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과목명 : 양자역학 2009 . 12 . 23 시행

1. Let us consider a one-dimensional model for the two-electron system resembling the helium atom, taking δ-function potentials between particles. Assuming a fixed nucleus, we may then write the Hamiltonian operator for the system as

$$H = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} - Ze^2\delta(x_1) - Ze^2\delta(x_2) + e^2\delta(x_1 - x_2).$$

Here, for the 'helium' (rather than helium-like ions), $Z=2$. We want to find the approximate ground state of this system by using the variational method. (Assume that the ground state has the spatial wave function which is exchange-symmetric).

(a) Give brief explanations for various potential terms appearing in the above Hamiltonian. (10 pts)

(b) Now, for the system defined by the Hamiltonian without the two-particle interaction term (but for arbitrary $Z>0$)

$$H_0 = -\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_1^2} - \frac{\hbar^2}{2m}\frac{\partial^2}{\partial x_2^2} - Ze^2\delta(x_1) - Ze^2\delta(x_2),$$

find the corresponding ground state energy $E_0(Z)$ and also ground state eigenfunction $\Psi_0(x_1, x_2; Z)$. (10 pts)

(c) Find the approximate ground state energy E of the original system (i.e., for the above Hamiltonian H and $Z=2$), by using the solution of (b), i.e., $\Psi_0(x_1, x_2; Z)$ as the variational solution possessing a variational parameter. (10 pts)

(d) In (c), what is the physical significance of taking Z as a variational parameter? (10 pts)

2. H' is a perturbation to the unperturbed Hamiltonian H^0 (the energy levels are assumed to be nondegenerate). The total Hamiltonian can be written as $H^0 + \lambda H'$, where λ is assumed to be a small real number. The first order correction to the nth energy eigenvalue can be obtained as $E_n^1 = \langle \Psi_n^0 | H' | \Psi_n^0 \rangle$, where Ψ_n^0 is the nth energy eigenstate of H^0, while the first order correction to the nth energy eigenstate is

$$\Psi_n^1 = \sum_{m \neq n} \frac{\langle \Psi_m^0 | H' | \Psi_m^0 \rangle}{E_n^0 - E_m^0} \Psi_m^0,$$

where E_n^0 is the nth energy eigenvalue of H^0.

(a) Starting from the above conditions and results, derive the second order correction to the nth energy eigenvalue (10 pts).

A particle of mass m moves (nonrelativistically) in the three-dimensional potential

$$V = \frac{1}{2}k(x^2 + y^2 + z^2 + \lambda xy),$$

where k is a real positive constant.

(b) Find the ground state energy to the first order in the perturbation theory (10 pts).

(c) Find the ground state energy to the second order in the perturbation theory (10 pts).

(d) Find the first excited energy levels to the first order in the perturbation theory (10 pts).

*The standard solution of the one-dimensional harmonic oscillator may be used without proof together with

$$a = \frac{1}{\sqrt{2\hbar m\omega}}(m\omega x + ip),$$

$$a\phi_n = \sqrt{n}\,\phi_{n-1}, \quad a^\dagger\phi_n = \sqrt{n+1}\,\phi_{n+1},$$

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where $\omega = \left(\frac{k}{m}\right)^{\frac{1}{2}}, a\,(a^{\dagger})$ is the annihilation (creation) operator, ϕ_n is the n^{th} energy eigenstate of the one-dimensional harmonic oscillator, and p is the x-component momentum operator.

3. A spin-$\frac{1}{2}$ particle is placed in a time-dependent magnetic field

$$\vec{B} = B_0 \cos(\omega t)\hat{k},$$

where B_0 and ω are real constants. The spin operators for $x-$, $y-$, and $z-$components are

$$S_x = \frac{\hbar}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},\quad S_y = \frac{\hbar}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},\quad S_z = \frac{\hbar}{2}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

(a) Find the Hamiltonian in the matrix form assuming that the gyromagnetic ratio of the particle is γ. (10 pts)

(b) The particle was initially in the spin-up state with respect to the x-axis. Find the time-dependent spinor at time t. (10 pts)

