소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 양자역학 **2018 . 07. 18 시행**

1. Consider a two-level system. An operator $\hat{A}$ representing the observable A has normalized eigenstates ψ_1 and ψ_{-1} with eigenvalues +1 and −1, respectively. Another operator $\hat{B}$ for the observable B has eigenstates $(\psi_1 + \sqrt{3}\,\psi_{-1})/\sqrt{2}$ and $(\sqrt{3}\,\psi_1 - \psi_{-1})/\sqrt{2}$ with eigenvalues +1 and −1.

(a) [10 pts] Calculate $[\hat{A}, \hat{B}]$. Express it as a 2×2 matrix in the basis $\{\psi_{1,}\psi_{-1}\}$.

(b) [10 pts] Suppose that a system is in the state $(\psi_1 + \psi_{-1})/\sqrt{2}$. . Calculate the variances $(\Delta\hat{A})^2 = <\hat{A}^2> - <\hat{A}>^2$ and $(\Delta\hat{B})^2$. Do they satisfy the uncertainty relation?

(c) [10 pts] Suppose the observable A is measured at some time. Immediately after this, the expectation value of B is measured to be 1/2. What was the measured value of A?

(d) [10 pts] For some normalized state $\overline{\psi} \equiv a\psi_1 + b\psi_{-1}$ with a and b real and $|a|,|b| < 1$, find all the values of a that minimize $(\Delta\hat{A})^2$. Explain why those values do so.

2. [10 pts] Consider a simple harmonic oscillator in one-dimensional space with the Hamiltonian given by

$$\hat{H} = \frac{\hat{p}^2}{2m} + \frac{m\omega^2}{2}\hat{x}^2$$

for some mass m and frequency ω. Find the minimum possible energy of the oscillator, limited to be non-zero by the uncertainty relation $[\hat{x}, \hat{p}] = i\hbar$.

소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 양자역학 **2018 . 07. 18 시행**

3. Consider a charged particle moving on (x,y)-plane, subject to a perpendicular magnetic field $B_z = B_0\theta(x)\theta(d-x)$. That is, the magnetic field is constant in the strip region of width d, and zero outside. The Hamiltonian of this system is given by $H = \frac{1}{2m}(\vec{p} - e\vec{A})^2$, where $\vec{A}$ is the vector potential.

(a) [10 pts] Choose the gauge so that $A_x = A_z = 0$ everywhere on (x,y)-plane, and $A_y(x,y) = 0$ for $x < 0$. Determine $A_y(x,y)$ for all x, y. Substitute your answer to the Hamiltonian

$$H = \frac{1}{2m}[-\hbar^2\frac{\partial^2}{\partial x^2} + (-i\hbar\frac{\partial}{\partial y} - eA_y)^2]$$

and obtain the explicit form of H.

(b) [20 pts] Consider an electron incident from $x < 0$, For an incident wave $\exp(ikx)$ in the x direction, and the transmitted wave $T\exp(i\tilde{k}x)$ in the region $x > d$. [You can take the whole wavefunction to be y-independent.] $\tilde{k}$ is determined by simple kinematics in terms of k and $B_0 d$. What is that relation?

(c) [20 pts] Show that, below a critical energy E_0, $\tilde{k}$ is imaginary. Compute E_0, and give a classical interpretation.

(d) Compute the transmission coefficient T (defined in problem (b) above), in the limit $d \to 0$, $B_0 \to \infty$ while holding $B_0 d$ fixed.

소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 통계역학 **2018 . 07. 18 시행**

1. [60 pts] Consider a one-dimensional harmonic oscillator of mass m, momentum p, displacement x, and the classical angular frequency of oscillation ω, immersed in a heat reservoir of absolute temperature T. The Boltzmann constant is k_B.

(a) [5 pts] What is the total energy of such an oscillator, expressed in terms of m, p, x, and ω?

(b) [5 pts] What is the "equipartition theorem" of classical statistical mechanics? Use one or more of the following keywords if necessary: independent, quadratic, mean, energy, $\frac{1}{2}k_B T$.

(c) [5 pts] Applying the equipartition theorem in the classical approximation, what is the "mean" total energy of the oscillator described in (a)?

Now let us turn to quantum mechanics in order to check the limits of the classical description above. The energy levels of this harmonic oscillator are given by $E_n = (n + \frac{1}{2})\hbar\omega$ where n is the quantum number and $\hbar$ is the reduced Planck constant.

