## Physics II (Fall 2024): Homework #1 Solution

Department of Physics and Astronomy, SNU

10 problems; 100 points in total; Posted Sep. 13; Due Sep. 27, 2024

1. [10 pt] Two identical conducting spheres are fixed in space, with their center-to-center separation r = 1.0 m (much larger than the sphere's radius). Initially, they attract each other with an electrostatic force magnitude of  $F_1 = 0.027$  N. The spheres are then briefly connected by a conducting wire. When the wire is removed, the spheres repel each other with a force magnitude of  $F_2 = 0.0090$  N. What were the initial charges  $q_1$  and  $q_2$  on the two spheres? (Note: Assume  $q_1 > q_2$  and  $q_1 + q_2 > 0$ .)



• From Eq.(21-2) of Halliday & Resnick,  $q_1q_2 = -\frac{r^2F_1}{k} = -\frac{(1.0 \text{ m})^2 \times 0.027 \text{ N}}{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2} = -3.0 \times 10^{-12} \text{ C}^2$ , and  $\frac{q_1+q_2}{2} = r\sqrt{\frac{F_2}{k}} = 1.0 \text{ m}\sqrt{\frac{0.0090 \text{ N}}{9.0 \times 10^9 \text{ N} \cdot \text{m}^2/\text{C}^2}} = 1.0 \times 10^{-6} \text{ C}.$ 

2. [10 pt] Recall the electric dipole in Chapter 22-3 of Halliday & Resnick. Now, consider a point P at distance r from the dipole's midpoint, measured perpendicular to the dipole axis (see figure). Find the magnitude and direction of the electric field due to the dipole at point P.



• The y-component magnitude of the electric field induced by +q is  $E_{(+)}\sin\theta = \frac{1}{4\pi\epsilon_0}\frac{q}{r^2+(d/2)^2} \cdot \frac{d/2}{sqrtr^2+(d/2)^2}$ , where  $\theta$  is the angle between  $\vec{E}_{(+)}$  and the x-axis. The x-component of this field is cancelled out by symmetry when considering the other electric field  $\vec{E}_{(-)}$  induced by -q.

3. [10 pt] A plastic rod forms a semicircle of radius r. Charge -Q is uniformly distributed along the left half of the rod, and +Q along the right half (see figure). Find the magnitude and direction of the electric field due to the rod at point P, the center of the semicircle. (Note: You may want to review the Sample Problem in Chapter 22-4 of Halliday & Resnick.)



• Similar to Eq.(22-21) of Halliday & Resnick, the magnitude of the electric field due to the right half is  $E_{\text{right}} = \frac{1}{4\pi\epsilon_0 r} \frac{2Q}{\pi r} [\sin\theta]_{-\pi/4}^{+\pi/4} = \frac{\sqrt{2}Q}{2\pi^2\epsilon_0 r^2}$ , which is towards  $-\frac{1}{\sqrt{2}}(\hat{x}+\hat{y})$  direction. The *y*-component of this field is cancelled out by symmetry when considering the other electric field  $E_{\text{left}}$  induced by the left half with -Q.

4. [10 pt] An electron of mass m is constrained to move along the central axis of a fixed ring of uniform positive charge (total charge q and radius R; see Chapter 22-4 of Halliday & Resnick). After the electron is displaced vertically by a very small z (i.e.,  $z \ll R$ ) from the ring's center and released, it oscillates up and down. Determine the period of these small oscillations. (Note: You are asked to first derive the electron's equation of motion explicitly. But then, you may utilize the fact that the equation of motion in the form of  $a(t) = -\omega^2 x(t)$  depicts an oscillatory motion of angular frequency  $\omega$ , as you learned in Chapter 15-1 of Halliday & Resnick.)

• From Eq.(22-16) of Halliday & Resnick,  $F = ma = \frac{eqz}{4\pi\epsilon_0(z^2 + R^2)^{3/2}} \simeq \frac{eqz}{4\pi\epsilon_0 R^3}$ .

5. [10 pt] *Triboluminescence* is a type of luminescence that occurs when a material is mechanically rubbed, fractured, or crushed. While the phenomenon is yet to be fully understood, physicists have some ideas to explain it. After reviewing the relevant part of your textbook (Chapter 21-1 of Halliday & Resnick) and watching a demonstration with a high-speed camera at youtube.com/watch?v=tW8q\_JfmcbU&t=80s (or scan the QR code below with your cellphone camera), describe how we can see blue light when a candy is crushed. In particular, explain how this could be an indirect evidence for the existence and interaction of charges. Two or more paragraphs of 3-4 sentences each are expected to clearly convey your understanding.



