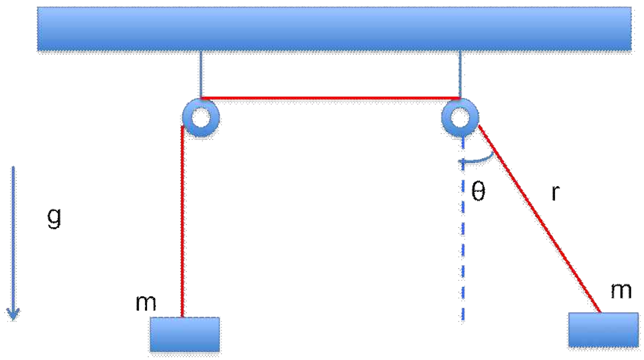
소속대학 학과(부)	자연과학대학 물리·천문학부	학번		성 명		감독교수 확 인	(인)

자격시험 문제

과목명 : 고전역학 **2015 . 06. 26 시행**

1. [50 pts] Two bricks are connected each other by a massless string (length of the string is ℓ) and their masses are equal (m). These bricks hang over two pulleys (of negligible size), as shown in the figure. The left one moves in a vertical line, but the right one is free to swing back and forth in the plane of the masses and pulleys.



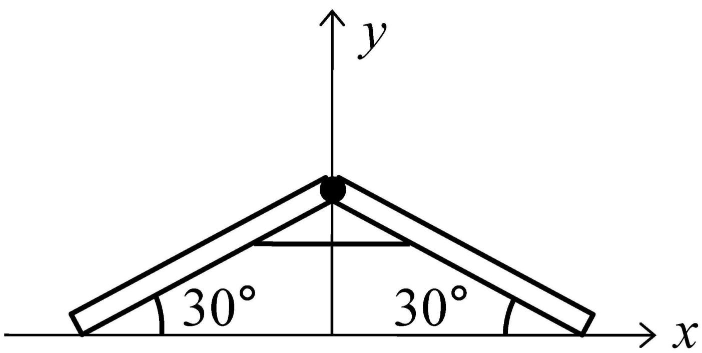
(a) [15 pts] Find the Lagrangian and its equations of motion for r and θ.

(b) [15 pts] Assume that the left mass starts at rest and the right mass undergoes small oscillations ($\sin\theta \cong \theta, \cos\theta \cong 1-\theta^2/2$) with angular amplitude A (with A << 1). Assume that r is constant, and find a frequence of the right brick and a solution for θ under the initial condition.

(c) [10 pts] Now let us assume that r is not a constant. Obtain the equation of motion with respect to r. What is the initial "average" acceleration (averaged over a few periods) of the left brick?

(d) [10 pts] In which direction does the left brick move?

2. [50 pts] Two thin rods of mass m and length ℓ are connected by frictionless hinge and a thread. The two rods are uniform. The system rests on a frictionless surface as shown in the figure. The hinge and the thread are massless. The two rods are initially at rest under the constant gravity along the vertical y-axis. Let us cut the thread using a scissor at $t=0$.



(a) [15 pts] Find the generalized coordinates and obtain the Lagrangian.

(b) [15 pts] Obtain the canonical momentum and Hamiltonian. Obtain the equations of motion for the Lagrangian.

(c) [10 pts] Obtain the speed of hinge when it hits the floor.

(d) [10 pts] How much time does it take for the hinge to hit the floor? You may express the answer in terms of a concrete definite integral which you need not evaluate explicitly.

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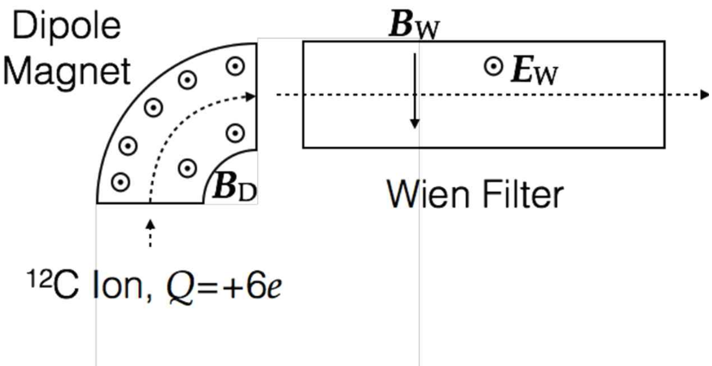
자격시험 문제

