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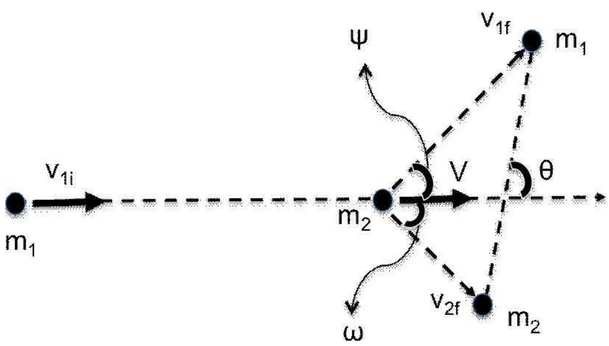
2017학년도 석사과정/석사·박사통합과정 전기모집 면접·구술고사 전공시험

2016.10.24. 시행

1. (20 pt) A particle of mass m_1 collides head-on with a stationary particle of mass m_2 as shown in the figure. The collision is elastic and the initial and final velocities of m_1 are v_{1i} and v_{1f}., and the velocity of m_2 in final state is v_{2f}. V is the velocity of the center of mass (CM) in the lab frame.



1) (7 pt) Using the CM frame of the particle system, derive the relation between Ψ and Θ. Use the momentum and energy conservation under non-relativistic condition. Show Ψ~Θ when $m_1 << m_2$ and Ψ=Θ/2 when $m_1 = m_2$.

2) (7 pt) The m_1 particle is accelerated up to 0.8 times the speed of light and it collides head-on again with the m_2 particle (stationary). After the collision, the m_1 particle is embedded in the m_2 particle. Determine the rest mass and velocity of the composite particle after the collision.

3) (6 pt) Assume two protons collide head-on (proton rest mass is about 1 GeV). Here, the head-on collision means that the two protons have the same kinetic energy just before the collision. Let us assume that each proton has a kinetic energy T. When we change the set up of the experiment as above figure: one proton (m_1) is moving and the other proton ($m_2 = m_1$) is at rest. Find the relativistic total energy (E) of the m_1 in terms of T to get the same result (do not consider the internal structure of the proton).

2. (20 pt) A perfect conductor is filling the region $x < 0$ of space, and a uniform magnetic field $B > 0$ is applied along the z direction. A particle with mass m and electric charge $q > 0$ is placed at $x(t=0) = d > 0$, with zero initial velocity.

(a) (6 pt) Write down the particle's Newton equations for $x(t), y(t), z(t)$, and show that $z(t)$ is constant with the above initial condition. Also, show that

$$m\frac{dy(t)}{dt} = -qB(x(t)-d) \quad .... (*)$$

by solving one of the equations of motions.

(b) (6 pt) Obtain the expression for the conserved energy E, in terms of $x(t), y(t), \frac{dx(t)}{dt}, \frac{dy(t)}{dt}$. By substituting the formula (*), show that one can write the conserved energy in the following form,

$$E = \frac{m}{2}(\frac{dx}{dt})^2 + V_{eff}(x) \ .$$

What is the effective potential $V_{eff}(x)$?

(c) (8 pt) We would like to understand whether the particle will hit the boundary of the perfect conductor or not. Show that the particle hits the conductor if

$$B^2 < \frac{m}{2\pi\epsilon_0 d^3} \ ,$$

and changes its velocity to the +x direction before hitting the conductor if

$$B^2 > \frac{m}{2\pi\epsilon_0 d^3} \ .$$

What happens if $B^2 = \frac{m}{2\pi\epsilon_0 d^3}$?

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3. (13 pt) In fig. 1, magnetic field in z direction can be turned off and on, and exists only inside the very thin, long tube which can completely shield out the magnetic field. Electrons are moving outside the tube as free particles.