(Note: You are welcomed to study various resources on the web; but surely, your answer must be your own work *in your own words*, and must reference your sources appropriately with a proper citation convention. To access academic journals off-campus via SNU library's proxy service, see library.snu.ac.kr/using/proxy. You may visit t.ly/M2C8D or t.ly/Y6dgA to read an anecdote involving Richard Feynman and triboluminescence — the latter link is for a book *"Surely You're Joking, Mr. Feynman!"*; it may not work well depending on your browser cache.)

• Useful pages include: sciencenotes.org/triboluminescence-definition-and-examples-cold-light/.

6. [10 pt] The figure below shows an edge-on view of two intersecting planar surfaces that are mutually perpendicular. Surface 1 has an area of  $1.6 \text{ m}^2$ , while surface 2 has  $3.5 \text{ m}^2$ . The electric field E = 250 N/C is uniform and  $\theta = 35^{\circ}$ . Compute the magnitude of the electric flux  $\Phi$  through each surface.



• From Eq.(23-2) of Halliday & Resnick,  $\Phi = EA \cos \theta$ , where  $\theta$  is the angle between  $\vec{E}$  and  $\vec{A}$ .

7. [10 pt] A small circular hole of radius  $R = 2.0 \,\mathrm{cm}$  has been cut out of an infinite, nonconducting sheet of uniform surface charge density  $\sigma = 4.5 \,\mathrm{pC/m^2}$  (see figure). Find the magnitude and direction of the electric field due to the sheet at point P at a distance  $z = 3.0 \,\mathrm{cm}$  from the center of the hole. (Note: You are welcome to make use of any of the several electric field formulae you learned in Chapters 22 and 23 of Halliday & Resnick.)



• From Eqs.(22-26) and (23-13) of Halliday & Resnick,  $\vec{E} = \frac{\sigma}{2\epsilon_0}\hat{z} + \frac{-\sigma}{2\epsilon_0}\left(1 - \frac{z}{\sqrt{z^2 + R^2}}\right)\hat{z} = \frac{\sigma z}{\epsilon_0\sqrt{z^2 + R^2}}\hat{z}.$ 

8. [10 pt] Two spherical shells have a common center C. A charge  $q_1 = -1.7 \times 10^{-6}$  C is uniformly spread over the inner shell of radius  $R_1 = 0.050$  m, and a charge  $q_2 = 5.1 \times 10^{-6}$  C is uniformly spread over the outer shell of radius  $R_2 = 0.15$  m. Determine the magnitude and direction of the electric field due to the shells at distances (a) 0.25 m, (b) 0.10 m, and (c) 0.020 m from C.

• From Eqs.(23-15) and (23-16) of Halliday & Resnick, (a)  $E_1 = \frac{1}{4\pi\epsilon_0} \frac{q_1+q_2}{(0.25 \text{ m})^2}$ , (b)  $E_2 = \frac{1}{4\pi\epsilon_0} \frac{q_1}{(0.10 \text{ m})^2}$ , (c)  $E_3 = \frac{1}{4\pi\epsilon_0} \frac{0}{(0.02 \text{ m})^2}$ .

9. [10 pt] Two solid spheres of radius R with charges  $q_1$  and  $q_2$  uniformly distributed throughout their volumes are located as shown in the figure below. Point P lies on a line connecting the centers of the spheres, at distance 0.48R from the center of sphere 1. If the net electric field due to the spheres at point P is zero, calculate the ratio  $q_2/q_1$ .



• From Eqs.(23-16) and (23-20) of Halliday & Resnick, the magnitude of the electric field at P is  $\frac{1}{4\pi\epsilon_0} \frac{q_1(0.48R)}{R^3} - \frac{1}{4\pi\epsilon_0} \frac{q_2}{(1.52R)^2} = 0$ , from which one can obtain  $\frac{q_2}{q_1} = 0.48 \times 1.52^2 \simeq 1.1$ .

10. [10 pt] An *electrostatic* generator is a type of an electrical generator that produces static electricity. A manual electrostatic generator typically develops electrostatic charges of opposite signs on two conductors, thereby converting mechanical work into electrical energy. Two famous examples of such generators are the *van de Graaff generator* and the *Wimshurst machine*, which can be used to create the static electric field that we deal with in this problem set. After studying the books and web resources on these topics, including the videos at youtube.com/watch?v=Zilvl9tS0Og and at youtube.com/watch?v=nA4aCd5qFWs (or scan the QR codes below with your cellphone camera), describe how the Wimshurst machine works. In particular, explain how it develops, collects, and stores the opposite electrostatic charges. Two or more paragraphs of 3-4 sentences each are expected to clearly convey your understanding.



(Note: You are welcomed to study various resources on the web; but surely, your answer must be your own work *in your own words*, and must reference your sources appropriately with a proper citation convention. Always be skeptical and double check, as not everything you find on the web is scientifically accurate. To access academic journals off-campus via SNU library's proxy service, see library.snu.ac.kr/using/proxy.)

• Useful pages include: demos.smu.ca/index.php/demos/e-n-m/43-wimshurst-machine.