Physics I (Spring 2023): Final Examination

June 9, 2023

[total 25 pts, closed book, 90 minutes]

• First, make sure you have all 6 answer sheets. Write down your name and student ID on each of all 6 answer sheets. Then, number the sheets from 1 to 6 on the top right corner. Your answer to each problem must *only* be in the sheet with the matching number (e.g., your answer to Problem 2 must *only* be in sheet 2). After the exam, you will separately turn in all 6 answer sheets, even if some sheets are still blank.

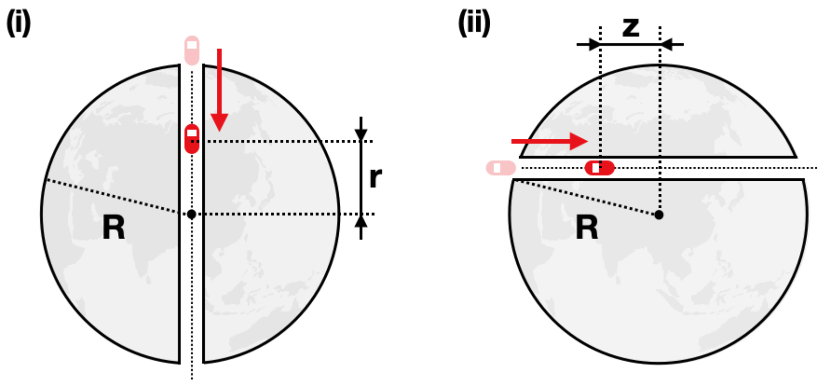
• Make sure you have all 6 problems. Have a quick look through them all and portion your time wisely. If you have any issue or question on the problem itself or on English expressions, you *must* raise it in the first 45 minutes. You have to stay in the room for that 45 minutes even if you have nothing to write down.

• Exhibit all intermediate steps to receive full credits. Make your writing easy to read. Illegible answers will *not* be graded. You are welcomed to use a scientific calculator — physical one or the one on your cellphone. Obtain numerical results accurate to *two* significant figures.

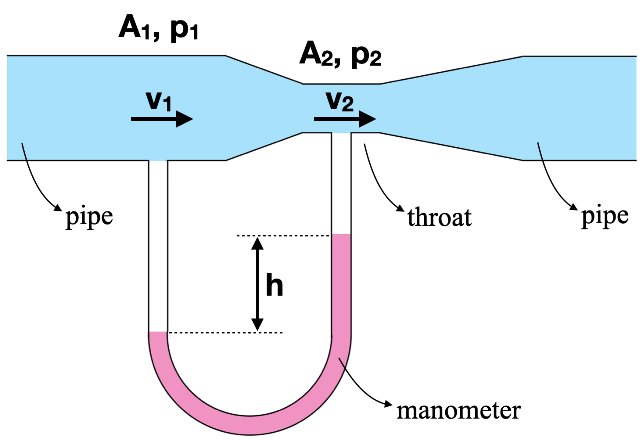
• Gravitational acceleration $g = 9.8\,\mathrm{m/s^2}$. Assume negligible friction and air resistance, unless stated otherwise. Any cord connecting one object to another is massless, unless stated otherwise.

1. (a) [1 pt] By modeling Earth as a uniform solid sphere of mass M and radius R, find the magnitude of the gravitational force acting on a particle of mass m inside Earth at a distance r from its center.

(b) [2 pt] A passenger capsule is dropped from rest into a narrow tunnel drilled straight through the center of Earth (see figure). Neglecting frictions and rotational effects, show that the capsule's motion is described as simple harmonic oscillations. Find the period of these oscillations as a function of R, M and the gravitational constant G. *(ii)* Explain how your answer changes if the tunnel is not through Earth's center, but obliquely through Earth (see figure; with z as a displacement along the tunnel from the tunnel's mid-point), in which the capsule slides without friction. (Note: You are asked to first derive the capsule's equation of motion explicitly. But then, you may utilize the fact that the equation of motion in the form of $a(t) = -\omega^2 x(t)$ depicts an oscillatory motion of angular frequency ω.)



