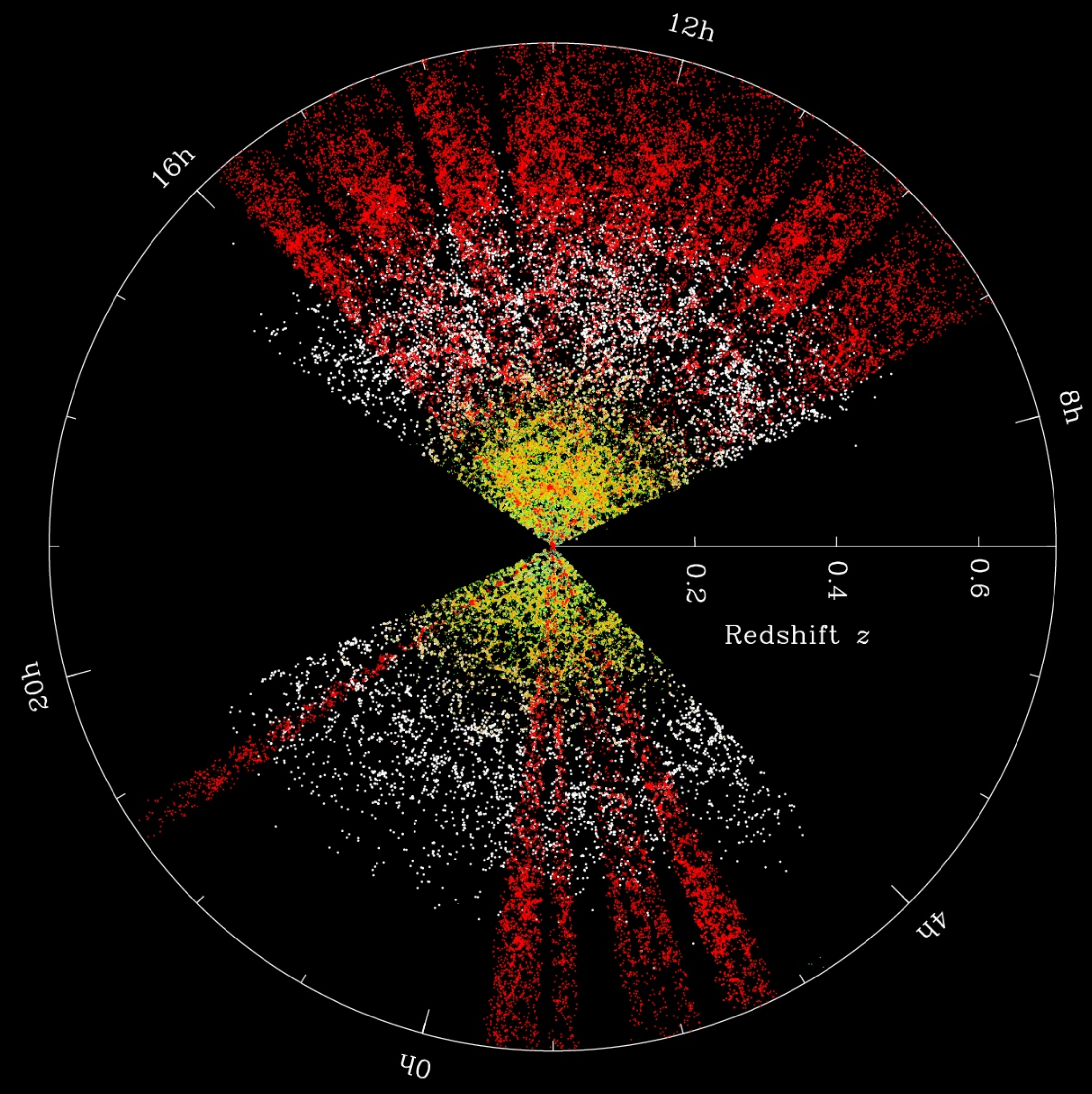


Credit: SDSS

Galaxy Surveys

- **Baryonic Oscillation Spectroscopic Survey**
  - **2009 ~ 2014**
  - **1/4 sky**
  - **2 million galaxies**
  - **160,000 quasars**
  - ***z ~ 0.8***
- **precise distance by BAO**
  - **low redshift anchor to CMB**
  - ***~1% to z=0.3, 0.5, 0.7***

Beutler et al. 2017



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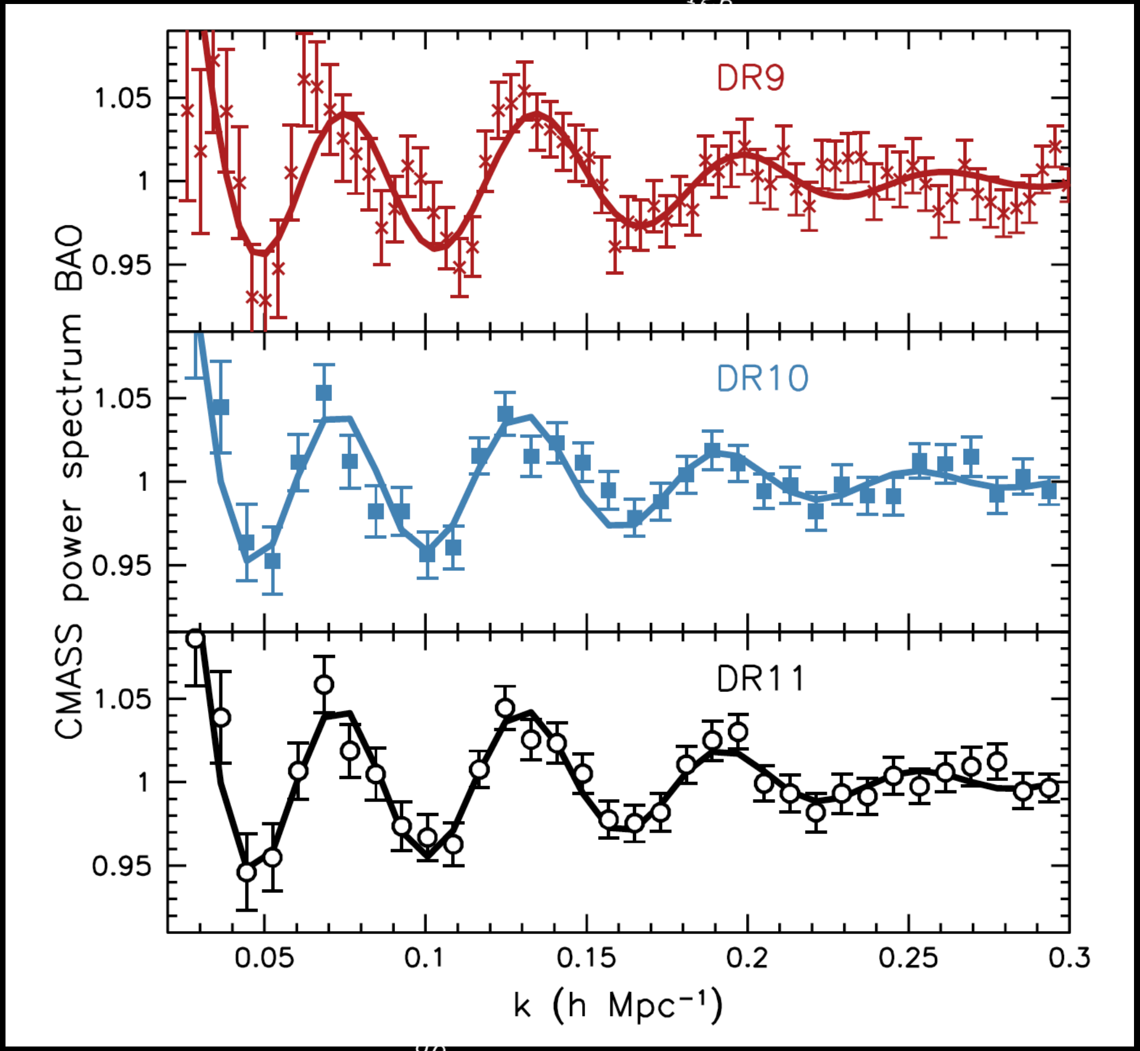


Anderson et al. 2013

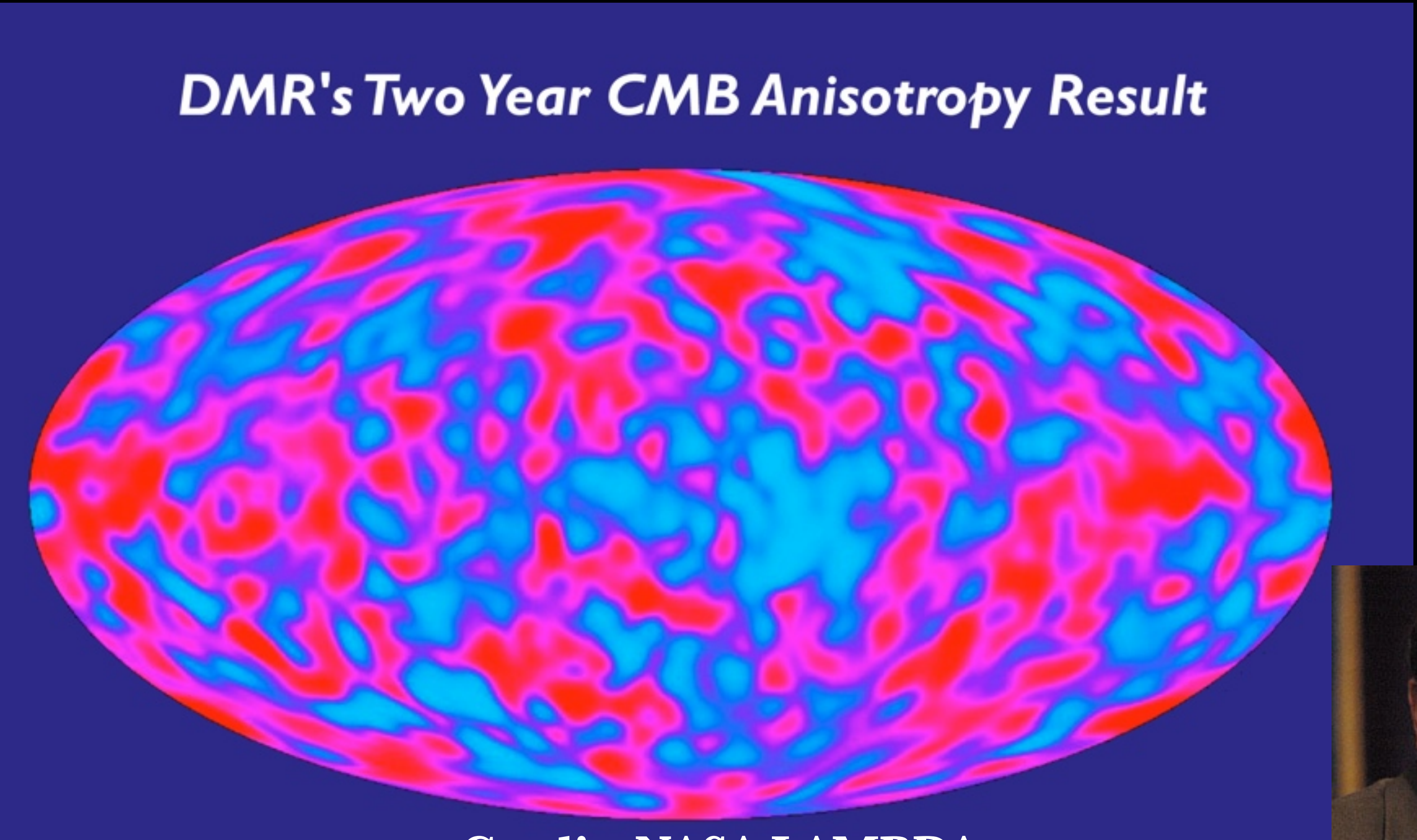
Cosmic Microwave Background

- Arno **Penzias** & Robert **Wilson** at Bell laboratory
  - detected the *3.5K* radiation from Big Bang in 1964
  - *noise* (systematic errors) for their experiments

Nobel prize 1978
for its discovery

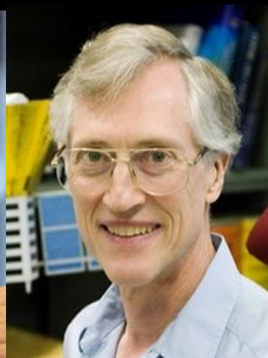
Cosmic Microwave Background

- **Cosmic Background Explorer in 1989 ~ 1993**
  - **large-scale anisotropies (up to 6 degree)**



Credit: NASA LAMBDA

Nobel prize 2006
for its discovery



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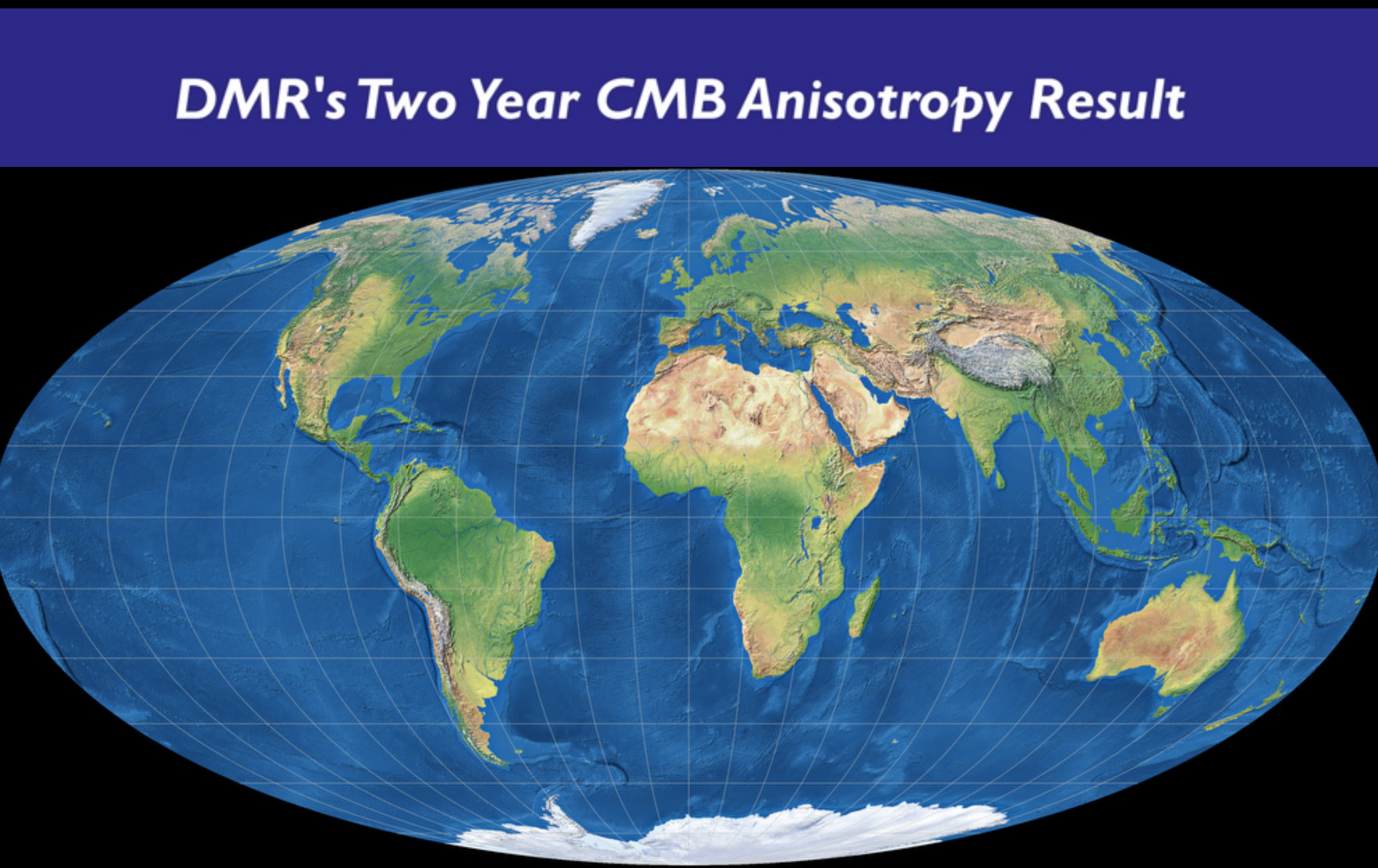

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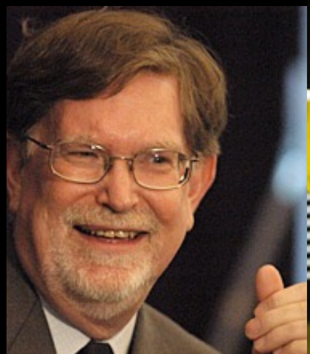
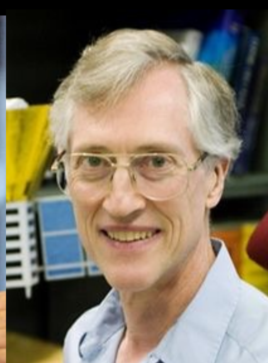
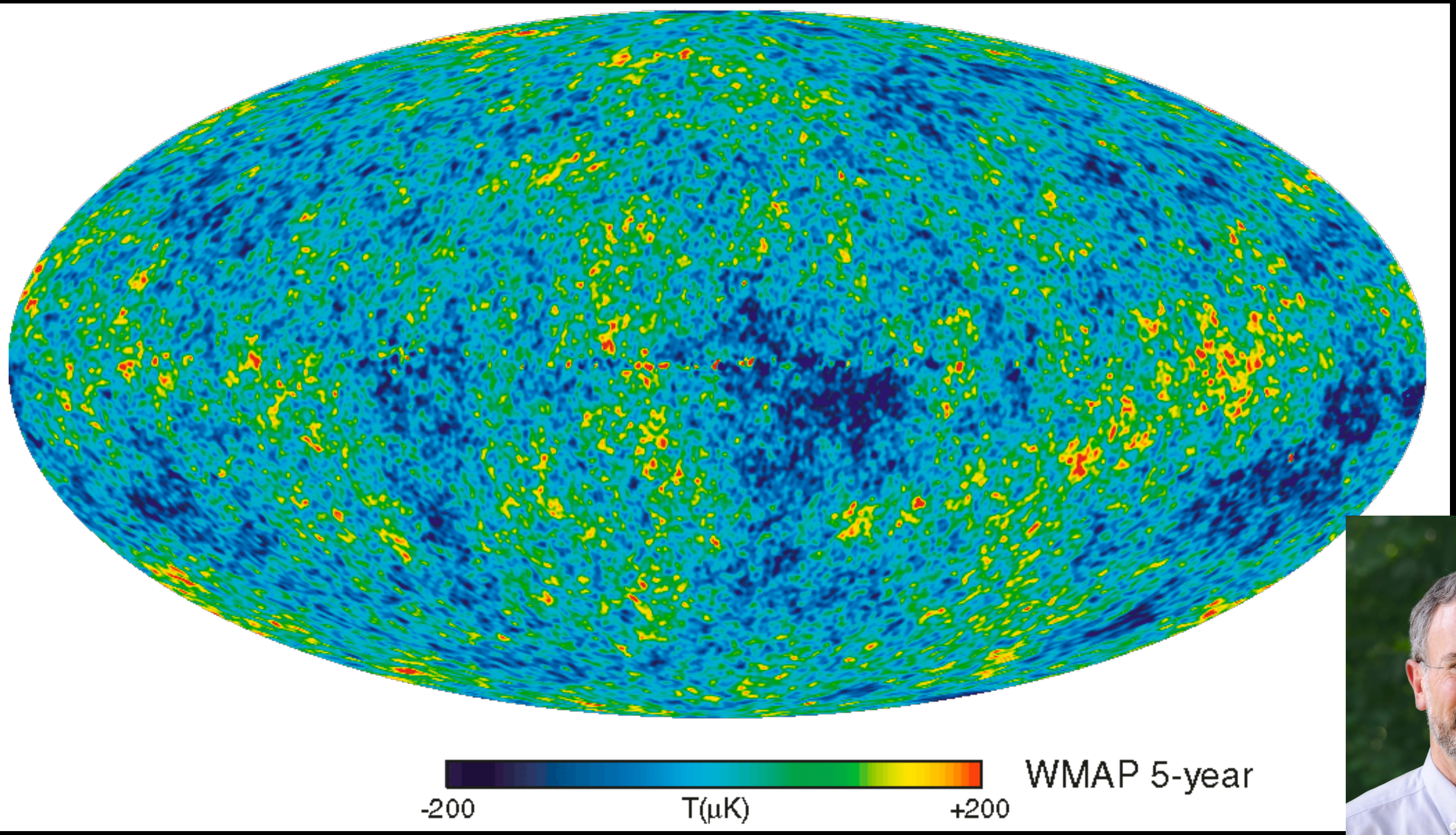




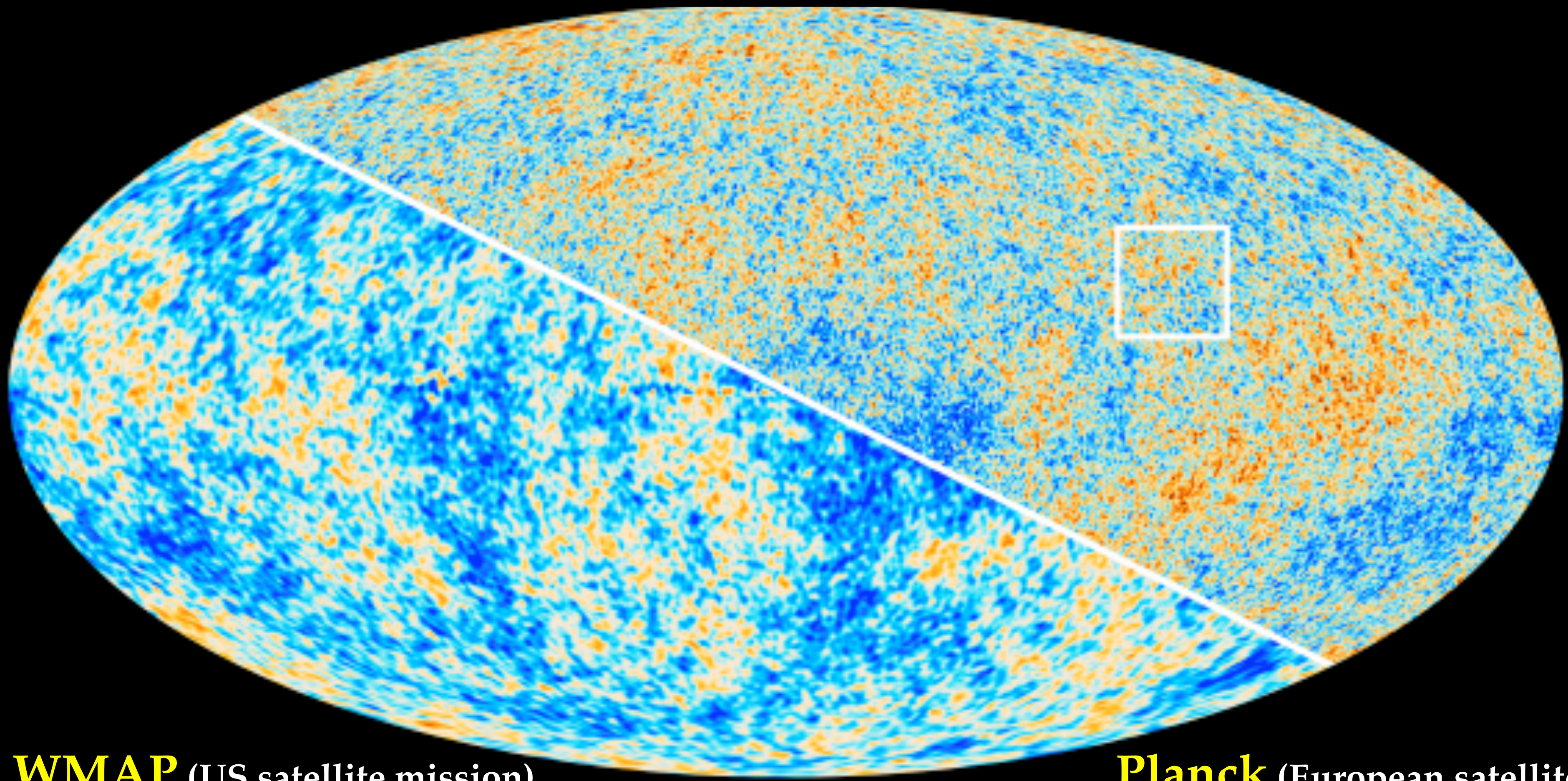
Cosmic Microwave Background

- **Wilkinson Microwave Anisotropy Probe in 2001~ 2010**
  - ***precision cosmology!*** **(resolution: ~25 arc minute)**

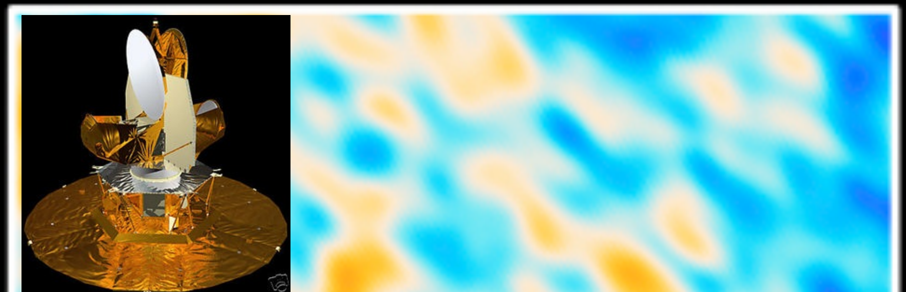


Planck satellite in 2009~ 2013

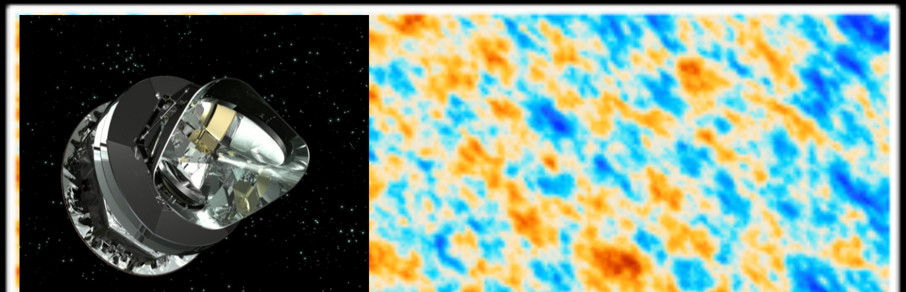
precision cosmology! **(resolution: ~10 arc minute)**



WMAP (US satellite mission)



