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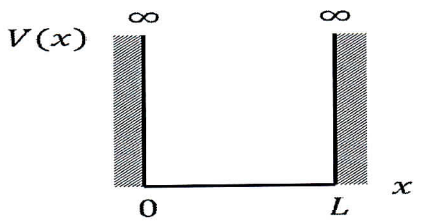
# 물리학부 자격시험 구술시험
## 시험지 및 답안지

**과목명 : 양자역학** 2012 . 6 . 22 시행

1. Suppose a one-dimensional infinite square well potential of width $L$ as shown below.



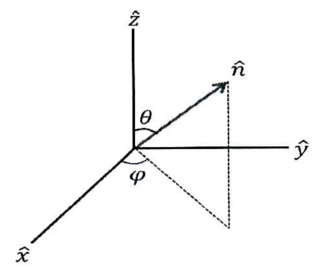

Consider a particle of mass $m$ confined in the potential which has the initial wavefunction of

$\psi(x,0) = A\left(2\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right)$ for $0 \leqq x \leqq L$.

(a) Find the energy eigenvalues and corresponding eigenstates which satisfy the boundary condition. (5 pts)
(b) Determine $A$ and find $\psi(x,t)$ at a time $t > 0$. (10 pts)
(c) What is the expectation value of the energy at $t$? (5 pts)
(d) Calculate the standard deviation of the momentum operator, $\sigma_p = \sqrt{<p^2> - <p>^2}$ at $t$. If $L$ is reduced or increased, how does $\sigma_p$ change? Explain this from the uncertainty principle. (10 pts)

2. Consider a bound state of two spin 1/2 particles with the same mass. The spin interaction of two particles gives the Hamiltonian $H_s = A\,\vec{S_1}\cdot\vec{S_2}$.
(a) Find out the eigenstates and the eigenvalues of the system. (10 pts)
(b) When an external magnetic field is applied, the Hamiltonian is modified to $H = H_s + H_B$ with $H_B = a\vec{S_1}\cdot\vec{B} + b\vec{S_2}\cdot\vec{B}$. If two particles have the same charge, what is the relation between $a$ and $b$? ($a > 0$) (5 pts)
(c) Find out the eigenstates and the eigenvalues in the strong magnetic field limit, $|aB|, |bB| \gg |A|$? (10 pts)
(d) For the angular momentum operator $\vec{J}$, the raising and lowering operators satisfy the following relation:
$J^{\pm}|j\ m> = \hbar\sqrt{(j \mp m)(j \pm m + 1)}\,|j\,m \pm 1>$
where $J^{\pm} = J_x \pm iJ_y$. Using this result, find out the relation between the eigenstates of $H_s$ and $H_B$. (10 pts)
(e) Draw the spectrum line as a function of the external magnetic field : (i) weak field limit, $|aB|, |bB| \ll |A|$, (ii) strong field limit, $|aB|, |bB| \gg |A|$, and (iii) the interpolation regime. (15 pts)

3. Imagine a particle of spin 1/2 under a uniform magnetic field along a direction $\hat{n}$, then we can write down the Hamiltonian as the following form: $H = \omega\hat{n}\cdot\vec{S}$ where $\omega$ is a real positive number and $\vec{S}$ is the spin operator with spin 1/2. Set spin up and spin down states along $S_z$ as $\begin{pmatrix}1\\0\end{pmatrix}$ and $\begin{pmatrix}0\\1\end{pmatrix}$, respectively.




(a) An arbitrary direction can be specified by $\hat{n} = (\sin\theta\cos\phi, \sin\theta\sin\phi, \cos\theta)$ in spherical coordinates, as shown above.

Prove that $\chi_+^{\hat{n}} = \begin{pmatrix}\cos\frac{\theta}{2}\\ e^{i\phi}\sin\frac{\theta}{2}\end{pmatrix}$ and $\chi_-^{\hat{n}} = \begin{pmatrix}-\sin\frac{\theta}{2}\\ e^{i\phi}\cos\frac{\theta}{2}\end{pmatrix}$ are the eigenstates of the Hamiltonian. Find the corresponding energy eigenvalues. (5 pts)
(b) Next, suppose that $\hat{n} = \hat{z}$ and an initial state is given by $\chi(0) = \begin{pmatrix}\cos\frac{\theta}{2}\\ \sin\frac{\theta}{2}\end{pmatrix}$. Find $\chi(t)$ at a time $t > 0$. (10 pts)
(c) For the case of (b), calculate $\langle S_x\rangle, \langle S_y\rangle$, and $\langle S_z\rangle$ at $t$. (10 pts)
(d) From the time-dependent Schrödinger equation, show that $i\hbar\frac{d}{dt}\langle\vec{S}\rangle = \langle[\vec{S}, H]\rangle$. For a general $\hat{n}$, derive $\frac{d}{dt}\langle\vec{S}\rangle = \omega\hat{n}\times\langle\vec{S}\rangle$. Using this result, interpret the result of (c). (15 pts)

