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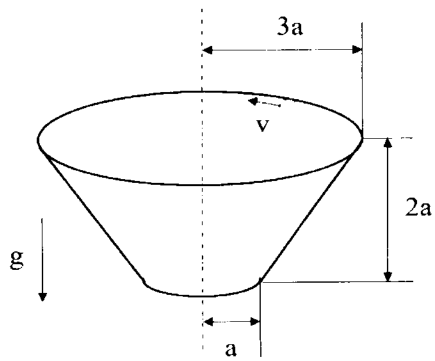
2019학년도 석사 및 석·박통합과정 전기모집 면접·구술고사 전공시험

과목명 : 역학 2018.10.19. 시행

Consider a ball which is moving without friction on the inner wall of a funnel as shown below. The funnel axis is aligned to the gravity direction and the ball starts its motion at the upper rim of the funnel with velocity of v along the azimuthal direction. Assume that the ball is negligibly small to the funnel size and ignore the ball's rolling motion. The ball's mass is given by m and the funnel is fixed with repect to the ground.



(a) [5 pts] What is the velocity v_1 required for the ball to keep rotating along the upper rim of the funnel?

(b) [5 pts] When the initial velocity v is slightly smaller than v_1, the ball oscillates up and down in the funnel. Calculate the oscillation period T for the small oscillations.

(c) [5 pts] When v is smaller than a certain critical velocity v_2, the ball can exit down out of the funnel. Obtain the critical velocity v_2.

(d) [5 pts] For $v_2 < v < v_1$, calculate the depth d that the ball reaches into the funnel. Check if your answer gives $d(v) = 2a$ for $v = v_2$.

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과목명 : 전자기학

2018.10.19. 시행

1. (12 pts) Consider an ideal gas of particles of mass m and charge q moving in a two-dimensional plane under a force $\vec{F}$. Then the equation of motion for a particle is given by

$$m\left(\frac{d\vec{v}}{dt}+\frac{\vec{v}}{\tau}\right)=\vec{F}.$$

Here we introduce the relaxation time τ to include the effect of impurity scattering.

(a) (2 pts) Assume that an electric field $\vec{E}=E_x\hat{x}+E_y\hat{y}$ is applied along the in-plane direction and a magnetic field $\vec{B}=B\hat{z}$ is applied perpendicular to the plane. Write down the equation of motion for each direction.

(b) (4 pts) From now on, assume that the system is in the steady state. Obtain the resistivity tensor ρ_{ij} $(i,j=x,y)$ defined by $E_i=\sum_j \rho_{ij}J_j$ where $\vec{J}=nq\vec{v}$ is the current density and n is the particle density.

(c) (3 pts) Draw schematically ρ_{xx} and ρ_{xy} as a function of B. If τ is increased to 2τ, how do ρ_{xx} and ρ_{xy} change? Here assume that $q=-e<0$.

(d) (3 pts) Discuss what we can learn by measuring the sign and magnitude of ρ_{xy} as a function of B.

2. (8 pts) A dielectric sphere of radius R and dielectric constant ϵ is placed in vacuum in a uniform electric field $\vec{E}=E\hat{z}$ along the z direction. Let the electrostatic potentials inside and outside of the sphere be $\Phi_{\rm in}$ and $\Phi_{\rm out}$, respectively.

(a) (3 pts) Write down the boundary conditions at $r=0$, $r=R$ and $r=\infty$, respectively.

(b) (5 pts) Determine $\Phi_{\rm in}$ and $\Phi_{\rm out}$.

Note that the general form of the solution to the Laplace's equation for an axially symmetric situation is given by

$$\Phi=\sum_{n=0}^{\infty}\left(A_n r^n+\frac{B_n}{r^{n+1}}\right)P_n(\cos\theta)$$

where $P_n(x)$ are the Legendre polynomials.
The first few polynomials are given by

$P_0(x)=1,\quad P_1(x)=x,\quad P_2(x)=\frac{1}{2}(3x^2-1),$

$P_3(x)=\frac{1}{2}(5x^3-3x),\ldots$ with the recursion relation $(n+1)P_{n+1}(x)=(2n+1)xP_n(x)-nP_{n-1}(x)$.