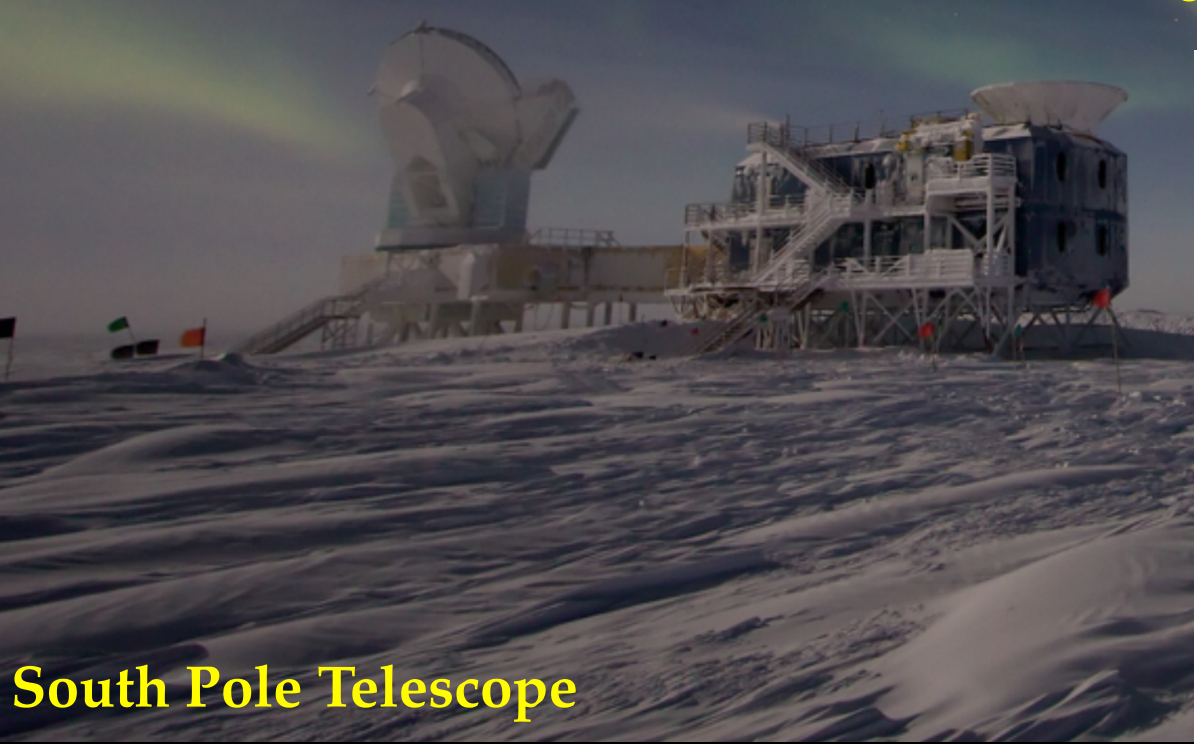
Planck (European satellite)

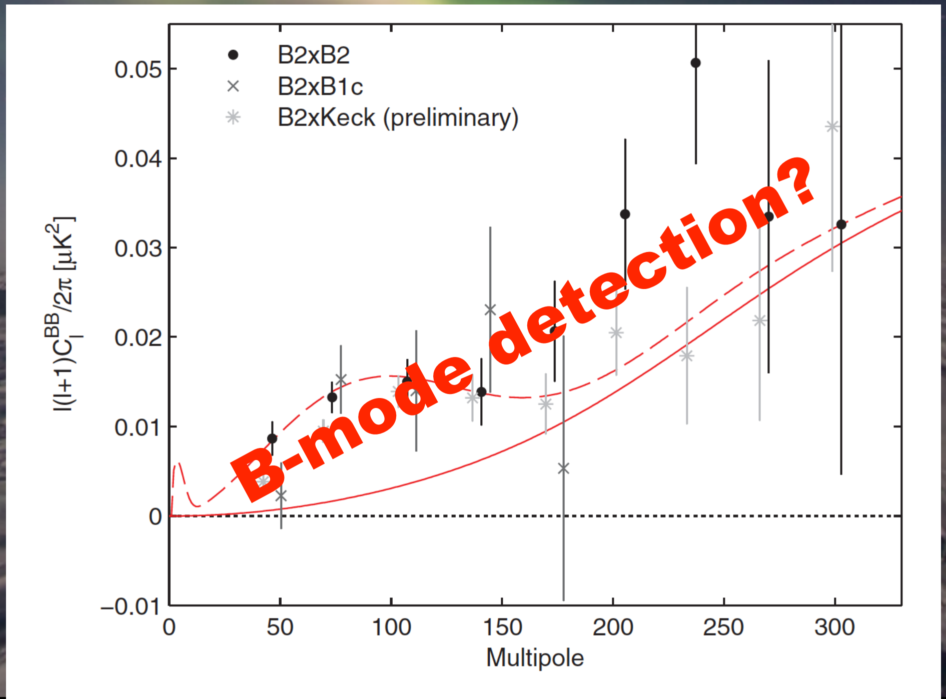


The Experiment

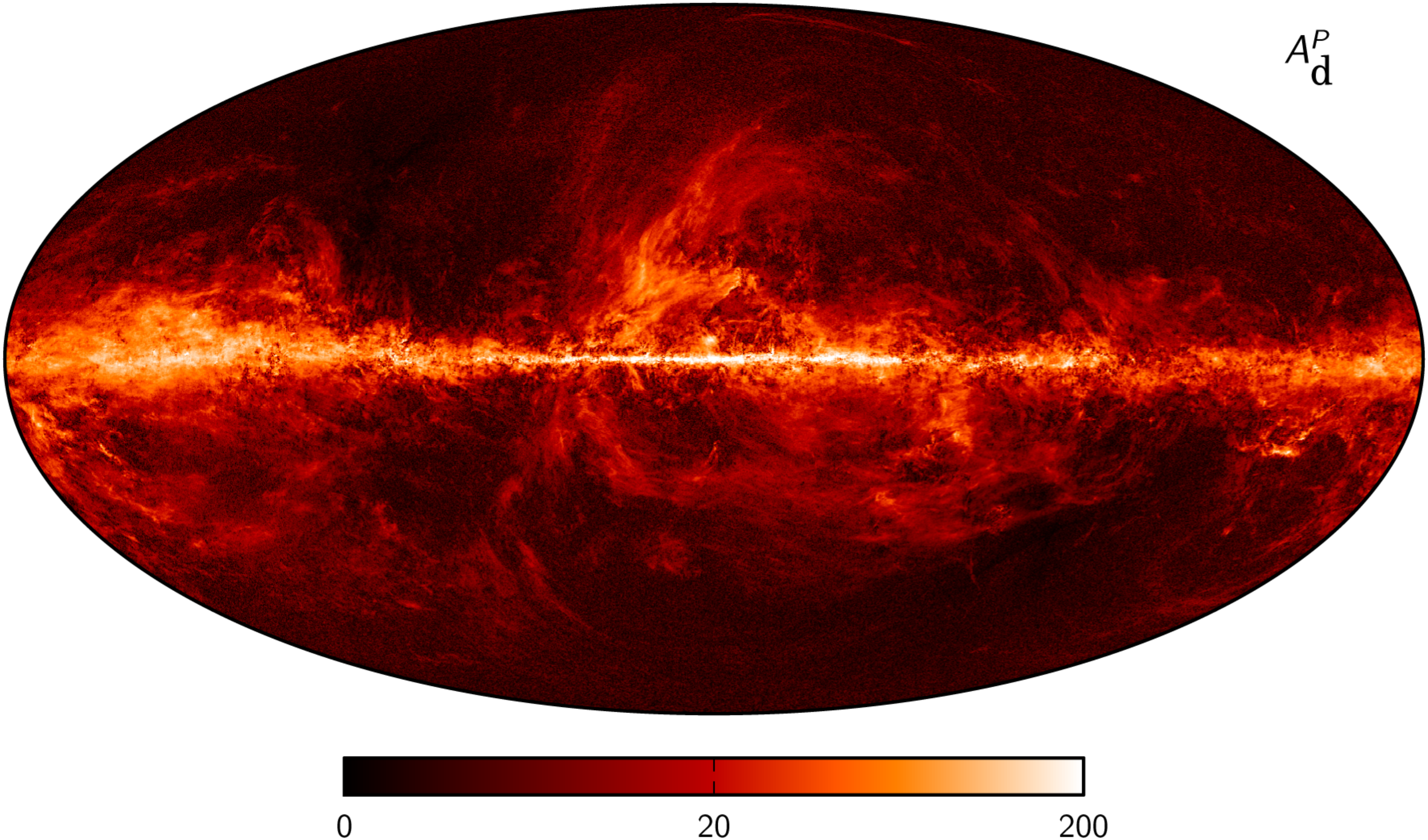
BICEP and Keck Array

cosmological gravity waves 2014





- **Planck *dust* polarization map**




B-mode in the Sky?
Vincent van Gogh

Standard Candle

- *intrinsic* brightness: well-known
- apparent brightness: cosmological distance

farther away, dimmer

closer, brighter

Type-Ia SN: Standard Candle

- white dwarf in binary accrete material from companion
- runaway carbon nuclear fusion in few secs
- same amount of carbon & easy to calibrate

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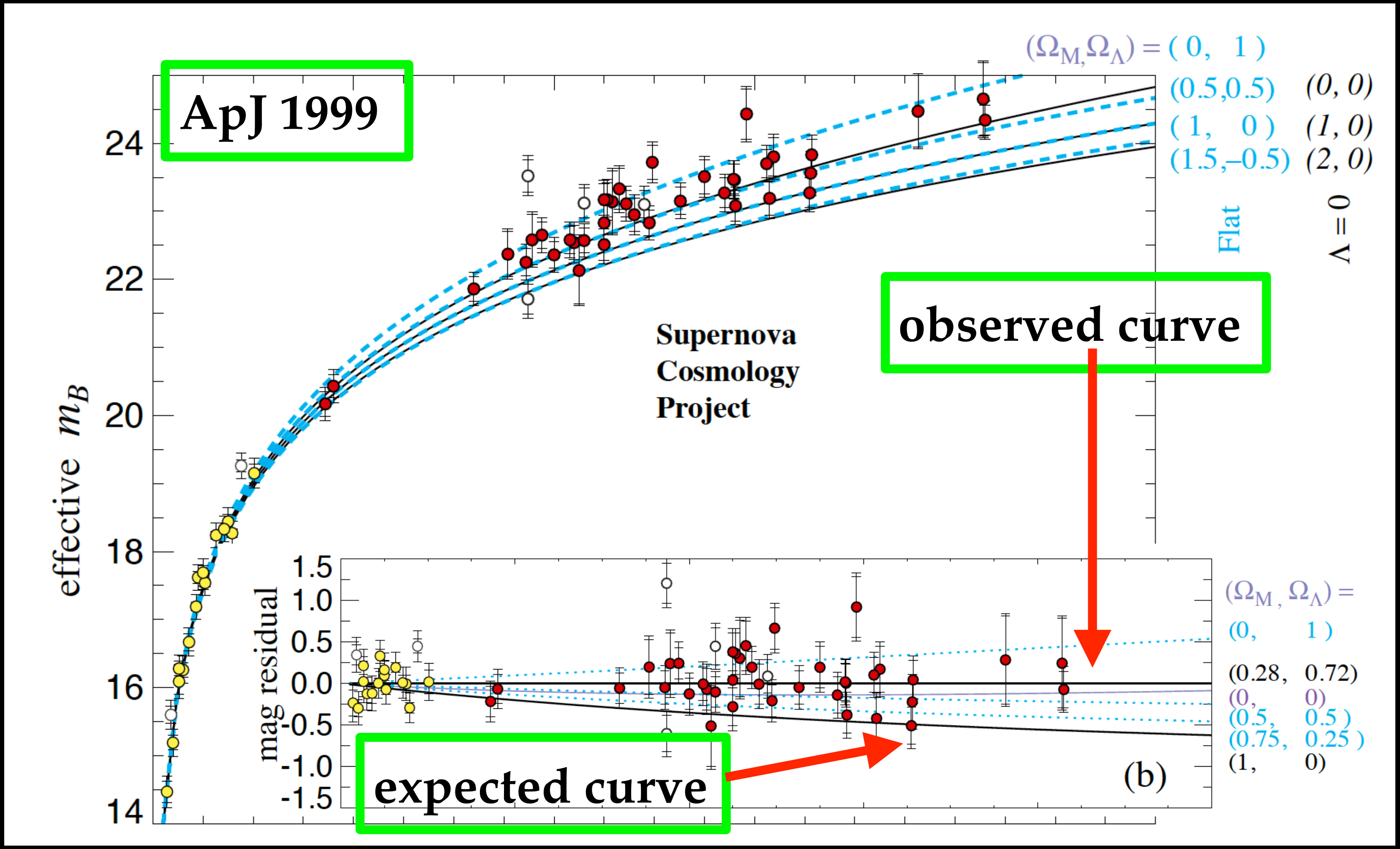
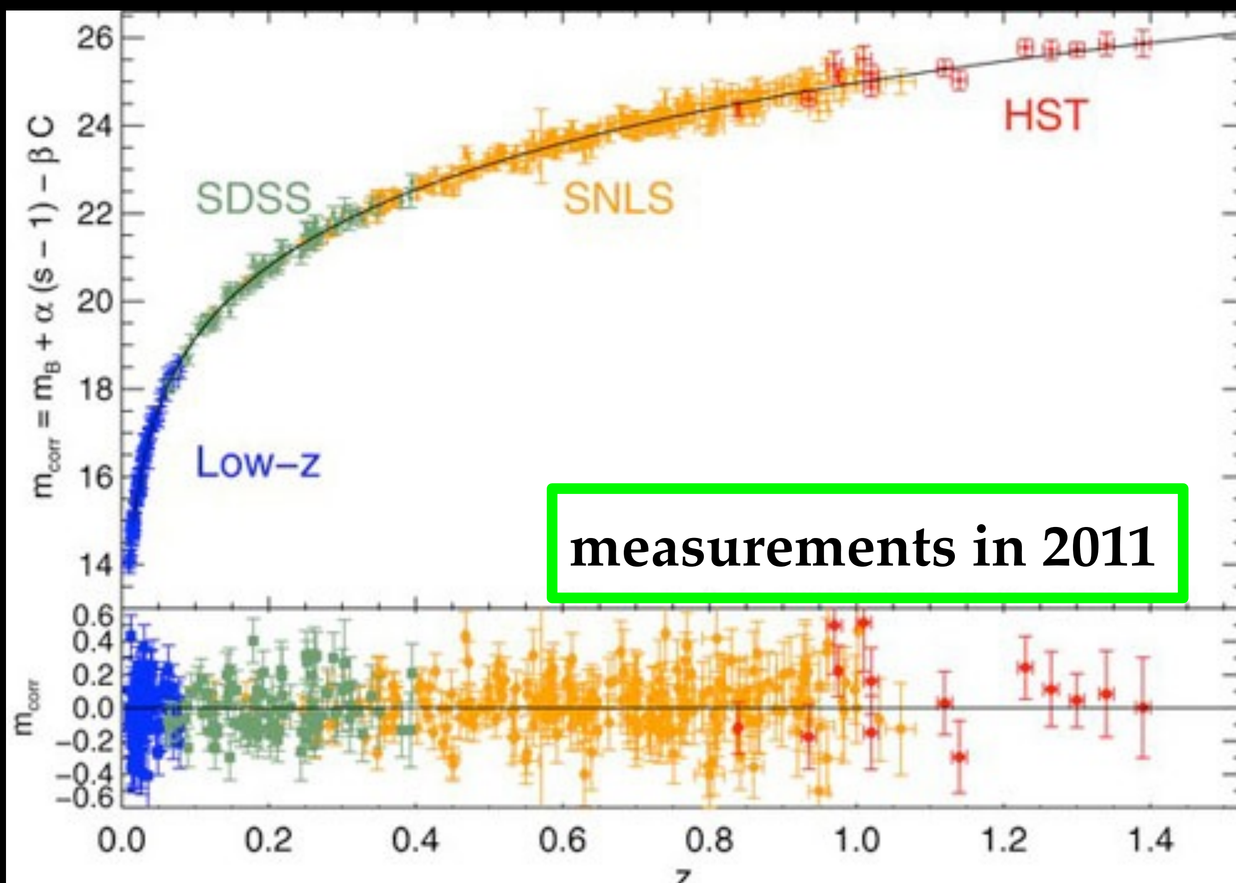
Supernova Observations

- distant galaxies are *farther away* (dimmer!)
  - larger magnitude for fainter objects (z<1.0)



Dark Energy

- **acceleration of the Universe: repulsive gravity**
- ***what is it?*** **very mysterious in nature**
- ***cosmological observations*** **can shed light**



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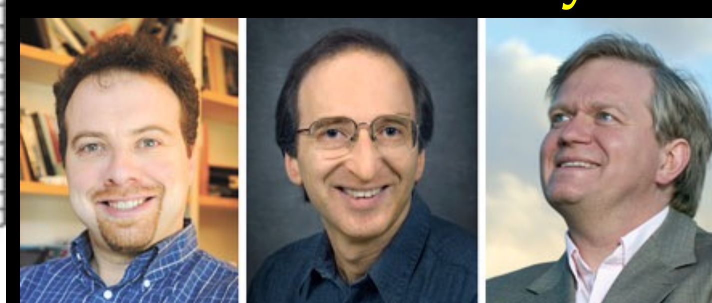
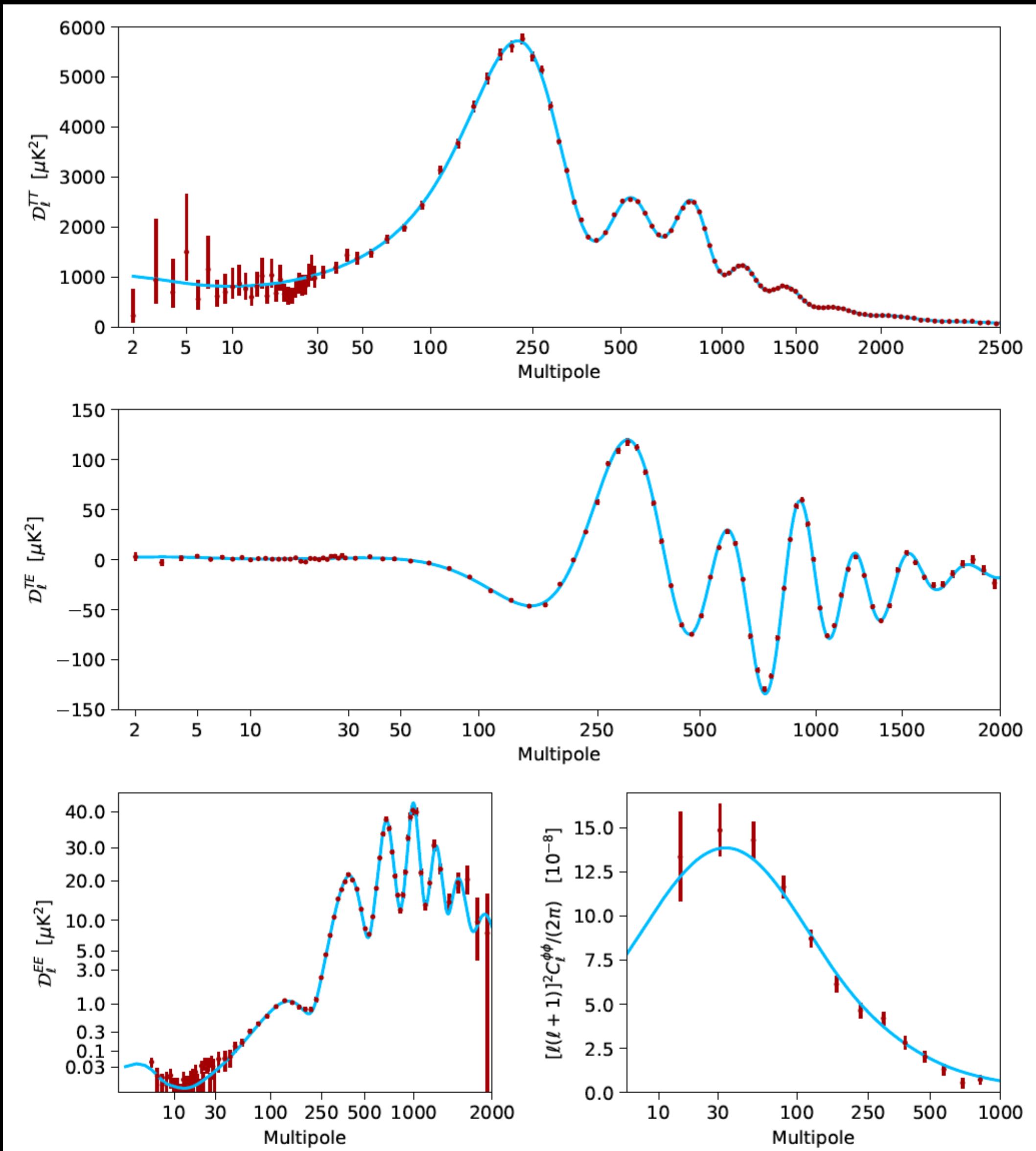


I. PRECISION COSMOLOGY: PAST, TODAY, and FUTURE

- **Planck collaboration 2018:** ***precision measurements!***



Parameter	*Planck* alone	*Planck* + BAO
$\Omega_b h^2$	0.02237 ± 0.00015	0.02242 ± 0.00014
$\Omega_c h^2$	0.1200 ± 0.0012	0.11933 ± 0.00091
$100\theta_{MC}$	1.04092 ± 0.00031	1.04101 ± 0.00029
τ	0.0544 ± 0.0073	0.0561 ± 0.0071
$\ln(10^{10} A_s)$	3.044 ± 0.014	3.047 ± 0.014
n_s	0.9649 ± 0.0042	0.9665 ± 0.0038
H_0	67.36 ± 0.54	67.66 ± 0.42
Ω_Λ	0.6847 ± 0.0073	0.6889 ± 0.0056
Ω_m	0.3153 ± 0.0073	0.3111 ± 0.0056
$\Omega_m h^2$	0.1430 ± 0.0011	0.14240 ± 0.00087
$\Omega_m h^3$	0.09633 ± 0.00030	0.09635 ± 0.00030
σ_8	0.8111 ± 0.0060	0.8102 ± 0.0060
$\sigma_8(\Omega_m/0.3)^{0.5}$	0.832 ± 0.013	0.825 ± 0.011
z_{re}	7.67 ± 0.73	7.82 ± 0.71
Age[Gyr]	13.797 ± 0.023	13.787 ± 0.020
r_*[Mpc]	144.43 ± 0.26	144.57 ± 0.22
$100\theta_*$	1.04110 ± 0.00031	1.04119 ± 0.00029
r_{drag}[Mpc]	147.09 ± 0.26	147.57 ± 0.22
z_{eq}	3402 ± 26	3387 ± 21
k_{eq}[Mpc^{-1}]	0.010384 ± 0.000081	0.010339 ± 0.000063
Ω_K	-0.0096 ± 0.0061	0.0007 ± 0.0019
Σm_ν [eV]	< 0.241	< 0.120
N_{eff}	$2.89^{+0.36}_{-0.38}$	$2.99^{+0.34}_{-0.33}$
$r_{0.002}$	< 0.101	< 0.106

Standard Model of Cosmology

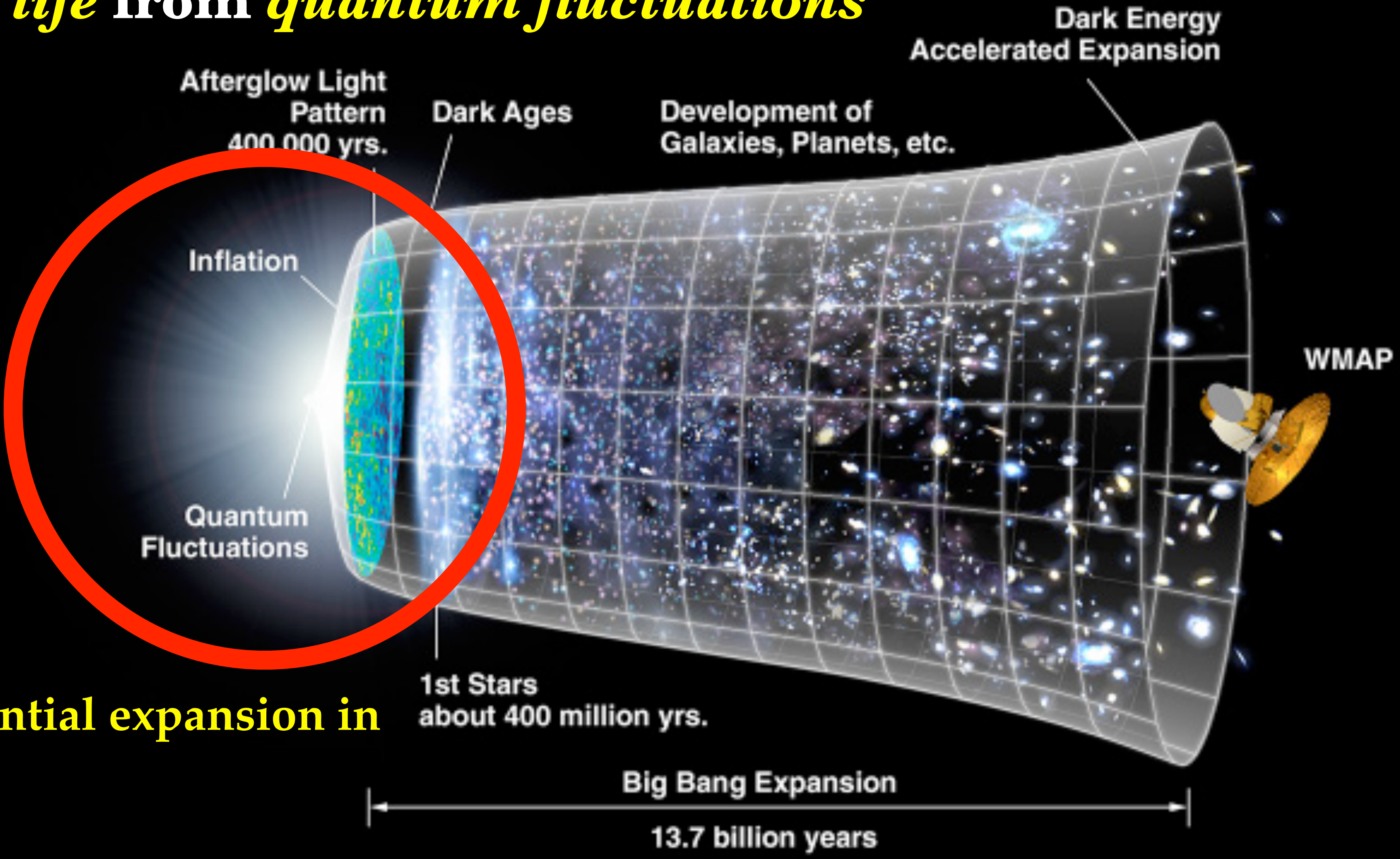
- I. *inflationary epoch* in early Universe
  - seed fluctuations for formation of galaxies & life

- II. matter & energy content of Universe today
  - exotic particles: *dark matter* (22%)
  - repulsive gravity: *dark energy* (74%)
  - ordinary matter: only 4%!