과목명 : 전기역학 **2015 . 06. 26 시행**

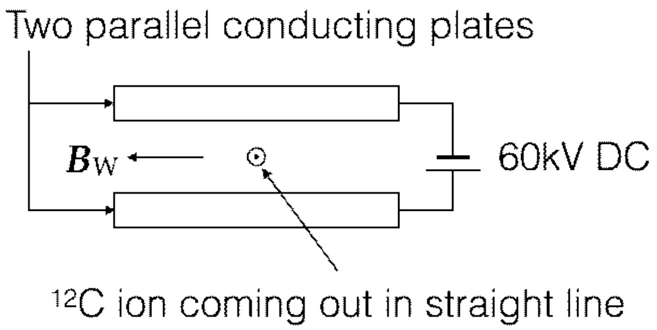
1. [40 pts] In high energy experiments, charged ions are steered using electric or magnetic fields. Let us consider the following system. The system has one dipole magnet which provides a uniform magnetic field. Another device is called Wien filter and provides electric and magnetic field in perpendicular configuration. ($E_W \perp B_W$). Answer to the following questions.



(a) [10 pts] Charged ions enter into the entrance of the dipole magnet. The ions are characterized by their charge Q, mass M and speed v. Find the necessary condition for some ions to exit the dipole magnet through the circular trajectory. The magnetic field of the magnet is B_D, and the radius of the curvature of the trajectory inside the magnet is ρ.

(b) [10 pts] Now consider a carbon ion, whose mass M is 12GeV/c^2 with charge Q=+6e. If the speed of the carbon ion is v=0.1c, what should be the value of $B_D\rho$ for the dipole in order to have the carbon ion to pass the dipole? Give a numerical value in unit of T·m with 1 significant digit. (1GeV=10^9 eV and c is the speed of light.)

(c) [10 pts] Now the ions coming out of the dipole magnet enters into the Wien filter, where they meet electric and magnetic field in perpendicular configuration ($E_W \perp B_W$). Find the condition that the ion(s) exit the Wien filter via straight line trajectory. The following figure shows the cross section of the Wien filter seen from its exit.



(d) [10 pts] Now the same carbon ion (M=12 GeV/c^2, Q=+6e, v=0.1c) is going straight through the Wien filter. In the Wien filter, a voltage difference of 60kV is applied between two parallel conducting plates separated by 10cm. What should be the magnetic field to be applied in perpendicular to the electric field? Give a numerical value in unit of T with 1 significant digit.

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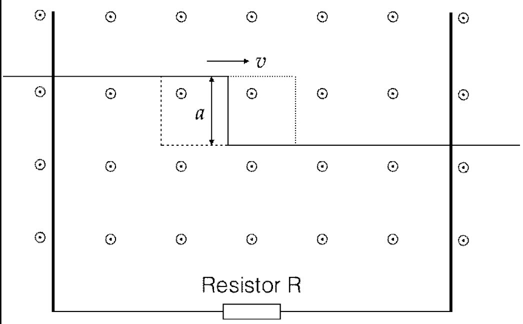
과목명 : 전기역학 **2015 . 06. 26 시행**

2. [40 pts] Consider a wire which is stretched horizontally. This wire can conduct current, and can also be easily bent (like a string). The string is pulled from the two ends with a tension, and a transverse wave is made from the left and propagates to the right, with speed v. The wave profile is given by a step function with height a as shown in the figure. The position of the step moves toward the right. There are two vertical wires at the left and right end which make contacts with the stretched wire. Finally a resistor R is connected to these two vertical wires to form a closed circuit.

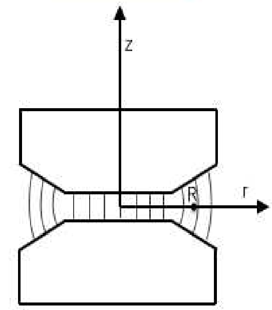


(a) [15 pts] The wave front (the position of the step) enters a region with uniform magnetic field B, as shown on the Figure. (B is applied `upwards' on the paper.) Compute the electromotive force on the wire.

(b) [5 pts] Connect a resistor R to the two ends of the wire, outside the region of applied B field, as shown in the figure. Compute the current I flowing through the wire. Explain the direction of the current I.

