Physics II (Fall 2024): Midterm Examination

Oct. 25, 2024

[total 25 pts, closed book, 90 minutes]

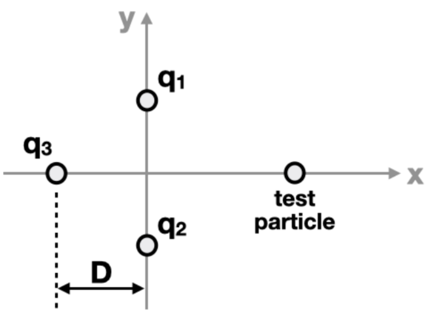
• First, make sure you have all 6 answer sheets. Write down your name and student ID on each of all 6 answer sheets. Then, number the sheets from 1 to 6 on the top right corner. Your answer to each problem must *only* be in the sheet with the matching number (e.g., your answer to Problem 2 must *only* be in sheet 2). After the exam, you will separately turn in all 6 answer sheets, even if some sheets are still blank.

• Make sure you have all 6 problems. Have a quick look through them all and portion your time wisely. If you have any issue or question on the problem itself or on English expressions, you *must* raise it in the first 45 minutes. You have to stay in the room for that 45 minutes even if you have nothing to write down.

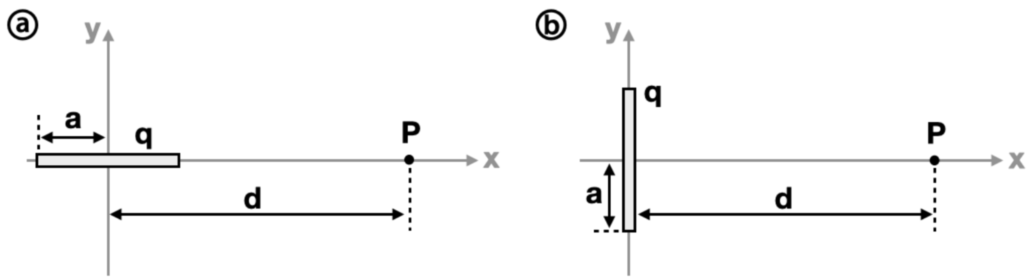
• Exhibit all intermediate steps to receive full credits. Make your writing easy to read. Illegible answers will *not* be graded. You are welcomed to use a scientific calculator; if you wish to use one on your cellphone, declare it when prompted by the TA. Obtain numerical results accurate to *two* significant figures.

• Elementary charge $e = 1.6 \times 10^{-19}\,\mathrm{C}$, the permittivity constant $\epsilon_0 = 8.9 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$, the permeability constant $\mu_0 = 1.3 \times 10^{-6}\,\mathrm{T \cdot m/A}$, and gravitational acceleration $g = 9.8\,\mathrm{m/s^2}$. Assume negligible friction and air resistance, unless stated otherwise.

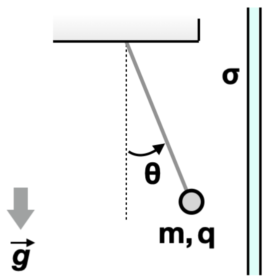
1. (a) [2 pt] In the figure below, particles 1 and 2 are located at (0, 1.0 cm) and (0, −1.0 cm), respectively, with charges $q_1 = q_2 = -e$. Particle 3 is located on the negative x-axis with $q_3 = +5e$, and the test particle is at (2.0 cm, 0). *(i)* What is the distance D between the origin and q_3 if the net electrostatic force on the test particle due to the other particles is zero? *(ii)* If particles 1 and 2 were moved straight towards the origin but maintained their symmetry about the x-axis, would the required value of D be greater than, less than, or the same as in *(i)*?



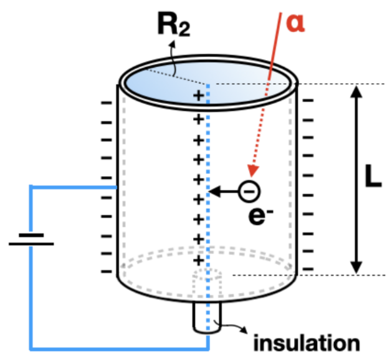
(b) [2 pt] Positive charge q is uniformly distributed on a thin, nonconducting rod of length $2a$ along the x-axis between $[-a, +a]$ (case "a"), or along the y-axis between $[-a, +a]$ (case "b"; see figure). *(i)* For each case, find the magnitude and direction of the electric field at point P on the x-axis at distance d from the rod's midpoint. *(ii)* What form do your answers in *(i)* reduce to when P is very far from the charged rod, or if the rod is very short (i.e., $d \gg a$)? Explain why your answer is reasonable. *(iii)* What form does your answer for case "b" in *(i)* reduce to when P is very close to the charged rod, or if the rod is very long (i.e., $a \gg d$)?