- III. *general relativity*: Einstein
  - describe evolution of matter & energy

Standard Model: Inflation

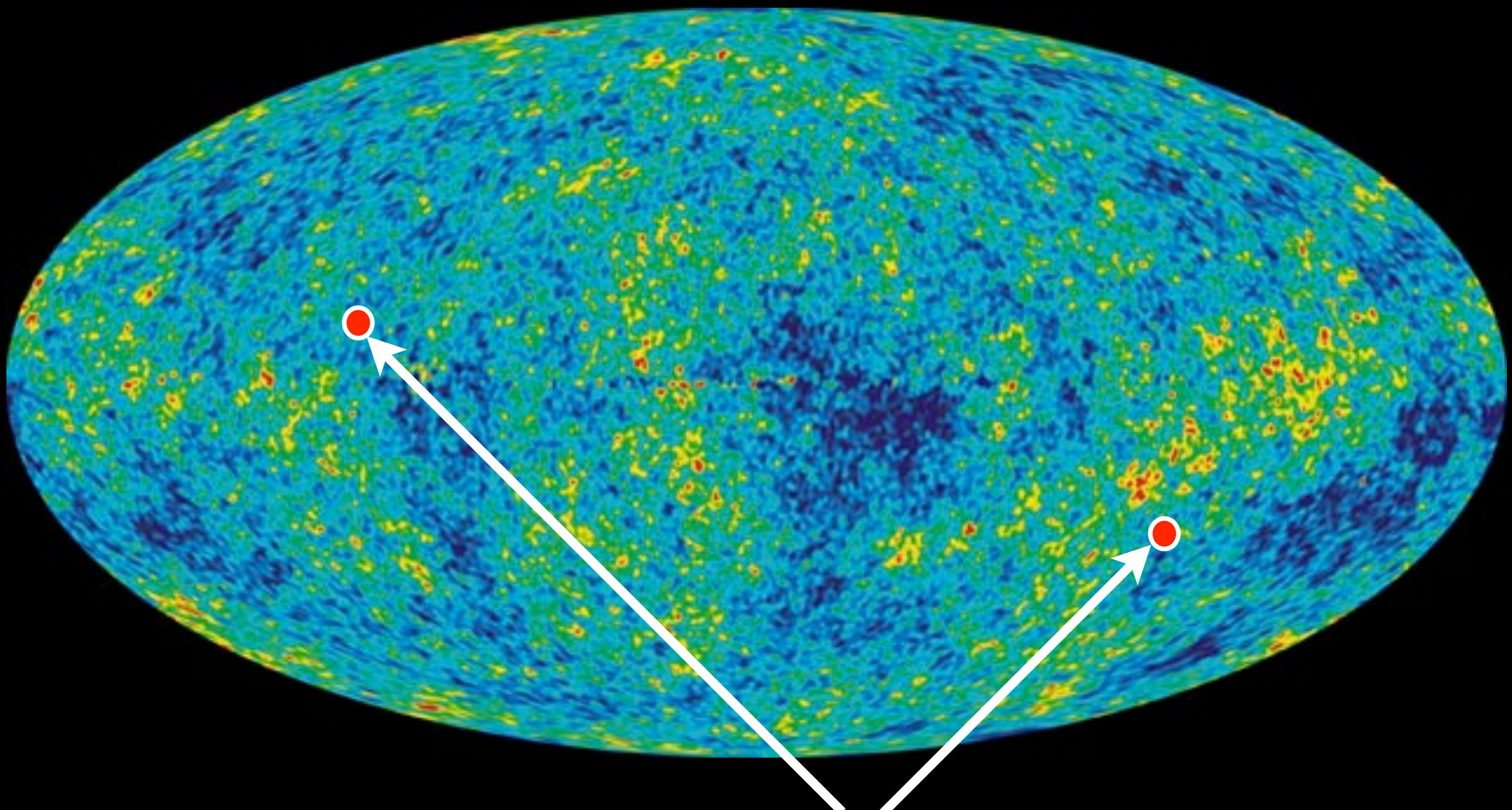
- ***initial perturbation generation*** **in early Universe**
- **seed for *life* from *quantum fluctuations***



exponential expansion in space

Horizon Problem

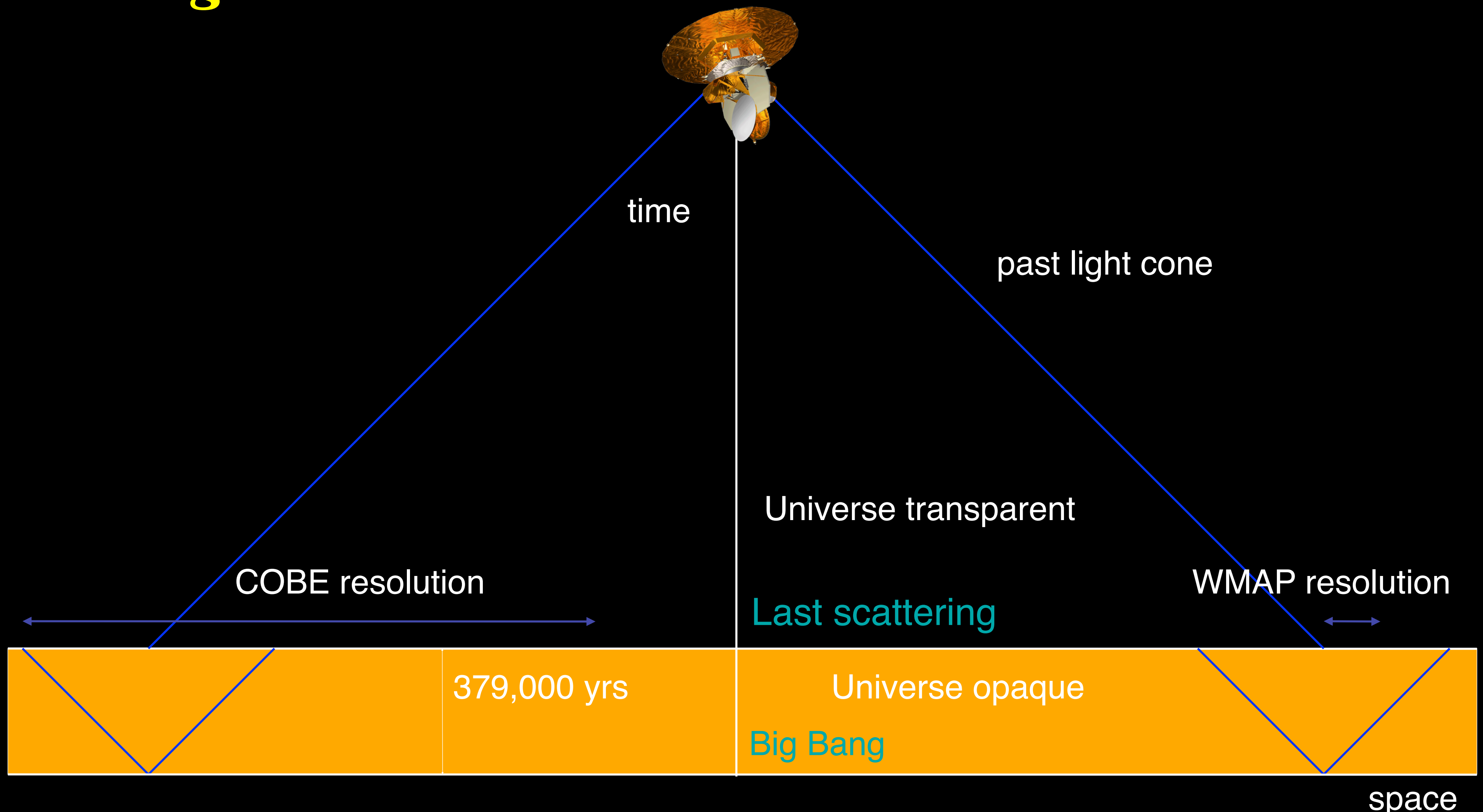
- *same temperature*: two patches of sky
- *not enough time* to communicate with each other



temperature ~ 3K up to 1/100,000

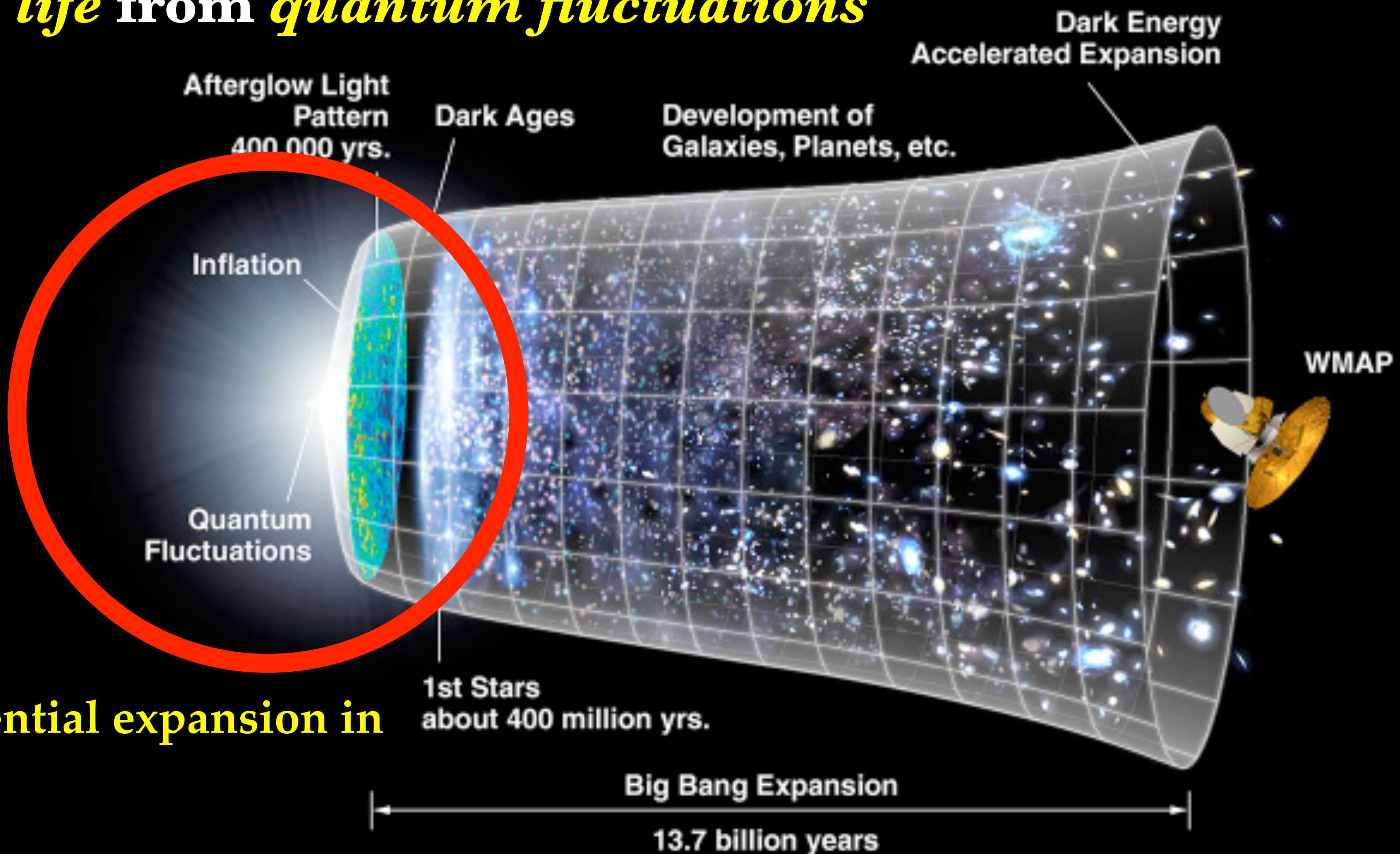
Horizon Problem

- ~ 400,000 years old, ~ 10 billion light years across
- *not enough time* to communicate with each other



Inflation: Horizon Problem

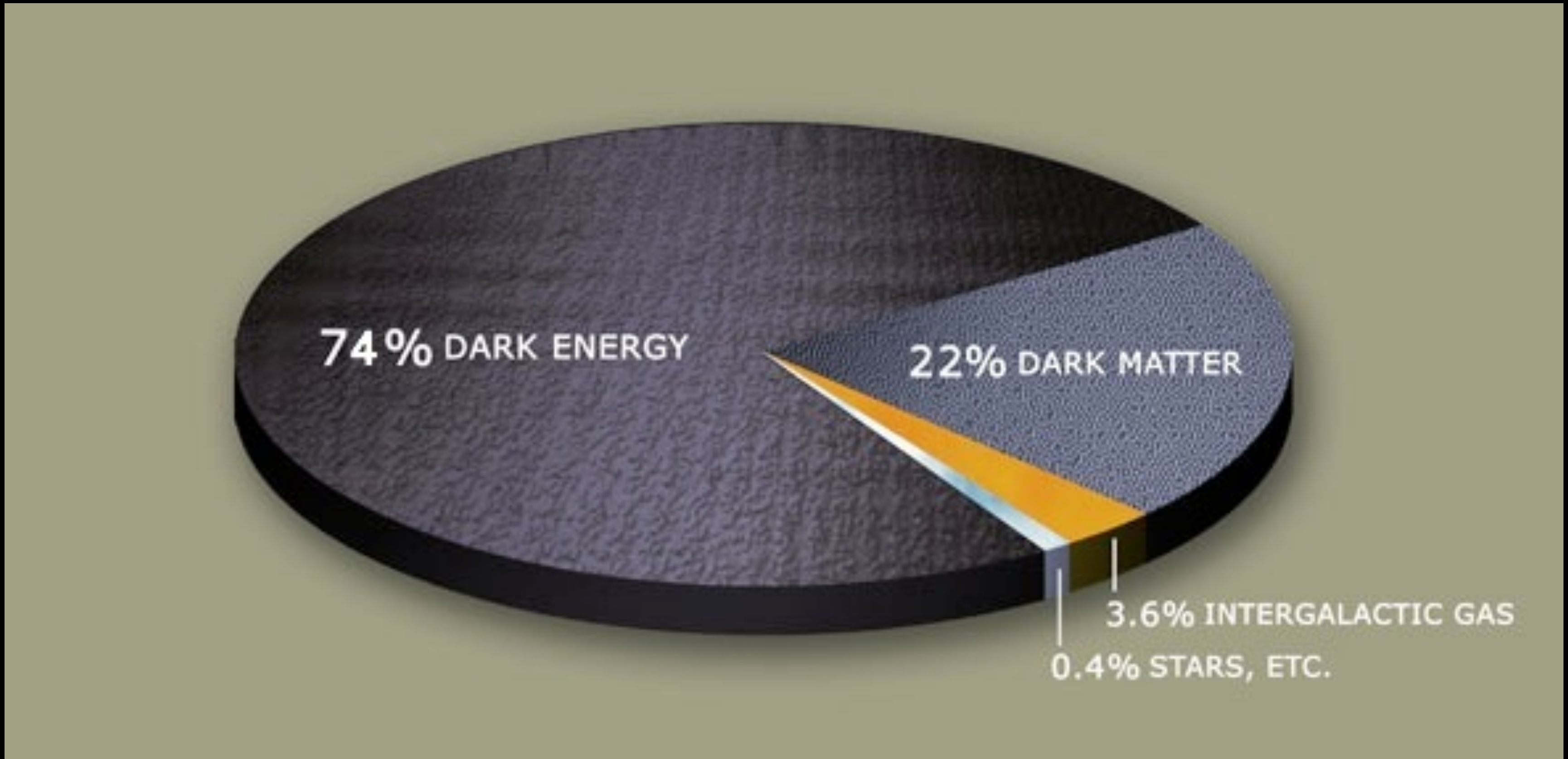
- *solve* the Horizon problem
- seed for *life* from *quantum fluctuations*



exponential expansion in space

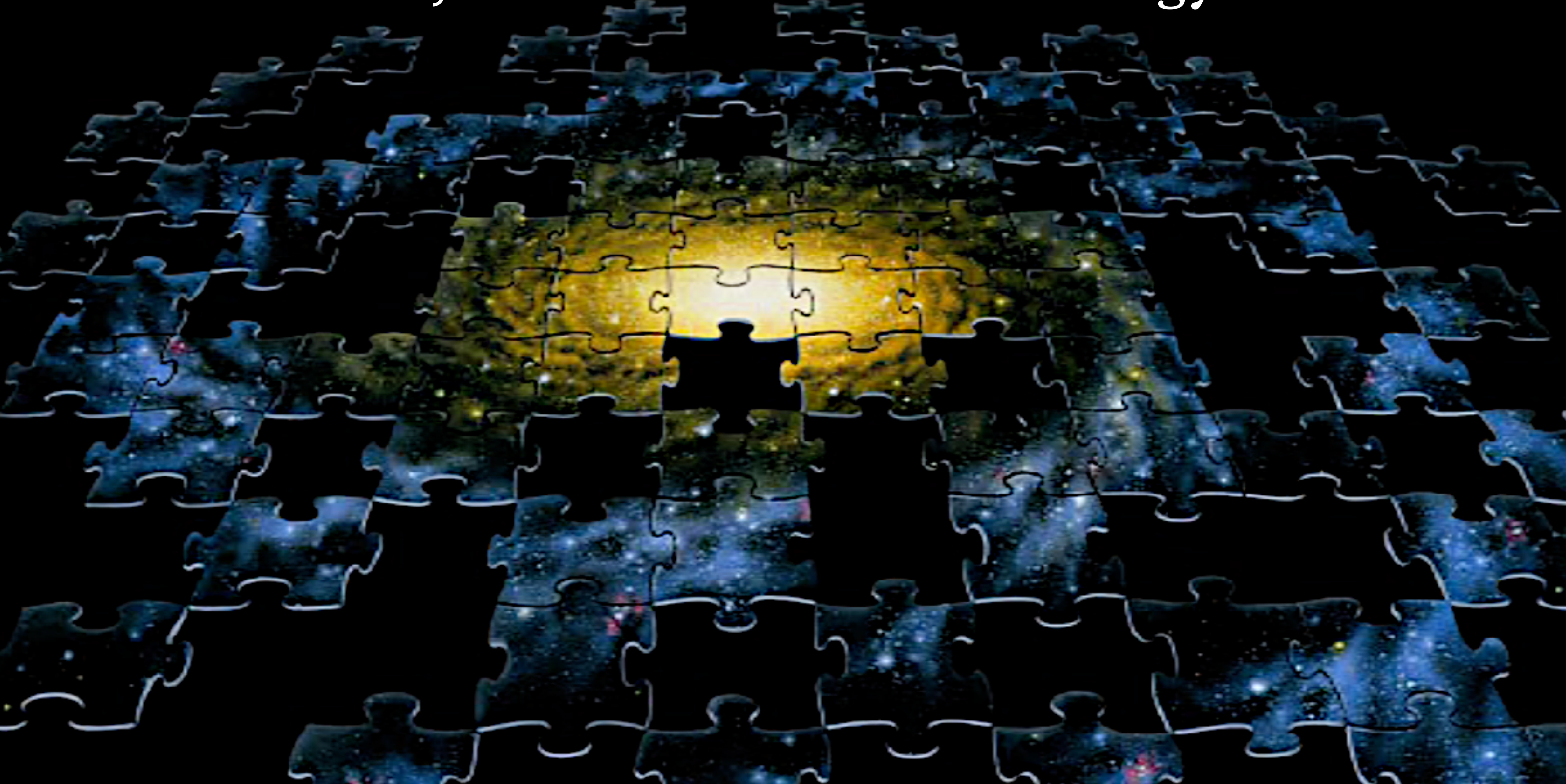
Standard Model: Dark Sector

- matter & energy content of Universe **today**
  - exotic particles: *dark matter* (22%)
  - repulsive gravity: *dark energy* (74%)
  - ordinary matter: **only 4%!**

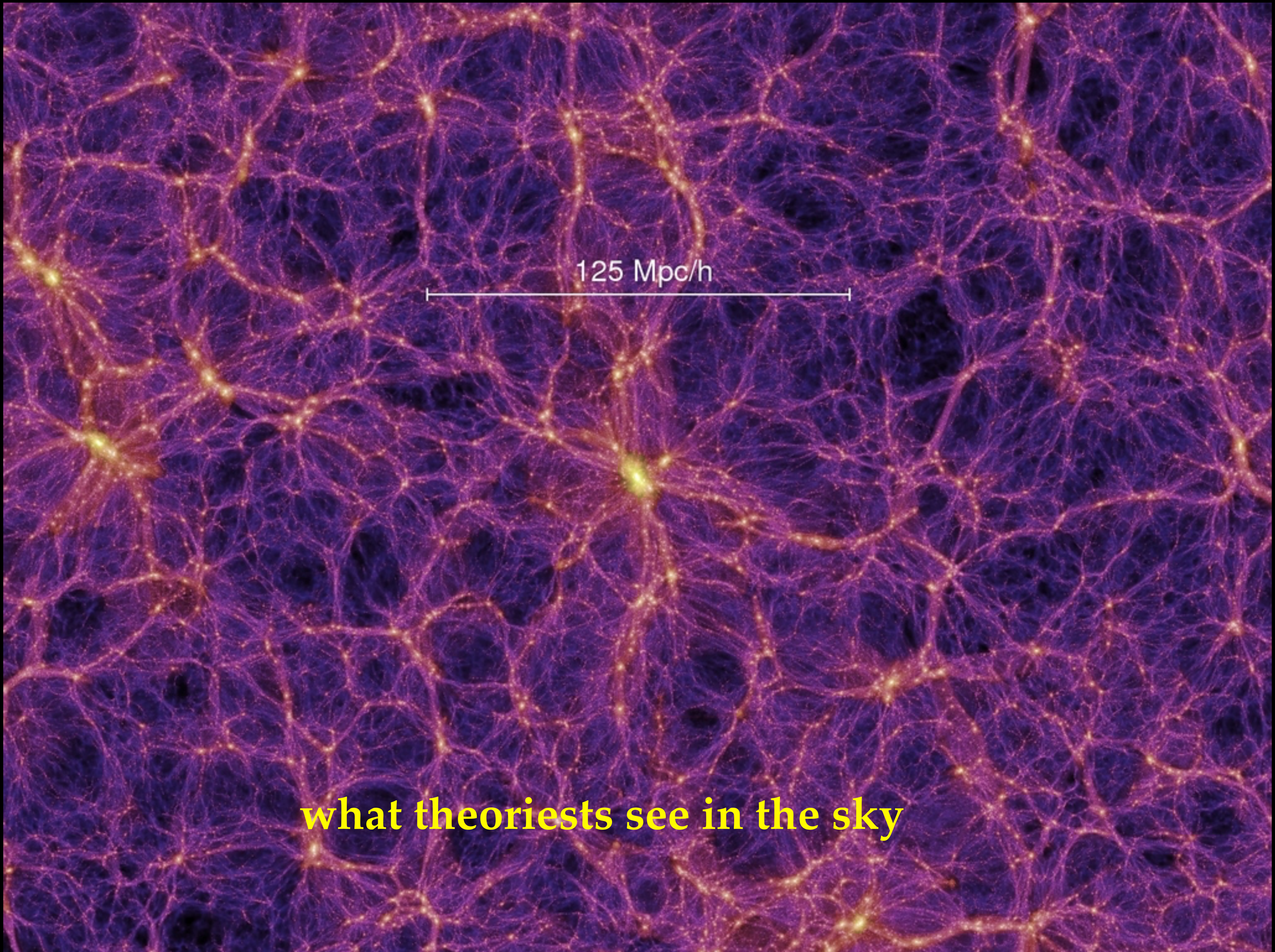


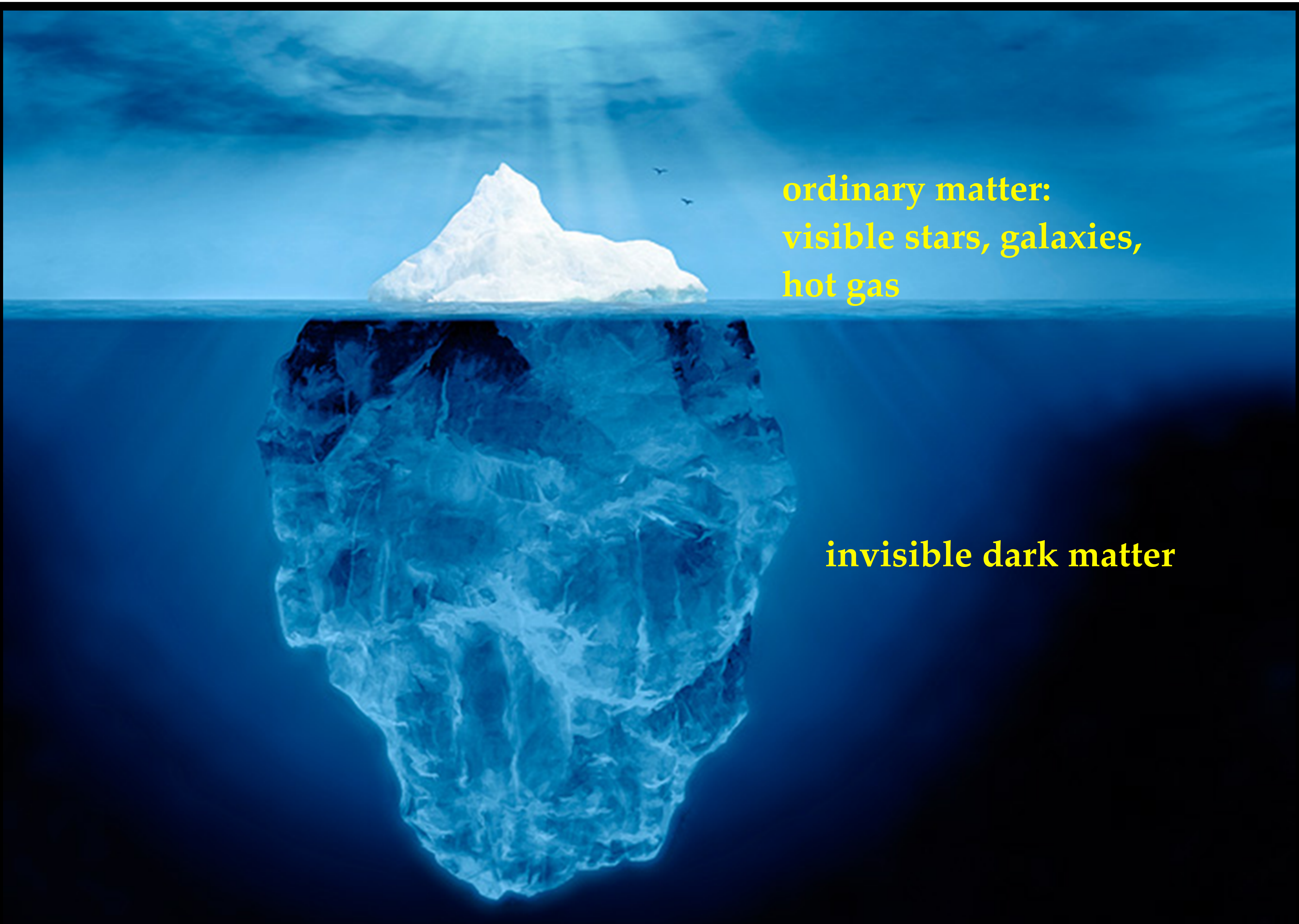
Dark Matter

- *invisible* matter, exotic particles
- 85% of matter, 22% of total matter & energy




what observers see in the sky


125 Mpc/h
what theoriests see in the sky


ordinary matter:
visible stars, galaxies,
hot gas
invisible dark matter

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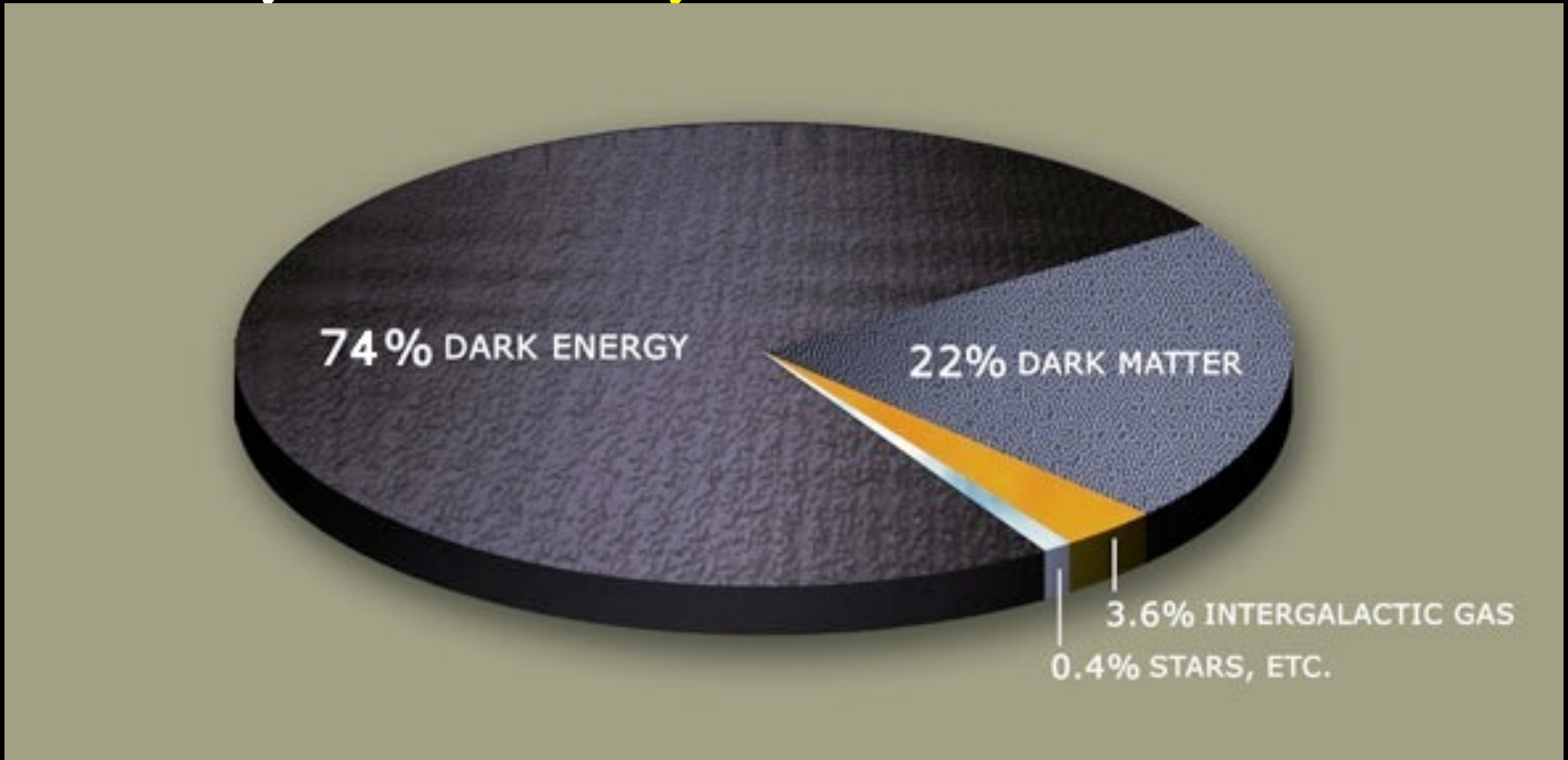

Dark Energy

- *repulsive gravity*, negative pressure
- accelerating expansion of Universe today

Dark Energy

- *repulsive gravity*, negative pressure
- accelerating expansion of Universe today

- cosmological constant? it should be infinite from loop corrections
- extra scalar field? why now? we have not seen it
- modification of gravity? do we even have a consistent theory of modified gravity?