(c) Find the expectation value when the z-component of the spin is measured at time t. Comment on the result. (10 pts)

(d) Find the probability of getting "spin-down" when the x-component of the spin is measured at time t. (5 pts)

(e) What is the condition for the magnetic field to cause a *complete* flip of the spin in S_x at time t? (5 pts)

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과목명 : 통계물리 2009 . 12 . 23 시행

1. We consider the photon gas in three dimensions. Answer the following questions.

(a) The first law of thermodynamics is $dU = TdS - PdV$, where U is the average energy, T is the temperature, S is the entropy, P is the pressure, and V is the volume. Show that the entropy S and the pressure P can be obtained from the Helmholtz free energy $A \equiv U - TS$ by the relations

$$S = -\left(\frac{\partial A}{\partial T}\right)_V, \qquad P = -\left(\frac{\partial A}{\partial V}\right)_T. \quad \text{(7 pts)}$$

(b) The Helmholtz free energy A of the photon gas is given by

$$A = -\frac{\pi^2 V (k_B T)^4}{45 \hbar^3 c^3}$$

in three dimensions. Find the entropy S and the pressure P of the photon gas. (7 pts)

(c) Find the heat capacity at constant volume, C_V. (8 pts)

(d) Show that in the adiabatic process the photon gas exhibits the pressure-volume relation $PV^\gamma =$ constant. Find the value of γ. (8 pts)

2. For a particle of mass m in a two-dimensional periodic box of area L^2.

(a) Find the density of state, $g(E) \equiv \frac{dN(E)}{dE}$, where $N(E)$ is the number of states with energy less than E. (10 pts)

(b) What is the spatial density of particles, i.e., $n \equiv \frac{N}{L^2}$ in a two dimensional Fermi-Dirac gas as a function of temperature and chemical potential μ? (10 pts)

(c) The same question as (b) for bosons. (10 pts)

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1. This is an electrostatic problem with a conducting sphere and/or a conducting plane.

a) Find the capacitance C_o of an isolated conducting sphere of radius a. (10)

b) A point charge q_o is at distance d away from an infinite grounded conducting plane. Find the surface charge density induced on the plane using the image method. (10)

c) If the point charge q_o in the above problem is replaced by a charge q_o at the center of a sphere of radius a $(< d)$, it produces a constant potential over the surface of the sphere while the image charge $-q_o$ beneath the plane does not. Find the magnitude q_1 and the position of the second image charge inside the sphere necessary to cancel the potential due to $-q_o$. (10)

d) The capacitance C between a conducting sphere and the infinite conducting plane may be found by successive images placed inside the sphere. When $a << d$, a few iteration of successive images is enough to produce an accurate value of the capacitance. Find the expression of C correct to $O((\frac{a}{d})^2)$. (10)

2. Let us picture an electron as a uniformly charged spherical shell, with the charge e and the radius R, spinning at the angular velocity ω. Assume the spinning direction is in the z-direction.

a) Find the electric field for points inside and outside the shell. (5)

b) The vector potential is given by $\vec{A}(\vec{r}) = \frac{\mu_o R\sigma}{3}(\vec{\omega}\times\vec{r})$ for a point inside and $\vec{A}(\vec{r}) = \frac{\mu_o R^4\sigma}{3r^3}(\vec{\omega}\times\vec{r})$ for a point outside, where $\sigma = \frac{e}{4\pi R^2}$, the surface charge density. Find the total magnetic field for points (r,θ) inside and outside the shell. Prove that the field inside is uniform. Obtain the magnetic dipole moment from the field outside. (10)

c) Calculate the total energy contained in the electromagnetic fields. Express the result in terms of e, ω, R.(10)

d) Show the total angular momentum contained in the fields is given by $\vec{L} = \frac{\mu_o e^2 R}{18\pi}\vec{\omega}$.