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# 물리학부 자격시험 구술시험
## 시험지 및 답안지

과목명 : 통계역학　　　　　　2012 . 6 . 22 시행

1. Consider N number of Ar molecules with mass m, that is the molecular weight M = $N_A$ m = 40 g ($N_A$ : Avogadro number). Suppose that they are in equilibrium at a temperature of T = 300 K and the density of the molecules is low enough to be considered as an ideal gas.

(a) When the molecules are confined in a volume V. Write down the speed distribution using a phase space argument and a Boltzmann factor. By using the speed distribution, calculate the root-mean-square speed, $v_{rms}$ and discuss the results in view of the equipartition theorem. (8 pts)

( Hint: $\int_0^{\infty} x^{2n}\exp(-ax^2) = \frac{1\times 3\times 5\times \ldots \times(2n-1)}{2^{n+1}a^n}\sqrt{\frac{\pi}{a}}$ )

(b) Calculate the ratio of the potential energy and kinetic energy for the height of the box is 1 m and g = 10 m/s$^2$. (Gas constant R = k $N_A$ = 8.31 J mol$^{-1}$ K$^{-1}$ and k = Boltzmann constant) (7 pts)

(c) For the case of (b), write down the partition function Z and derive the Helmholtz free energy F. (8 pts)

( Hint : $\int_0^{\infty} \exp(-ax^2) = \frac{1}{2}\sqrt{\frac{\pi}{a}}$ ,

$\ln N! \cong N\ln N - N$ )

(d) Show that the chemical potential is given as follows: (7 pts)

$$\mu = kT\log\left[\frac{p}{kT}\left(\frac{h^2}{2\pi mkT}\right)^{3/2}\right]$$

2. We have a gas of photons in volume V and at temperature T.

(a) Show that the partition function is given by

$$\ln Z = -\frac{V}{\pi^2 c^3}\int_0^{\infty} \omega^2 \ln(1-e^{-\hbar\beta\omega})\, d\omega$$

and calculate Z by integration. Note that we ignore the zero point energy here. (8 pts)

( Hint: $\int_0^{\infty} \frac{x^3}{e^x - 1}dx = \zeta(4)\Gamma(4) = \frac{\pi^4}{15}$ )

(b) Calculate the chemical potential of photons and demonstrate that it has a zero chemical potential. (7 pts)

(c) Derive the total number of photons $N = \frac{2\,\zeta(3)}{\pi^2}\left(\frac{kT}{\hbar c}\right)^3 V$

where $\zeta(3) = 1.2$ is a Riemann zeta function. (8 pts)

( Hint: $\int_0^{\infty} \frac{x^2}{e^x - 1}dx = \zeta(3)\Gamma(3)$ ,

where $\Gamma(n) = (n-1)!$ )

(d) Evaluate the average energy per photon. (7 pts)

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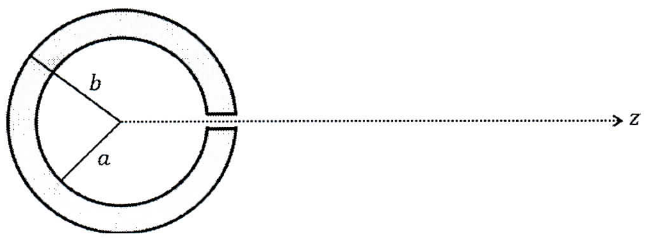
## 시험지 및 답안지

**과목명 : 전기역학** 2012 . 6 . 22 시행

1. A metal spherical shell (inner radius $a$ and outer radius $b$) is located with its center at the origin. There is a very small pinhole, which is negligible for calculations, at one point of the shell, as shown in the figure below.