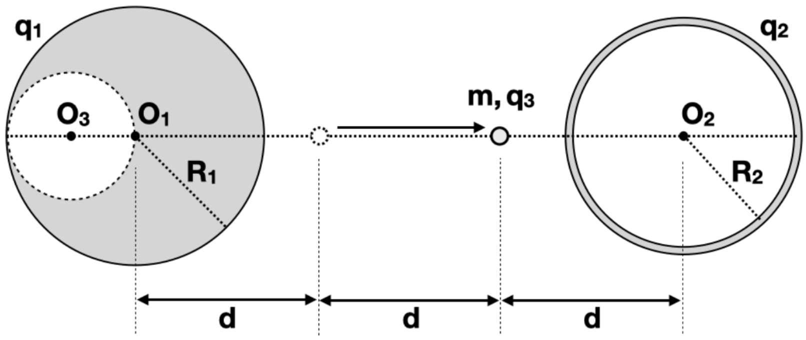
2. (a) [1 pt] A small, nonconducting ball of mass $m = 2.0\,\text{mg}$ and charge $q = +4.8 \times 10^{-8}\text{C}$ hangs from an insulating thread near a uniformly charged nonconducting sheet (shown in cross-section in the figure below; the sheet extends far vertically and into and out of the page). If the thread makes an angle $\theta = 20°$ with the vertical, find the surface charge density σ of the sheet.



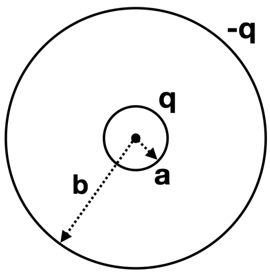
(b) [2 pt] A *Geiger counter* detects radiation such as α particles by using the fact that the radiation ionizes the air along its path. A thin, positively charged wire lies on the axis of a hollow, concentric cylindrical metal shell with an equal negative charge (see figure). A large potential difference is established between the wire and the shell, creating a strong electric field directed radially outward. When ionizing radiation enters the device, it ionizes a few air molecules. The resulting free electrons are accelerated towards the central wire by the electric field and become more energetic. As a result, they ionize many more air molecules on their way to the wire, producing even more electrons. The "avalanche" of electrons is collected by the wire, generating a current pulse and a clicking sound. Suppose that the radius of the central wire is $R_1 = 25\,\mu\text{m}$, the inner radius of the shell $R_2 = 1.5\,\text{cm}$, and the length of the shell $L = 16\,\text{cm}$. If the electric field at the shell's inner wall is $E = 2.4 \times 10^4\,\text{V/m}$, what is the total charge Q on the central wire? (Note: You may assume that L is much larger than R_2.)



3. (a) [2 pt] The figure below shows an arrangement of three objects. On the left, a nonconducting sphere of radius $R_1 = 0.7\,\mathrm{cm}$ has been hollowed out so that the surface of the spherical hollow passes through the center of the sphere (O_1) and "touches" the left side of the sphere. On the right, a nonconducting spherical shell of radius $R_2 = 0.6\,\mathrm{cm}$ is placed so that its center (O_2) lies on the straight line connecting O_1 and the center of the hollow (O_3; see figure). Charges $q_1 = +3.5 \times 10^{-9}\mathrm{C}$ and $q_2 = -3.0 \times 10^{-9}\mathrm{C}$ are uniformly distributed throughout the volume of the left and right object, respectively. O_1 and O_2, both fixed in space, are separated by $3d = 3.0\,\mathrm{cm}$. Then, a particle of mass $m = 4.0\,\mathrm{mg}$ and charge $q_3 = +2.0 \times 10^{-9}\mathrm{C}$ is placed at a distance d from O_1, on a common line with all the centers on it. The particle starts from rest and moves along the common line. What is its speed v when it is $d = 1.0\,\mathrm{cm}$ away from O_2?