(d) [15 pts] Using a new variable $\beta = \frac{1}{k_B T}$ and the definition of a partition function for a canonical distribution, $Z(\beta) = \sum_{n=0}^{\infty} e^{\beta E_n}$, show that the mean energy of the oscillator is given by $\overline{E} = \hbar\omega\left(\frac{1}{2} + \frac{1}{e^{\beta\hbar\omega} - 1}\right)$. (Hint: You may need to derive $\overline{E} = -\frac{\partial \ln Z}{\partial \beta}$ and/or use the relationship $\frac{1}{1-x} = 1 + x + x^2 + x^3 + \cdots$ if $|x| < 1$.)

(e) [10 pts] Discuss the two limiting cases of $\overline{E}$ and their implications: when (1) $\hbar\omega \ll k_B T$ and (2) $\hbar\omega \gg k_B T$. For (1), you will want to compare it with the result found in (c). For (2), you may want to use the following keyword: zero-point energy.

Now let us consider N_A independent one-dimensional harmonic oscillators where N_A is the Avogadro number.

(f) [10 pts] Show that the heat capacity of this system at constant volume, $C_V = \left(\frac{\partial \overline{E}}{\partial T}\right)_V$, is given by $N_A k_B (\beta\hbar\omega)^2 \frac{e^{\beta\hbar\omega}}{(e^{\beta\hbar\omega} - 1)^2}$.

(g) [10 pts] Discuss the two limiting cases of C_V and their implications: when (1) $\hbar\omega \ll k_B T$ and (2) $\hbar\omega \gg k_B T$. For (1), you will want to compare it with the classically derived C_V. For (2), explain how your result is consistent with the third law of thermodynamics.

소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 통계역학 **2018 . 07. 18 시행**

2. [40 pts] Consider a classical ideal gas composed of N identical massless particles moving with speed of light c in three-dimensions, which is described by the following phase space Hamiltonian

$$H(q,p) = \sum_{i=1}^{N} c\sqrt{p_{i,x}^2 + p_{i,y}^2 + p_{i,z}^2} \quad ,$$

where $q_i = (q_{i,x}\ ,\ q_{i,y}\ ,\ q_{i,z})$ is the spatial coordinate and $p_i = (p_{i,x}\ ,\ p_{i,y}\ ,\ p_{i,z})$ is the momentum of the i -th particle.

(a) [10 pts] Find the partition function of the system at temperature T, considering that the particles are indistinguishable. You may want to use the integral $\int_0^\infty x^n e^{-ax} dx = \frac{\Gamma(n+1)}{a^{n+1}}$ when $a > 0$, where $\Gamma(n+1) = n!$ if n is an integer.

(b) [10 pts] Show that the Helmholtz free energy A can be written as

$$A = -Nk_B T f\left[\frac{(V/N)^{1/3}}{\lambda}\right] \quad ,$$

where $f(x) = 3\ln(x) + 1$ is a known function, and V is the volume of the system. Find the expression of λ which is often called the "thermal de Broglie wavelength." Estimate the temperature at which the quantum interference between particles becomes important. You may need to use the Stirling's formula, $\ln N! \approx N\ln N - N$, when N is a large number.

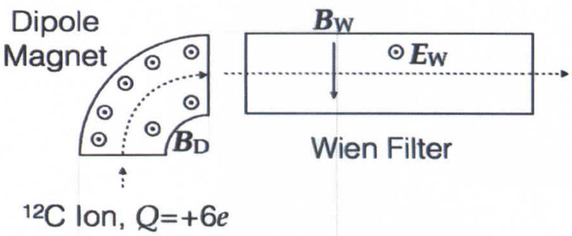
(c) [20 pts] Determine the chemical potential μ, the pressure P, the entropy S, and the specific heat at constant volume C_V .