(c) [1 pt] In the class, we watched a video demo of a *Venturi tube* which allows us to determine the flow rate of a fluid in a pipe. Water flows from the "pipe" of cross-sectional area A_1 with speed v_1, and then through a narrow "throat" of cross-sectional area A_2 with speed v_2 (see figure). A manometer[1] connects the wider portion of the tube to the narrower portion, and measures the height difference h of the gauge liquid in its two arms caused by the difference in the water pressures p_1 and p_2. Show that the flow speed can be computed as $v_1 = \left[\frac{2(p_1-p_2)}{\rho\{(A_1/A_2)^2-1\}}\right]^{1/2}$, where $\rho = 1.0\,\mathrm{g/cm^3}$ is the water density.

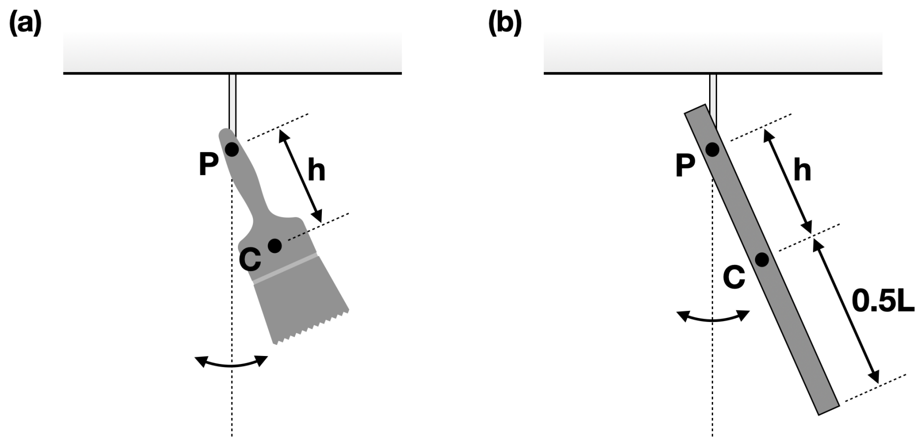


(d) [1 pt] Suppose that the cross-sectional areas are $A_1 = 50\,\mathrm{cm^2}$ in the pipe and $A_2 = 10\,\mathrm{cm^2}$ in the throat. *(i)* If the pressures are $p_1 = 1.5\,\mathrm{atm}$ in the pipe and $p_2 = 1.2\,\mathrm{atm}$ in the throat, what is the volume flow rate of water in $\mathrm{m^3/s}$? *(ii)* If the pressure in the pipe is $p_1 = 2.0\,\mathrm{atm}$, what is the speed v_1 that makes the pressure p_2 equal to (nearly) zero? (Note: When this condition is met, the water vaporizes into small bubbles in the throat, a phenomenon known as *cavitation*. $1\,\mathrm{atm} = 1.013 \times 10^5\,\mathrm{Pa}$. $1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$.)

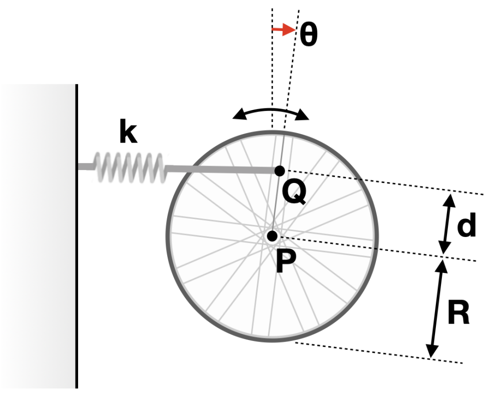
[1] manometer: 액주압력계

2. (a) [1 pt] Consider small angle oscillations of a physical pendulum (see figure). Show that its motion can be described as simple harmonic motion with a period $T = 2\pi\sqrt{\frac{I}{mgh}}$ where I is the pendulum's rotational inertia about the pivot, m is the pendulum's mass, and h is the distance between its center of mass C and the pivot P. (Note: You are asked to first derive the pendulum's equation of motion explicitly. But then, you may utilize the fact that the equation of motion in the form of $a(t) = -\omega^2 x(t)$ depicts an oscillatory motion of angular frequency ω.)

(b) [1 pt] A uniform thin rod of mass $m = 1.0\,\text{kg}$ and length $L = 2.0\,\text{m}$ oscillates as a physical pendulum (see figure). Assuming small oscillations, what value of distance h between the rod's center of mass C and its pivot P gives the shortest period? What is that shortest period? (Note: The table of various rotational inertias is in the last page of this exam.)



