소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 고전역학 **2020. 07. 31 시행**

1. [50 pts] The potential energy of a particle of mass m moving in a plane is given by $U(r)=kr$, where r is the distance from the origin of the force, and k is a positive constant.

(a) [10 pts] First, by using the concept of an effective potential (denoted by V_{eff}) in plane polar coordinates (r,θ), find the radius r_0 at which a particle with constant angular momentum l is in a circular orbit.

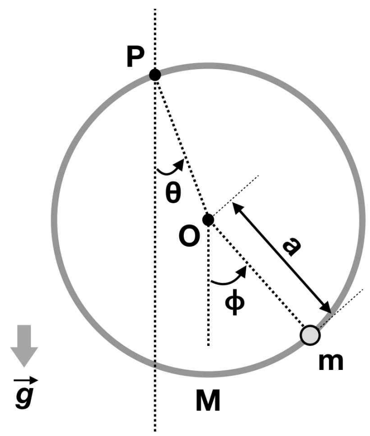
(b) [10 pts] Write the Lagrangian and Lagrange's equations of motion of the particle.

(c) [10 pts] Find the Hamiltonian and Hamilton's equations of motion. Here use p_r and $p_\theta\ (=l)$ to denote the particle's momentum in r and θ, respectively. Using the equation(s) you find here, verify the answer acquired in (a).

(d) [10 pts] Show that for the circular orbit of radius r_0, a slight radial nudge x (i.e., $r=r_0+x$ with $x\ll r_0$) causes only small radial oscillations. You will need to derive the equation of motion for these oscillations to the first order in x. Obtain the ratio $T_{\text{orb}}/T_{\text{osc}}$, where T_{orb} is the particle's orbital period, and T_{osc} is the period of small radial oscillations.

(e) [10 pts] Generalize your results for $U(r)=k_2 r^n$ with constants k_2 and n. In particular, determine the ratio of two oscillation periods $T_{\text{orb}}/T_{\text{osc}}$ under the conditions $k_2 n>0$ and $n>-2$.

2. [50 pts] A bead of mass m is threaded on a frictionless, circular wire hoop of mass M and radius a. The hoop is pivoted on its rim at point P (see the figure) so it can swing freely in its plane in the presence of the uniform gravitational field g.



(a) [10 pts] Determine the Lagrangian of the system using the generalized coordinates θ and ϕ shown in the figure.

(b) [10 pts] Find Lagrange's equations of motion of the system assuming small oscillations about the equilibrium points $\theta=\phi=0$.

(c) [20 pts] From your answer in (b), determine the eigenfrequencies and describe the normal mode motion. You will need to write the angular displacements as functions of time, $\theta(t)$ and $\phi(t)$.

(d) [10 pts] When $m\ll M$, the system becomes a simpler physical pendulum of mass M. Assuming small oscillations about the equilibrium point $\theta=0$, find $\theta(t)$ and its oscillation frequency. Verify that, in the limit $m\ll M$, the displacement $\theta(t)$ found in (c) reduces to what you acquired here in (d).

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자격시험 문제

과목명 : 양자역학 **2020. 07. 31 시행**

1. [50 pts] Consider a 1D infinite potential well $V(x)$ such that

$$V(|x|<a)=(\hbar^2/m)\,V_0\delta(x)$$
$$V(|x|>a)=\infty.$$

(a) [10 pts] Write down the boundary conditions of the wave function $\psi(x)$ of a particle to satisfy the Schrodinger eq.

(b) [10 pts] Show the parity operator $\hat{P}$, which inverts the x coordinate, commutes with the Hamiltonian $\hat{H}$. Argue that the non-degenerate eigenfunctions of $\hat{H}$ are also eigenfunctions of $\hat{P}$ with eigenvalues of 1 (even-parity) or -1 (odd-parity).

(c) [10 pts] Solve for even-parity wave functions $\psi_n^e(x)$ and find the conditions satisfied by their energy eigenvalues E_n. [Here, you can ignore wave function normalization.]

(d) [10 pts] Following (b), consider two extreme cases of $V_0=0$ and $V_0=\infty$. Find E_n for both cases and plot the ground state wave functions.

(e) [10 pts] Now, consider odd-parity wave functions $\psi_n^o(x)$. Write down $\psi_n^o(x)$. Show that their energy eigenvalues are independent of the value V_0 and explain why in words.