소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 전기역학 **2018 . 07. 18 시행**

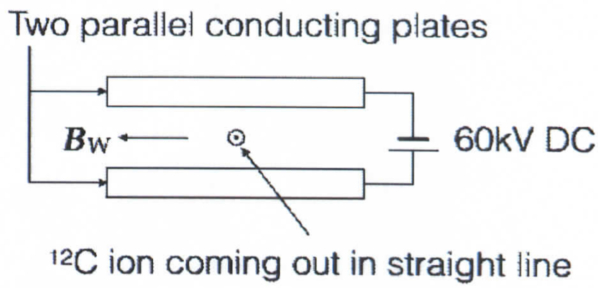
1. [60 pts] In high energy experiments, charged ions are steered using electric or magnetic fields. Let us consider the following system. The system has one dipole magnet which provides a uniform magnetic field. Another device is called Wien filter and provides electric and magnetic field in perpendicular configuration. ($E_W \perp B_W$). Answer to the following questions.



(a) [15 pts] Charged ions enter into the entrance of the dipole magnet. The ions are characterized by their charge Q, mass M and speed v. Find the necessary condition for some ions to exit the dipole magnet through the circular trajectory. The magnetic field of the magnet is B_D, and the radius of the curvature of the trajectory inside the magnet is ρ.

(b) [15 pts] Now consider a carbon ion, whose mass M is $12 GeV/c^2$ with charge Q = +6e. If the speed of the carbon ion is v = 0.1c, what should be the value of $B_D\rho$ for the dipole in order to have the carbon ion to pass the dipole? Give a numerical value in unit of $T{\cdot}m$ with 1 significant digit. (1 GeV=10^9 eV and c is the speed of light.)

(c) [15 pts] Now the ions coming out of the dipole magnet enter into the Wien filter, where they meet electric and magnetic field in perpendicular configuration ($E_W \perp B_W$). Find the condition that the ions exit the Wien filter via straight line trajectory. The following figure shows the cross section of the Wien filter seen from its exit.



(d) [15 pts] Now the same carbon ion (M = $12 GeV/c^2$, Q = +6e, v = 0.1c) is going straight through the Wien filter. In the Wien filter, a voltage difference of 60kV is applied between two parallel conducting plates separated by 10cm. What should be the magnetic field to be applied in perpendicular to the electric field? Give a numerical value in unit of T with 1 significant digit.

소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 전기역학 **2018 . 07. 18 시행**

2. [80 pts] Consider a plane wave propagating in a conducting medium with dielectric constant ϵ, permeability μ, and conductivity σ. Assume that there are no external free charges.

(a) [30 pts] Write down Maxwell's equations in the absence of external free charges [15 pts] and show that the electric field satisfies [15 pts]

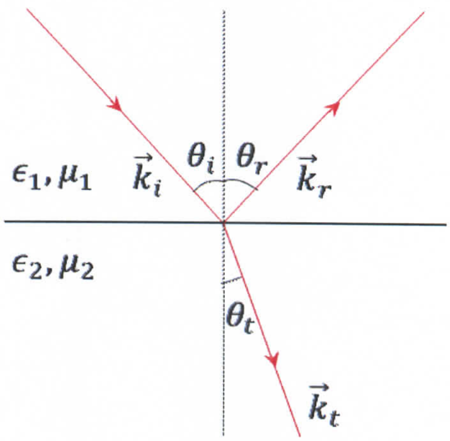
$$\nabla^2 \vec{E} = \mu\sigma \frac{\partial \vec{E}}{\partial t} + \mu\epsilon \frac{\partial^2 \vec{E}}{\partial t^2}.$$

(b) [20 pts] Find the expression for the wave number $k = \alpha + i\beta$ of the electromagnetic wave where α and β are real numbers. [You may use the result of the problem (a) even if you have not derived it.]

(c) [20 pts] Find the penetration depth δ in which the amplitude decreases by a factor of $1/e$ for 1) a good conductor and 2) a poor conductor, respectively.

(d) [10 pts] Find the skin depth for a metal which has $\sigma = 10^8 (\Omega m)^{-1}$ in the microwave range of 1 GHz.
(Here assume that $\epsilon = \epsilon_0 = 8.85 \times 10^{-12} C^2/Nm^2$ and $\mu = \mu_0 = 4\pi \times 10^{-7} N/A^2$.)

3. [60 pts] Consider two nonconducting media $(\sigma = 0)$ described by ϵ_1, μ_1 for a medium 1 and ϵ_2, μ_2 for a medium 2, respectively. A plane wave in medium 1 is incident on medium 2 at an angle θ_i. The incident, reflected, and transmitted electric fields are given by $\vec{E_i} = \vec{E_{0i}} e^{i(\vec{k_i} \cdot \vec{x} - \omega_i t)}$, $\vec{E_r} = \vec{E_{0r}} e^{i(\vec{k_r} \cdot \vec{x} - \omega_r t)}$ and $\vec{E_t} = \vec{E_{0t}} e^{i(\vec{k_t} \cdot \vec{x} - \omega_t t)}$, respectively.



