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자격시험 문제

과목명 : 양자역학 2023 . 07. 28 시행

1. [50pts] Consider a harmonic oscillator system with following Hamiltonian

$$\hat{H} = \frac{1}{2}(\hat{x}^2 + \hat{p}^2)$$

where $\hat{x}$ and $\hat{p}$ satisfy the commutation relation $[\hat{x}, \hat{p}] = i\hbar$.

(a)[10pts] For $\hat{a} = \frac{1}{\sqrt{2\hbar}}(\hat{x} + i\hat{p})$, show that

$$\hat{H} = \hbar\left(\hat{a}^\dagger \hat{a} + \frac{1}{2}\right)$$

also show that allowed eigenvalues of $\hat{H}$ are $E_n = \hbar(n + \frac{1}{2})$ $(n = 0, 1, 2, \ldots)$.

(b)[10pts] Construct nth energy eigenstate $|n>$ in terms of the ground state $|0>$, $\hat{a}$ and $\hat{a}^\dagger$.
(The states are normalized : $<n|n> = 1$)

(c)[10pts] Get an explicit expression for the ground state wave function $\psi_0(x) = <x|0>$.

(d)[10pts] Construct the displacement operator $\hat{D}(z)$ using $\hat{a}$ and $\hat{a}^\dagger$ which has the following properties.

- $\hat{D}^\dagger(z)\hat{a}\hat{D}(z) = \hat{a} + z$
- $\hat{D}^\dagger(z)\hat{D}(z) = 1$

where z is a complex number.

(e)[10pts] Using the $\hat{D}(z)$ in (d), construct $|z>$ which is a normalized eigenstate of $\hat{a}$, i.e., $\hat{a}|z> = z|z>$.

2. [50pts] Suppose a one-dimensional infinite square well potential of width L:

$$V(x) = \begin{cases} 0, & 0 \le x \le L \\ \infty, & x < 0 \text{ or } x > L \end{cases}$$

Consider a particle of mass m confined in the potential.

(a)[20pts] Find the general solution (energy eigenvalues and normalized eigenfunctions) of the time-independent Schrödinger equation.

Let the initial wavefunction be

$$\psi(x,0) = A\left(2\sin\frac{\pi x}{L} - \sin\frac{3\pi x}{L}\right).$$

(b)[10pts] Determine the positive real number A by requiring $<\psi|\psi> = 1$ and find $\psi(x,t)$ at a time $t > 0$.

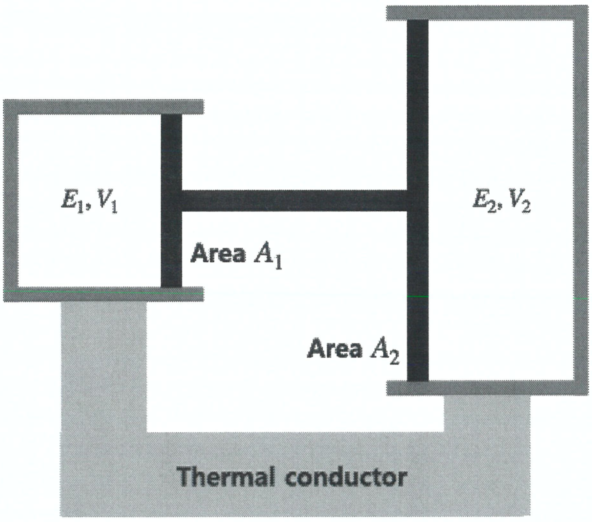
(c)[10pts] What is the expectation value of the energy at $t > 0$?

(d)[10pts] Calculate the standard deviation of the momentum operator, $\sigma_p = \sqrt{<p^2> - <p>^2}$ at $t > 0$. If L is reduced or increased, how does σ_p change? Explain this from the uncertainty principle.

자격시험 문제

과목명 : 통계역학 2023. 07. 28 시행

1. [50 pts] Consider an isolated system consisting of two cylindrical chambers. The chambers are connected via a thermal conductor and a rigid piston so that they can exchange both heat and volume (see the figure below). The energy, volume, and cross-sectional area of chamber i are denoted by E_i, V_i, and A_i, respectively, where $i = 1, 2$.



(a) [6 pts] Assume that all interactions in the system are short-ranged so that the two chambers are statistically independent. Express the number of possible microstates Ω of the whole system in terms of those of the two chambers, denoted by $\Omega_1(E_1, V_1)$ and $\Omega_2(E_2, V_2)$.

