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과목명 : 고전역학

2019.04.29. 시행

1. [5 pts] Consider a non-relativistic collision process in a free space, where a probe particle of mass m moves at velocity v in the $+x$ direction and collides with a target particle of mass M which is initially at rest. A particle detector is placed far from the collision site, and it detects the probe particle which are scattered from the target particle with an deflection angle of θ from the incoming x-axis. The probe particle's kinetic energy $E_{k,final}$ is measured at the detector. The ratio of $E_{k,final}$ to the initial kinetic energy of the probe particle is denoted by ϵ. Obtain the expression of M/m as a function of ϵ and θ.

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과목명 : 고전역학 2019.04.29. 시행

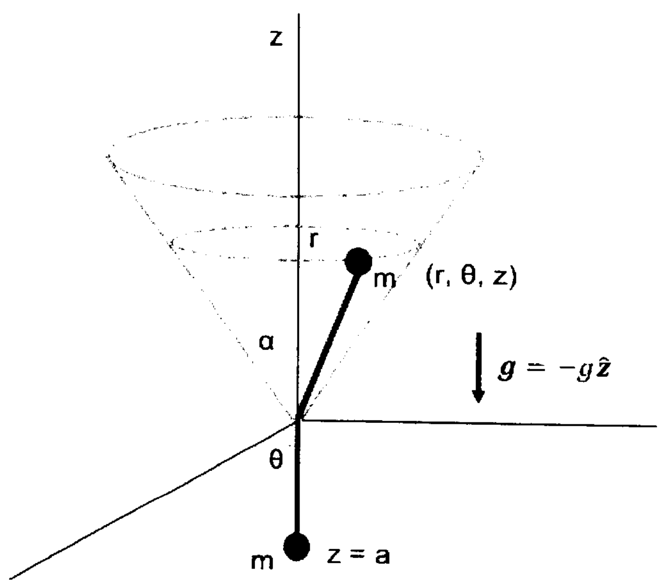
2. [15 pts] Two particles with mass m are connected by a massless string of fixed length b. The particles are subject to a gravitational force. One particle slides along the frictionless surface of a cone of half-angle α with vertex at the origin. The other particle is hung at the position $z = a$ ($a < 0$) by the string threaded through a small hole at the bottom of the cone.

(a) [2 pts] Using a generalized coordinates of (r, θ, z, a) as shown in the figure, write down the Lagrangian.

(b) [4 pts] Write down all equations of constraints among the coordinates (r, θ, z, a).

(c) [7 pts] Write down all equations of motion with the Lagrange multipliers.

(d) [2 pts] Show how the normal Force F_n from the surface and the tension T from the string are related to your Lagrange multipliers.



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과목명 : 양자역학 2019.04.29. 시행

1. [10 pts] Consider a particle of mass m that satisfies the Schrodinger equation with the following potential,

$$V(x) = \infty \quad (for\ x < 0)$$
$$= U\,\delta(x-a) \quad (for\ x \geq 0)$$

, where $a > 0$ and $U < 0$.

(a) Write down the wave functions and boundary conditions for the particle. [4 pts]

(b) Find the condition to have a bound state, in terms of a and U. [2 pts]

Now consider the different potential as follows,

$$V(x) = U\,\{\delta(x+a) + \delta(x-a)\}$$

, where $a > 0$ and $U < 0$.

(c) Assume the condition in (b) is met, then there are two bound states for this potential. Draw a *qualitative* sketch of the two bound states. (*Hint*: Consider the symmetry of the potential.) [2 pts]

(d) Explain by words, why the condition to have the second bound state for this potential is the same as the condition found in (b). [2 pts]

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과목명 : 양자역학 2019.04.29. 시행

2. [15pt]

(a) [4pt] Consider an one-dimensional simple harmonic oscillator centered at $x=0$. The Hamiltonian is given by

$$H_0 = \frac{p^2}{2m} + mw^2\frac{x^2}{2} \ .$$

This can be solved by quantizing it

$$[p, x] = -i\hbar$$

and by introducing ladder operators

$$a = \sqrt{\frac{1}{2\hbar}}\left(\sqrt{mw}\,x + i\frac{p}{\sqrt{mw}}\right),$$

$$a^\dagger = \sqrt{\frac{1}{2\hbar}}\left(\sqrt{mw}\,x - i\frac{p}{\sqrt{mw}}\right).$$

Work out $[a, a^\dagger]$ and express H_0 in terms of the ladder operators.