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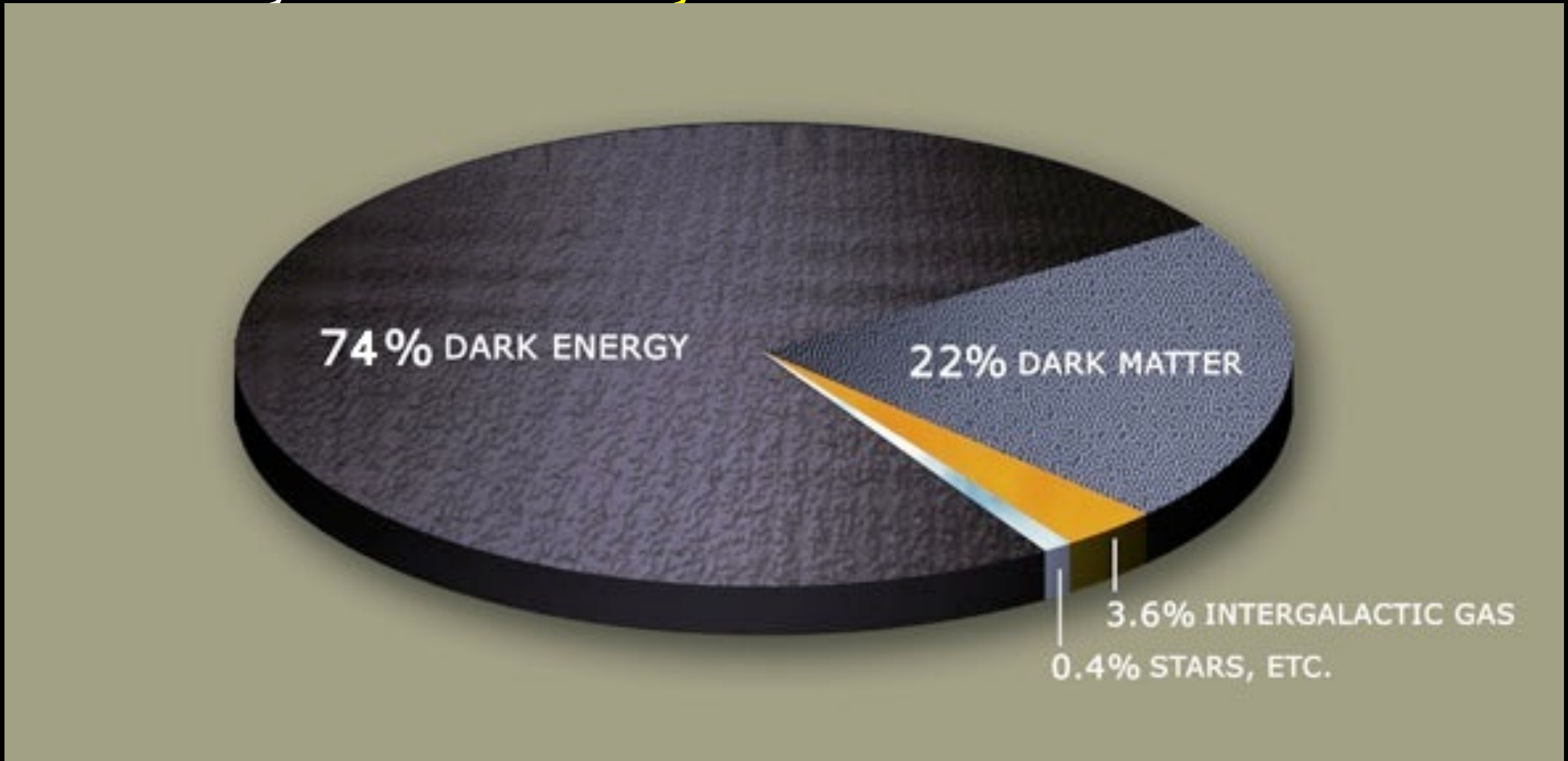
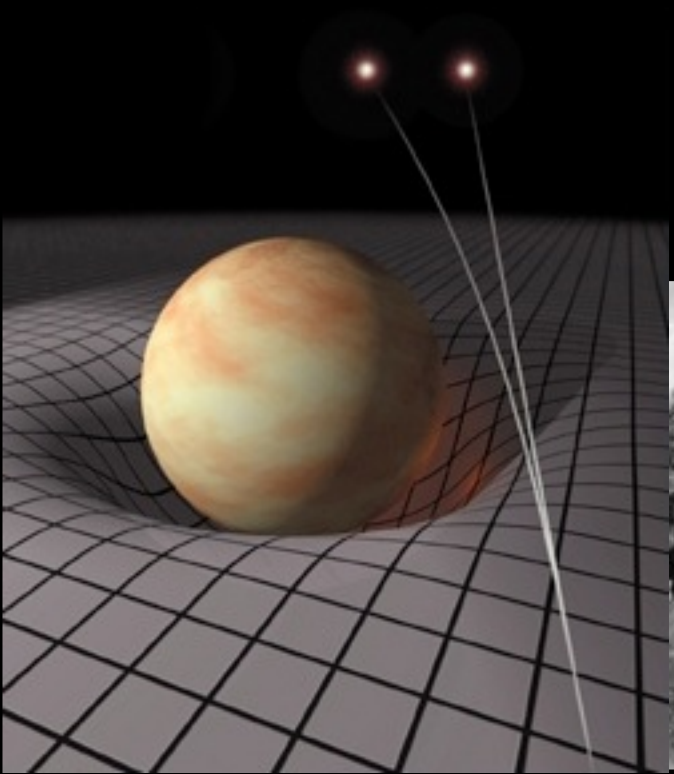


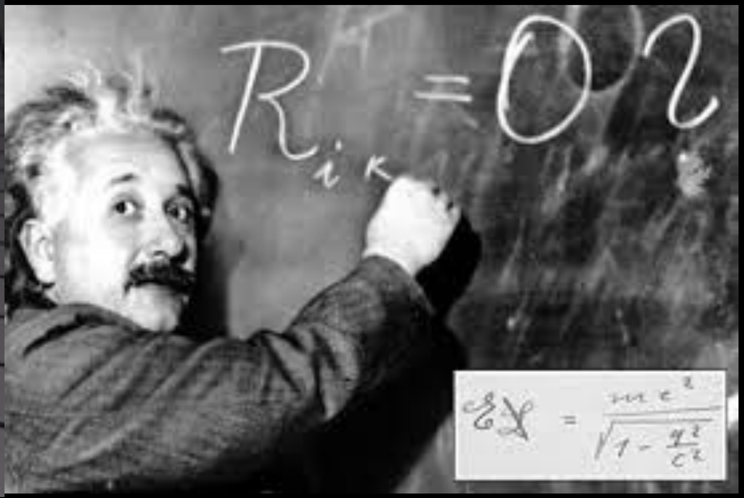
Standard Model: Einstein Gravity

- ***general relativity*: Albert Einstein**
  - **describe evolution of matter & energy**
  - ***from micrometer to hundred billion light years***

spacetime tells matter how to move;

matter tells space-time how to curve





Einstein equation

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = 8\pi G T_{\mu\nu}$$

Rich History at Zürich

- ***PhD*** **in 1905 at University of Zürich**
- ***Professor*** **at Institute for Theoretical Physics, Zürich**

UNTER DER OBERHOHEIT DER BEHORDEN UND DES VOLKES DES KANTONS

UND IM NAMEN DER

UNIVERSITÄT ZÜRICH

HAT DIE

MATHEMATISCH-NATURWISSENSCHAFTLICHE SEKTION

DER

PHILOSOPHISCHEN FAKULTÄT

IN IHRER SITZUNG VOM 27. JULI 1905

DEM HERRN

ALBERT EINSTEIN

VON ZÜRICH

AUF GRUND SEINER DISSERTATION, BETITELT:

„EINE NEUE BESTIMMUNG DER MOLEKÜLDIMENSIONEN"

UND DER VORSCHRIFTSMÄSSIGEN PRÜFUNGSAUSWEISE

DIE RECHTE UND WÜRDEN

EINES

DOCTOR DER PHILOSOPHIE

VERLIEHEN

UND STELLT ZUM ZEUGNIS DESSEN DIESE MIT DEM UNIVERSITÄTSSTEMPEL VERSEHENE URKUNDE AUS

GEGEBEN IN ZÜRICH

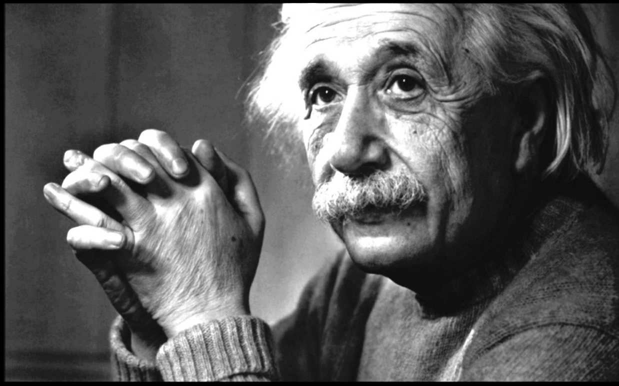
15. JANUAR 1906

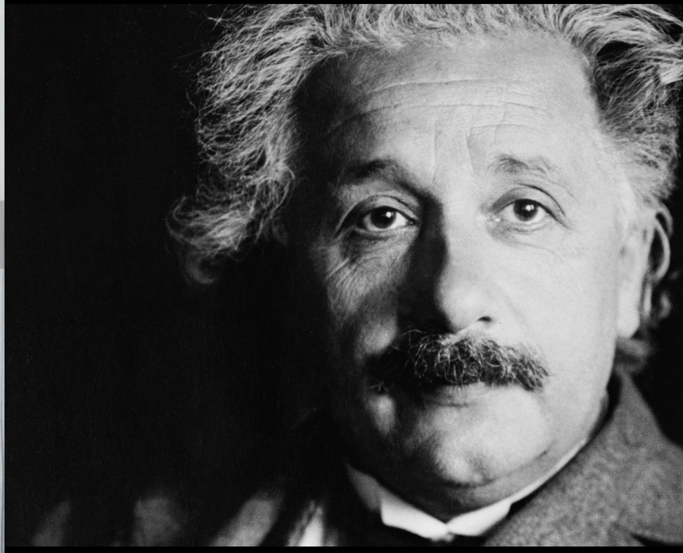
FÜR DEN AKADEMISCHEN SENAT

DER REKTOR:

FÜR DIE II. SEKTION DER PHILOSOPHISCHEN FAKULTÄT

DER DEKAN:





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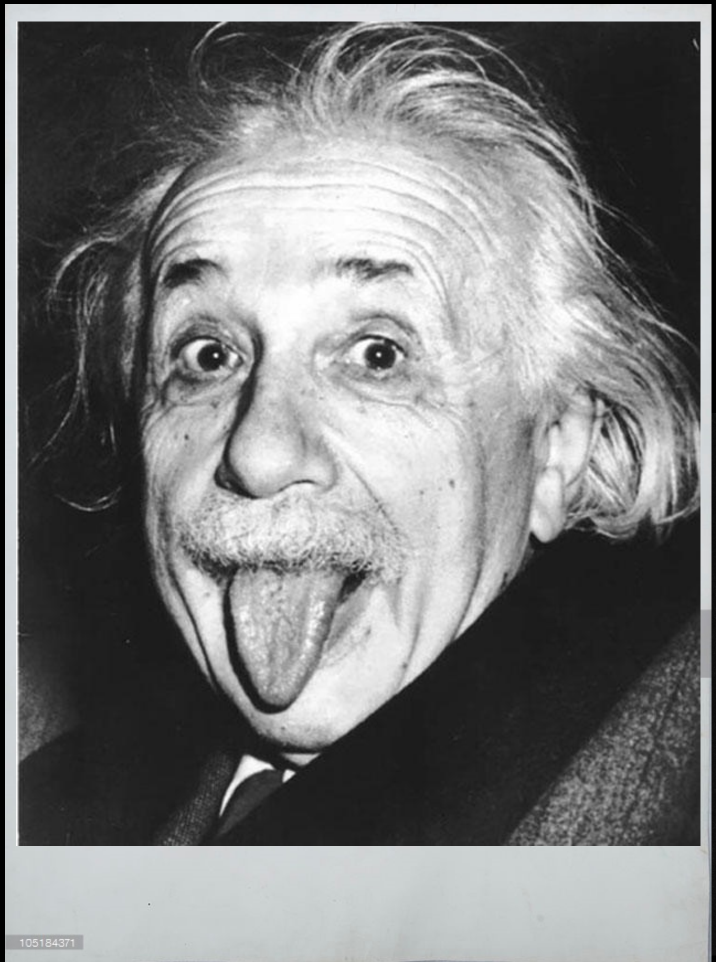

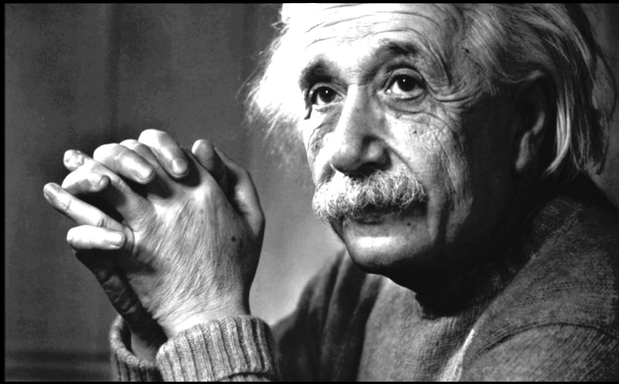

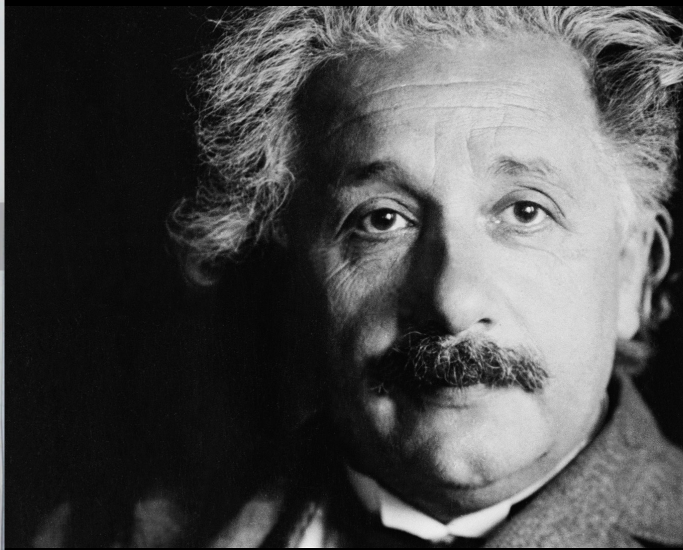


Standard Model of Cosmology
Inflation
Dark Sector
Relativity

Problems in Cosmology

- **I. inflationary epoch: *Not Understood***
  - **what generates initial perturbations?**
- **II. dark sector (96%): *Not Understood***
  - **dark matter (*22%*) and dark energy (*74%*)**
  - **what are the nature of dark sector?**
  - **ordinary matter (*4%*): *Understood, check!***
- **III. Einstein's general relativity: *Not Sure***
  - ***valid* on cosmological scales*?* modified gravity*?***
  - ***well tested* in Solar System**

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Problems in Cosmology

It is the *most compelling* of
all outstanding problems in physical science!

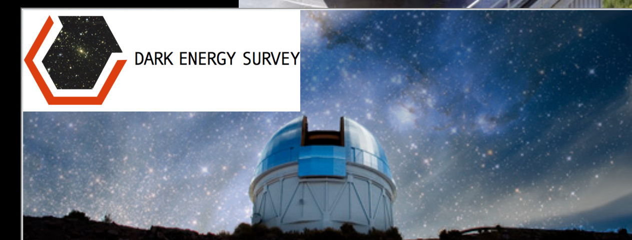
Dark Energy Task Force 2006

The Birth of the Universe: *one of the big issues for 21st century particle physicists*

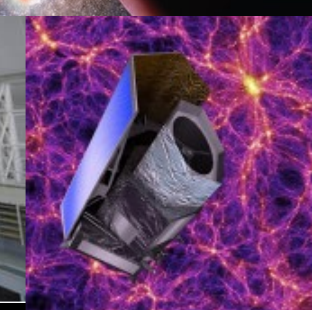
Quantum Universe Report 2010

Large-Scale Surveys

- current and future ground-based surveys:
  - **B**aryonic **O**scillation **S**pectroscopic **S**urvey
  - **D**ark **E**nergy **S**urvey
  - **D**ark **E**nergy **S**pectroscopic **I**nstrument
  - **L**arge **S**ynoptic **S**urvey **T**elescope



- future space missions:
  - **Euclid**
  - **W**ide-**F**ield **I**nfrared **S**urvey **T**elescope



- <u>sub-percent level</u>

precision measurements!

More Ambitious Surveys

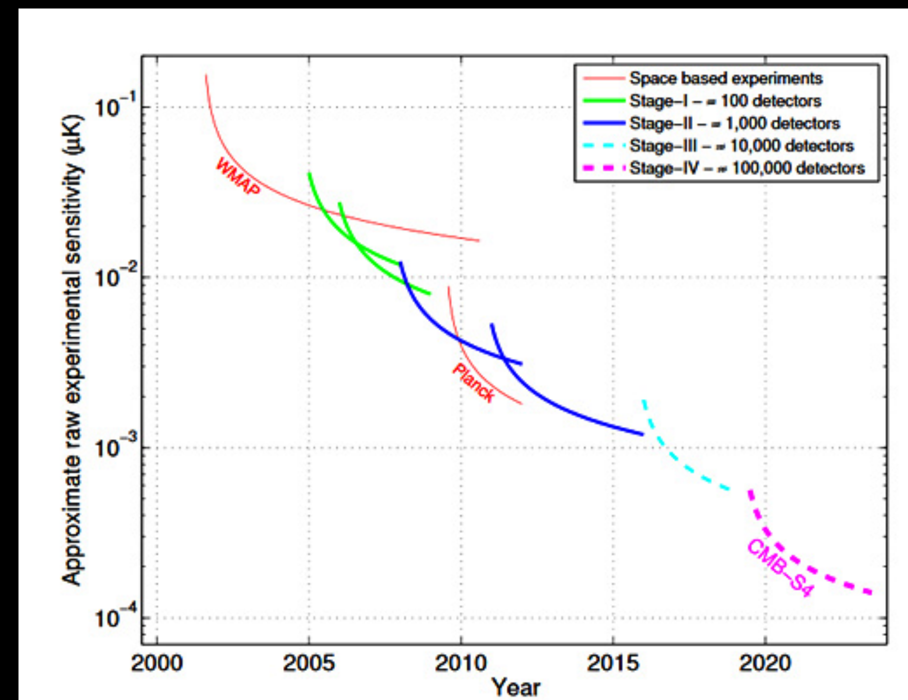
- future radio surveys:
  - Murchison Wide-field Array Phase-II
  - Square Kilometer Array

- redshifted 21cm lines:
  - from hyperfine transition in neutral hydrogen
  - probe redshift 10 ~ 30
  - *more statistical power than CMB*

Even more *precise* measurements!

CMB Stage IV (S4)

- next-generation CMB experiment:
  - dedicated telescopes
  - South Pole & Chile Atacama
  - and more telescopes?
  - inflation $r < 0.002$
  - neutrino mass $\sum m_\nu$
  - relativistic species $\sigma(N_{\rm eff}) = 0.02$



Challenges

- **precision measurements demand:**
  - ***substantial advances*** **in theoretical modeling of cosmological observables**
- **standard theoretical descriptions:**
  - **sufficiently accurate to describe precision measurements*?*** **answer:** ***No!***
  - **galaxy clustering, weak lensing, Boltzmann eq. etc**
  - **incomplete and limited to linear theory due to** ***gauge dependence*** **&** ***missing observer specification***

Research Program

- re-write theoretical descriptions of all cosmological observables:
  - in proper *relativistic framework*
  - check *gauge-invariance*
  - work out *impact of missing physics* on observables
  - relativistic effects as *novel probes* of cosmology
  - work in progress!

Research Program

- re-write theoretical descriptions of all cosmological observables:
    - in proper *relativistic framework*
    - check *gauge-invariance*
    - work out *impact of missing physics* on observables
    - relativistic effects as *novel probes* of cosmology
    - work in progress!

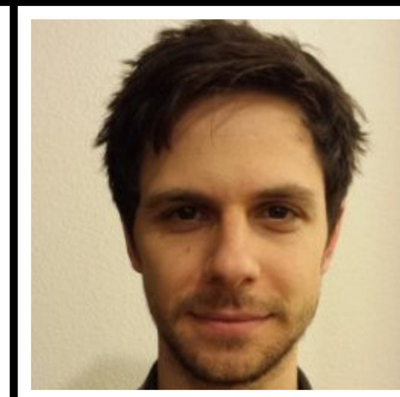
warning:

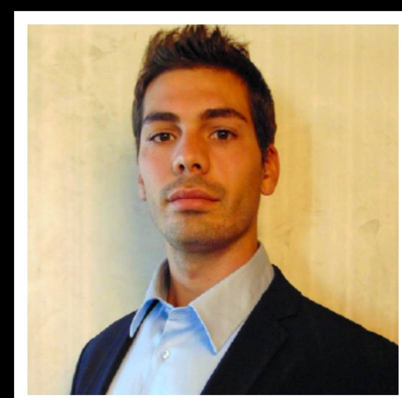
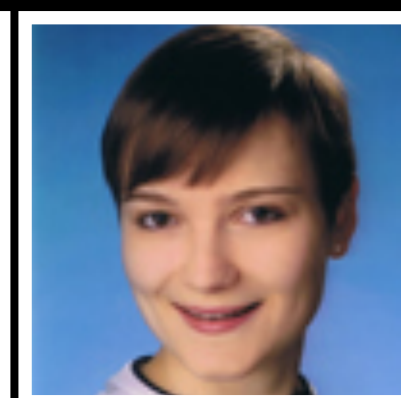
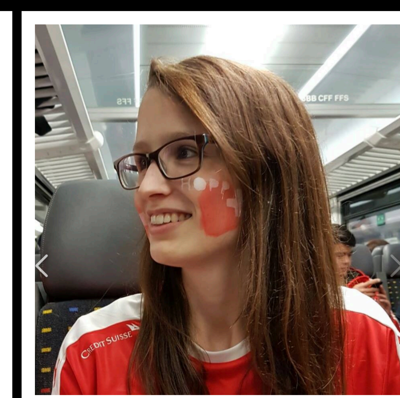
They work well!