(c) [20 pts] Compute the work done by the B field per unit time on the wire with current I. Compare this with the power dissipated through the resistor R.

3. [60 pts] Azimuthal symmetric magnetic field B=B(r,z) is formed in a space between two iron poles, as shown in the figure below. This shape of magnetic field has been used for a charged particle accelerator called betatron. Answer the following questions.



(a) [10 pts] A charged particle(charge=e, mass=m) of <u>relativistic speed</u> is circulating on the z=0 plane with a constant radius R. (B_r = 0 at z=0.) Find out the relation between the momentum p_ϕ of the particle and R. (ϕ is the angle coordinate around the z axis.)

(b) [10 pts] If the magnetic field varies as a function of time, an electric field E_ϕ along ϕ direction is generated. Obtain the expression for E_ϕ in terms of the magnetic flux Φ in the region $r \le R$.

(c) [20 pts] In the setting of problem (b), charged particles at rest at $r=R$ can be accelerated. In a betatron, the particle stays in a fixed circular orbit of radius, R, even with particle acceleration to higher momentum. In such a condition, obtain the expression of p_ϕ in terms of Φ. (Let $B(t=0)=0$.)

(d) [20 pts] For the motion of problem (c) to be possible, show that magnetic field at R should be half of the average field within the area covered by the orbit of the particle.

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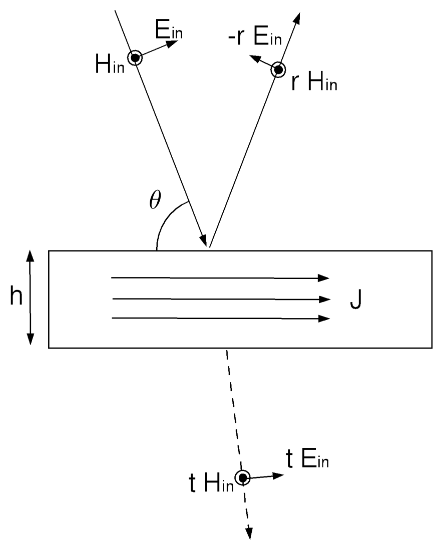
과목명 : 전기역학 **2015 . 06. 26 시행**

4. [60 pts] Consider the multiple reflections and transmissions of electromagnetic waves through a very thin metal film. See the figure below for the setting. All the summations of multiple transmissions and reflections are denoted by t and r, and J is the current inside the metal film induced by the electric field. Assume the normal incidence of the wave, i.e. $\theta = \frac{\pi}{2}$, for simplicity.



(a) [5 pts] Looking at the Maxwell's equation

$$\nabla \times H = J + \epsilon_0 \frac{\partial E}{\partial t} = \sigma E + \epsilon_0 \frac{\partial E}{\partial t},$$

show that both $\frac{\sigma}{\epsilon_0 \omega}$ and $\frac{\epsilon_0 c}{\sigma h}$ are dimensionless.

(b) [10 pts] When $\frac{\sigma}{\epsilon_0 \omega} \gg 1$ is satisfied, we can ignore the displacement current and use the Ampere's law $\nabla \times H = J$ inside the conductor. Let us assume that both J and E are constant inside the conductor because of the thinness of the metal. From the Ampere's law, show that

$$H_{\text{in}} + r H_{\text{in}} - t H_{\text{in}} = Jh = \sigma E h.$$

(c) [10 pts] From the continuity of tangential components of the electric field across the boundaries, show that $1 - r = t$.

(d) [15 pts] From (b) and (c), and from vacuum impedance relationship $E_{\text{in}} = \sqrt{\frac{\mu_0}{\epsilon_0}} H_{\text{in}} = \frac{1}{\epsilon_0 c} H_{\text{in}}$, show that

$$t = \frac{\frac{2\epsilon_0 c}{\sigma h}}{1 + \frac{2\epsilon_0 c}{\sigma h}} \approx \frac{2\epsilon_0 c}{\sigma h}$$

for good metal films that are not too thin; here, specify the meaning of `too thin' for gold with $\sigma = 4 \times 10^7$ Siemens. (The vacuum permitivity is given by $\epsilon_0 = 8.85 \times 10^{-12} F/m$.)

