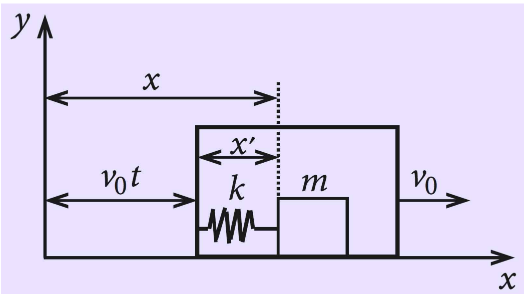
수험번호		성 명		감독교수 확 인	(인)

물리학부 자격시험 문제

과목명 : 고전역학 2014 . 6 . 20 시행

1. (50 pts) For (a)−(c): A container truck moves along the x axis at a constant speed v_0. Inside the container, a small mass m oscillates along the x axis held by a massless spring with a spring constant k and an equilibrium length d.



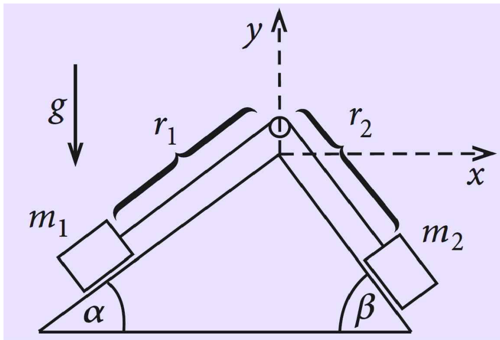
(a) (15 pts) Specify the Hamiltonian $H(x,p,t)$ of the system composed of the spring and the mass m in the lab frame. Is H equal to the energy E of the system? Is H a conserved quantity? Also obtain the Hamilton equations of motion.

(b) (5 pts) Specify the Hamiltonian $H'(x',p',t)$ in the comoving frame in terms of the comoving coordinate $x' = x - v_0 t$ (see figure) and obtain the Hamilton equations of motion.

(c) (10 pts) Solve the equation of motion in (b), and then also solve the equation of motion in (a).

(d) (20 pts) For (d) only, suppose that the mass of the container truck is M and the container truck now glides smoothly on the ground (without any external driving force). Also, consider the truck as part of the system, i.e., now x and x' are independent variables. Set up the Hamiltonian and the equations of motion (no need to solve). Finally, find the non−zero angular frequency of the system from your Hamiltonian equation of motion.

2. (50 pts) Two masses m_1 and m_2 move smoothly on a fixed wedge under the influence of gravity g. They are connected by a massless thread of length l together.



(a) (5 pts) Write down the constraints in terms of x_1, y_1, x_2, y_2 (For convenience one might use r_1, r_2 also). What is the number of degrees of freedom in this system?

(b) (5 pts) Choose generalized coordinate(s) from the result of (a) and express x_1, y_1, x_2, y_2 in terms of the generalized coordinate(s) and l, α, β.

(c) (10 pts) Formulate the Lagrangian and obtain the Lagrange equation(s) of motion.

(d) (10 pts) Solve the Lagrange equation(s) for boundary conditions $r_1(t=0) = r_0$, $\dot{r}_1(t=0) = 0$, and find the equilibrium condition.

(e) (20 pts) Set up the Lagrangian in terms of r_1, r_2. This time, introduce a Lagrangian multiplier λ to deal with constraint related to r_1, r_2. Obtain Lagrange equations of motion and find the tension on the thread. What is the value of the tension using the equilibrium condition of (d)?

소속대학원	물리·천문학부	학번		성 명		감독교수 확 인	(인)

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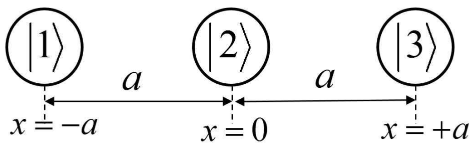
1. (90 pts) Consider the following system composed of normalized states $|1\rangle$, $|2\rangle$ and $|3\rangle$ whose on-site energies are all zero. The states $|1\rangle$, $|2\rangle$ and $|3\rangle$ are localized at $x=-a$, $x=0$, and $x=+a$, respectively.



Here, nearest-neighbor hopping is allowed and hence the Hamiltonian is $H_0=\kappa(|2\rangle\langle 1|+|1\rangle\langle 2|+|3\rangle\langle 2|+|2\rangle\langle 3|)$, where κ is a positive real number.