(a) [20 pts] For the given incident angle θ_i, find the reflected angle θ_r and the transmitted angle θ_t. Consider the two cases that the electric field is 1) perpendicular and 2) parallel to the plane of incidence, respectively.

(b) [20 pts] Find the condition that the incident wave is totally reflected.

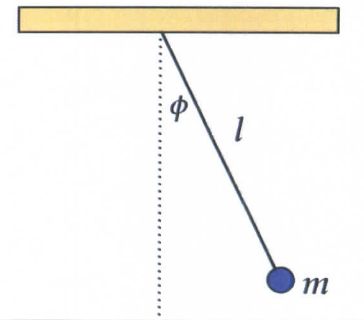
(c) [20 pts] Finally, assume that an incident wave in a nonconducting medium is normally incident on a perfectly conducting medium $(\sigma = \infty)$. For a given $\vec{E_{0i}}$, find $\vec{E_{0r}}$ and $\vec{E_{0t}}$.

소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 고전역학 **2018 . 07. 18 시행**

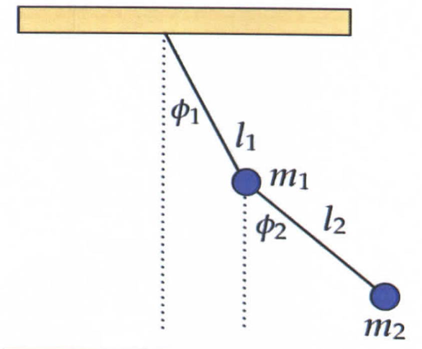
1. Consider a simple pendulum under the gravity as in the figure below. The string is massless and extensionless, and its length is l. The mass of the pendulum is m, and the gravitational field strength is g. Suppose that the pendulum has been released from a small angle φ.



1) [10 pts] Find the equation of motion and the period of the pendulum.

2) [15 pts] When there is the retarding force $2m\sqrt{gl}\,\dot{\phi}$, acting opposite to the moving direction, find the modified equation of motion for the pendulum and its solution under the retarding force. Discuss the pendulum motion.

3) [20 pts] Now the system is changed by double pendulum shown in the figure below (the retarding force is disappeared). Find the equations of motion for each mass, m_1 and m_2. Suppose that φ_1 and φ_2 are small and the motion is only in the plane.



4) [15 pts] When $l_1 = l_2 = l$ and $m_1 = m_2 = m$, the equation of motion can be simplified as follows:

$$l\ddot{\phi}_1 = -2g\phi_1 + g\phi_2$$
$$l\ddot{\phi}_2 = 2g\phi_1 - 2g\phi_2$$

Find the two angular frequencies ω_1 and ω_2 for the normal modes of this double pendulum system.

소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

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과목명 : 고전역학 **2018 . 07. 18 시행**

2. Let us consider a particle of mass m moving in three-dimensional space. This particle is acted on by a force whose potential is given by $U(r) = -k/r^2$. The particle is coming from infinity with an initial velocity V_∞.

(a) [10 pts] Let us define the contributions from the potential energy $U(r)$ and the angular momentum part to the Hamiltonian collectively as an effective potential $U_{eff}(r)$. What is the effective potential for the radial motion of the particle when its angular momentum is L? Express the angular momentum L with the impact parameter b.

(b) [15 pts] Depending on its initial condition, the particle undergoes one of the following two motions: 1) it approaches to a minimum distance $r_{\min} > 0$ and then goes away back to the infinite distance, or 2) it collides with the center $(r = 0)$ of the potential. Let us consider the second case of 2). What is the maximum impact parameter $b_{\max}$ for the particle to fall to the force center at $r = 0$? Write down the total cross section for this falling-in process.

(c) [15 pts] Show that the particle moves in a logarithmic spiral orbit given by $r = r_0 e^{\alpha\theta}$ for $b < b_{\max}$, where θ is the azimuthal angle of the particle's position vector in the plane normal to its angular momentum vector. Express α in terms of L, m, and k.