(b) [10 pts] Suppose that the position of the piston is fixed, allowing only the heat exchange between the two chambers. Derive the condition for thermal equilibrium in terms of $\Omega_1(E_1, V_1)$, $\Omega_2(E_2, V_2)$, and their partial derivatives.

(c) [10 pts] Express the temperature T_i of chamber i in terms of $\Omega_i(E_i, V_i)$ and its partial derivatives. Rewrite the condition for thermal equilibrium obtained in (b) in terms of T_1 and T_2.

(d) [12 pts] Now the piston is allowed to move. Assuming that the condition for thermal equilibrium is maintained, derive the condition for mechanical equilibrium in terms of $\Omega_1(E_1, V_1)$, $\Omega_2(E_2, V_2)$, their partial derivatives, and the cross-sectional areas A_1 and A_2.

(e) [12 pts] Express the pressure P_i of chamber i in terms of $\Omega_i(E_i, V_i)$ and its partial derivatives. Rewrite the condition for mechanical equilibrium obtained in (b) in terms of P_i and A_i.

자격시험 문제

과목명 : 통계역학 2023. 07. 28 시행

2. [50 pts] Consider a classical ideal gas consisting of N indistinguishable molecules moving freely on a one-dimensional line of length L. Each molecule consists of two distinguishable atoms, whose Hamiltonian is given by

$$H(p_1, p_2, x_1, x_2) = \frac{p_1^2 + p_2^2}{2m} + \frac{K}{2}(x_1 - x_2)^2.$$

Here x_i and p_i denote the position and momentum of the i-th atom in the molecule, respectively. The system is closed and kept at temperature T, with macroscopically large values of N and L whose ratio N/L is finite. Using the relations

$$\ln N! \simeq N\ln N - N, \quad \int_{-\infty}^{\infty} e^{-at^2} dt = \sqrt{\frac{\pi}{a}},$$

answer the following questions.

(a) [10 pts] Derive the Helmholtz free energy $A(T, L, N)$ of the system.

(b) [10 pts] Derive the mean internal energy $U(T, L, N)$, and show that the result is in agreement with the equipartition theorem.

(c) [10 pts] Find the mean square interatomic distance, $\langle (x_1 - x_2)^2 \rangle$.

(d) [20 pts] Now, suppose that the molecules are in a two-dimensional square of size $L \times L$, with each molecule's Hamiltonian given by

$$H(\vec{p}_1, \vec{p}_2, \vec{x}_1, \vec{x}_2) = \frac{\vec{p}_1^{\,2} + \vec{p}_2^{\,2}}{2m} + \frac{K}{2}(\vec{x}_1 - \vec{x}_2)^2,$$

where $\vec{p}_i$ and $\vec{x}_i$ are two-dimensional vectors. Repeat the calculations of (a), (b), and (c) for this case.

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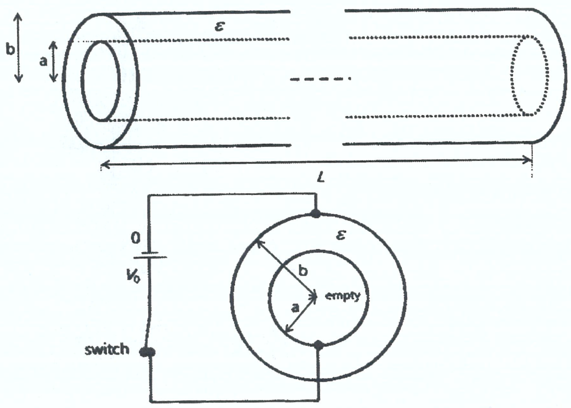
자격시험 문제

과목명 : 전기역학 2023 . 07. 28 시행

1. [50 pts] Two long highly conducting coaxial tubes (radii a and b with total length L and negligible thickness) are separated by a material of dielectric constant ϵ. The tubes have potential difference V_0 maintained by a battery. Let us neglect the fringing effects due to the edges.



(a) [10 pts] Using the Gauss's law, obtain the electric field between the tubes as a function of the axial distance ρ. You can set the charge per unit length of the inner tube as λ.

(b) [10 pts] Evaluate the capacitance C between the tubes.

(c) [10 pts] Suppose that the dielectric material between the tubes is slightly conductive with conductivity σ. This conductivity is much smaller than the conductivity of the tube, so each tube can be considered as an equipotential surface. Evaluate the resistance R between the tubes. What is the relationship between R and C?