(a) (20 pts) Show that the energy eigenvalues of the system are $+\sqrt{2}\kappa$, 0, and $-\sqrt{2}\kappa$. Also, find the normalized eigenstates $|+\rangle$, $|0\rangle$, and $|-\rangle$ satisfying $H_0|+\rangle=+\sqrt{2}\kappa|+\rangle$, $H_0|0\rangle=0|0\rangle$, and $H_0|-\rangle=-\sqrt{2}\kappa|-\rangle$ in terms of $|1\rangle$, $|2\rangle$ and $|3\rangle$.

(b) (30 pts) If state $|\psi(t)\rangle$ satisfies $|\psi(0)\rangle=|1\rangle$, draw a single graph showing $|\langle 1|\psi(t)\rangle|$, $|\langle 2|\psi(t)\rangle|$ and $|\langle 3|\psi(t)\rangle|$ as a function of time t. Provide as much important information as possible on the graph. Also, if assume that this state is moving $|1\rangle\to|2\rangle\to|3\rangle\to|2\rangle\to|1\rangle\to|2\rangle\to\ldots$, what is its average velocity?

For (c) and (d), suppose that we add n electrons to this system. Assume that the n electrons are (1) not interacting among themselves and (2) are in the ground state.

(c) (10 pts) Draw a graph showing the total energy of the system, E_0, as a function of n for n from 0 to the highest value. Note that electrons are fermions. (Neglect any energy that has not been mentioned, e.g., rest mass energy of electrons.)

(d) (30 pts) Now suppose that we apply a very weak electric field E to the system. Assume that the system still remains in the ground state. The effect of the electric field is incorporated by a perturbation Hamiltonian $H_1=eEa(-|1\rangle\langle 1|+|3\rangle\langle 3|)$. (The charge of an electron is $-e$.) Draw a graph showing the induced electric dipole moment of this n-electron system (only the leading order term of E) as a function of n (from 0 to the highest value).

소속대학원	물리·천문학부	학번		성 명		감독교수 확 인	(인)

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2. (110 pts)

(a) (20 pts) First, consider gauge transformations in classical electromagnetism and quantum mechanics. The electric field $\vec{E}$ and the magnetic field $\vec{B}$ can be expressed in terms of the scalar and vector potentials as follows:

$$\vec{E} = -\nabla\phi - \frac{1}{c}\frac{\partial \vec{A}}{\partial t}, \quad \vec{B} = \nabla \times \vec{A}.$$

The Schrödinger equation for a particle of mass m and charge q in the presence of ϕ and $\vec{A}$ is given by

$$i\hbar\frac{\partial \Psi}{\partial t} = \left[\frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}\right)^2 + q\phi\right]\Psi.$$

Here we are using the cgs units. Note that in the SI units, c is absent in the above equations.

a1) (10 pts) Consider the following gauge transformation:

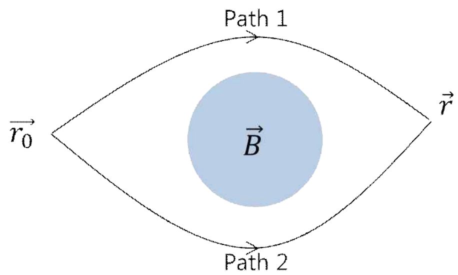
$$\phi' = \phi - \frac{1}{c}\frac{\partial \Lambda}{\partial t}, \quad \vec{A}' = \vec{A} + \nabla\Lambda, \quad \Psi' = e^{i\frac{q\Lambda}{\hbar c}}\Psi,$$

where Λ is any function of position and time. Show that under this transformation, $\langle\psi|\vec{p} - \frac{q}{c}\vec{A}|\psi\rangle = \langle\psi'|\vec{p} - \frac{q}{c}\vec{A}'|\psi'\rangle$ is unchanged, and explain the reason.

a2) (10 pts) Show that $\Psi' = e^{i\frac{q\Lambda}{\hbar c}}\Psi$ satisfies the Schrödinger equation with the gauge transformed ϕ' and $\vec{A}'$:

$$i\hbar\frac{\partial \Psi'}{\partial t} = \left[\frac{1}{2m}\left(\vec{p} - \frac{q}{c}\vec{A}'\right)^2 + q\phi'\right]\Psi'.$$