(b) [2pt] Consider a similar simple harmonic oscillator, but this time centered at $x=c$. Shifting $x \rightarrow x-c$ in the expressions in (a), obtain new ladder operators a', $a'^\dagger$, and write H_0' in terms of these new ladder operators.

Now, we introduce a perturbation term in the Hamiltonian so that we want to solve:

$$(H_0 + \lambda H_I)|n> \ = E_n|n> \quad \text{with } \lambda \ll 1.$$

The original energy eigenvalues and eigenvectors without perturbation are

$$H_0|n_0> \ = E_n^{(0)}|n_0> \ , \quad <n_0|n_0> \ = 1 \ ,$$

and $<n_0|m_0>=0$ for $n \neq m$ (non-degenerate).

Expanding perturbed eigenvalues and eigenvectors in terms of original ones in powers of λ

$$|n> \ = |n_0> + \lambda|n_1> + O(\lambda^2) \ ,$$

$$<n_0|n_1> = 0 \ ,$$

$$E_n = E_n^{(0)} + \lambda E_n^{(1)} + O(\lambda^2) \ ,$$

the first-order energy correction that we are interested in is $E_n^{(1)} = \ <n_0|H_I|n_0>$.

(c) [5pt] We turn on the perturbation $H_I = x$ in addition to the H_0 in (a)

$$H = H_0 + \lambda H_I = \frac{p^2}{2m} + mw^2\frac{x^2}{2} + \lambda x \ .$$

Using the results obtained so far, show that $E_n^{(1)} = 0$ and explain this using the symmetry of the system.
(Hint: Consider parity.)

(d) [4pt] If we add $H_I = x$ to H_0' in (b), what would be $E_n^{(1)}$? Is it also zero or not; explain this using the symmetry, in comparison with the answer (c).

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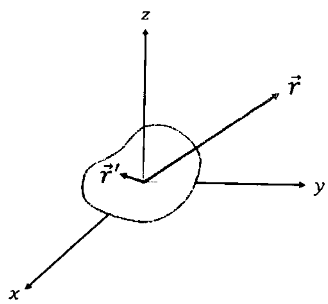
과목명 : 전기역학

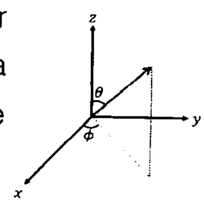
2019.04.29. 시행

1. [10 pts] Consider a system with the charge density $\rho(\vec{r}')$ as shown below. Then the potential at $\vec{r}$ is given by $\phi(\vec{r}) = \int d^3r' \frac{\rho(\vec{r}')}{4\pi\epsilon_0 |\vec{r}' - \vec{r}|}$.



(a) [2 pts] From the multipole expansion of the potential $\phi(\vec{r})$ for $|\vec{r}| \gg |\vec{r}'|$, find the expressions for the monopole and dipole terms, respectively.

From now on, let us consider a sphere of radius R with a charge density $\rho = \rho_0 \cos\theta$ where ρ_0 is a constant.



(b) [3 pts] Calculate the monopole and dipole terms.

(c) [3 pts] Calculate the electric field at $(r,0,0)$, $\frac{1}{\sqrt{2}}(r,0,r)$ and $(0,0,r)$, respectively for $r \gg R > 0$.

(d) [2 pts] Draw the equipotential lines and electric field lines, respectively, in the xz plane. Here, assume $R \to 0$.

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과목명 : 전기역학 2019.04.29. 시행

2. [10 pts] The wave number k of a electromagnetic wave packet in an isotropic, dispersive medium is given by $k=w(\mu\epsilon)^{1/2}$ as a function of frequency w, permittivity of the medium ϵ, and permeability of the medium μ. The permeability and the permittivity are functions of frequency, $\mu=\mu(w)$, and $\epsilon=\epsilon(w)$.

(a) Find the group velocity v_g of a wave packet in this medium [3 pts].