Assume that the shell is isolated and has no net charge, answer the following questions:

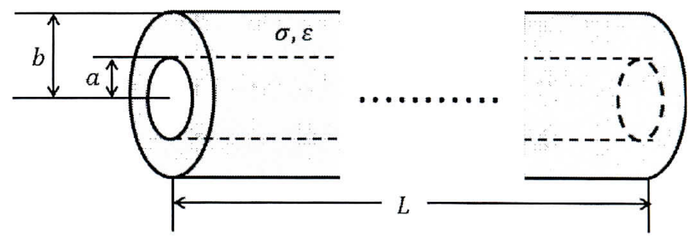
(a) A point charge $q$ is placed outside of the shell at $\vec{r}=(0,0,d)$ $(d>b)$. In such a case, electric field outside of the shell can be derived by assuming two image charges $q'$ and $-q'$ at $(0,0,\frac{b^2}{d})$ and at $(0,0,0)$, respectively. Determine $q'$ in terms of given parameters. (10 pts)

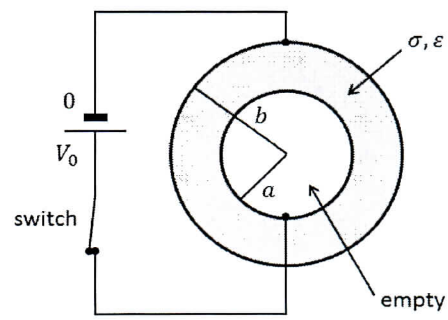
(b) Calculate the potential $V(\vec{r})$ everywhere for the case above. (10 pts)

(c) Calculate the force $\vec{F}$ the point charge feels when $d<a$, i.e., the charge is inside of the shell. (10 pts)

(d) How much work $W$ is required to bring the point charge $q$ from infinity, through the pinhole, to the origin? (10 pts)

2. Two long highly conducting coaxial tubes (radii $a$ and $b$ with total length $L$ and negligible thickness) are separated by a material of conductivity $\sigma$ and dielectric constant $\epsilon$. [The conductivity of the tube is much higher than $\sigma$, so each tube can be considered as an equipotential surface. Let us neglect the fringing effects due to the edges and thickness of the tubes.] The tubes have potential difference $V_0$ maintained by a battery.




(a) Evaluate the capacitance $C$ between the tubes. (10 pts)

(b) Evaluate the resistance $R$ between the tubes. What is the relationship between $R$ and $C$ ? (10 pts)

(c) If you disconnect the switch at $t=0$, the charge will gradually leak off. What is the potential difference $V(t)$ across the tubes as a function of time? (10 pts)

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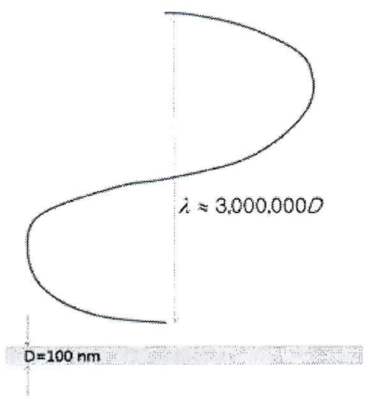
과목명 : 전기역학 2012 . 6 . 22 시행

3. A microwave of frequency $f=\dfrac{\omega}{2\pi}=1$ GHz ($\lambda=30$ cm) is incident normally on a gold film of $D=100$ nm thick. Calculate the transmission of the microwave through the gold film, which has a good conductivity $\sigma=4\times10^{7}\,\Omega^{-1}\mathrm{m}^{-1}$, in the following way:




(a) In the Maxwell's equations

$$\nabla\times\vec{B}=\mu_0\vec{J}+\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}=\mu_0\sigma\vec{E}+\mu_0\epsilon_0\frac{\partial\vec{E}}{\partial t}$$

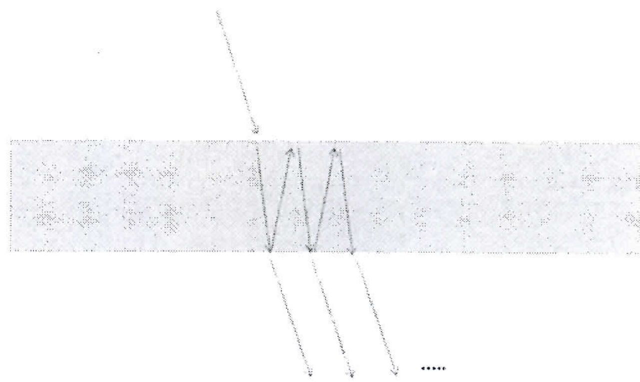
$$\nabla\times\vec{E}=-\frac{\partial\vec{B}}{\partial t},\ \nabla\vec{E}=0,\ \nabla\vec{B}=0$$

show numerically that the displacement current $\epsilon_0\dfrac{\partial\vec{E}}{\partial t}$ is negligibly small compared to the real current $\vec{J}$ in the present case. ($\epsilon_0=8.85\times10^{-12}$ F/m.) (8 pts)