BUT not quite so at the percent level or better

Team at Zürich

- **postdoctoral fellows**:
    - Yves Dirian
    - Ermis Mitsou
    - Enea Di Dio
- **PhD students**:
    - Fulvio Scaccabarozzi
    - Nastassia Grimm
    - Sandra Baumgartner


Universität Zürich UZH

CONTENTS

I. Cosmology: Past & Future

II. Improving Standard Cosmology

III. Impact of the Relativistic Effects

IV. Summary and Future Work

II. IMPROVING STANDARD COSMOLOGY

General Relativistic Effects

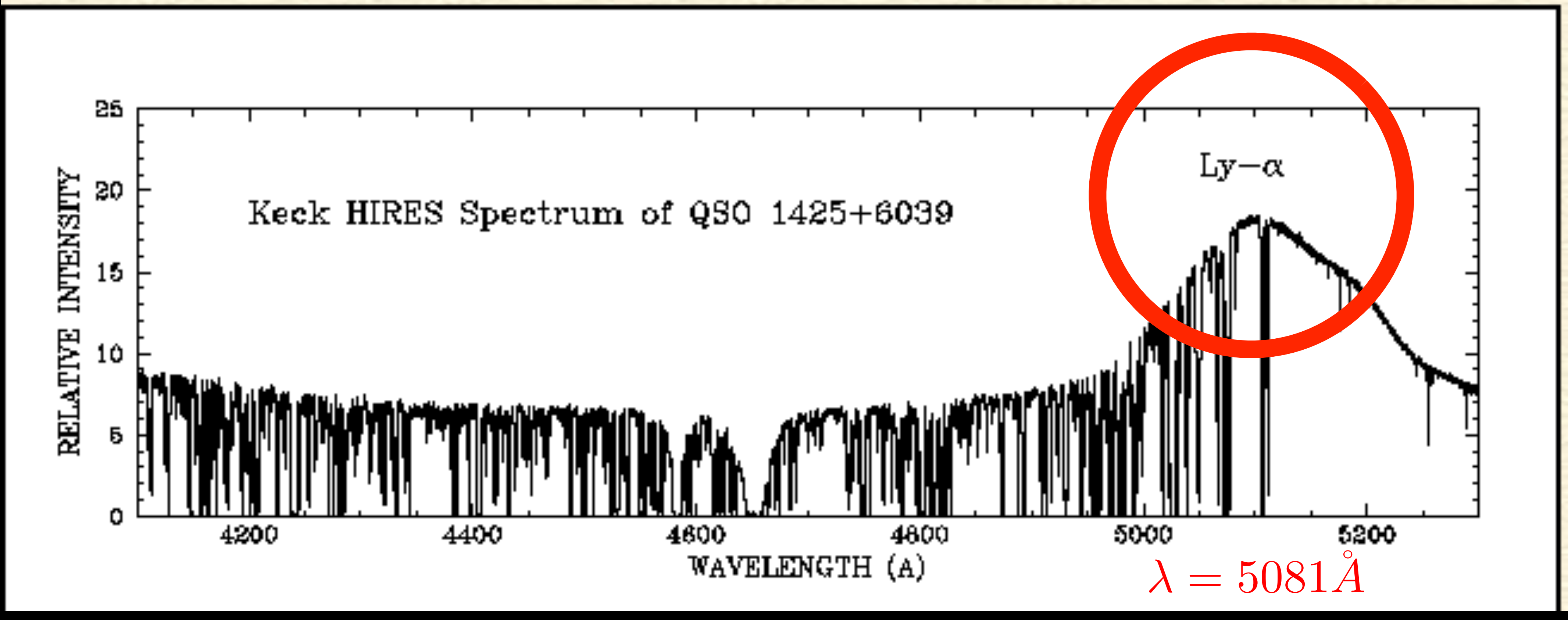
(a) Relativistic Effects

- all cosmological observations by measuring photons:
  - well known, but often *ignored!*
  - null geodesic for light path (vs instantaneous prop.)
  - light cone observation (vs same time volume)

- missing relativistic effects:
  - gravitational redshift, gravitational lensing
  - frame distortion, etc
- more subtle relativistic effects:
  - primordial non-Gaussianity, inflationary fossils
  - dark energy fluctuations, modified gravity, and so on

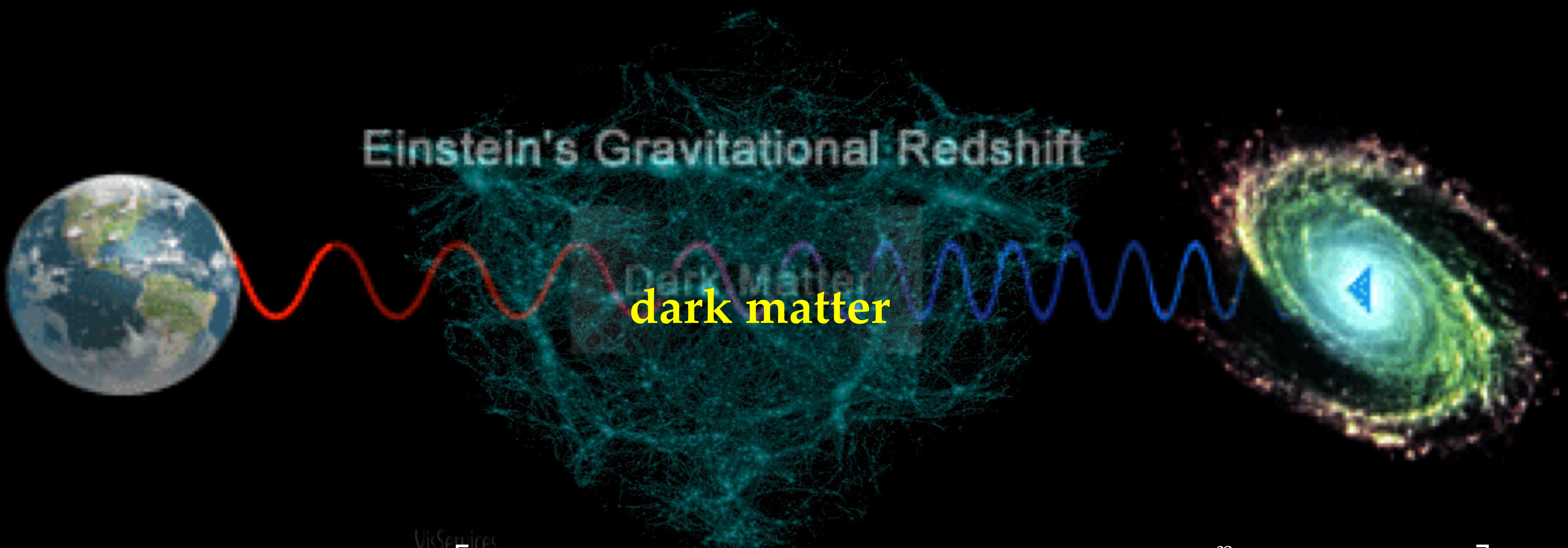
Observed Redshift

- *LyA* **emission from Quasar at** $\lambda = 5081\mathring{A}$
- **cosmological expansion & other relativistic effects**
- **observed redshift z=3.18 (= 5081/1216 —1)**



Gravitational Redshift

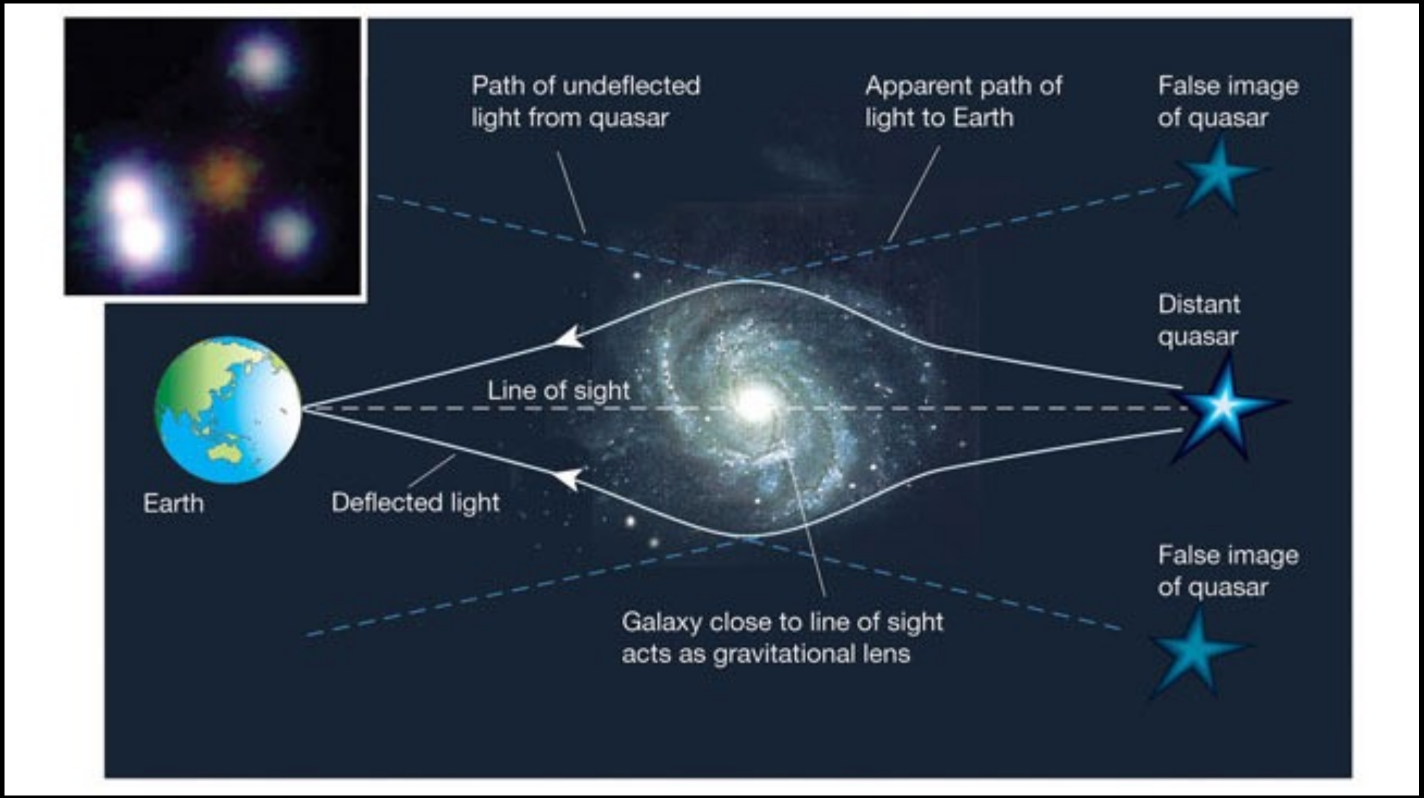
- **photon energy is affected**
- **due to gravity at source and observer (*Sachs-Wolfe*)**
- **also change in gravity during propagation (*iSW*)**

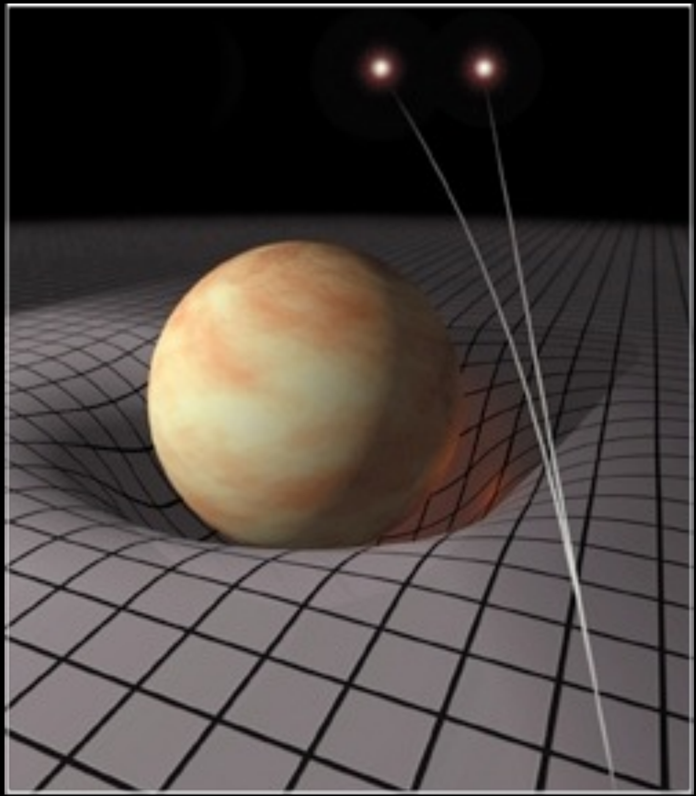


$$1 + z_{\rm obs} = (1+z)\left[1 + V(z) - V(0) - \psi(z) + \psi(0) - \int_0^r dr' \; (\dot{\psi} - \dot{\phi})\right] .$$

Gravitational Lensing

- *observed* angular position is *not* real position
- matter distribution *deflect* light propagation





$$\hat{n}_{\rm obs} = (\theta_{\rm obs},\ \phi_{\rm obs}) = \hat{n}_{\rm true} + \delta n, \qquad \delta n = (\delta\theta,\ \delta\phi)$$

Gravitational Lensing

- *observed* angular position is *not* real position
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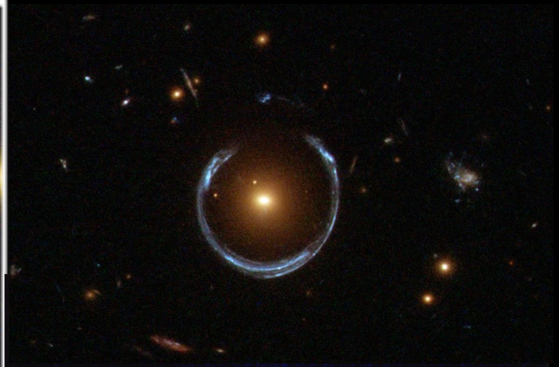
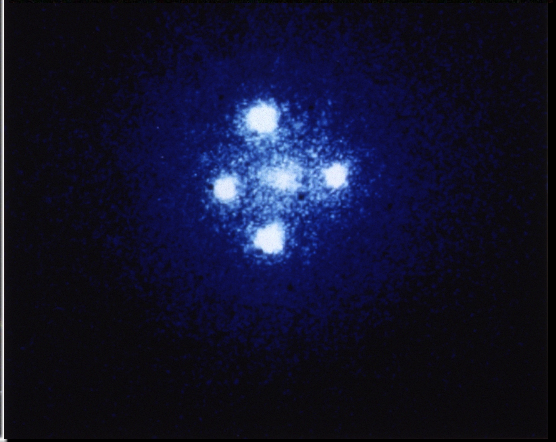
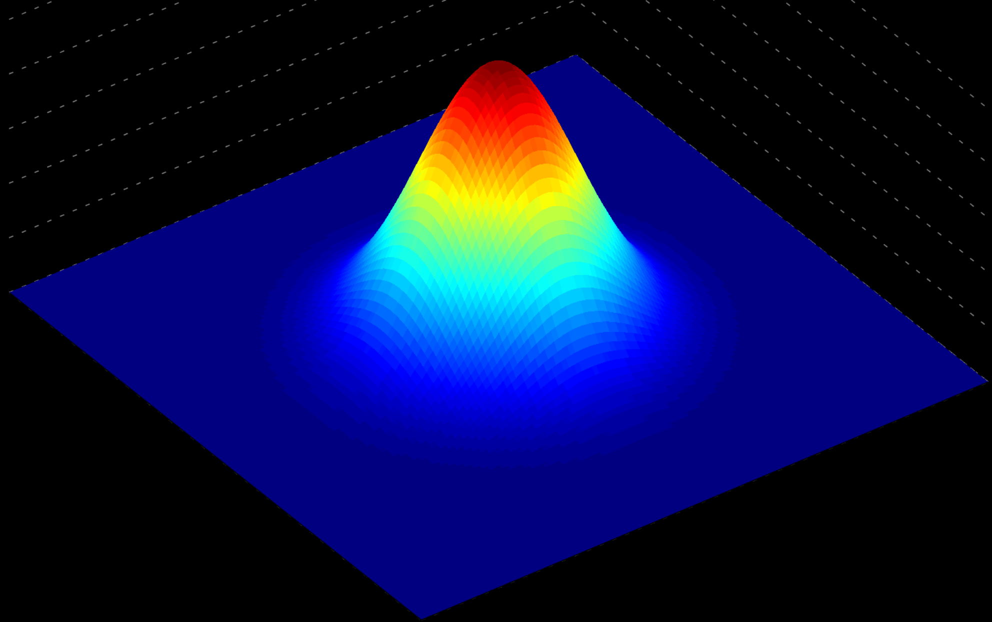


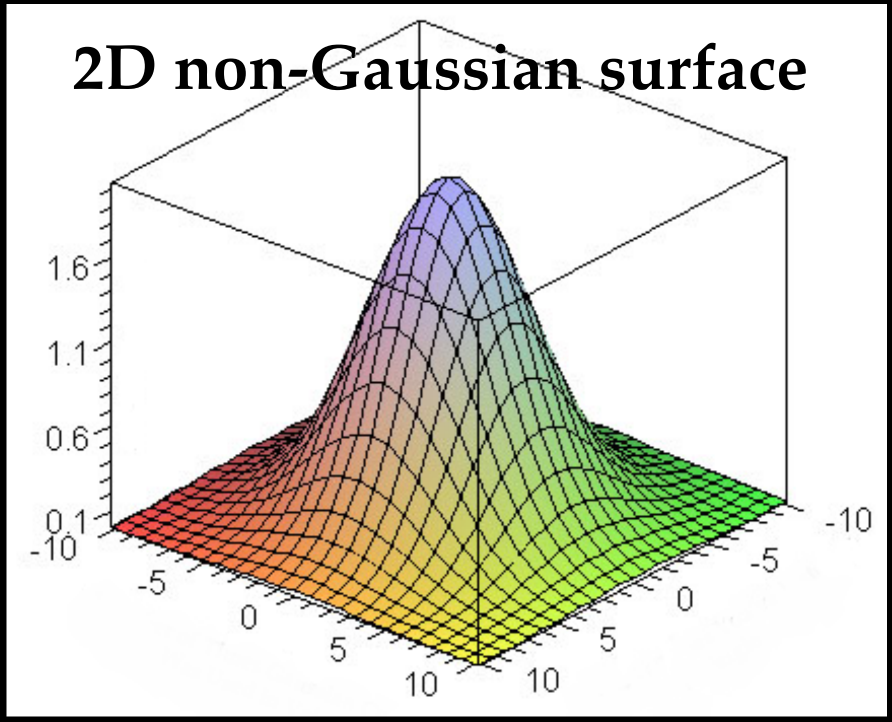


Primordial non-Gaussianity

- *deviation* in Gaussian fluctuations in early Universe
- **beyond** standard inflationary models
- status of early Universe

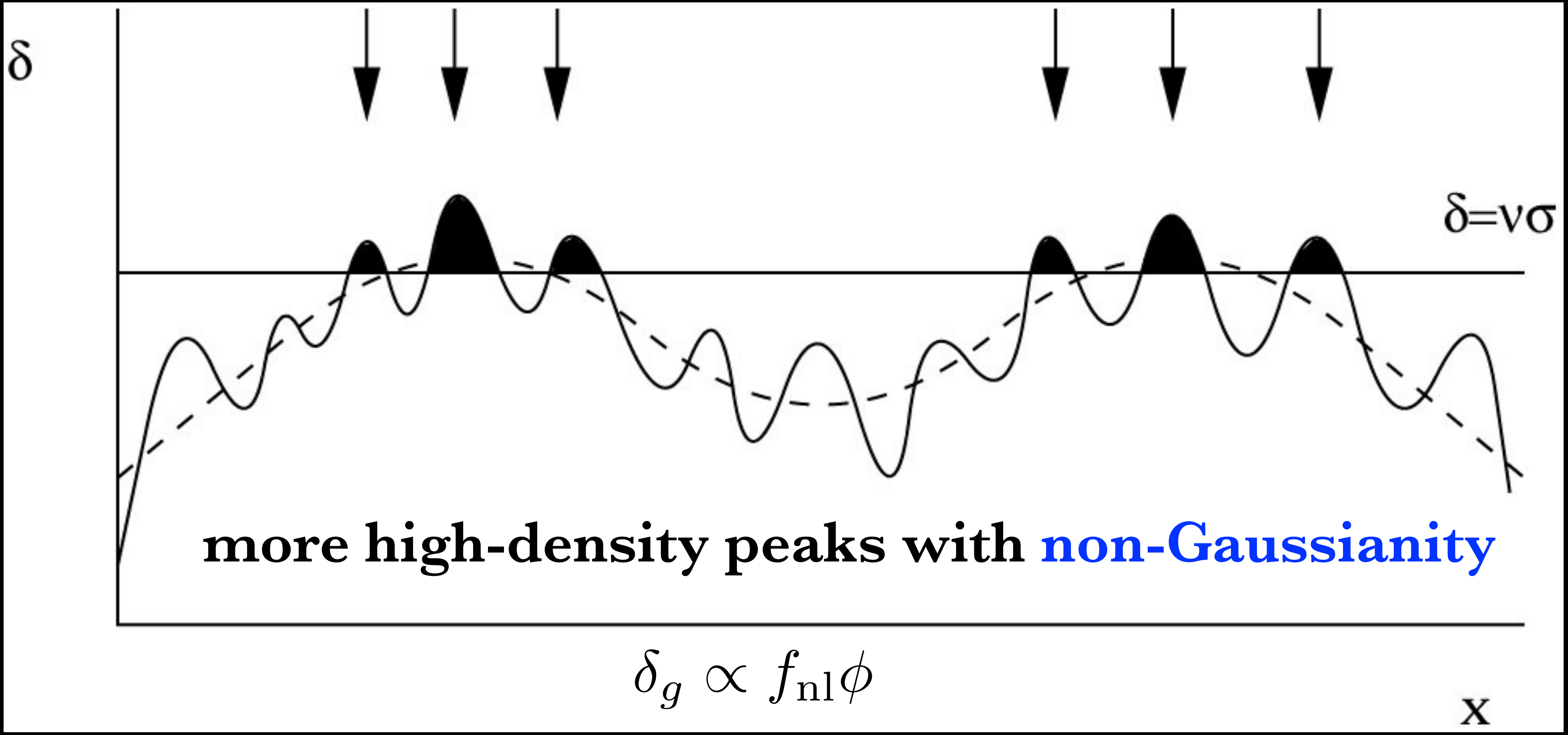
2D Gaussian surface



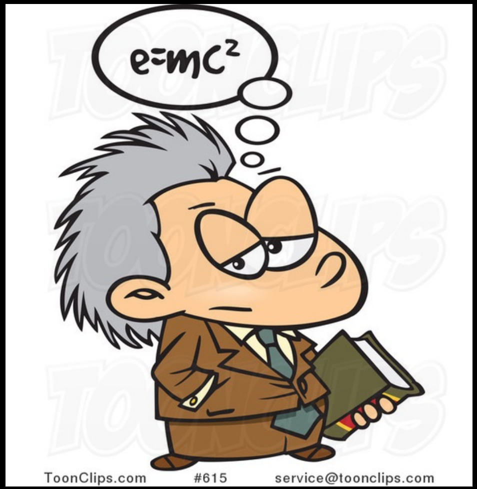


Primordial non-Gaussianity

- large-scale fluctuations *modulate* small-scale dynamics
- high density peaks: *sensitive probe!*

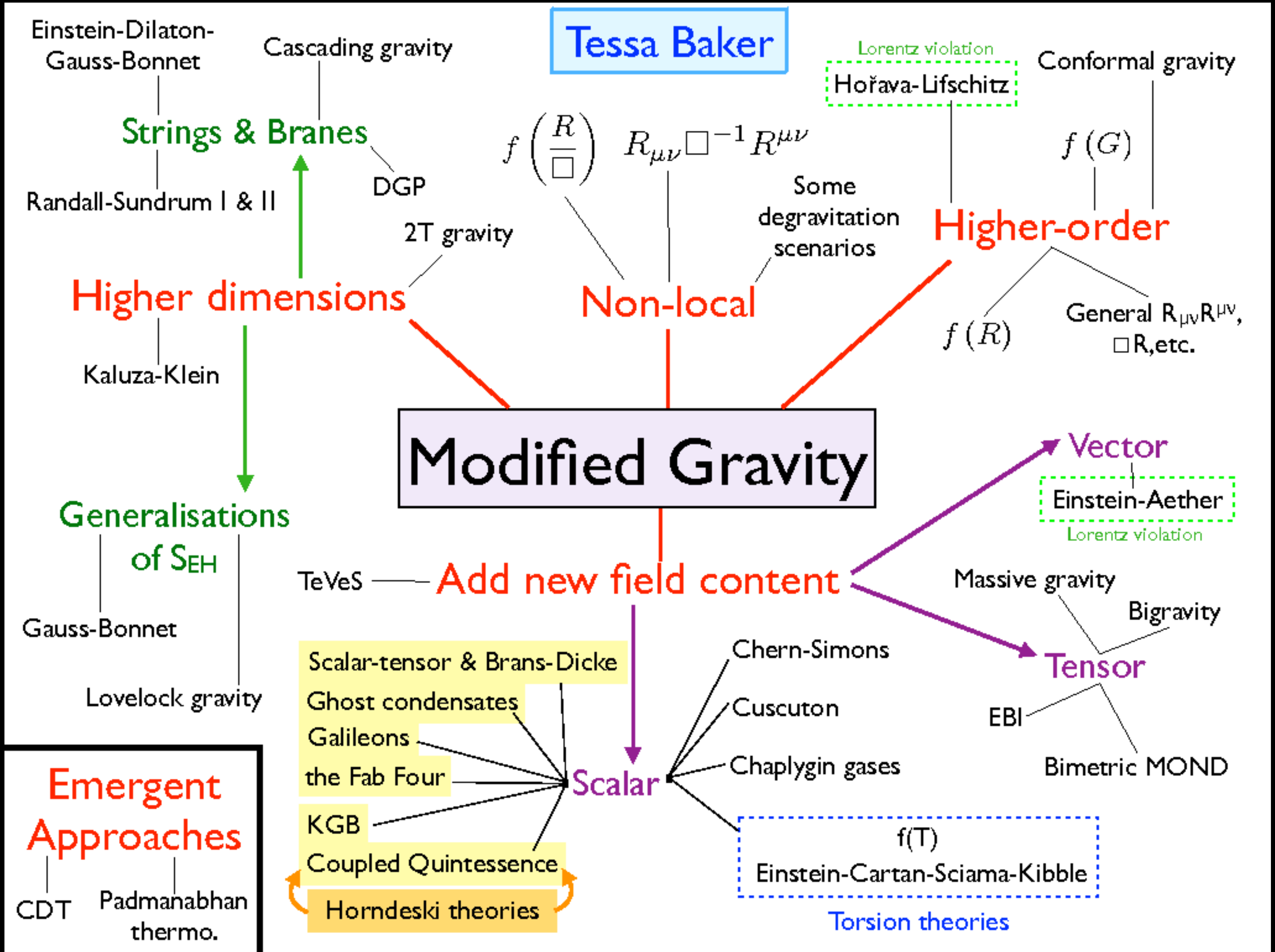


Modified Gravity

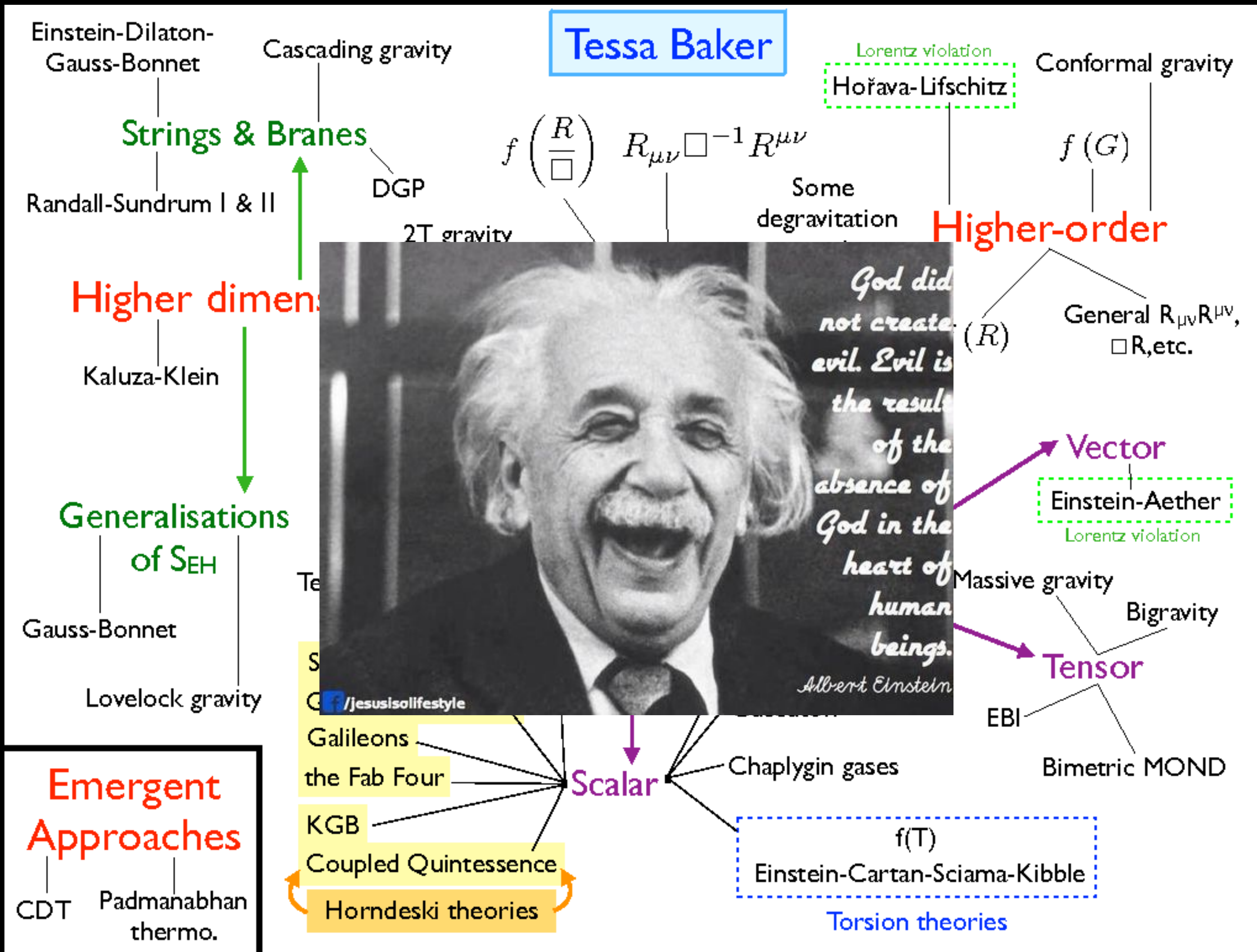


- motivation for different gravity theories:
  - *no* dark energy: explain it away!
  - why not? *smarter* than Einstein
- *main* character:
  - different from general relativity (infinite possibilities)
  - *highly constrained* by various tests & experiments
  - *no* convincing guiding principle
- *main* problems:
  - free parameters, no difference, incompleteness (ghost)

Modified Gravity Theories



Modified Gravity Theories



Significance

- relativistic effects are prevalent!
- encode *extra* and *critical* information:
  - initial condition of Universe
  - nature of gravity on large scales

key information!