The Hamiltonian of an electron when the magnetic field is turned on is given by $H=\frac{1}{2m_e}\left(\vec{p}+\frac{e\vec{A}}{c}\right)^2$, where $\vec{A}$ ($\vec{B}(\vec{r})=\nabla\times\vec{A}(\vec{r})$) is a vector potential. Then, the quantum mechanical Hamiltonian can be written as $\hat{H}=\frac{1}{2m_e}\left(\hat{p}+\frac{e\vec{A}}{c}\right)^2$ where $\hat{p}=-i\hbar\nabla$. The time independent Schrodinger's equation from the Hamiltonian above is given by $\frac{1}{2m_e}\hat{\Pi}^2\psi=\frac{1}{2m_e}\left(\frac{\hbar}{i}\nabla+\frac{e\vec{A}}{c}\right)^2\psi=\varepsilon\psi$,where $\hat{\Pi}=\left(\frac{\hbar}{i}\nabla+\frac{e\vec{A}}{c}\right)$ is a newly defined kinetic momentum.

(a) (3 pt) When $\vec{B}$ = 0, an electron's wave function ψ has an eigen momentum $\hbar\vec{k}$ or $\hat{p}\psi=\hbar\vec{k}\psi$. When the $\vec{B}$ field is turned on, since there is no $\vec{B}$ field outside the tube and no Lorentzian force is exserted on the electrons, hence we can assume the electron's wave function with $\vec{B}$ field ψ_A will have the same momentum eigenvalue $\hbar\vec{k}$ as when $\vec{B}$ = 0 as long as we use $\hat{\Pi}$ as a new momentum operator. Namely, $\hat{\Pi}\psi_A=\left(\frac{\hbar}{i}\nabla+\frac{e\vec{A}}{c}\right)\psi_A=\hbar\vec{k}\psi_A$. Assuming $\psi=Ce^{i\vec{k}\cdot\vec{r}}$, $\psi_A=Ce^{i\vec{k}\cdot\vec{r}}e^{i\phi(\vec{r},\vec{A})}$ (C is a normalization constant), find the proper expression for the phase $\phi(\vec{r},\vec{A})$ to satisfy this condition (hint : you can use an integral ($\int d\vec{r}\,\cdot$) form).

(b) (3 pt) We have freedom of choosing the gauge $\vec{A}(\vec{r})=\vec{A}(\vec{r})+\nabla\chi(\vec{r})$. Prove that the Schrodinger's equation $\frac{1}{2m_e}\hat{\Pi}^2\psi=\varepsilon\psi$ is invariant under this gauge transformation.

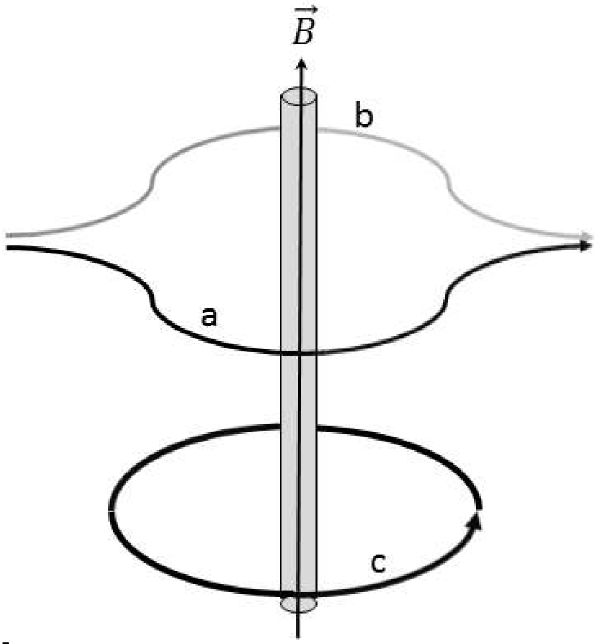

Fig.1

(c) (3 pt) $\vec{B}$ field is turned on and two electrons with the same momentum and phase travel along two different trajectories from the same point (fig.1). One electron moves along the grey trajectory b and the other electron travels along the black trajectory a. When the two electrons travel the same distance and end up at the same position on the right side of the tube, they show a modulated interference pattern as a function of the $\vec{B}$. What is the period of this modulation in terms of the flux $\Phi=\int\vec{B}\cdot d\vec{a}$?

(d) (4 pt) An electron travels along a closed curve c around the tube when there is a $\vec{B}$ field inside the tube. After one complete revolution, the wave function needs to be the same as when the electron was at the starting point. What condition does the phase of the wave function need to satisfy? What's the physical implication regarding the magnetic flux Φ?