(b) (40 pts) Next, imagine an infinitely long, impenetrable cylinder with a radius R. Assume that magnetic field inside the cylinder is $\vec{B}$ along the cylinder axis while zero outside.



b1) (10 pts) Show that a transformation $\Lambda(\vec{r}) = -\int_{\vec{r}_0}^{\vec{r}} d\vec{r}' \cdot \vec{A}(\vec{r}')$ (along a path connecting $\vec{r}_0$ and $\vec{r}$ outside the cylinder) makes $\vec{A}' = 0$. Then what is the corresponding equation that $\Psi' = e^{i\frac{q\Lambda}{\hbar c}}\Psi$ satisfies?

b2) (20 pts) Suppose that a beam of particles with mass m and charge q is split in two and passes either side of the cylinder. Using the result of b1), show that the phase difference between the two beams which arises due to the magnetic field inside the cylinder is given by $2\pi\frac{\Phi_B}{\Phi_0}$ where $\Phi_B = \pi R^2 B$ and $\Phi_0 = hc/q$.

b3) (10 pts) Discuss that this effect cannot be explained by classical electrodynamics and is purely quantum mechanical. Also deform slightly the paths in b2) and compare the phase difference for the two split beams due to the magnetic field, and then discuss the topological nature of this effect.

소속대학원	물리·천문학부	학번		성 명		감독교수 확 인	(인)

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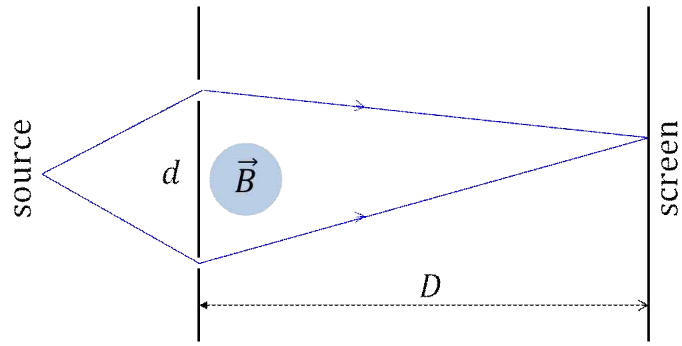
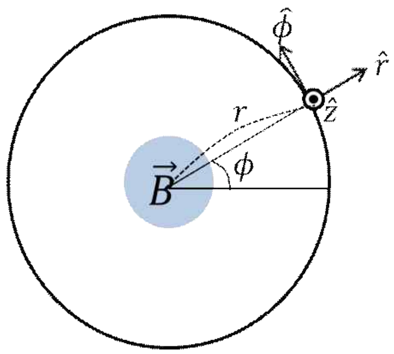
(c) (20 pts) Next, consider a double slit experiment as shown below, in which a beam of particles with mass m and charge q is emitted from the source after being accelerated by the voltage V. Then the beam reaches a detection screen by passing the double slit. Assume that the distance D between the screen and the slit is much larger than the slit separation $d \ll D$. An infinitely long, impenetrable cylinder with a radius $R \ll d$ is located just next to the slit so that the classical trajectories do not pass through the cylinder.



c1) (10 pts) First, assume that the magnetic field inside the cylinder is zero. Find the distance between the interference pattern on the screen.

c2) (10 pts) Now magnetic field inside the cylinder along the cylinder axis is increased from zero. Discuss how the interference patterns change with increasing the magnetic field.

(d) (30 pts) Finally, consider a particle of mass m and charge q confined in a circular ring with the radius r, as shown below. Along the axis there is an infinitely long, impenetrable cylinder with a radius $R < r$, and the magnetic field $\vec{B} = B\hat{z}$ is applied inside the cylinder along the cylinder axis. Here neglect spin degrees of freedom in this problem.



d1) (15 pts) First, consider that the magnetic field inside the cylinder is zero. Then the Hamiltonian of the system is given by $H = \frac{\vec{p}^2}{2m}$ where $\vec{p} = -i\hbar \frac{\partial}{r\partial\phi}\hat{\phi}$. Considering a periodic boundary condition, find the energy spectrum and corresponding eigenstates of the Hamiltonian.

d2) (15 pts) Now magnetic field $\vec{B} = B\hat{z}$ is applied inside the cylinder. Find the vector potential at the ring and the energy spectrum of the system. At which magnetic field is the ground state energy degenerate?