- *Not* in Newtonian descriptions:
- need *proper relativistic descriptions!*

(b) Who Measures What?

- **cosmological observables:**
  - **photons: frequency, polarization, flux, position**
  - **derivables: redshift, shape, luminosity, number density, lensing shear, etc.**

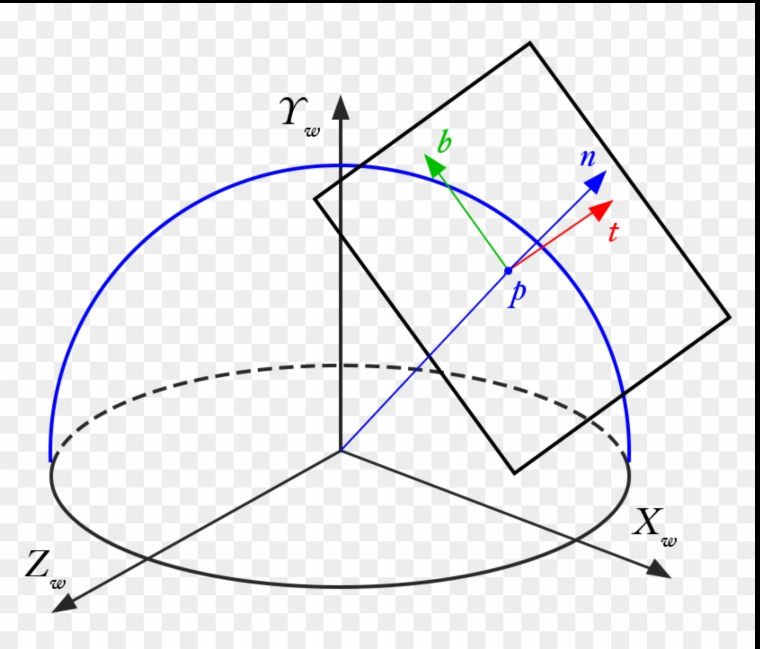
- ***observers* (*us*) in rest frame (Minkowski):**
  - **observer dependent, but so trivial, often ignored!**
  - **(FRW) coordinate independent! (diffeo. invariant)**
  - ***scalar* under diffeomorphism: FRW vectors, tensors, *all against our local basis***
  - **same for physical quantities in the *source* rest frame**

Symmetries

- general relativity (in cosmology):
  - diffeomorphism symmetry: any coordinates work
  - FRW metric with any gauge choice
  - cosmological observables: *gauge invariant*

Symmetries

- coordinates in observer rest frame (Minkowski):
  - (local) Lorentz symmetry (indep. of FRW coordinates) in the tangent space at observer position
  - boost is fixed, only rotational freedom
  - cosmological observables: *not invariant under Lorentz*
  - direct connection to *QFT* calculations



Tetrad Formalism

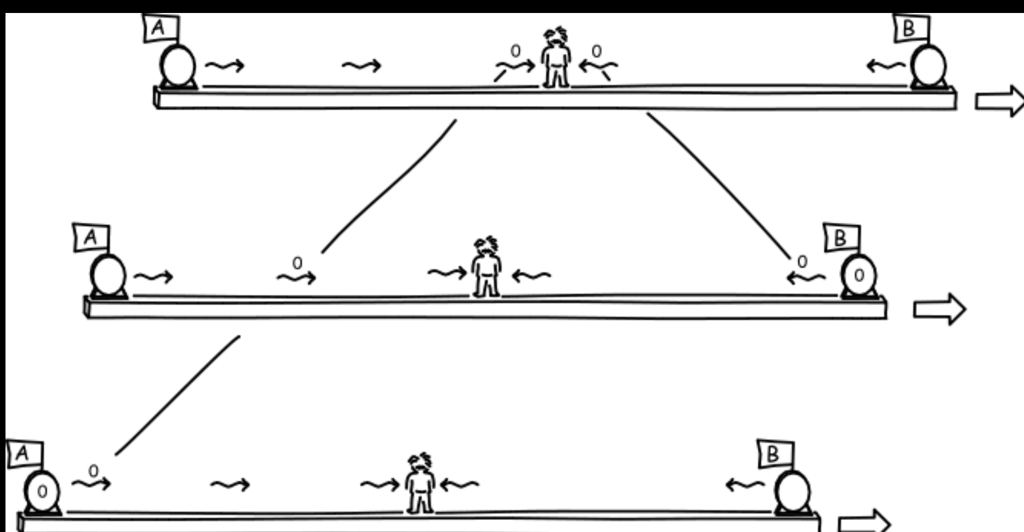
- tetrad fields: e_a^μ (four directional vector fields)
    - metric is fully contained in tetrad $g_{\mu\nu} = \eta_{ab} e_\mu^a e_\nu^b$
    - transparency: diffeo. & LLT (internal gauge symm.)
    - spinors: a representation of SL(2,C), not in diffeo.
    - already well developed in general relativity

- application to cosmology: natural generalization
    - not only at observer or src, but fields (*everywhere!*)
    - observer family: all possible observers everywhere
    - when projected, "*observables*" & *gauge invariant*
    - natural connection to QFT in Minkowski spacetime

Mitsou & Yoo 2019

Simultaneity

- <u>no absolute simultaneity</u>
- any choice of hypersurface is ok (*gauge freedom*)
- perturbations depend on *choice of hypersurface (gauge)*



simultaneity is relative!

hypersurface

Correspondence

- **perturbations:**
    - **inhomogeneous universe and spacetime**
    - **homogeneous *fictitious* background**

Correspondence

- perturbations:
  - inhomogeneous universe and spacetime
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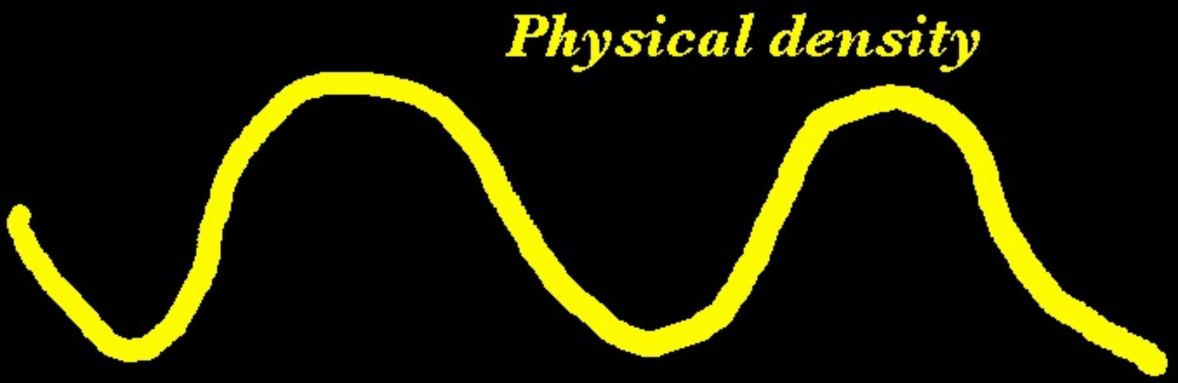

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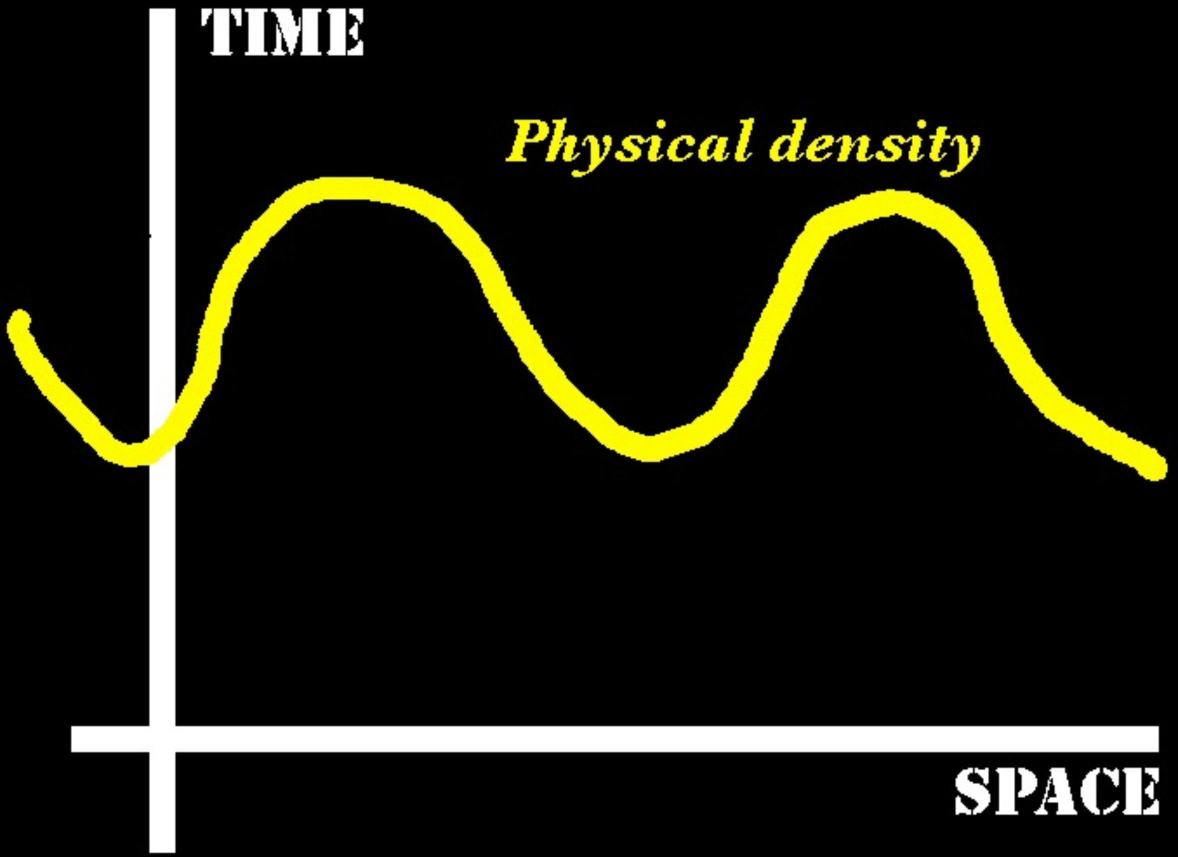

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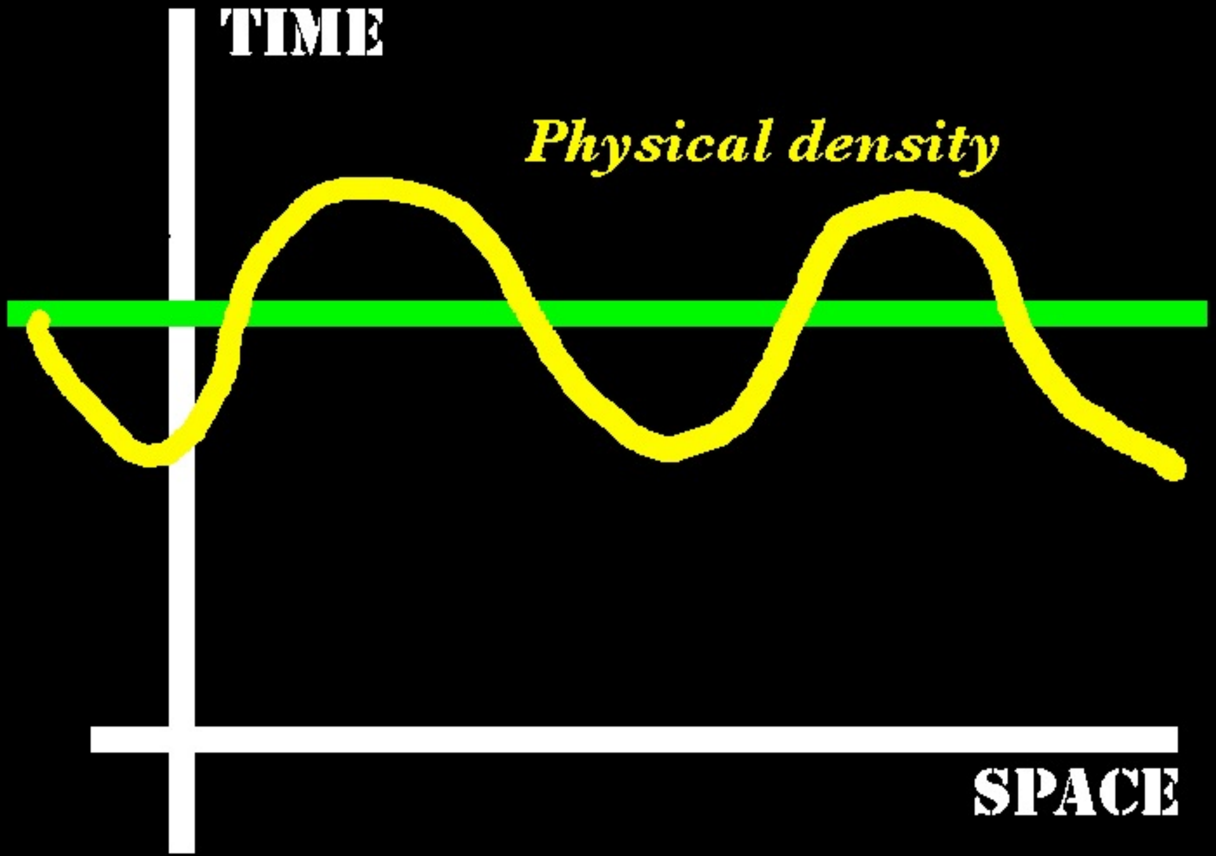

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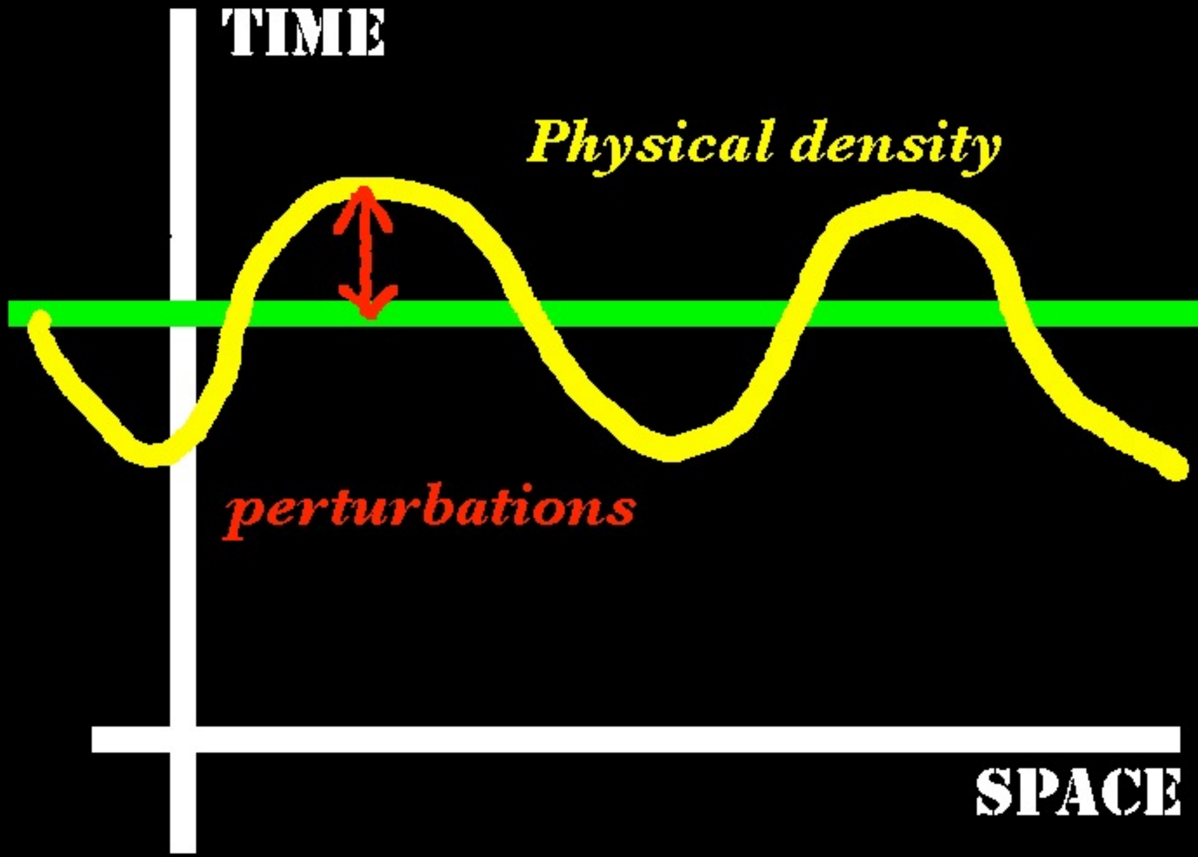
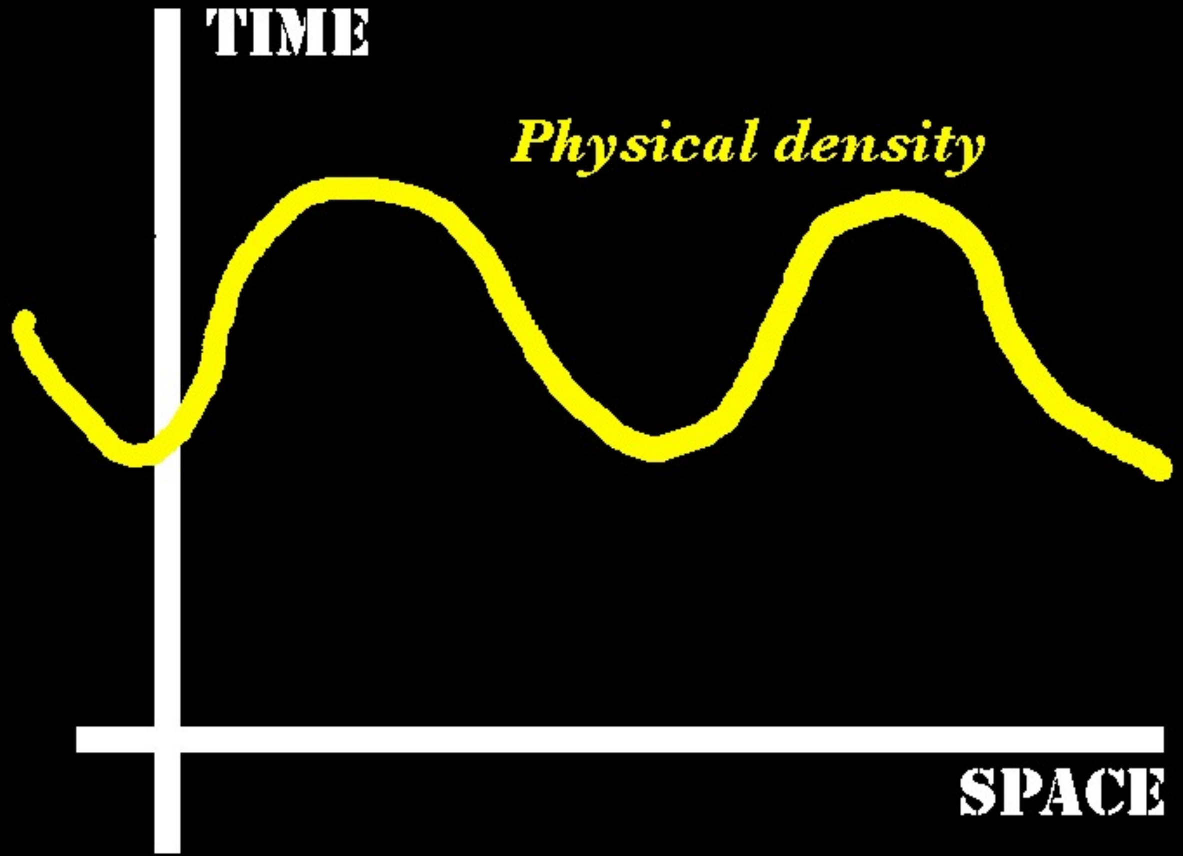


Gauge Freedom

- **general covariance** in GR:
  - free to choose a coordinate system
  - change in *correspondence* to background



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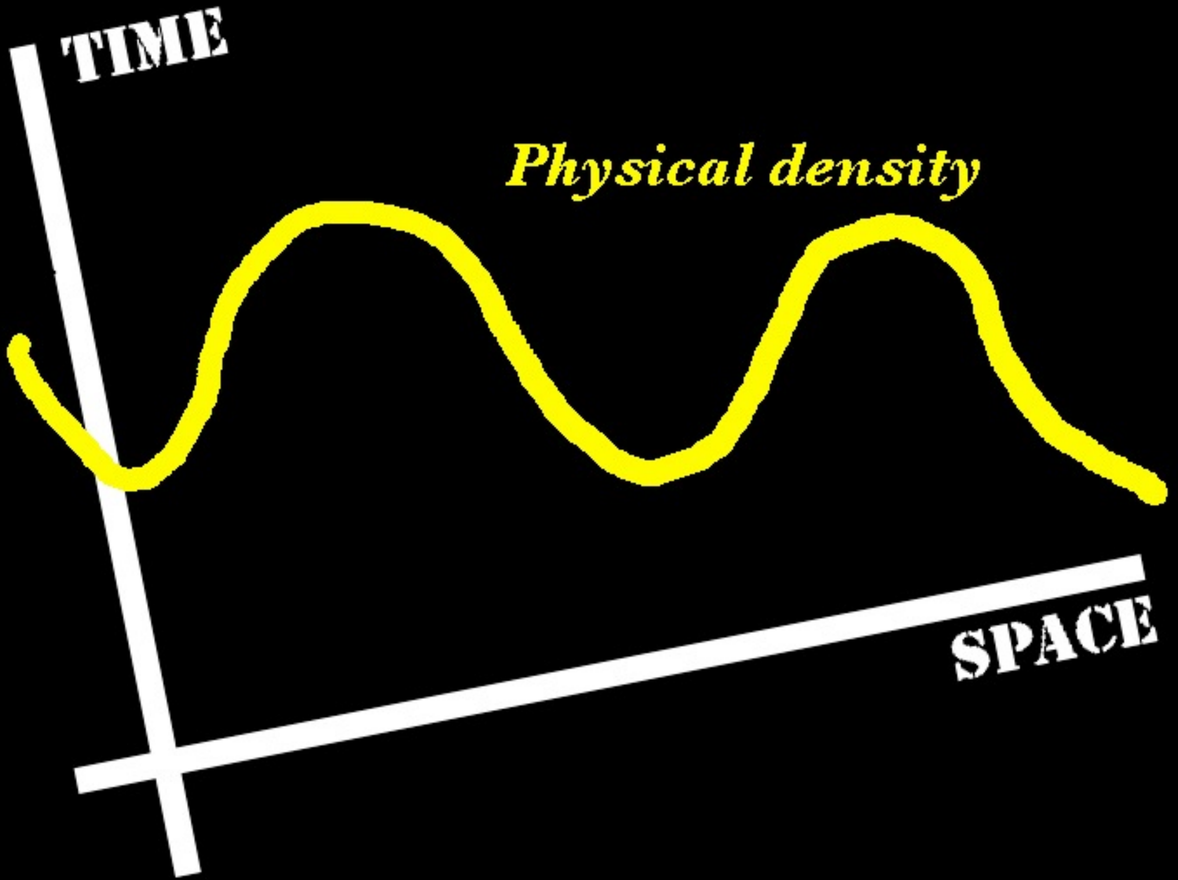

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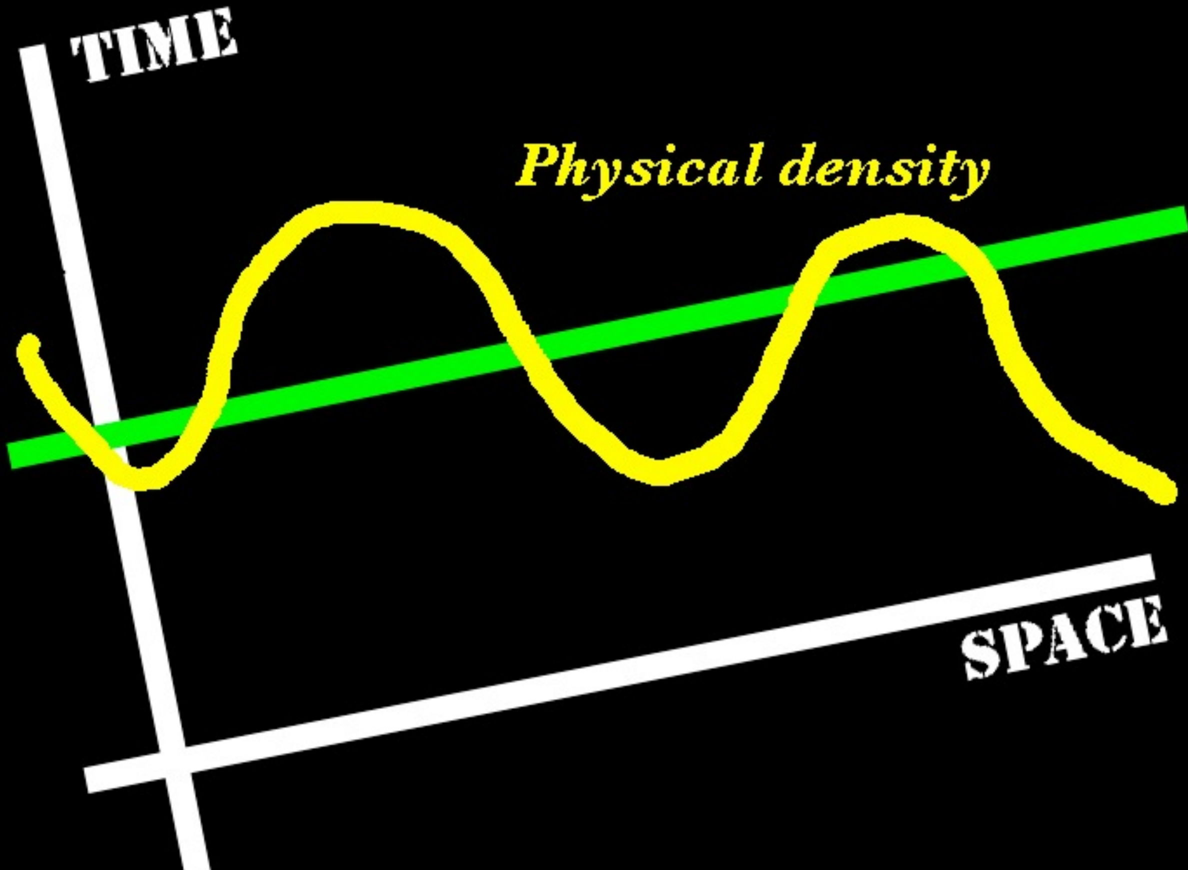

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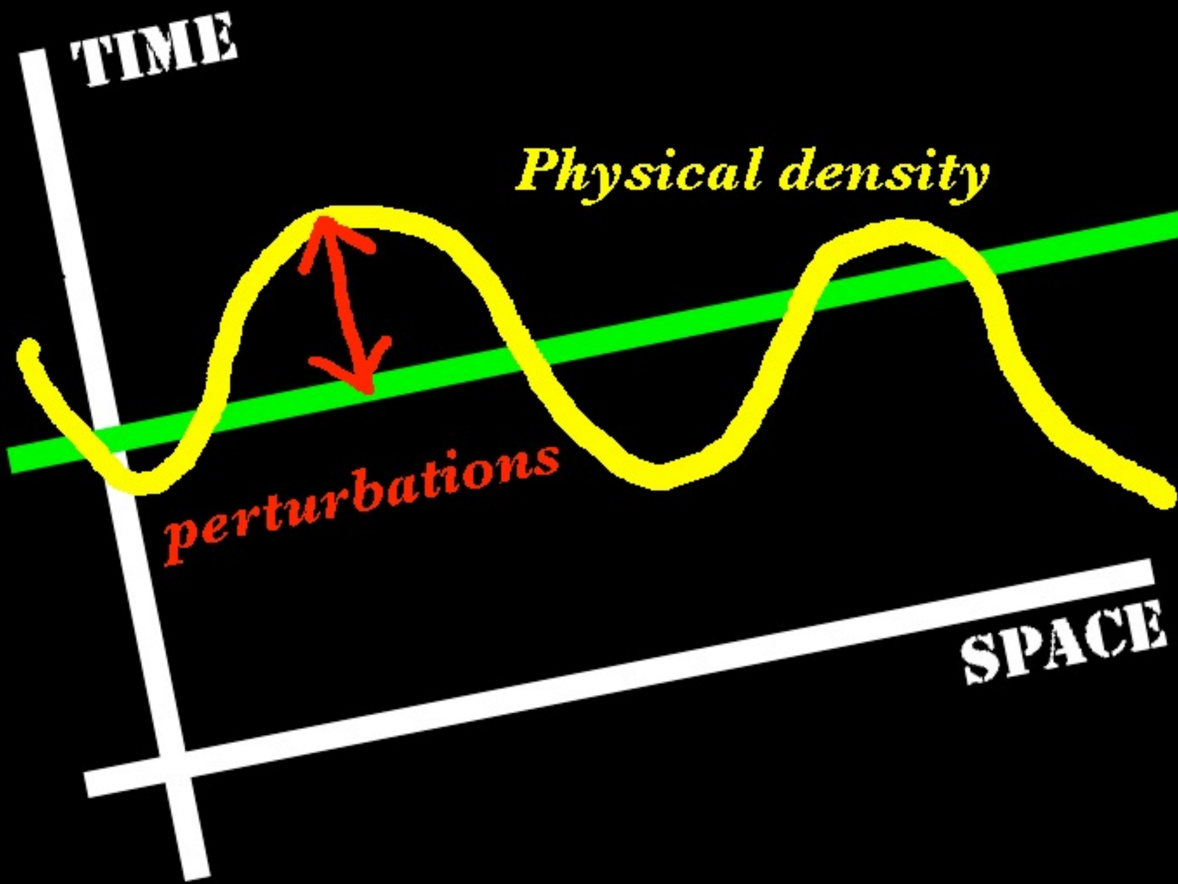


Gauge Issues

- theoretical predictions in cosmology
  - perturbations are *gauge-dependent*
  - *cannot* be directly associated with *observables!*

- observable quantities:
  - gauge-invariance is a *necessary* condition, but *not a sufficient* condition
  - explicit check is *needed*

(c) Gauge-Invariance

- cosmological observables:
  - should be gauge-invariant, but std description: *no!*
- standard theoretical descriptions:
  - chose one gauge, e.g., conformal Newtonian gauge
  - complete gauge fixing: gauge-invariant, *not enough!*
  - *gauge-dependent*: in general representations
- lessons learned:
  - gauge fixing: easier, but lose ability to verify
  - explicit check of gauge transformations

Take-Home Message

- standard descriptions: incomplete
  - gauge dependent: *different values* in different gauges
  - *no* specification of observer & source
  - *nor* frames in which physical events take place

- limited to linear order:
  - rely on background FRW metric
  - similar to Minkowski metric in observer rest frame
  - *only* at 1st order, not generally valid

need to *re-write* cosmology: what we do in Zürich!