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4. (14 pts) Consider the Schrodinger's equation for a particle with mass m^* in a spherically symmetric attractive potential $V(r) = -aV_0\delta(r-a)$ with positive a. The general form of the wavefunction is given by $\psi_{nlm}(r,\theta,\phi) = \left[\frac{1}{r}\chi_{nl}(r)\right]Y_{lm}(\theta,\phi)$ where $Y_{lm}(\theta,\phi)$'s are the spherical harmonics. The radial form of the Schrodinger's equation then is given by $-\frac{\hbar^2}{2m^*}\frac{d^2}{dr^2}\chi_{nl}(r) + \left[V(r) + \frac{\hbar^2}{2m^*}\frac{l(l+1)}{r^2}\right]\chi_{nl}(r) = E\chi_{nl}(r)$.

(a) (4 pts) The boundary condition for $\chi_{nl}(r)$ at r=0 is $\chi_{nl}(0) = 0$. What are the boundary conditions at r=a?

(b) (6 pts) Assuming that the potential has one or more bound states, find the equation for the (negative) energy E of the lowest-energy bound state.

(c) (4 pts) Find the condition on V_0 to have at least one bound state (1 pt). <u>You should draw a relevant graph of your choice (2 pts)</u> in order to prove why V_0 should be lower and / or higher than certain values.

5. (13 pt) Consider a three-dimensional system composed of N identical, localized (hence, distinguishable), mutually non-interacting classical dipoles. In the presence of external magnetic field $\vec{H}$, the total energy of the system is given by $E = -\sum_{i=1}^{N}\vec{m_i}\cdot\vec{H} = -mH\sum_{i=1}^{N}\cos\theta_i$ where θ_i is the angle between the i-th dipole with the magnetic moment $\vec{m_i}$ and the external magnetic field $\vec{H}$.

(a) (5 pt) Show that the partition function of the system is given by

$Q_N(\beta) = [Q_1(\beta)]^N = [4\pi\frac{\sinh(\beta mH)}{(\beta mH)}]^N$ where $\beta = 1/k_BT$.

(b) (5 pt) Show that the mean magnetization of the system $M = N\langle m\cos\theta\rangle$ is given by $M = Nm[\coth(\beta mH) - \frac{1}{(\beta mH)}]$.

(c) (3 pt) Show that in the limit of high temperature or small magnetic field where $\beta mH \ll 1$, the mean magnetization follows $M_z \approx \frac{Nm^2}{3k_BT}H$. (Hint: use the small x expansion of the function coth(x).)

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6. (20 pt) Light intensity measurement

We would like to measure the intensity of light. One way to do it is to use photo-conductivity in the following way. When light is shone on an electrical component X made of a photo-conductive channel, charge carriers are generated in the channel and the conductance C_X of the component increases. When the variation of the light intensity I is small, we can assume that the conductance change ΔC (= C - C_0) of the component X is linearly proportional to the light intensity change ΔI (= I - I_0), $C-C_0=\alpha$ x (I - I_0) where a is a constant. Thus, one can measure the intensity of the incident light by measuring the conductance C_X of the component X.

1) (7 points) Design a circuit to measure the intensity of the incident light using the component X. You may use all or some of the following components : a constant voltage source E_{const}, a constant current source I_{const}, a voltmeter V_m, an ammeter A_m, an Op-Amp, and a resistance R. You can also use other components if necessary.

2) (5 points) Assume that all electrical instruments are ideal instruments such that all input resistances are infinite and all output resistances are zero. Then, write down the expression for the light intensity in terms of the measured electrical signals (e.g. voltage v, electrical current i etc.)

3) (8 points) In real world, instruments are not ideal and have finite values of input and output resistances. These finite input and output resistances can generate measurement errors. Let's consider the non-ideal instruments with finite values of input and output resistances. Assume that the output resistance of your instruments is R_{out} and their input resistance is R_{in}. Draw an equivalent circuit diagram for your intensity measurement circuit including the input and output resistances of the used instruments. Estimate the possible measurement errors caused by the finite values of the input and output resistances.