수험번호		성 명		감독교수 확 인	(인)

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과목명 : 전기역학 2014 . 6 . 20 시행

1. (50 pts)

(a) (10 pts) A lump of copper with a charge q hangs from an insulating thread as shown in Fig 1 (A). Describe the charge distribution if the lump of copper has a cavity within it.

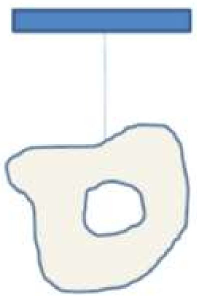

Fig 1 (A)

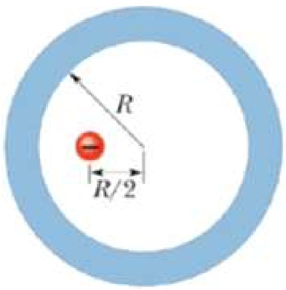

Fig 1 (B)

(b) (40 pts) Fig 1 (B) shows a cross section of a spherical metal shell of inner radius R. A negative point charge of −q1 is located at a distance R/2 from the center of the shell. If the shell is electrically neutral, what are the (induced) charges on its (i) inner and (ii) outer surfaces?

(iii) Are those charges uniformly distributed?

(iv) What is the field pattern inside and outside the shell?

(v) If the spherical metal shell is not neutral but has positive charge +q2, what is the electric field pattern inside and outside the shell?

2. (60 pts) Consider the plane electromagnetic waves in conductors of the dielectric permittivity ε, the magnetic permeability μ, and the conductivity σ.

(a) (15 pts) Show that the electric field $\vec{E}$ satisfies

$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\varepsilon \frac{\partial^2 \vec{E}}{\partial t^2} \quad .$$

(b) (20 pts) Find the expression for the wave number $K = k + i\kappa$ of the electromagnetic wave, where k and κ are real numbers. [You may use the result of problem (a) even if you have not derived it.]

(c) (10 pts) Show that the skin depth in a poor conductor (σ << ωε) is (2/σ)(ε/μ)1/2.

(d) (15 pts) Show that the skin depth in a good conductor (σ >> ωε) is λ/2π (where λ is the wave length in the conductor). Find the skin depth for a sea water of conductivity $\sigma = 4.3\ (\Omega m)^{-1}$ in the microwave range of 1GHz. (Here $\mu \sim \mu_0 = 4\pi \times 10^{-7}$ N/A^2)

수험번호		성 명		감독교수 확 인	(인)

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3. (45 pts) This is a problem on possible modifications to Coulomb's law for the electric field.

(a) (10 pts) The usual expression for the electric field at point $\vec{x}$, due to a point charge(at rest) at the location $\vec{x_0}$, is $\dfrac{q(\vec{x}-\vec{x_0})}{|\vec{x}-\vec{x_0}|^3}$. Explain in what sense this is a consequence of Maxwell's equations for electromagnetism.

Now consider the theory where the electric field due to the point charge is given by the form

$$\vec{E}(\vec{x}) = \frac{q(\vec{x}-\vec{x_0})}{|\vec{x}-\vec{x_0}|^{3+\delta}}$$

where δ is some small nonzero number. In this case, answer followings:

(b) (20 pts) Calculate $\vec{\nabla}\cdot\vec{E}$ and show that $\vec{\nabla}\times\vec{E}=0$ at $\vec{x}\neq\vec{x_0}$, for the given electric field $\vec{E}(\vec{x})$. Also find the corresponding electric potential $\Phi(\vec{x})$ in space due to the point charge.

(c) (15 pts) Consider a thin spherical shell of radius a that carries a uniform surface charge density $Q/4\pi a^2$. Taking the coordinate origin at the center of the shell, compute the potential $\Phi(\vec{x})$ due to this spherical charge distribution for both $|\vec{x}|>a$ and $|\vec{x}|<a$. [How do the results differ from the Maxwell theory predictions?]