III. IMPACT OF THE RELATIVISTIC EFFECTS

Predictions of Gauge-Invariant Formalism

(a) Luminosity Distance

- **standard candle:**
  - **intrinsic luminosity: L_{SN} known**
  - **observables: flux, redshift, position** $f_{\rm obs}\,,\,z_{\rm obs}\,,\,n_{\rm obs}$
  - **luminosity distance:** $\mathcal{D}_L = \left(\frac{L_{\rm SN}}{4\pi f_{\rm obs}}\right)^{1/2}$

- **inhomogeneities:**
  - **all observables are affected**
  - **perturbations:** $\mathcal{D}_L = \bar{D}_L(z_{\rm obs})(1+\delta\mathcal{D})$
  - **LHS: observable, gauge-invariant**
  - **RHS: should be gauge-invariant**

Fluctuations in Luminosity Distance

- **linear-order calculations:**
  - **with respect to observed redshift:** z_{obs}
  - **luminosity distance:** $\mathcal{D}_L = \bar{D}_L(z_{\mathrm{obs}})(1+\delta\mathcal{D})$
    perturbations: ***gauge invariant*** $\delta\mathcal{D} = \delta z + \frac{\delta r}{\bar{r}} - \kappa + \Xi$
  - **individual terms:** **gauge dependent**

Sasaki 1987, Schmidt & Jeong 2014, Biern & Yoo 2017

- **physical interpretation:**
  - **distortion in redshift:** δz $\quad 1 + z_{\mathrm{obs}} = \frac{1+\delta z}{a(\eta)}$
  - **radial & angular distortions of src position:** δr $\quad \kappa$
  - **distortion in local frame:** $\Xi = \frac{1}{2}\left(C^i_i - C_{ij}n^in^j\right)$

No Dark Energy?

- SN Ia observations **without Dark Energy**:
  - super-horizon fluctuations give large variance in LD
  - *good chance* to be in under-dense region

- if in *under-dense region*:
  - acceleration of expansion
  - explain SN Ia with ordinary cosmology
  - *no need* for dark energy

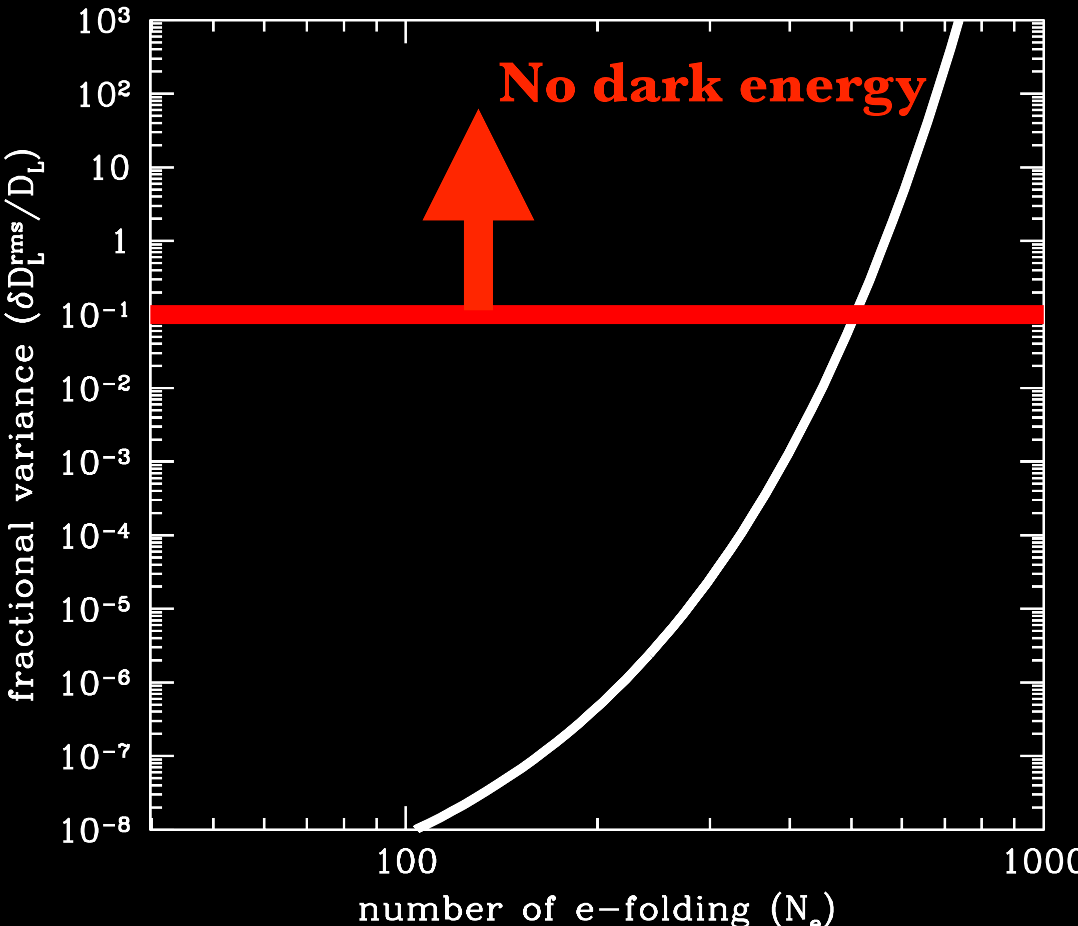
if we live here!?

Barausse et al. 2005, Kolb et al. 2005

Infrared Divergences

- **standard calculations:**
  - **order unity variance with *N*~500 e-folding**
  - ***no need* for dark energy:**
  - **no upper limit on *N***

Barausse et al. 2005, Kolb et al. 2005



- **many groups:**
  - **found same results**
  - **impose ad hoc IR cutoff k_{IR}~H_0 is imposed**
  - **conformal Newtonian gauge**

Equivalence Principle

- *no impact* of long modes on *observables*:
  - local observables independent of such long modes
  - uniform gravitational potential & acceleration
  - long-mode contribution $\propto k_L^2$ to the least

  Hirata & Seljak 2005, Green & Wald 2011, Biern & Yoo 2017

- what's wrong in many calculations?
  - *errors* made when choosing gauge condition
  - mostly in conformal Newtonian gauge
  - some terms are ignored, break gauge-invariance
  - cause pathology

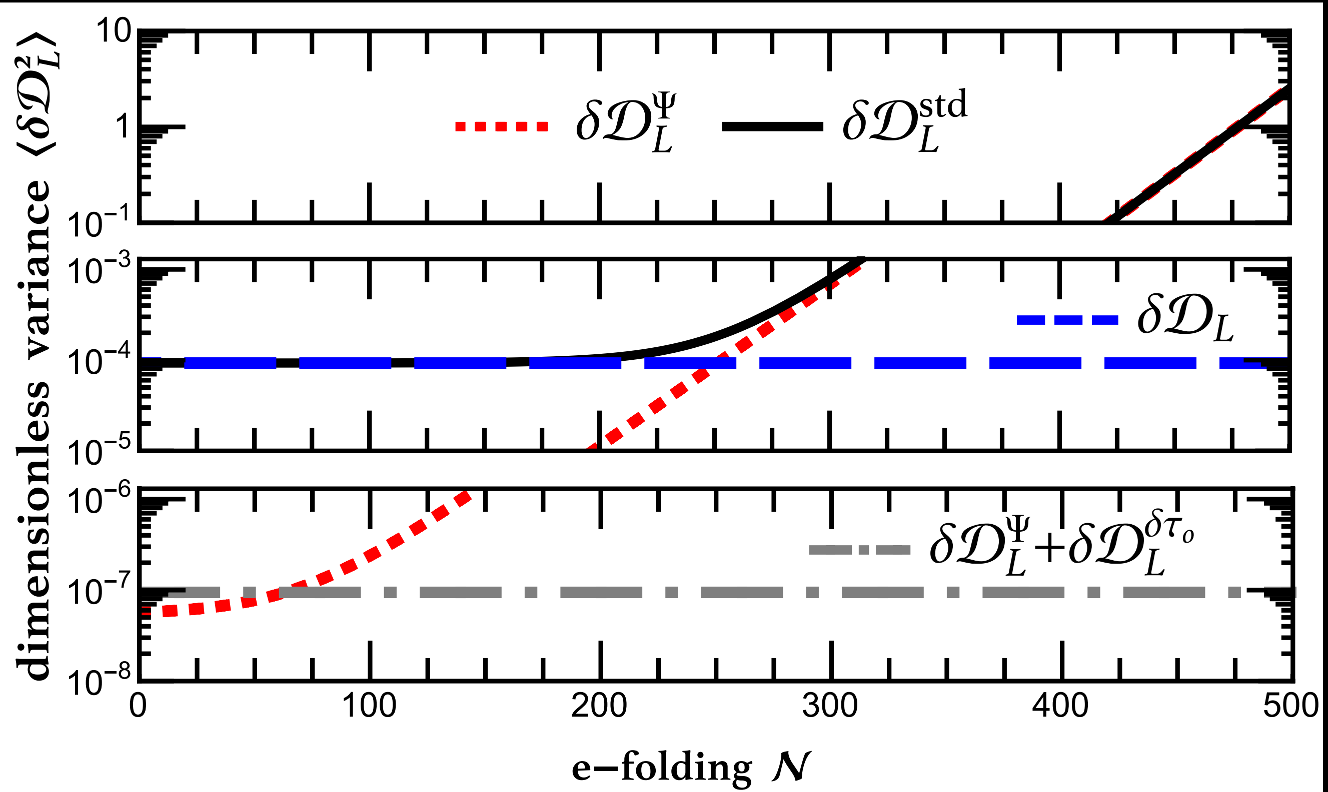
Impact on Luminosity Distance

- **linear-order expressions:**

$$\mathcal{D}_L = \bar{D}_L(z_{\rm obs})(1+\delta\mathcal{D}) \qquad \delta\mathcal{D} = \delta z + \frac{\delta r}{\bar{r}} - \kappa + \Xi$$

  - **black**: standard calculation (IR divergence)
  - **blue**: correct gauge-invariant calculation
- **grey**: missing component
- **cancellation**: potential terms balanced

Biern & Yoo 2017 JCAP



(a) Luminosity Distance

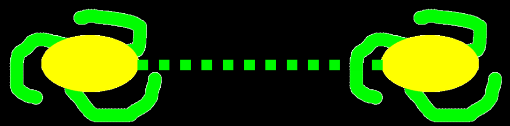
- **summary of new findings:**
  - ***no IR-divergences*** **in variance: we need dark energy!**
  - ***no ad hoc IR cutoff*** **is needed**
  - **definitive & explicit end to controversy** **Biern & Yoo 2017**

- **lessons learned:**
  - **use** ***correct gauge-invariant*** **expression**
  - **make sure to explicitly check** ***gauge-invariance***
  - **shift in mean LD from background (2nd order; in progress)**
  - **suspect more missing terms in other calculations**

(see Yoo & Scaccabarozzi 2016: compare 4 methods for computing luminosity distance)

(b) Galaxy Clustering

- **measure of how galaxies are distributed:**



- **construct fluctuation in galaxy counts:**
  - **total number of observed galaxies** $dN_{\rm tot}$
  - **observed volume** $dV_{\rm obs}$ **given** $(z_{\rm obs}, \theta_{\rm obs}, \phi_{\rm obs})$
  - **fluctuation field** $\delta_g^{\rm obs} = \frac{n_g^{\rm obs}}{\langle n_g^{\rm obs} \rangle} - 1$

- **relation to physical number density:**
  - **number conservation** $dN_{\rm tot} = n_g^{\rm phy} dV_{\rm phy} = n_g^{\rm obs} dV_{\rm obs}$
  - **observed number density** $n_g^{\rm obs} = n_g^{\rm phy} \frac{dV_{\rm phy}}{dV_{\rm obs}}$

$$z_{\rm obs} \neq z \ , \quad f_{\rm obs} \neq f_{\rm phy} \ , \quad dV_{\rm obs} \neq dV_{\rm phy}$$

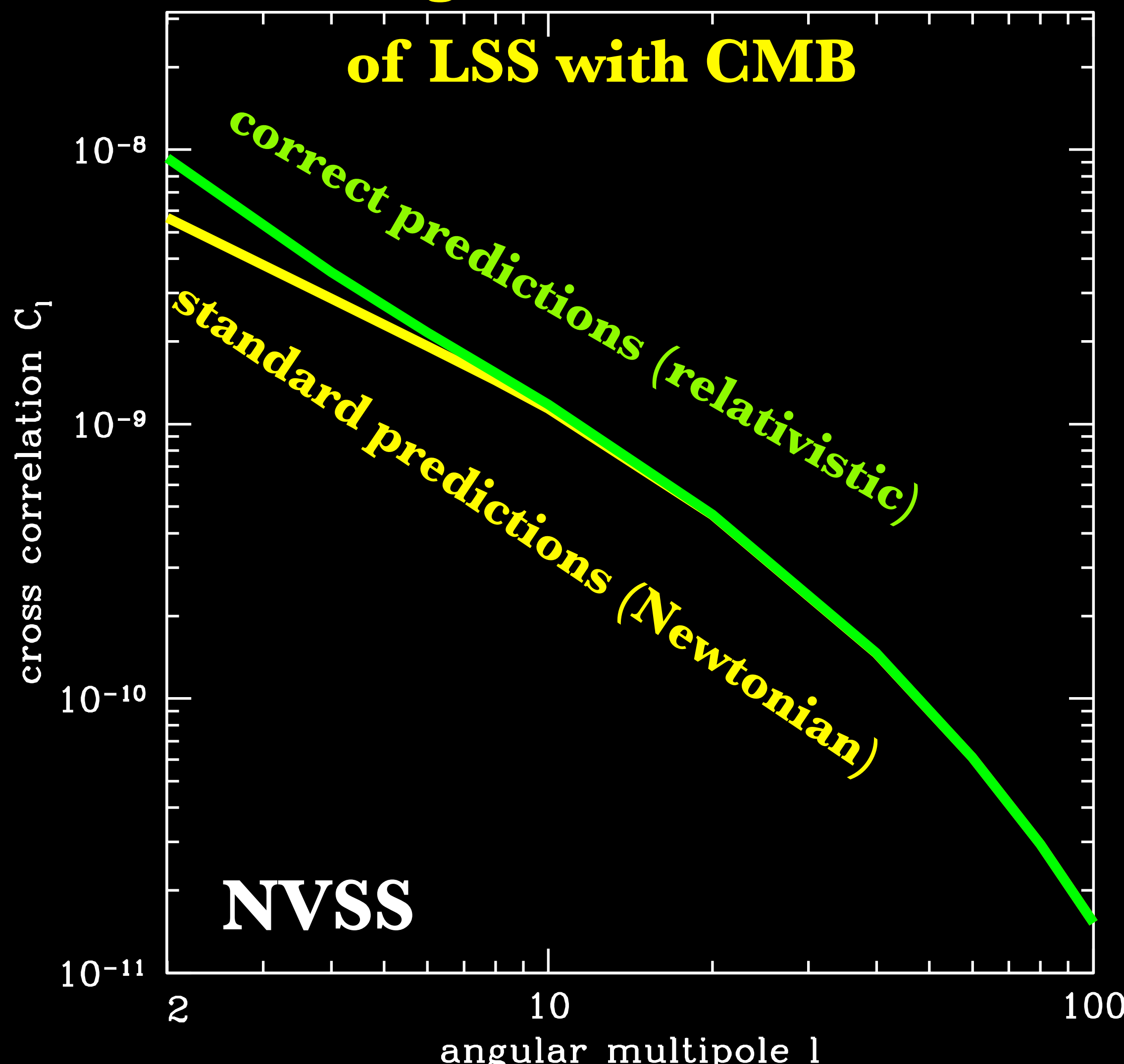
Systematic Errors

- theoretical predictions:
  - relativistic (*correct*)
  - standard Newt. (*incorrect*)
- standard method:

$$\delta_g = b\ \delta_m^{sync} + (5p-2)\ \kappa$$

- *underestimate* the observed signals at low multipoles
- large cosmic variance

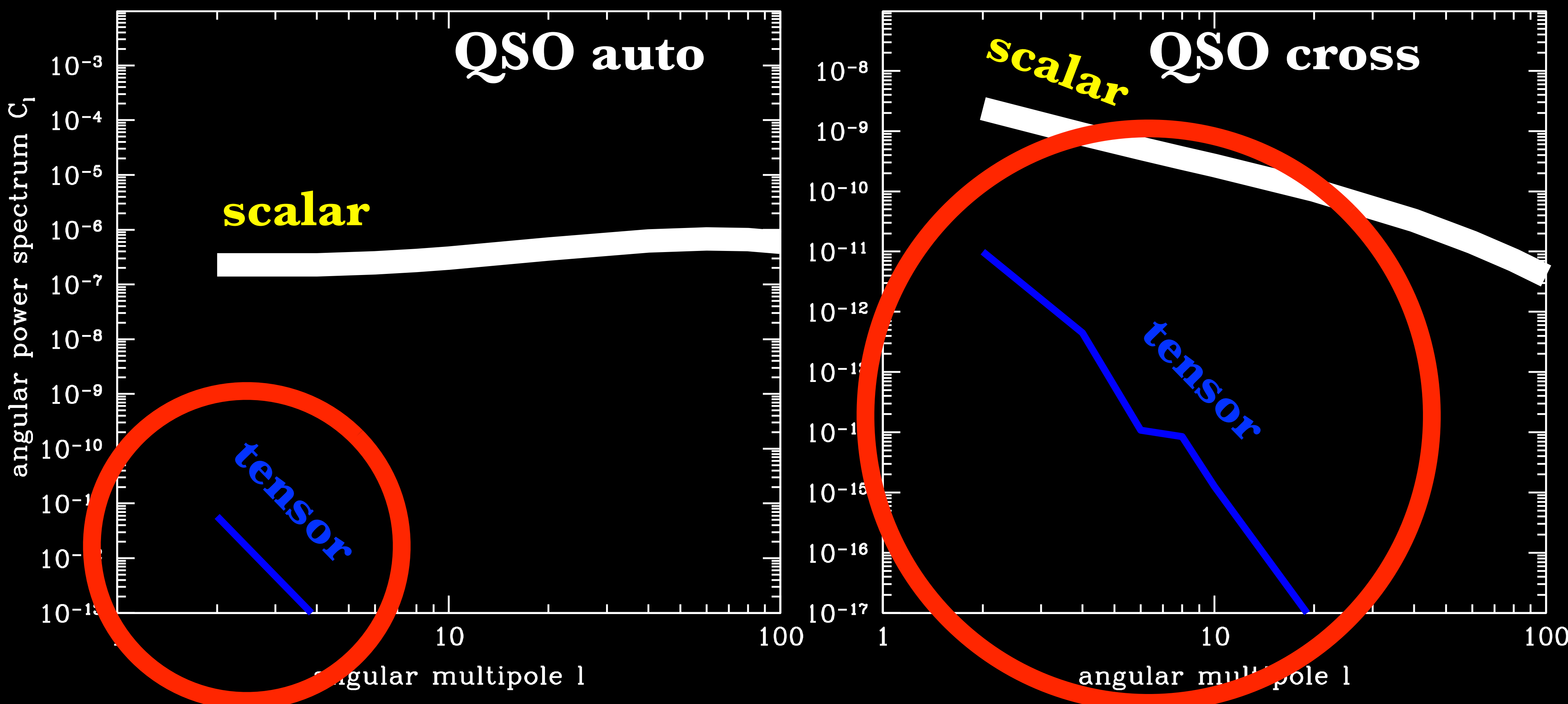
Angular cross-correlation of LSS with CMB



Yoo, Fitzpatrick, Zaldarriaga 2009 PRD

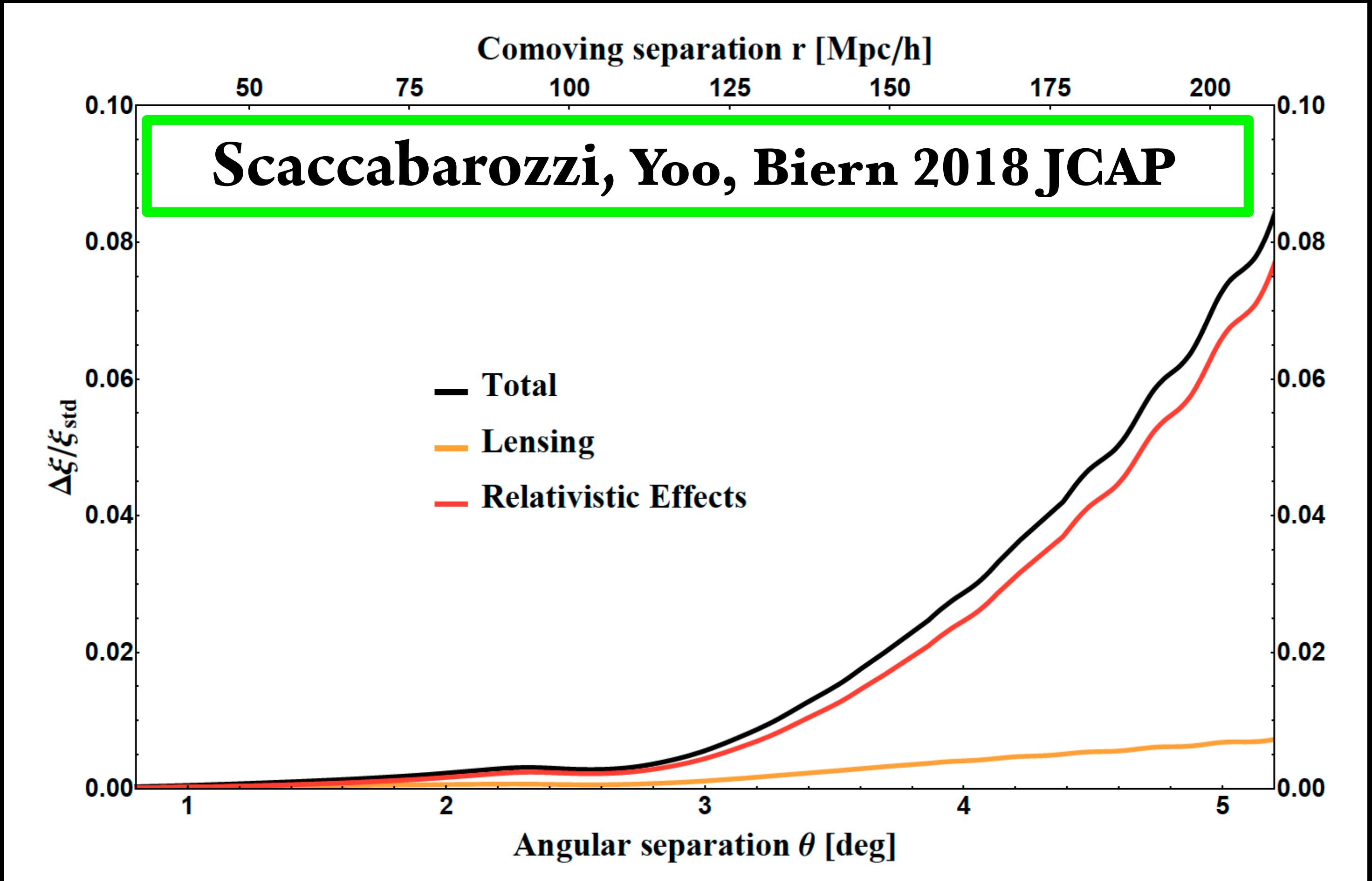
Primordial Gravity Waves

- *gravity waves* in galaxy clustering (spin 0 vs spin 2)
  - tensor-to-scalar ratio *r=0.1*



Correlation Function

- relativistic effects:
  - beyond BAO: a few percent level corrections
  - lensing and velocity contributions



Effective non-Gaussianity?