(b) The Poynting vector is given by,

$$\vec{S}=v_g(u_E+u_H)\hat{n}=\vec{E}\times\vec{H}$$

where u_E and u_H are electric and magnetic energy densities. Show that the electromagnetic wave energy density is given by,

$$u=u_E+u_H=\frac{|\vec{E}|^2}{2}(\epsilon+w\frac{d\epsilon}{dw})+\frac{|\vec{H}|^2}{2}(\mu+w\frac{d\mu}{dw})$$ [4 pts]

(c) Show that in a medium in which both $\epsilon=\epsilon(w)$ and $\mu=\mu(w)$ are negative, the group velocity and the phase velocity of the wave packet are opposite in direction. [3 pts]

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과목명 : 통계역학 2019.04.29. 시행

1. [10 pts] Consider an one-dimensional chain of N Ising magnetic dipoles S_i $(i=1,2,...,N)$ in an external magnetic field h. Each dipole can either point up ($S_i = +1$, parallel to h) or point down ($S_i = -1$, anti-parallel to h). We adopt a periodic boundary condition $S_{i+N} = S_i$. In this Problem, let us assume that each dipole does not interact with any other dipoles or any other degrees of freedom of the substance. The system is immersed in a heat reservoir of absolute temperature T.

(a) [2 pts] Using a partition function for a canonical distribution, $Z(T) = \sum_{n=0}^{\infty} e^{-E_n/k_B T}$ with the Boltzmann constant k_B , find the mean energy in the system, $\overline{E}$. One may simplify the result by using $\tanh x = \frac{e^x - e^{-x}}{e^x + e^{-x}}$. What value does $\overline{E}$ approach when T is very large?

Now let us acquire the above expression for $\overline{E}$ from microscopic considerations.

(b) [3 pts] Show that the total number of states of the system with its total energy E lying in the range of $[E, E+\delta E]$ is given by $\Omega(E) = \frac{N!}{\left(\frac{N-\frac{E}{h}}{2}\right)!\left(\frac{N+\frac{E}{h}}{2}\right)!}\frac{\delta E}{2h}$. Assume that δE is very small compared to E but is still larger than $S_i h$ (i.e., $S_i h < \delta E \ll E$).

(c) [2 pts] Using the approximation $\ln N! = N \ln N - N$ for large N, write $\ln \Omega(E)$ as a function of N, E and h. One may eliminate the term containing δE by utilizing the assumption $\delta E \ll E$.

(d) [3 pts] Finally, using the definitions of absolute temperature and entropy, find the relation between T and $\overline{E}$. Compare the result with (a).

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과목명 : 통계역학 2019.04.29. 시행

2. [10 pts] Now we consider the same system as in Problem 1, but assume that there exists interaction between dipoles. Assuming that it is energetically favorable to have nearest-neighbor spins aligned in parallel, the system is now described by the following Hamiltonian,

$$H = -J\sum_{i=1}^{N} S_i S_{i+1} - h\sum_{i=1}^{N} S_i, \quad J > 0, \quad h > 0.$$

(a) [2 pts] To treat the interaction J between magnetic dipoles, let us use a mean field approximation

$S_i S_{i+1} \approx m(S_i + S_{i+1}) - m^2$ where $m = \frac{1}{N}\sum_{i=1}^{N} < S_i >$ indicates the average magnetic moment. Show that, in the mean field approximation, the Hamiltonian is given by

$$H \approx H_m = -(2Jm + h)\sum_{i=1}^{N} S_i + JNm^2.$$

(b) [2 pts] Using the mean field Hamiltonian H_m given above, compute the partition function.

(c) [2 pts] Show that the average magnetization m is given by $m = \tanh\left[\frac{2Jm + h}{k_B T}\right]$.

(d) [2 pts] From now on let $h = 0$. Find T_c that satisfies

$$m = \begin{cases} nonzero, & T < T_c \\ 0, & T > T_c \end{cases}$$

(e) [2 pts] Find m when m is very small, by using the approximation $\tanh x \approx x - \frac{1}{3}x^3$.

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과목명 : 실험 2019.04.29. 시행

1. [15 pts] Particles that are hazardous to a human body are categorized as fine dust less than 10 microns (PM10) in diameter, and ultrafine dust less than 2.5 microns (PM2.5) in diameter. Provide your answer to the following questions in English or Korean.

(a) [10 pts] Design a device that can measure the concentration of PM10 and PM2.5. Provide a schematic of your device, a list of necessary equipment or materials, and an explanation how to measure the concentration of the particles.

(b) [5 pts] Describe any error sources of your measurement.