(d) [10 pts] With the condition given in (c), if you disconnect the switch at $t=0$, what will happen?

(e) [10 pts] With the condition given in (d), calculate the potential difference $V(t)$ across the tubes as a function of time.

2. [50 pts] Consider a particle of mass m and a point charge q located at the origin at time $t=0$. A magnetic field B along the $-z$ direction is applied. Assume that the particle moves with a velocity $\vec{v}$.

(a) [5 pts] Write down the equation of the motion for the particle.

(b) [5 pts] For the motion in the xy plane, show that the two components of the velocity vector, v_x and v_y, both of which are real, obey the equations: $\frac{dv_x}{dt}=-\frac{qB}{m}v_y$ and $\frac{dv_y}{dt}=\frac{qB}{m}v_x$.

(c) [10 pts] For the motion in the xy plane, introduce $R=x+iy$ and rewrite the equation of motion using R.

(d) [10 pts] The motion can be interpreted as a cyclotron motion. Find the cyclotron frequency.

(e) [10 pts] By taking the real and imaginary parts of the above solution, show that the solution of the equation of the motion is

$$v_x = v_x(0)\cos wt - v_y(0)\sin wt,$$
$$v_y = v_x(0)\sin wt + v_y(0)\cos wt.$$

(f) [10 pts] Draw the trajectory of the particle. Assume that the particle initially moves along $+x$ axis.

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자격시험 문제

과목명 : 고전역학 **2023. 07. 28 시행**

1. [50 pt] Consider a particle of mass m moving within a central force potential.

(a) [10 pt] First, consider a potential $U(r) = -\frac{G}{r}$ where G is a constant, and r is the distance between the particle and a central body of mass M that produces the potential ($M \gg m$). Assuming that the particle's orbit is in a plane, write down its Lagrangian and Hamiltonian in the polar coordinates (r, θ). For the Hamiltonian, use p_r and p_θ to denote the angular momentum conjugate to the coordinate r and θ, respectively.

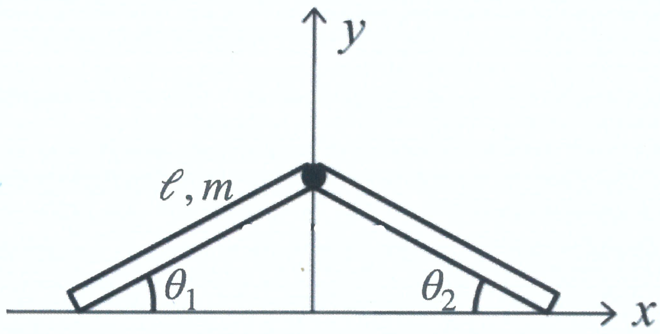
(b) [10 pt] Find the Hamiltonian equation of motion for the coordinate r. Then, determine the radius r_0 at which a particle with angular momentum $l\ (= p_\theta)$ can move in a circular orbit.

(c) [10 pt] Show that this circular orbit is stable. That is, demonstrate that for this orbit, a slight radial nudge (i.e., $r = r_0 + \Delta r$ with $\Delta r \ll r_0$) causes only small radial oscillations about the circular motion. Derive the equation of motion for these oscillations, and show that their period, T_{osc}, is equal to the particle's orbital period, T_{orb}.

(d) [15 pt] Now repeat the above analysis (a) to (c) for a different potential $U(r) = Gr$. Here, show that the circular orbit in this potential is again stable, and the ratio of the two oscillation periods is $T_{\text{orb}}/T_{osc} = \sqrt{3}$.

(e) [5 pt] Finally, generalize your result for $U(r) = Gr^n$. Here, show that under the conditions $Gn > 0$ and $n > -2$, the ratio is $T_{\text{orb}}/T_{osc} = \sqrt{n+2}$.

2. [50 pt] Consider two uniform rods of length l and mass m, connected by a hinge and placed on the frictionless floor. Let $\theta_{1,2}$ be the angle between each rod and the floor. At $t = 0$, the rods are released at rest with $\theta_1 = \theta_2 = 30^\circ$.



(a) [20pt] Write the Lagrangian.

(b) [5pt] Write the equation(s) of motion.

(c) [10pt] Find the speed of the hinge when it hits the floor.

(d) [10pt] What is the time $t = t_h$ when the hinge hits the floor. [Write the answer in a closed integral form.]

(e) [5pt] Suppose there is no second rod and no floor, describe the motion of a single rod for $t > 0$ qualitatively.