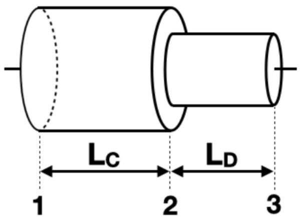
(b) [2 pt] A metal sphere of radius a and charge q (> 0) is on an insulating stand at the center of a larger, spherical metal shell of radius b and charge $-q$ (see figure). Find the electric field as a function of distance r from the sphere's center, $\vec{E}(r)$, for all values of r ($0 < r < \infty$). Calculate and sketch $V(r)$, while taking $V(\infty) = 0$. What is the potential difference V_{ab} between the spheres? Finally, show that the capacitance of this two-sphere system is $C = 4\pi\epsilon_0 \left(\frac{ab}{b-a}\right)$.



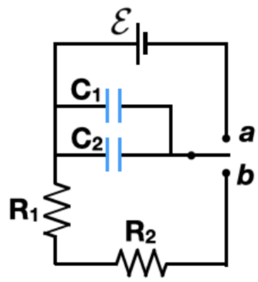
(c) [1 pt] Using your findings in (b), verify that the following equality holds for the energy stored in the electric field between the spheres: $\frac{1}{2}CV_{ab}^2 = \int_{\mathcal{V}} \frac{1}{2}\epsilon_0 \{E(r)\}^2 \, d\mathcal{V}$. (Note: In the class, we discussed this equality for the case of a parallel-plate capacitor. You may utilize $\int_{\mathcal{V}} f(r) d\mathcal{V} = \int_a^b f(r) 4\pi r^2 dr$ for a spherically symmetric function $f(r)$ and volume $\mathcal{V}$.)

4. (a) [2 pt] Wire C of length $L_C = 1.0\,\mathrm{m}$ and wire D of length $L_D = 0.80\,\mathrm{m}$ are joined as shown in the figure below, but are made from different materials. The resistivity and diameter of wire C are $2.0 \times 10^{-6}\Omega \cdot \mathrm{m}$ and $1.0\,\mathrm{mm}$, and those of wire D are $1.0 \times 10^{-6}\Omega \cdot \mathrm{m}$ and $0.60\,\mathrm{mm}$. If a current $I = 2.0\,\mathrm{A}$ is set up in them, what is the electric potential difference between points 1

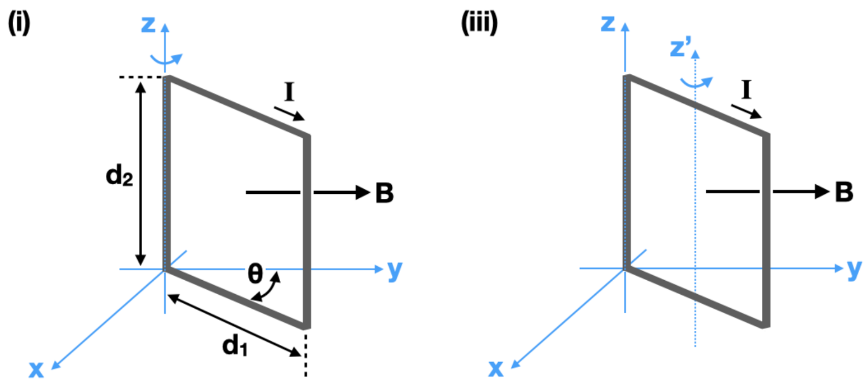
and 2, and between points 2 and 3? How about the rate at which energy is dissipated between points 1 and 2, and between points 2 and 3?



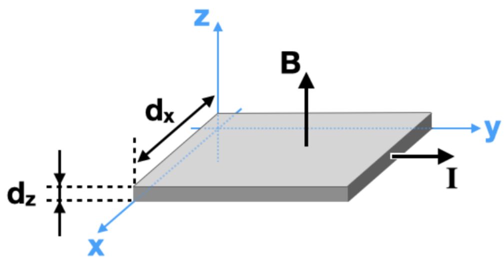
(b) [2 pt] In the circuit below, the resistances are $R_1 = 30\,\Omega$, $R_2 = 50\,\Omega$, and the capacitances are $C_1 = 15\,\mu\text{F}$, $C_2 = 20\,\mu\text{F}$, and the ideal battery has an emf of $\mathcal{E} = 45\,\text{V}$. First, the switch was held in position "*a*" for a long time; then, it was quickly thrown to position "*b*". *(i)* How long after the switch is moved to position "*b*" will the potential across C_1 be reduced to 10 V? *(ii)* What will be the current in the circuit at that time?