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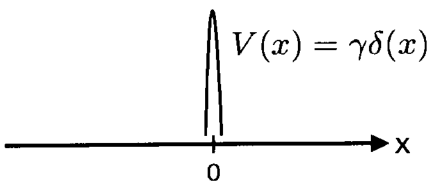
과목명 : 양자역학 2018.10.19. 시행

1. [15pt] A particle of mass m and wavenumber k is incident from the left ($x < 0$) upon a delta-function potential, $V(x) = \gamma\delta(x)$ at $x = 0$. Its incident wave function is $\psi(x) = e^{ikx}$, which is transmitted with wave function te^{ikx} or reflected with wave function re^{-ikx}, where t and r are complex coefficients.



(a) [3pt] Write the form of the wave functions, $\psi_-(x)$ for $x < 0$ and $\psi_+(x)$ for $x > 0$, in terms of t and r.

(b) [3pt] Find the condition on t and r, from the number (or flux) conservation.

(c) [5pt] Find the boundary conditions of the wave functions at $x = 0$.

(d) [4pt] Calculate the probabilities of the transmission and reflection. Also, check whether they satisfy the number conservation found in (b).

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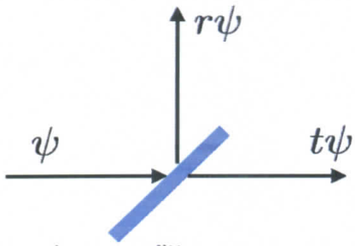
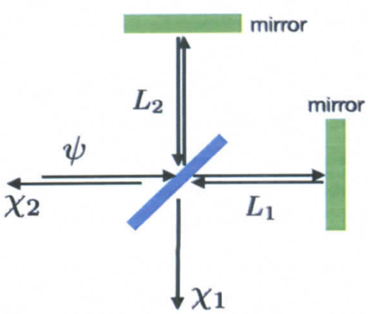
과목명 : 양자역학 2018.10.19. 시행

2. [10pt] A particle with wave function ψ is incident upon a beam splitter. It is transmitted with wave function $t\psi$ and reflected with $r\psi$, where t and r are complex coefficients.



(a) [2pt] Find the condition on t and r from the number (or flux) conservation.

(b) [2pt) Now, both waves are reflected back to the beam splitter by a perfect mirror in each direction. Mirrors are located by the same distance $L_1 = L_2 = L$ away from the beam splitter. After these waves pass the beam splitter again, two outgoing waves χ_1 and χ_2 are produced.



Write χ_1 and χ_2, in terms of t, r and ψ. Find the condition on t and r from the number conservation of outgoing waves. (Ignore any extra phases from mirror reflections.)

(c) [2pt] Calculate t and r by combining the conditions obtained in (a) and (b) and by assuming that $|t| = |r|$ and r is real.

(d) [4pt] The propagation phase $\phi_{\text{prop}}(x) = kx$ shall be additionally multiplied, with wavenumber k and the distance x traveled from the beam splitter. (Ignore other contributions to the propagation phase.)

Suppose that the length L_2 changes by $L_2 = L(1+\delta)$ with $\delta \ll 1$ a small dimensionless number, while $L_1 = L$ remains unchanged.

Calculate the intensity of χ_1, using the t and r values found in (c).