2. [50 pts] Consider two electrons (i=1,2) located at two lattice sites (k=1,2). The Hamiltonian of the system is effectively written as $\hat{H}=-J(\sigma_x^{(1)}\sigma_x^{(2)}+\sigma_y^{(1)}\sigma_y^{(2)})$, where $J>0$. Here, $\sigma^{(i)}$ is the Pauli-operator for 1/2 spin for i-th electron.

[Hint:

$$\sigma_x^{(1)}\sigma_x^{(2)}+\sigma_y^{(1)}\sigma_y^{(2)}=\vec{\sigma}^{(1)}\cdot\vec{\sigma}^{(2)}-\sigma_z^{(1)}\sigma_z^{(2)}$$

Total spin operator $\vec{S}=\frac{\hbar}{2}\vec{\sigma}=\frac{\hbar}{2}(\vec{\sigma}^{(1)}+\vec{\sigma}^{(2)})$]

(a) [10 pts] Find energy eigenvalues and degeneracies of this system.

(b) [15 pts] Suppose the spatial parts of the electron wave functions are written as $\phi_k(\vec{r}_i)$, where i,k=1,2, and spin parts are written as $|\uparrow>_i$, $|\downarrow>_i$, where i=1,2. Express the total wave functions of the system for eigenvalues obtained in (a).

(c) [10 pts] Now, imagine an external magnetic field $\vec{B}=B\hat{z}$ is applied. Write down the Hamiltonian of the system.

(d) [15 pts] Find energy eigenvalues of the system in (c), and obtain the magnetic field value when the ground state of the system changes.
[For electrons, assume g-factor is 2]

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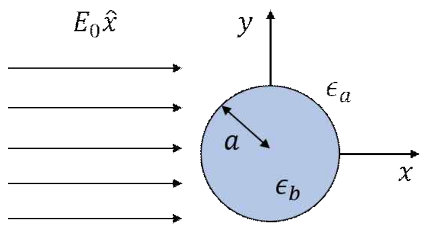
자격시험 문제

과목명 : 전기역학 **2020. 07. 31 시행**

1. [40 pts] A uniform electric field $E_0\hat{x}$ exists in a dielectric having permittivity ϵ_a. With its axis perpendicular to this field, a sufficiently long cylindrical dielectric rod (extended along the z-direction) having permittivity ϵ_b and radius a is introduced.



Using a cylindrical coordinate (ρ,θ,z), we will look for solutions of Laplace equation inside $(\phi_b(\rho,\theta))$ and outside $(\phi_a(\rho,\theta))$ of the rod with the following boundary conditions.

(i) $\phi_a = \phi_b$ at $\rho = a$

(ii) $\epsilon_b \dfrac{\partial \phi_b}{\partial \rho} = \epsilon_a \dfrac{\partial \phi_a}{\partial \rho}$ at $\rho = a$

(iii) $\phi_a \to -E_0 x = -E_0 \rho \cos\theta$ for $\rho \gg a$

(a) [10 pts] Justify the boundary conditions (ii) and (iii).

(b) [20 pts] The general forms of the potential outside and inside the rod satisfying the boundary conditions are

$$\phi_a = -E_0 \rho \cos\theta + \sum_{k=1}^{\infty} \rho^{-k}(C_k \cos k\theta + D_k \sin k\theta)$$

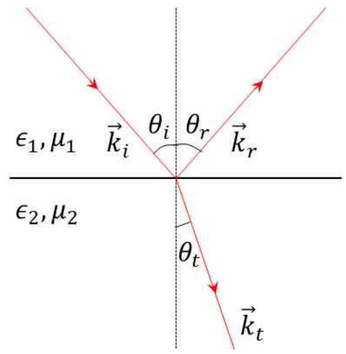
$$\phi_b = \sum_{k=1}^{\infty} \rho^{k}(A_k \cos k\theta + B_k \sin k\theta)$$

(we set $\phi = 0$ on y−z plane)

Determine the potential and the electric field at points outside and inside the rod, neglecting the end effects. [20pts]

(c) [10 pts] Sketch the electric field lines inside and outside the rod when $\epsilon_a > \epsilon_b$.