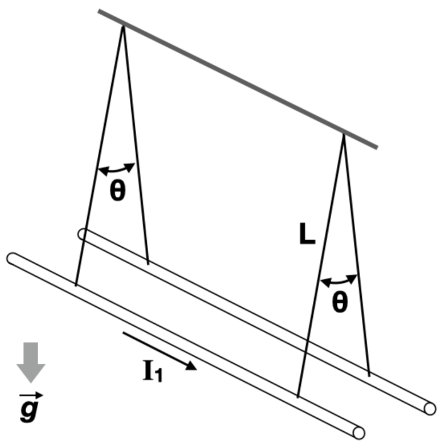
5. (a) [2 pt] The rectangular loop of $d_1 = 50\,\text{cm}$ and $d_2 = 70\,\text{cm}$ shown in the figure consists of 65 turns and carries a current of $I = 4.5\,\text{A}$. A uniform magnetic field of $B = 1.8\,\text{T}$ is directed along the $+y$-axis. The loop is pivoted about the z-axis (i.e., free to rotate about the z-axis) along one of the long sides. The angle θ shown in the figure is 30°. *(i)* Determine the magnitude and direction of the net torque exerted on the loop (clockwise or counterclockwise when viewed from the top). (Note: You may want to be reminded that, when expressed in SI base units, $1\,\text{T} = 1\,\text{kg/A}\cdot\text{s}^2$.) *(ii)* State whether the angle θ will increase or decrease. *(iii)* If the loop were pivoted about an axis through the center of the loop, parallel to the z-axis (see figure), how would your answer in *(i)* change?



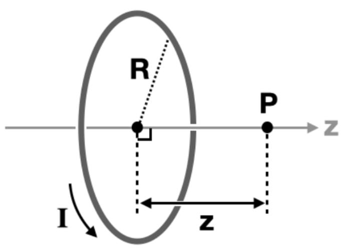
(b) [2 pt] A thin, rectangular silver strip of width $d_x = 1.2\,\text{cm}$ and thickness $d_z = 0.30\,\text{mm}$ carries a steady current $I = 120\,\text{A}$ in the $+y$-direction (see figure). The strip lies in a uniform magnetic field of $B = 1.0\,\text{T}$ along the $+z$-axis. The charge carrier density in silver is $n = 5.9 \times 10^{28}\,\text{electrons/m}^3$. *(i)* Find the magnitude of the electron drift velocity, v_d. *(ii)* Determine the magnitude and direction of the electric field due to the Hall effect. *(iii)* Prove that the Hall voltage (Hall potential difference) developed across the width of the strip is written as $V_\text{H} = \frac{BI}{ned_z}$. Find its numerical value. *(iv)* Discuss qualitatively how the Hall voltage would change if the material of the strip was a p-type semiconductor instead of silver.



6. (a) [2 pt] Two long parallel wires are suspended from a common axis by four cables of length $L = 40\,\text{cm}$ (see figure). The wires have a mass per unit length of $\lambda = 0.013\,\text{kg/m}$, and one of the wires carries a current of $I_1 = 30\,\text{A}$. What is the magnitude and direction of the current I_2 in the other wire if the angle between the cables holding the two wires is $\theta = 12°$?



(b) [2 pt] Consider a magnetic dipole, that is, a single circular current loop of radius R and current I. Find the the magnetic field (magnitude and direction) at point P on the symmetry axis of the loop, at distance z from the center of the loop (see figure), by explicitly integrating the differential magnetic field, $\vec{B} = \int d\vec{B}$. Explain which expressions your answer reduces to *(i)* when P is at the center of the loop, and *(ii)* when P is very far from the loop ($z \gg R$).



[Problem 6(c) next page.]

(c) [1 pt] Now consider an arrangement known as a *Helmholtz coil*, which was briefly discussed in the class. It consists of two circular coaxial coils, each of N turns and radius R, separated by a distance R. The two coils carry equal currents I in the same direction. Find the the net magnetic field $\vec{B}$ (magnitude and direction) at the midpoint P between the coils. Demonstrate that the field is reasonably uniform near P ($z = 0$; see figure) by showing that its first and second derivatives are zero, i.e., $\frac{\partial \vec{B}}{\partial z}\Big|_{z=0} = \frac{\partial^2 \vec{B}}{\partial z^2}\Big|_{z=0} = 0$.