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과목명 : 열 및 통계 물리 2018.10.19. 시행

1. Consider a system composed of N identical localized classical dipoles $\vec{\mu}_i$ (i=1,2,...,N) under magnetic field along the z-direction $\vec{H}=H\hat{z}$. When the interaction between dipoles is neglected, the total energy of the system is given by $E=-\sum_{i=1}^{N}\vec{\mu}_i\cdot\vec{H}=-\mu H\sum_{i=1}^{N}\cos\theta_i$ where θ_i is the relative angle between the i-th local magnetic moment $\vec{\mu}_i$ and $\vec{H}$. At a finite temperature, the directional fluctuation of each local magnetic moment $\vec{\mu}_i$ with a fixed magnitude μ can be described by using two angular variables $\theta_i\in(0,\pi),\phi_i\in(0,2\pi)$.

(1) (4 pts) At the temperature T, find the partition function of the system $Q_N(\beta)$ where $\beta=1/(kT)$ and k is the Boltzmann constant.

(2) (4 pts) Assuming that there are N_0 magnetic dipoles per unit volume, compute the average magnetic moment per unit volume M_0.

(3) (4 pts) Show that the isothermal magnetic susceptibility χ_T can be written as $\chi_T=C/T$ in the high temperature limit and find the expression of C.

(4) (8 pts) To treat the magnetic dipoles quantum mechanically, let us assume that $\vec{\mu}_i=\frac{g\mu_B}{\hbar}\vec{J}_i$, $J_i^2=J(J+1)\hbar^2$, $J_{iz}=m\hbar$ where g is the Lande g-factor, μ_B is the Bohr magneton, and $m=-J,-J+1,\ldots,J-1,J$. Find the partition function, the average magnetic moment per unit volume. Show that the isothermal magnetic susceptibility χ_T can be written as $\chi_T=\tilde{C}/T$ in the high temperature limit and find the expression of $\tilde{C}$.

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과목명 : 실험

2018.10.19. 시행

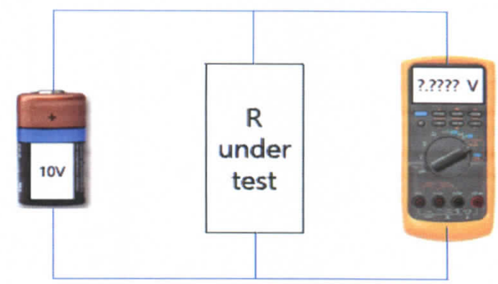
Performing correct electrical measurement is essential in any physics experiments. Consider a situation where a resistor is given. We know that its value is in range from 0.1 to 1 $M\Omega$, and we want to measure its resistance value more precisely.

In an ideal measurement, one flows some amount of *known* electrical current to the resistor, measure voltage drop *exactly* across the resistor, and figure out the resistance from the Ohm's law (current source – voltage measure). However, in this problem only limited equipments are available as listed below. The source and meter are non-ideal instruments having finite internal and input resistances.

Instruments	Example picture	Schematic circuit diagram
10V voltage source with internal resistance 1 $M\Omega$		You draw. Problem (a)
Ideal resistors of values 10, 100, 1k, 10 k, 100 k, 1M, 10M, and 100M Ohms.		You draw. Problem (a)
A voltmeter having input resistance 5 $M\Omega$ and 0.1 mV resolution		You draw. Problem (a)

(a) (4 points) Draw schematic circuit diagram of the each instrument including non-ideal effects like internal resistance.

(b) (4 points) One first tries the following wiring, which is voltage source – voltage measure. If the battery and the voltmeter were ideal instruments, can you measure the value of R ? Why or why not ?



Convert the diagram to circuit diagram with the result of (a), and briefly explain how to measure the value of R when the sources and meters are not ideal – you don't need to do the circuit analysis.

(c) (7 points) In practice, we want to perform current source – voltage measure or voltage source – current measure, rather than voltage source – voltage measure. Explain how you can convert either your battery to practical current source or your voltmeter to current meter within the following condition,

- The value of R is in range from 0.1 – 1 $M\Omega$
- The voltmeter has 0.1 mV resolution.

and estimate roughly (order of magnitude) how precisely you can measure the value of R in current source – voltage measure and voltage source – current measure mode. Which mode is better for measuring R ranging from 0.1 –1 $M\Omega$?