- **primordial non-Gaussianity:** $\zeta = \zeta_G + \frac{3}{5} f_{\rm nl} \zeta_G^2 + \cdots$
  - **local-type: super-horizon mode on small scales**
  - ***non-gravitational* initial condition**

- ***yes,* effective non-Gaussianity:**
  - ***no* primordial origin, but by *nonlinear gravity***
  - **due to nonlinear constraint equations in GR**

Bruni et al. 2014, Bruni, Hidalgo, Wands 2014, Bartolo et al. 2015

$$\delta_s = \left(1 - 2\zeta_l + 2\zeta_l^2 + \cdots\right) \delta_G \quad \rightarrow \quad \Delta f_{\rm nl} = -5/3 \ , \quad \Delta g_{\rm nl} = 50/27 \ , \cdots$$

Resolutions

- **answer:** ***no effective non-Gaussianity*** **due to gravity**
  - **equivalence principle: geometric gravity** $\propto (k/\mathcal{H})^2$
  - **coordinate re-scaling with SH mode**

$$d\tilde{x}^i \equiv e^{\zeta_l} dx^i \quad \rightarrow \quad \Delta\zeta = e^{2\zeta_l} \tilde{\Delta}\zeta \ , \quad \delta_s \approx \delta_G$$

  - **non-vanishing SH correlation from non-gravity**

Yoo & Gong 2016 PLB

- **nonlinearly generated?** ***no,*** **in the initial condition**
- **anything wrong in calculations? just** ***fine***
  - **all correct second-order calculations**

Bruni et al. ApJ 2014, Bruni, Hidalgo, Wands, ApJL 2014, Bartolo et al. 2015

  - **character of gauge choice, as in Maldacena 2003**

(c) Weak Gravitational Lensing

- **limitations in standard weak lensing:**
  - **intrinsically relativistic, but incomplete**
  - **extension beyond linear order: difficult**

- **problems in standard weak lensing:**
  - **true source angular position: *un-observable***

    $\hat{s}_{\rm true} = \hat{n}_{\rm obs} + \delta n, \qquad \hat{n}_{\rm obs} = (\theta,\ \phi)_{\rm obs}, \qquad \delta n = (\delta\theta,\ \delta\phi)$
  - ***un-observable* distortion matrix: angular size** $(d\theta, d\phi)$

$$\begin{pmatrix} ds_\theta \\ \sin\theta\ ds_\phi \end{pmatrix} \equiv \begin{pmatrix} \mathbb{D}_{11} & \mathbb{D}_{12} \\ \mathbb{D}_{21} & \mathbb{D}_{22} \end{pmatrix} \begin{pmatrix} d\theta \\ \sin\theta d\phi \end{pmatrix}, \quad \mathbb{D} \equiv \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2-\omega \\ -\gamma_2+\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$

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$$\begin{pmatrix} ds_\theta \\ \sin\theta\ ds_\phi \end{pmatrix} \equiv \begin{pmatrix} \mathbb{D}_{11} & \mathbb{D}_{12} \\ \mathbb{D}_{21} & \mathbb{D}_{22} \end{pmatrix} \begin{pmatrix} d\theta \\ \sin\theta d\phi \end{pmatrix}, \quad \mathbb{D} \equiv \begin{pmatrix} 1-\kappa-\gamma_1 & -\gamma_2-\omega \\ -\gamma_2+\omega & 1-\kappa+\gamma_1 \end{pmatrix}$$

observables to measure

(c) Weak Gravitational Lensing

- **limitations in standard weak lensing:**
  - **intrinsically relativistic, but incomplete**
  - **extension beyond linear order: difficult**

- **problems in standard weak lensing:**
  - **true source angular position: *un-observable***

    $\hat{s}_{\rm true} = \hat{n}_{\rm obs} + \delta n, \qquad \hat{n}_{\rm obs} = (\theta,\ \phi)_{\rm obs}, \qquad \delta n = (\delta\theta,\ \delta\phi)$
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gauge-dependent **observables to measure**

Gauge-Invariant Formalism

- **relation to physical length & shape at src:**
  - **trace back size to src (geodesic deviation)**
  - **source position: *gauge-dependent* (still!)**
  - **rest frame: physical length** $(dL_{d\theta},\ dL_{d\phi})$ **(*coord. ind.*)**
  - ***physical* distortion matrix**

$$\begin{pmatrix} dL_{d\theta} \\ dL_{d\phi} \end{pmatrix} \equiv \bar{D}_A \begin{pmatrix} \hat{\mathbb{D}}_{11} & \hat{\mathbb{D}}_{12} \\ \hat{\mathbb{D}}_{21} & \hat{\mathbb{D}}_{22} \end{pmatrix} \begin{pmatrix} d\theta \\ \sin\theta\ d\phi \end{pmatrix} ,$$

  - **all lensing observables: *gauge-invariant***

Yoo, Grimm, Mitsou, Amara, Refregier 2018 JCAP and Grimm & Yoo 2018 JCAP

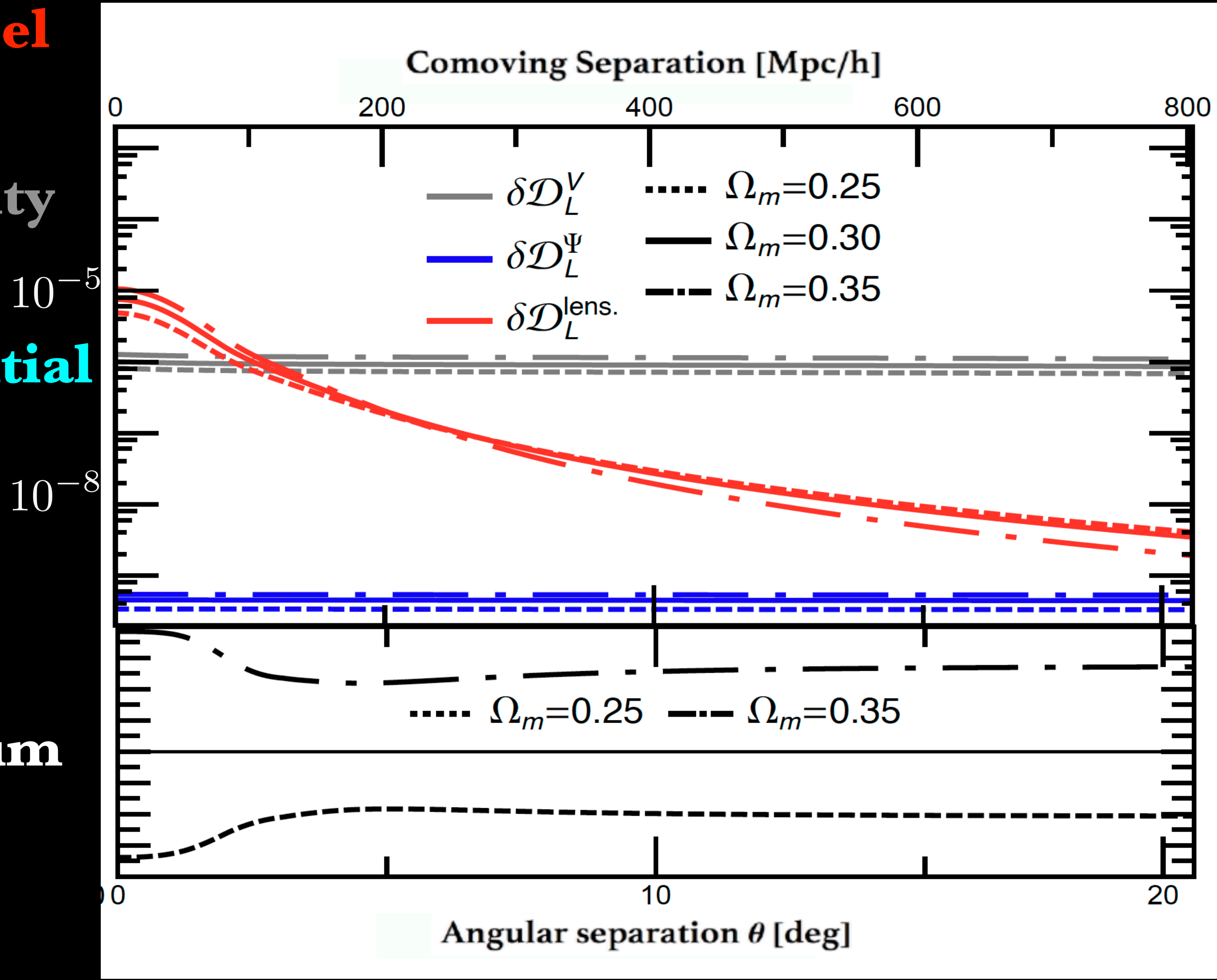
Systematic Errors

- standard model

  $\delta\mathcal{D}_L^{\text{lens.}} \equiv \kappa_{\text{std}}$
- missing velocity

  $\delta\mathcal{D}_L^V$
- missing potential

  $\delta\mathcal{D}_L^\Psi$
- significant systematic errors
- power spectrum in progress



Biern & Yoo 2017

Lensing Shear

- **standard formalism:** $_{\pm 2}\gamma \equiv \gamma_1 \pm i\gamma_2 = m^\alpha_\mp m^\beta_\mp \gamma_{\alpha\beta}$

$$\gamma_{\alpha\beta} = -\left(C_{\alpha\beta}\right)_o + \mathcal{G}_{\alpha,\beta} + \int_0^{\bar{r}_z} d\bar{r} \left(\frac{\partial}{\partial x^\beta}\right) 2C_{\alpha\gamma}n^\gamma + \int_0^{\bar{r}_z} d\bar{r} \left(\frac{\bar{r}_z - \bar{r}}{\bar{r}_z \bar{r}}\right) \left[\bar{r}^2 \left(\frac{\partial^2}{\partial x^\alpha \partial x^\beta}\right) \left(\alpha_\chi - \varphi_\chi - C_\parallel\right)\right]$$

  - **gauge dependent due to** $\mathcal{G}^\alpha \to \mathcal{G}^\alpha - \mathcal{L}^\alpha$
  - **correct shear in cN gauge w/ scalar only (*lucky!*)**
  - **incorrect shear in tensor & *IR divergence***

scalar: $\alpha_\chi,\ \varphi_\chi$ **tensor:** $C_{\alpha\beta}$ **(SVT decomposition of metric)**

- **gauge-invariant formalism:**

$$\hat{\gamma}_{\alpha\beta} = -\left(C_{\alpha\beta o} + C_{\alpha\beta}\right) + \int_0^{\bar{r}_z} d\bar{r} \left(\frac{\partial}{\partial x^\beta}\right) 2C_{\alpha\gamma}n^\gamma + \int_0^{\bar{r}_z} d\bar{r} \left(\frac{\bar{r}_z - \bar{r}}{\bar{r}_z \bar{r}}\right) \left[\bar{r}^2 \left(\frac{\partial^2}{\partial x^\alpha \partial x^\beta}\right) \left(\alpha_\chi - \varphi_\chi - C_\parallel\right)\right]$$

  - ***metric shear* or *FNC term*: tensor at source**

Dodelson et al. 2003, Schmidt & Jeong 2012

  - **rest frame: observer frame & source frame**

Yoo, Grimm, Mitsou, Amara, Refregier 2018 JCAP and Grimm & Yoo 2018 JCAP

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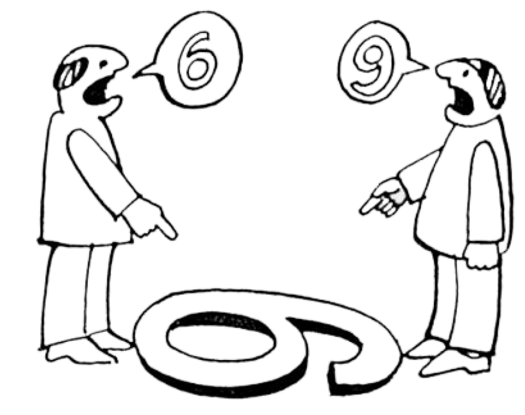
  - **rest frame: observer frame & source frame**

Yoo, Grimm, Mitsou, Amara, Refregier 2018 JCAP and Grimm & Yoo 2018 JCAP

To Rotate or Not to Rotate

- lensed images **rotate!**
  - gravitational lensing beyond linear order
  - 1st order gravity waves, too! **probe of gravity waves**

$$2\hat{\omega} = 2\left(\Omega_o^n - \Omega_s^n\right) - 2\ \cos\theta\ \Delta\phi - \int_0^{\bar{r}_z} d\bar{r}\ \mathrm{n}\cdot\nabla\times\left(\Psi^\alpha + 2C_\parallel^\alpha\right)$$

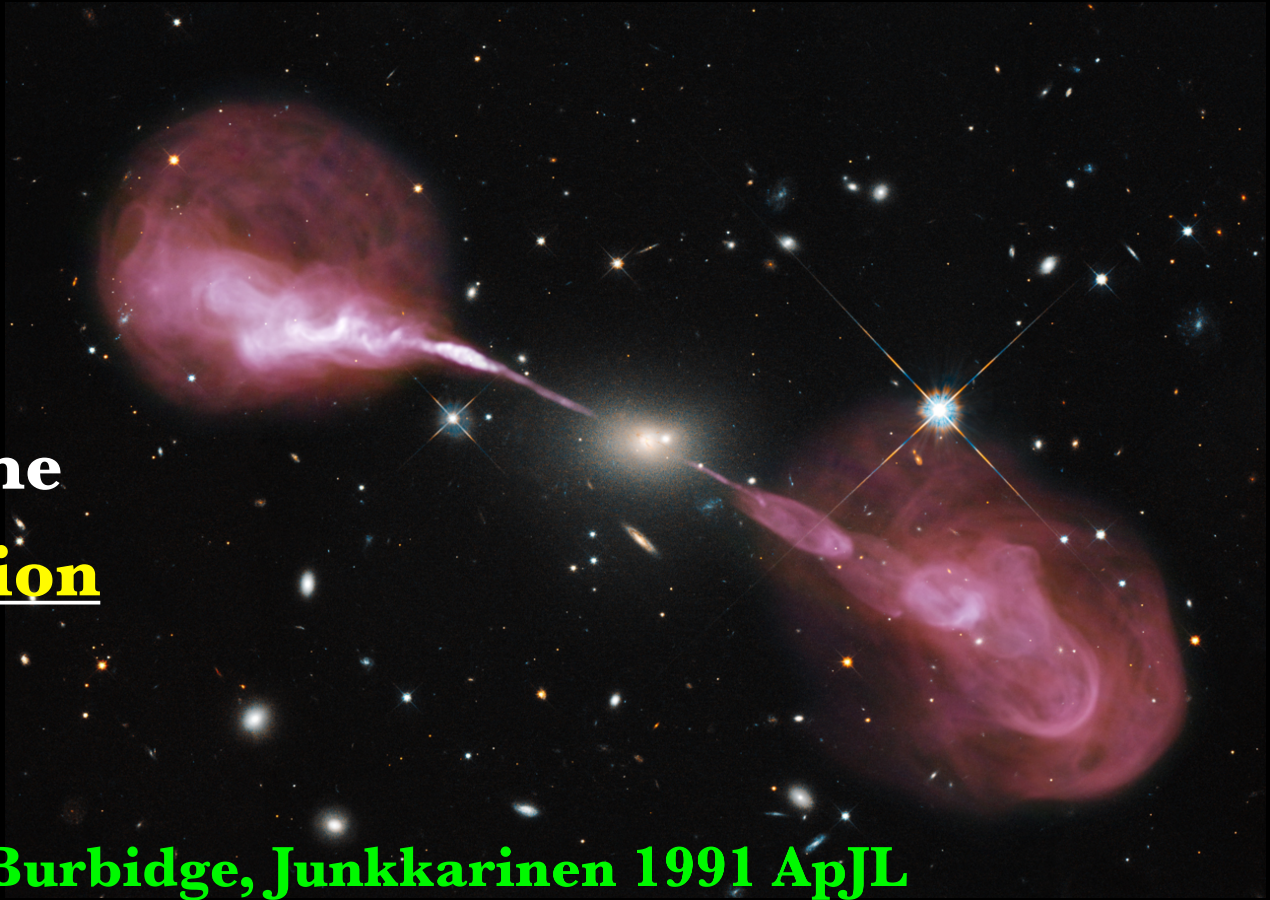


- how to measure *rotation*?
  - orientations should be **synchronized**
  - **parallel transport** along null path (*the only way!*)
  - orientation of src basis: *completely fixed*
  - *no* lensing rotation at all (no GW) at *1st order*
  - *fictitious rotation* against FRW coordinate (Skrotsky effect: **artifact**)

Radio Jets!

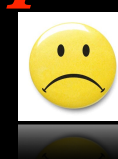
- orientation of radio jets:
  - polarization: central geodesic, *parallel transported*
  - extended images: geodesic deviation, *not PTed*

- intrinsic relation:
  - jet & polarization is perpendicular in source rest frame
  - infer lensing rotation



Kronberg, Dyer, Burbidge, Junkkarinen 1991 ApJL

(d) Cosmic Microwave Background

- **much of the work in progress:**
  - **Boltzmann equation in tetrad formalism**
  - **use of background metric (except Bond & Szalay 1983)**
  - **linear order: ok except *monopole* and *dipole*** ☺
  - **beyond linear order, not ok** 

- **future applications:**
  - ***CMB lensing* (2nd order): not complete**
  - **CMB spectral distortion**
  - **and more (stay tuned!)**

CMB Temperatures $\bar{T}$ and $\langle T \rangle^{\rm obs}$

- $\bar{T}$: cosmological parameter
  - determine background evolution
  - defined in background universe, *unique* number
  - influence perturbation evolution

- $\langle T \rangle^{\rm obs}$: observed CMB temperature (from FIRAS)
  - angle average CMB temperature over all sky
  - uncertainty in $\langle T \rangle^{\rm obs}$:
    - COBE FIRAS 1996: $\langle T \rangle^{\rm obs} = 2.728 \pm 0.004$ K (0.15%)
    - + WMAP 2009: $\langle T \rangle^{\rm obs} = 2.7255 \pm 5.7 \cdot 10^{-4}$ K (0.021%)

Yoo, Mitsou, Dirian, Durrer 2019 PRD

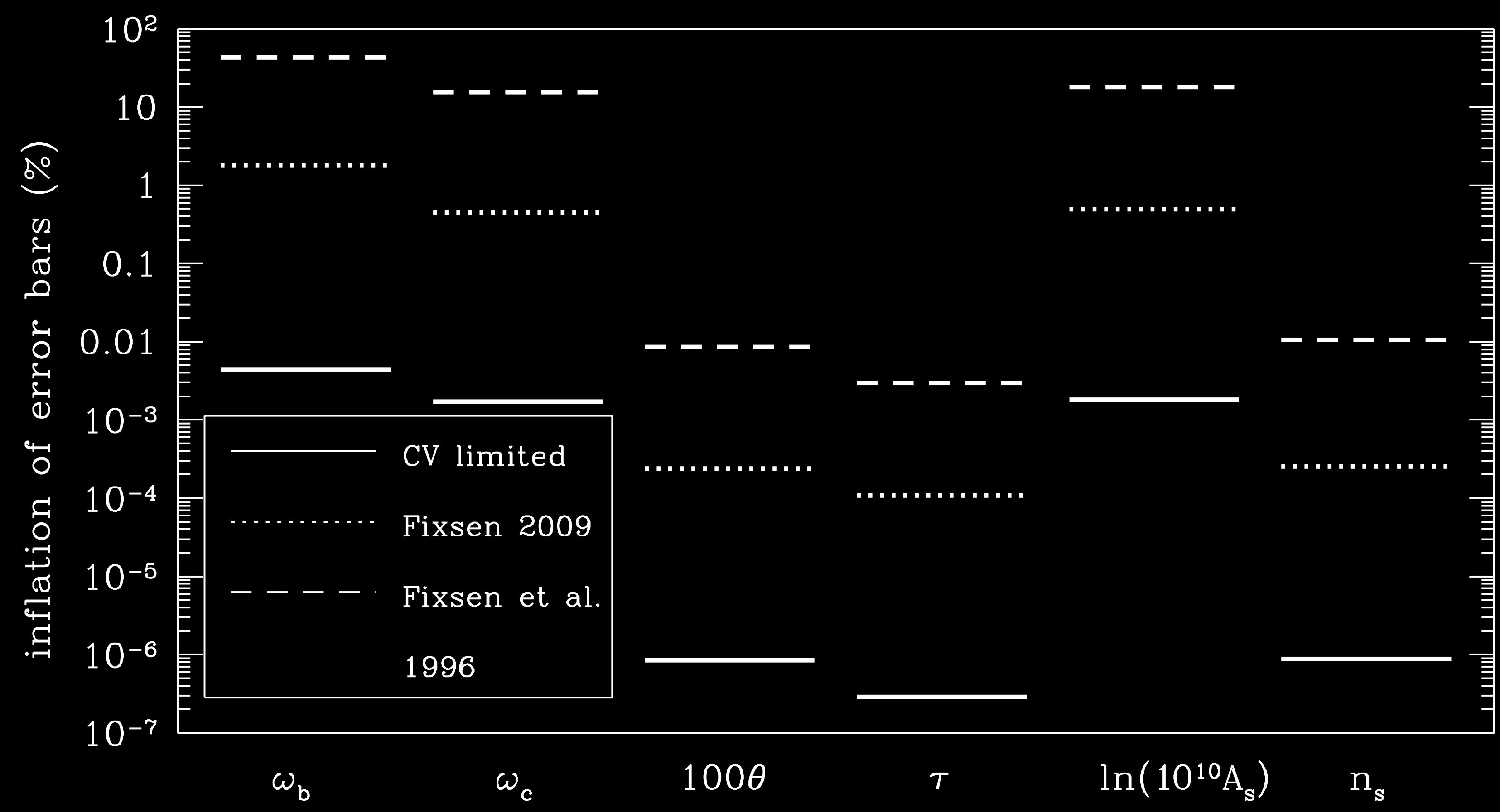
Problems in Standard Practice

- **impact of standard practice:** $\bar{T} \equiv \langle T \rangle^{\rm obs}$
  - **underestimation of error bars**
  - **systematic biases in the best-fit parameters**
  - ***not equal due to monopole!***
- **underestimation of error bars:** $\sigma_{\rm true} \gtrsim \sigma_{\rm std}$
  - **any models have *one less degree* of freedom:** $\bar{T}$
  - **error bars are always smaller than true error bars**

- **systematic biases:** $p_i^{\rm best} := p_i^{\rm true} + \delta p_i \,, \quad \delta p_i \neq 0$
  - ***bias*: in proportion to *monopole* at our position**
  - **monopole at our position: unknown**

Inflation of Error Bars

- **true error bars are larger:** $\sigma_{\rm true} \gtrsim \sigma_{\rm std}$
  - **depend on uncertainty in** $\langle T \rangle^{\rm obs}$ ($\bar{T}$ **will be unknown**)
  - ***irreducible*** **systematic errors, if unaccounted for**



IV. MORE WORK & FUTURE DIRECTION:

In the Next Five Years and the Coming Decade

Inflationary Fossils

- **standard inflation: (single clock)**
  - **drive *expansion* & generate *perturbations***

- **beyond the standard model:**
  - ***extra* degrees-of-freedom leave *trace***
  - **deviation from statistical isotropy**

$$\langle\zeta(k_1)\zeta(k_2)h_s(k)\rangle = (2\pi)^3\delta^D(k_1+k_2+k)P_{h_s}(k)f_h(k_1,k_2)\epsilon^s_{ij}(k)k_1^i k_2^j$$

  - **inflationary *fossils* (S,V,T)**
  - ***subtle relativistic effects!***

Probe of Dark Energy

- **dark energy models: (modified gravity included)**
  - **evolving spatial fluctuations (*relativistic effect*)**
  - ***no* convincing alternative**
  - **representative *Horndeski* theories (no ghost)**
  - **quantifying relativistic signatures**

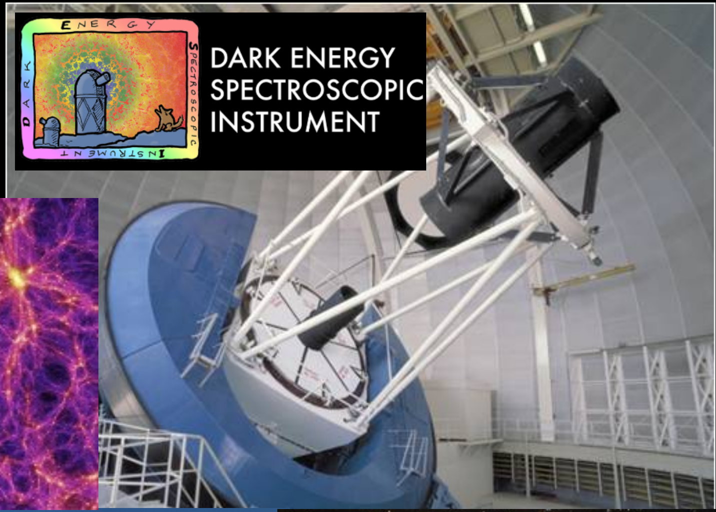
- **observational strategy:**
  - **deriving *generic observational strategy***
  - ***new probes*: relativistic signatures**

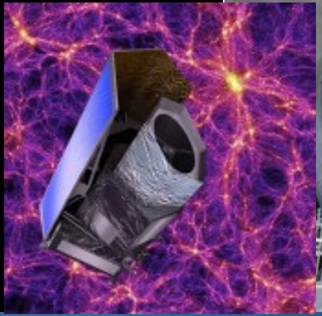
What can We Learn?

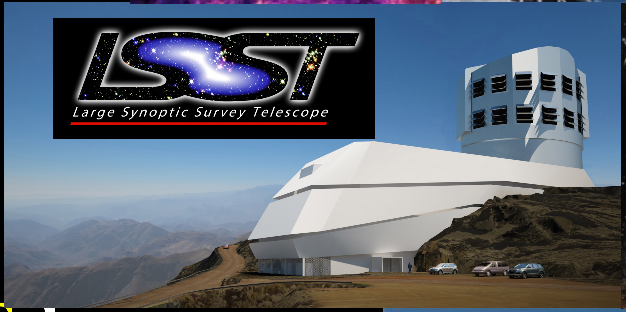
- **relativistic effects**: *small*, but *detectable!*
  - *extra* and *critical* information
  - difficult, but high gain (*new* opportunities)
- **key role**: *deviations* from standard cosmology
  - *higher-order signatures* (fossil fields, DE fluct.)
  - *not* present in Newtonian description
- **complementary role**: *enigmatic* standard cosmology
  - (better) *complementary* to CMB constraints
  - *convincing* constraints on dark energy

Future Surveys

- **future ground-based surveys:**
  - **D**ark **E**nergy **S**pectroscopic **I**nstrument
  - **L**arge **S**ynoptic **S**urvey **T**elescope
  - **S**quare **K**ilometer **A**rray
  - **CMB S**tage-**IV**
- **future space missions:**
  - **Euclid**
  - **W**ide-**F**ield **I**nfrared **S**urvey **T**elescope







Probing the Universe with Large-Scale Structure

General Relativistic Effects and Gauge-Invariant Formalism

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INSTITUTES for COMPUTATIONAL SCIENCE & PHYSICS, UNIVERSITY of ZÜRICH

2 October 2019

Universität Zürich[UZH]

Executive Summary

- **incomplete standard theoretical description of cosmological observables**
  - ***gauge dependent*** **& missing** ***relativistic effects***
  - ***no*** **frame specification for physical events and observables**
  - **limited to** ***linear order***

- **subtle relativistic effects in precision cosmology**
  - **a** ***new area*** **of research**
  - ***test*** **general relativity (or modified gravity)**
  - ***signatures*** **of inflationary models**
  - ***consistency*** **check &** ***complementary*** **constraints**