4. (45 pts) Consider an infinitely long straight conducting wire(along the z-axis), carrying a uniform, but time-dependent, current $I(t)$. At the position $\vec{r}$ at perpendicular distance ρ from the wire (see the figure), the retarded vector potential $\vec{A}(\vec{r},\vec{t})$ is given by

$$\vec{A}(\rho,t) = \frac{\hat{z}}{c}\int_{-\infty}^{\infty} dz' \frac{I\left(t-\dfrac{\sqrt{\rho^2+(z')^2}}{c}\right)}{\sqrt{\rho^2+(z')^2}} \quad .$$

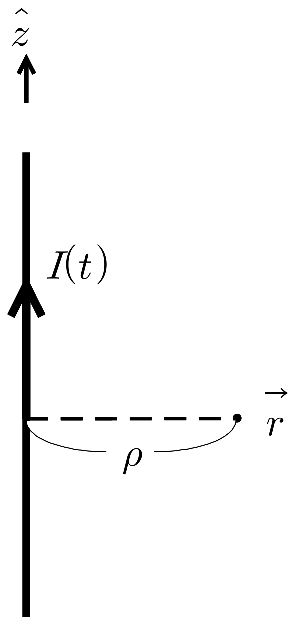
(a) (20 pts) Now, when a nonzero current I_0 is suddenly turned on at $t=0$, i.e.,

$$I(t) = \begin{cases} 0, & t<0 \\ I_0\,(:\text{const.}), & t>0, \end{cases}$$

find the vector potential $\vec{A}(\vec{r},\vec{t})$ in space explicitly.

(b) (15 pts) For the case of (a), determine the corresponding electric and magnetic fields.

(c) (10 pts) Using the results of (b), calculate the total radiation power emitted through the cylindrical surface of radius ρ and length L(along the direction parallel to the wire).



소속대학원	물리·천문학부	학번		성 명		감독교수 확 인	(인)

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과목명 : 통계역학 2014 . 06. 20 시행

1. (60 pts) The partition function for an interacting gas with mass m is assumed to be $Z = \left(\frac{V-nb}{N}\right)^N \left(\frac{mk_BT}{2\pi\hbar^2}\right)^{3N/2} \exp\left(\frac{n^2 a}{Vk_BT}\right)$, where a and b are constants while n is the number of moles, i.e. $N=nN_A$. And a is positive (negative) for attractive (repulsive) potentials, respectively. (N_A: Avogadro constant, k_B: Boltzmann constant, $\hbar$: Planck consant)

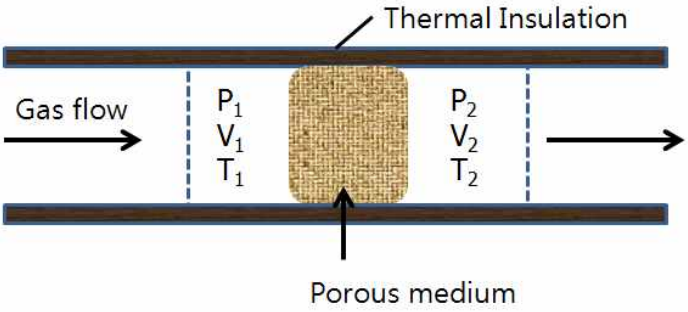
(a) (20 pts) Find out the relation between the volume V, temperature T and pressure P (van der Waals' equation).

(b) (10 pts) Calculate the internal energy and discuss the effects of the interaction parameter a.

(c) (20 pts) Show that the isobaric expansivity $\beta_P = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$ of the van der Waals gas is given by $\beta_P = \frac{1}{T}\left(1 + \frac{b}{V-b} - \frac{2a}{pV^2+a}\right)^{-1}$. For convenience, we assume $n=1$ here.

(d) (10 pts) What happens to this quantity close to the critical point?

2. (40 pts) The Joule-Thomson effect describes the temperature change of a gas when it is forced through a valve or a porous plug while kept insulated so that no heat is exchanged with the environment. See the figure for schematic description.



(a) (10 pts) Show that enthalpy $H=U+PV$ remains constant during the Joule-Thomson process.

(b) (20 pts) Express the Joule-Thomson coefficient $\mu_{JT} = \left(\frac{\partial T}{\partial P}\right)_H$ in terms of V, C_P, T and β_P (heat capacity at constant pressure $C_P = \left(\frac{\partial H}{\partial T}\right)_P$, isobaric expansivity $\beta_P = \frac{1}{V}\left(\frac{\partial V}{\partial T}\right)_P$).

(c) (10 pts) Calculate this Joule-Thomson coefficient for ideal gas. Once you have the results, interpret its physical meaning and discuss how it is different from a real gas.