(Hint: angular momentum density $\vec{l} = \epsilon_o\vec{r}\times(\vec{E}\times\vec{B})$) (10)

e) Suppose that the electron's spin angular momentum is entirely attributable to the electromagnetic fields: $L_{em} = \hbar/2$. On this assumption, what is ωR? Does this classical model of an electron make sense? (5)

$\hbar \approx 10^{-34}\ J\cdot s$

$\mu_0 = 4\pi\times 10^{-7} N/A^2$

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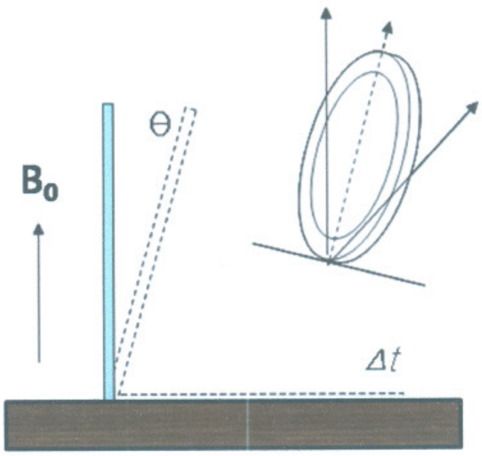
3. Consider a thin metallic circular loop (radius a) which is made of pure copper (mass M and resistance R). At $t=0$, the loop is released at θ_0, so that it falls on its face. After Δt, the loop comes to rest on the ground.

a) Two physically same loops are prepared; but in one, there is a strong applied magnetic field ($\vec{B}=B_o\hat{k}$), the other without any magnetic field. How can you discern the two? Why? (10)

b) Considering the coin in an applied magnetic field, find the expression for the magnetic torque applied to the loop as a function of the angle θ. (15)

c) Express the time Δt which takes for the loop to fall completely from its initial position of $\theta_0=\frac{\pi}{3}$. For simplicity, assume that the magnetic torques is always in equilibrium with the gravitational torque induced by $-g\hat{k}$. (15)



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과목명 : 역학　　　　　　　　　　　　　　　2009 . 12 . 23 시행

1. Consider a particle with mass m and charge e, moving in three dimensions in an external magnetic field given by a magnetic monopole: $\vec{B} = q_m \frac{\vec{r}}{r^3}$.

a) Write down the Newton's equation of motion. (7)

b) The usual angular momentum is not conserved in this situation. However, using the equation in a), show that

$$\frac{d}{dt}(m\vec{r} \times \dot{\vec{r}}\,) = eq_m \frac{\dot{\vec{r}}\, r^2 - \vec{r}\,(\dot{\vec{r}}\, \cdot \vec{r}\,)}{r^3} = eq_m \frac{d}{dt}\left(\frac{\vec{r}}{r}\right).$$

Therefore, we now have a new angular momentum, $\vec{J} \equiv m\vec{r} \times \dot{\vec{r}} - eq_m \frac{\vec{r}}{r}$, that is conserved. (7)

c) Let us take the spherical coordinate r, θ, ϕ, aligning $\vec{J}$ along the positive direction of z axis. Show that the particle moves along a constant θ cone. Find the angle θ. (7)

d) Show that the trajectory $r(\phi)$ is given by:

$$\frac{1}{r(\phi)} = \frac{\sqrt{2mE}}{J}\left(1 - \frac{e^2 q_m^2}{J^2}\right)^{-1/2} sin\left[\left(1 - \frac{e^2 q_m^2}{J^2}\right)^{1/2}(\phi - \phi_0)\right]$$

(9)

2. A mass m is tied to a massless, thin string of length L_o which is in turn tied to a stationary cylinder. There is no gravitational force acting in this system. The string makes a right angle with the radius vector of the cylinder at the point of contact. At time $t = 0$ an impulse I is delivered to the mass in a direction perpendicular to the string and perpendicular to the axis of the cylinder.

a) Find the Lagrangian to describe the motion of m for $t > 0$, in terms of the coordinate variable in the figure. (10)

b) Write down the equation of motion and calculate the time t_o, required for the string to wrap completely around the cylinder. (10)

c) Find the magnitude and direction of the instantaneous acceleration of m and the tension in the string as a function of time. (10)