(e) [20 pts] For a gold film of 30nm thickness and for microwaves, compute the energy loss by comparing the incident/transmitted/reflected energies. Also, compare this with the Ohmic energy loss

$$\frac{\int \sigma E^2 dV}{\text{incident energy}} .$$

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자격시험 문제지

과목명 : 양자역학 **2015 . 06. 26 시행**

1. [60 pts] A particle with energy E and mass m moves in one dimension along the x−axis in the positive direction. At x = 0, there is a very narrow potential wall like $V(x) = \frac{\hbar^2}{2m}\gamma\delta(x)$ where $d(x)$ is the Dirac delta function.

(a) [20 pts] For the case g > 0, write down the Schroedinger equation and calculate the wave function for the scattered state (E>0).

(b) [20 pts] For the case g > 0, calculate the reflection and transmission coefficients for the calculated scattered state.

(c) [20 pts] For the case g < 0, calculate the energy for the bound state (E<0).

2. [60 pts] Consider a two−level system. An operator $\hat{A}$ representing observable A, has two normalized eigenstates ψ_1 and ψ_{-1} with eigenvalues 1 and -1, respectively. Operator $\hat{B}$, representing observable B, has two normalized eigenstates $\left(\psi_1 + \sqrt{3}\psi_{-1}\right)/2$ and $\left(\sqrt{3}\psi_1 - \psi_{-1}\right)/2$ with eigenvalues 1 and -1, respectively.

(a) [20 pts] Compute $[\hat{A},\hat{B}]$ (find the corresponding 2×2 matrix in the basis $\{\psi_1,\psi_{-1}\}$).

(b) [20 pts] Assume that the system is in state $\left(\psi_1 + \psi_{-1}\right)/\sqrt{2}$. Compute the variances $\sigma_A^2 = <\widehat{A^2}> - <\hat{A}>^2$ and σ_B^2 and check the uncertainty relation.

(c) [20 pts] Suppose that observable A was measured once at a certain time. Right after the measurement, the expectation value of B for the system was 1/2. Then, what would be the expectation value of A for this system?

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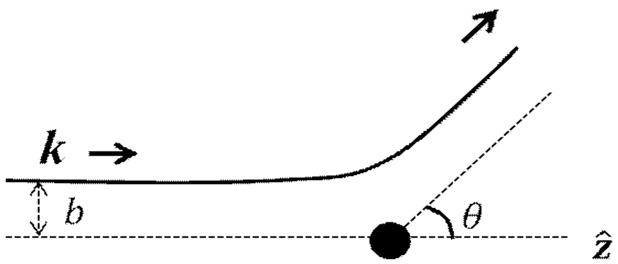
과목명 : 양자역학 **2015 . 06. 26 시행**

3. [80 pts] For a particle of mass m with energy E and angular momentum quantum number l in a central potential $V(r)$, the radial part of the wave function, $R(r)$ is given by

$$-\frac{\hbar^2}{2m}\frac{\partial^2 u}{\partial r^2}+\left(V(r)+\frac{\hbar^2 l(l+1)}{2mr^2}\right)u=Eu$$

where $u(r)=rR(r)$.



Imagine that an incident wave $\psi_{\rm in}=e^{ikz}$, where $k=\frac{\sqrt{2mE}}{\hbar}$, travelling along the z−direction is scattered by the central potential and an outgoing spherical wave of the form $\psi_{\rm sc}=f(\theta)\frac{e^{ikr}}{r}$ is produced. Particles incident within an infinitesimal patch of cross−sectional area $d\sigma$ will scatter into a corresponding infinitesimal solid angle $d\Omega$, then the differential cross−section is given by $\frac{d\sigma}{d\Omega}=|f(\theta)|^2$.

(a) [20 pts] Suppose that $f(\theta)$ is expressed as

$$f(\theta)=\sum_{l=0}^{\infty}(2l+1)f_l P_l(\cos\theta)$$

where $f_l=\frac{e^{i\delta_l}\sin\delta_l}{k}$, δ_l is the phase shift due to scattering and P_l is the lth Legendre polynomial. Show that the total cross−section is given by

$$\sigma_{\rm tot}=\int d\Omega\left(\frac{d\sigma}{d\Omega}\right)=\frac{4\pi}{k^2}\sum_{l=0}^{\infty}(2l+1)\sin^2\delta_l.$$

Let us consider a low−energy scattering by an attractive spherical potential well of depth $V_0>0$ and radius a:

$$V(r)=\begin{cases}-V_0 & (r<a)\\ 0 & (r>a)\end{cases}$$

(b) [20 pts] For the impact parameter b and incident momentum p, the angular momentum is given by $L=pb$. From the semiclassical point of view, explain that in the low−energy limit ($ka\ll 1$), s−wave ($l=0$) scattering is dominant.

