

물리학부 석사입시 구술시험 시험지 및 답안지

과목명 :

2011 . 10 . 21 시행

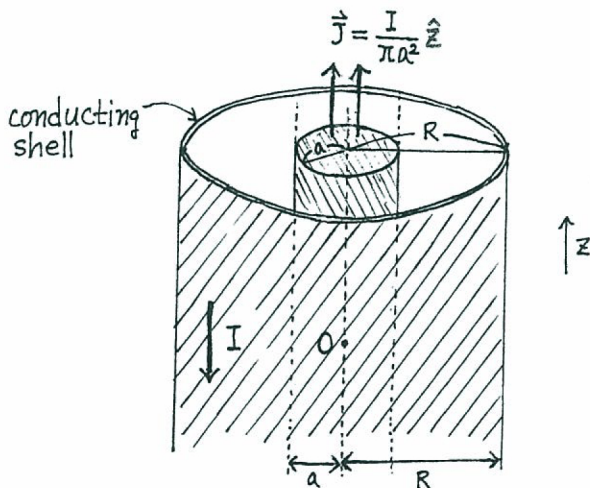
1. (25 points) An asteroid of mass m approaches the earth with initial velocity v_0 . The mass of the earth is M and its radius is R . The asteroid will strike the earth if the impact parameter b is smaller than certain value.

(a) (5 points) Write down the expression for initial energy and angular momentum relative to the Earth's center.

(b) (15 points) Find the relation between the impact parameter b and the distance of the closest approach to the earth, r_1 (measured from the Earth's center).

(c) (5 points) Find the largest impact parameter for which the asteroid strikes the earth.

2. (30 points) Consider a coaxial cable consisting of a cylindrical conductor of radius a surrounded by a concentric cylindrical conducting shell of radius R . (See the figure below). The inner conductor and the shell carry current I , flowing in opposite directions. Assume uniform current distributions in the inner conductor (i.e., the current density is equal to $\vec{J} = \frac{I}{\pi a^2} \hat{z}$ in the region $\rho < a$) and also in the shell.



(a) (6 points) Because of the symmetry of the given system, the magnetic field can be taken to be of the form $\vec{B}(\vec{r}) = B_\rho(\rho)\hat{\rho} + B_\phi(\rho)\hat{\phi}$, using cylindrical coordinates (ρ, ϕ, z) and the related set of unit vectors $(\hat{\rho}, \hat{\phi}, \hat{z})$. Then show that $B_\rho(\rho) = 0$ for all $\rho > 0$.

(b) (14 points) How is the function $B_\phi(\rho)$ given in each region of $0 < \rho < a$, $a < \rho < R$ and $\rho > R$.

(c) (10 points) Now imagine the situation that the current magnitude I varies slowly in time, i.e., $\frac{dI(t)}{dt} \neq 0$ in the same setup as above. Find the direction and magnitude of the induced electric field in the region $a < \rho < R$.

3. (30 points) Consider a nucleon inside a spherical nucleus. Assume that it moves in a spherical potential well of radius R

$$V_0(r) = \begin{cases} 0 & (r < R) \\ \infty & (r > R) \end{cases}.$$

The nucleon has mass m .

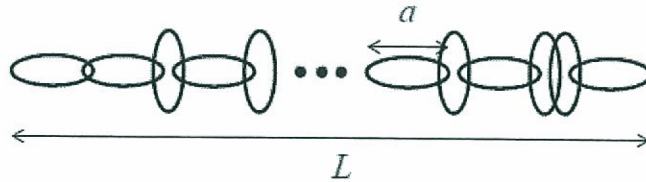
(a) (10 points) Obtain the energy eigenvalue E_0 and the corresponding normalized eigenfunction ψ_0 for the ground state of the nucleon. The Hamiltonian is $H_0 = T + V_0$, where T is the kinetic energy term.

[Hint: Note that $\nabla^2 = \frac{1}{r} \frac{\partial^2}{\partial r^2} r - \frac{\vec{L}^2}{r^2}$, where \vec{L} denotes the angular momentum operator.]

(b) (10 points) Let us now consider the Hamiltonian $H = H_0 + H_1$, where H_1 is quite small compared with H_0 . Then prove that, up to the first order in perturbation theory, the energy shift in the ground state is $\Delta E_0 = \langle \psi_0 | H_1 | \psi_0 \rangle$.

(c) (10 points) Suppose that the small perturbation H_1 is given by $H_1 = V_1(r) = v_0 \frac{r}{R}$ with $v_0 \ll E_0$ (for E_0 determined in (a)). Obtain the energy shift in the ground state, ΔE_0 , up to the first order in v_0 .

4. Consider a one-dimensional chain consisting of $N(\gg 1)$ segments as in the figure. Let the length of each segment be a when the long dimension of the segment is parallel to the chain and zero when the segment is vertical. Each segment has just two states, a horizontal orientation and a vertical orientation. There is no energy difference between the two states and the joints turn freely.



- (a) Find the entropy S of the chain as a function of the total length of the chain L . (6pt)
- (b) Using the energy conservation relation $dU = TdS + FdL$ (U is the internal energy of the chain), obtain a relation between the temperature T of the chain and the tension F which is necessary to maintain the length L . Use the Stirling's approximation ($\ln N! \simeq N \ln N - N$; $N \gg 1$). (6pt)
- (c) Show that your answer in (b) leads to the Hook's law at high temperature limit. (3pt)