(b) Indicate that modified Helmholtz equation $\nabla^2\vec{E}\approx-i\mu_0\sigma\omega\vec{E}$ holds. (8 pts)

(c) Show that the wavefront inside the conductor moves with $e^{i\tilde{k}z}\equiv e^{ik_0\tilde{n}z}\equiv e^{iz(1+i)/\delta}$ where $k_0=\dfrac{\omega}{c}$. Determine $\delta$. (8 pts)

(d) The microwave undergoes multiple reflections before it is transmitted.



When the amplitude of the initial wave is $E_0$, calculate the amplitude of singly reflected wave (reflected only once at the bottom of the film) at the top of the film. You may use $T(T')$ for transmission coefficients at the top (bottom) surface of the film, and $R$ for reflection coefficient. (10 pts)

(e) Noting $|\tilde{n}|\gg1$ and $|\tilde{k}D|\ll1$, show that the amplitude of transmitted wave is given by

$$\frac{E_t}{E_0}=\frac{TT'e^{i\tilde{k}D}}{1-R^2e^{2i\tilde{k}D}}\approx\frac{2}{2+D\mu_0\sigma c}$$

Here, you may use the following formulae for transmission and reflection coefficients: $T=\dfrac{2}{\tilde{n}+1}$, $T'=\dfrac{2\tilde{n}}{\tilde{n}+1}$, and $R=\dfrac{\tilde{n}-1}{\tilde{n}+1}$. Give the numerical value ($\mu_0=4\pi\times10^{-7}$ H/m). (10 pts)

(f) How thin the film must be for the transmission to be significant, say, $\dfrac{E_t}{E_0}=0.1$? Is it realistic? (6 pts)

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**과목명 : 고전역학** 2012 . 6 . 22 시행

1. Let us consider a particle of mass $m$ moving in 3-dimensional space. This particle is acted on by attractive force whose potential is given by $U(r) = -k/r^4$. The particle is coming from infinity with an initial velocity $V_\infty$.

(a) Let us define the contribution from the potential energy and the angular momentum part to the Hamiltonian collectively as the effective potential. What is the effective potential for the radial motion of the particle when its angular momentum is $L$. Express the angular momentum with the impact parameter $b$. (10 pts)

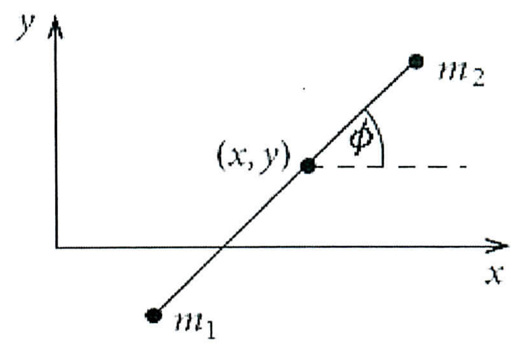
(b) Usually the particle can have two kinds of motion: 1) the particle can approach to a minimum distance $r_{\min} > 0$ and then go away to the infinite distance far, or 2) the particle may collide with the center ($r=0$) of the potential. Let us consider the second case of 2). What is the maximum impact parameter $b_{\max}$ for the particle to fall to the force center, i.e. $r=0$. Write down the total cross section for this falling-in process. (10 pts)

(c) Let's consider the case with $U(r) = -k/r^2$. Once again let us consider the case that the particle falls into the force center ($r=0$). What is the maximum impact parameter $b_{\max}$ in this case? What is the corresponding total cross section for the falling-in process? (10 pts)

2. Two point masses $m_1 = m_2 = m$ are connected by a massless rod of length $l$, forming a barbell. They move on the $xy$-plane (on the 2-dimensional surface) and are subject to a frictional force which is proportional to their velocity,

$F_i = -\alpha \dot{r}_i$, (i = 1, 2, $\alpha > 0$)

(Here (x,y) is coordinate for center of barbell)




(a) Formulate the constraints and choose the appropriate generalized coordinates. (10 pts)

(b) What are the equations of motion? (10 pts)

(c) Solve them with the initial conditions given: (10 pts)

★ Center of the barbell: $x(0) = y(0) = 0$; $\dot{x}(0) = v_{ox}$; $\dot{y}(0) = v_{oy}$

★ Angle: $\phi(0) = 0$; $\dot{\phi}(0) = \omega$