(c) [40 pts] For the s−wave scattering, the general solution of $u(r)$ can be expressed as

$$u(r)=\begin{cases}C_{\rm in}\sin(k_0 r) & (r<a)\\ C_{\rm out}\sin(kr+\delta_0) & (r>a)\end{cases}$$

where $k_0=\frac{\sqrt{2m(E+V_0)}}{\hbar}$. Find the condition on V_0 that in $E\to 0$ limit the total cross−section vanishes.

※ If necessary, use the following formula:

$$P_l(x)=\frac{1}{2^l l!}\left(\frac{d}{dx}\right)^l(x^2-1)^l$$

$$\int_{-1}^{1}dx P_l(x)P_{l'}(x)=\frac{2\delta_{l,l'}}{2l+1}$$

$$e^{i\vec{k}\cdot\vec{r}}=\sum_{l=0}^{\infty}i^l(2l+1)j_l(kr)P_l(\cos\theta)$$

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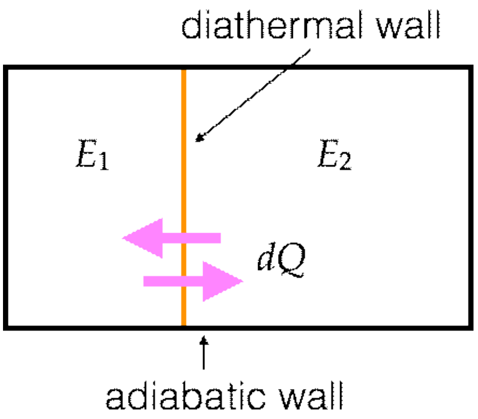
자격시험 문제지

과목명 : 통계역학 **2015 . 06. 26 시행**

1. [40pts] We have a chamber filled with ideal gas of monatomic molecules. The chamber is thermally isolated from the surroundings: this chamber is then separated into two parts by a diathermal wall through which heat can flow. The left chamber has a total energy E_1 while the right chamber has a total energy E_2.



(a) [5pts] If the number of possible states of the left chamber is $\Omega_1(E_1)$ and that for the right chamber is $\Omega_2(E_2)$, what is the total possible number of states Ω_{total} of the whole system.

(b) [15pts] Derive the condition for equilibrium in terms $\Omega_1(E_1)$ and $\Omega_2(E_2)$, i.e. number of states of each chamber, and its derivative with respect to E_1 and/or E_2. $E=E_1+E_2$ is the total energy of the whole system which is conserved.

(c) [10pts] Using the above results, you can now define one of the key thermodynamic quantities, temperature (T). State the equilibrium conditions using this quantity, T.

(d) [10pts] Examine how this newly-defined quantity control the flow of their conjugate quantity which is energy (E). For example, discuss the flow of energy when $T_1>T_2$.

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2. [60pts] Consider a system which can have a number of possible states specified with s and its energy E_s. The probability that the system is in a state s is given by p_s. Suppose that the entropy of the system is given by the Shannon entropy, $S = -k_B \sum_s p_s \ln p_s$. We are going to maximize the entropy under the condition that the mean energy of the system is given by E.

(a) [5pts] Write the formulas for the constraints of the system.

(b) [15pts] Obtain p_s by maximizing the entropy S under the constraints. You can use the Lagrange's method with undetermined multipliers for the constraints obtained in (a).

Now let us consider an ideal gas composed of N identical particles in a box with volume V_0. The mass of a single particle is given as m and the box is in thermal contact with heat reservoir with temperature T. We consider the so−called Joule expansion in which the volume of the box changes suddenly from V_0 to $2V_0$ isothermally.

(c) [15pts] Obtain the entropy change of the gas between before and after the Joule expansion using the formula for the Shannon entropy given above.

(d) [15pts] Obtain the partition functions before and after the Joule expansion. You can use the fact that the partition function is given as $Z_1 = \frac{V}{(h^2/2\pi m k_B T)^{3/2}}$ for a freely moving single particle of mass m in volume V.

(e) [10pts] Obtain the entropy change of the gas using the partition function obtained in (d).