2. [60 pts] Consider two nonconducting media $(\sigma = 0)$ described by ϵ_1, μ_1 for a medium 1 and ϵ_2, μ_2 for a medium 2, respectively. A plane wave in medium 1 is incident on medium 2 at an angle θ_i. The incident, reflected, and transmitted electric fields are given by $\overrightarrow{E_i} = \overrightarrow{E_{0i}} e^{i(\vec{k_i}\cdot\vec{x} - \omega_i t)}$, $\overrightarrow{E_r} = \overrightarrow{E_{0r}} e^{i(\vec{k_r}\cdot\vec{x} - \omega_r t)}$ and $\overrightarrow{E_t} = \overrightarrow{E_{0t}} e^{i(\vec{k_t}\cdot\vec{x} - \omega_t t)}$, respectively.



(a) [20 pts] For the given incident angle θ_i, find the reflected angle θ_r and the transmitted angle θ_t. Consider the two cases that the electric field is 1) perpendicular and 2) parallel to the plane of incidence, respectively.

(b) [20 pts] Find the condition that the incident wave is totally reflected.

(c) [20 pts] Finally, assume that an incident wave in a nonconducting medium is incident on a perfectly conducting medium $(\sigma = \infty)$ with $\theta_i = 0$. For a given $\overrightarrow{E_{0i}}$, find $\overrightarrow{E_{0r}}$ and $\overrightarrow{E_{0t}}$.

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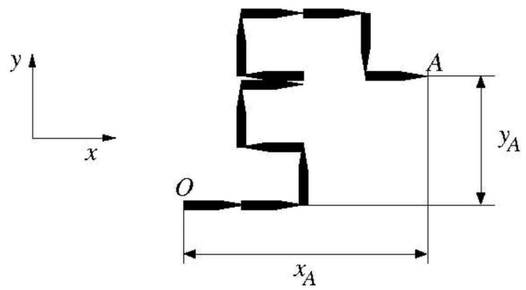
자격시험 문제

과목명 : 통계역학 **2020. 07. 31 시행**

1. [50 pts] Consider a polymer composed of N rod-like monomers of length a. The polymer is laid on a plane so that each monomer can be aligned only in four directions: up, down, left, and right. One end of the polymer is fixed to the origin O, and the other end A is allowed to move freely. If A is located at $(x_{\mathrm{A}}, y_{\mathrm{A}})$, the energy of the polymer is given by $E = \epsilon_x x_{\mathrm{A}} + \epsilon_y x_{\mathrm{B}}$. The polymer stays in equilibrium at temperature T.



(a) [10 pts] Calculate the partition function.

(b) [10 pts] Calculate the mean energy per monomer and the entropy per monomer.

(c) [15 pts] Calculate the mean square distance between O and A.

(d) [15 pts] Now assume that N is even, and that A is fixed to O so that the polymer is shaped like a ring. In the limit $N \to \infty$, calculate the entropy per monomer and compare it to the corresponding result obtained in (b) when $\epsilon_x = \epsilon_y = 0$.

Hint: The following formulas may be useful.

$$\sum_{k=0}^{n} \binom{n}{k} = \binom{2n}{n} \text{ for any integer } n \ge 0,$$

$$\ln(n!) = n \ln n - n + O(\ln n) \text{ for } n \gg 1.$$

2. [50 pts] Consider an ideal gas at temperature T and chemical potential μ in contact with an adsorbing surface consisting of N_{s} adsorbing sites. Each site can bind with at most one gas molecule and, upon binding, changes the energy of the gas molecule by $\epsilon < 0$.

(e) [10 pts] Obtain the grand partition function describing adsorption.

(f) [10 pts] Using the above result, calculate the adsorption coefficient $\theta \equiv \langle N_{\mathrm{a}} \rangle / N_{\mathrm{s}}$, where $\langle N_{\mathrm{a}} \rangle$ is the mean number of adsorbed gas molecules.

(g) [15 pts] Note that the partition function of the ideal gas is given by

$$Z = \frac{V^N}{N!\lambda^{3N}},$$

where N is the number of gas molecules, V the gas volume, and λ is the thermal wavelength (determined by temperature). Given these, express the chemical potential μ of the gas in terms of temperature T, pressure p, and thermal wavelength λ.

(h) [15 pts] Using the above results, show that the pressure dependence of the adsorption coefficient can be written as

$$\theta \approx \frac{p}{p + p_0(T)}.$$

Also, explicitly calculate $p_0(T)$.