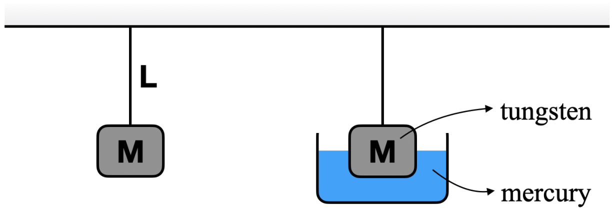
(c) [2 pt] A wheel is free to rotate about its axle fixed at point P, and is modeled as a hoop of mass m and radius R (see figure). A massless spring of spring constant k connects a rigid wall with one of the wheel's massless spokes[2] at point Q (at a distance d from the axle). Initially when $\theta = 0$, the spring is at its rest length. If the wheel is rotated by $\theta = 2°$ and released, what is the period T of small oscillations of this system in terms of m, R, d, and k? What is T if $d = R$? (Note: The table of various rotational inertias is in the last page of this exam.)



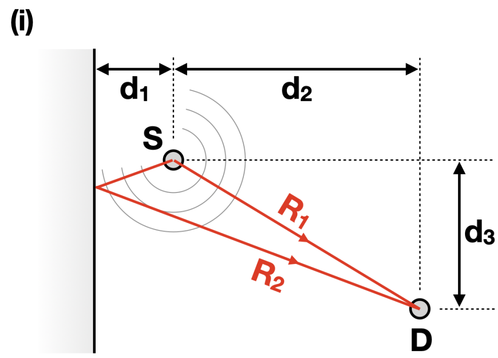
[2]spoke: 바퀴살

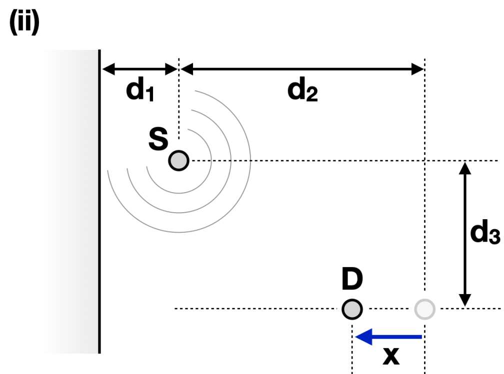
3. (a) [1 pt] Consider two sound waves of equal amplitude s_m but slightly different frequencies, f_1 and f_2 ($f_1 > f_2$), being measured together. By writing the displacements due to the two waves as $s_m \cos \omega_1 t$ and $s_m \cos \omega_2 t$, prove that the beat frequency is $f_{\text{beat}} = f_1 - f_2$.

(b) [2 pt] A tungsten block of $M = 20\,\text{kg}$ is suspended from a wire. In another configuration next to it, an identical tungsten block is suspended from an identical wire, but the lower half of the block is submerged in a container of mercury (see figure). The wires are each $L = 75\,\text{cm}$ long and have a mass of $m = 5.0\,\text{g}$. If both wires are simultaneously plucked at the center, what is the frequency of the beats that they will produce when vibrating in their fundamental frequencies? (Note: The density of tungsten is $\rho_{\text{W}} = 19.3\,\text{g/cm}^3$ while that of mercury is $\rho_{\text{Hg}} = 13.5\,\text{g/cm}^3$. You may assume that the mass of the wire is negligible compared to that of the tungsten block.)



(c) [2 pt] A speaker S lies near a reflecting wall. A sound detector D intercepts the sound ray R_1 traveling directly from S, and the sound ray R_2 that reflects from the wall and then travels to D (see figure; seen from above). The reflection by the wall causes a phase shift of 0.5λ, and the angle of incidence is equal to the angle of reflection. Distances shown are $d_1 = 2.0\,\text{m}$, $d_2 = 20\,\text{m}$, and $d_3 = 10\,\text{m}$. *(i)* What is the second lowest frequency at which R_1 and R_2 are in phase at D? *(ii)* Now, as the speaker S emits a sound wave of this frequency, the detector D slowly moves towards the wall and observes that the sound intensity gradually decreases. How far, denoted as distance x, should D move to find that the intensity reaches the minimum for the first time? For *(ii)*, you only need to write down the equation; you need not evaluate to find x. (Note: The speed of sound $v_s = 340$ m/s. Assume that S and D are at the same height above the ground.)

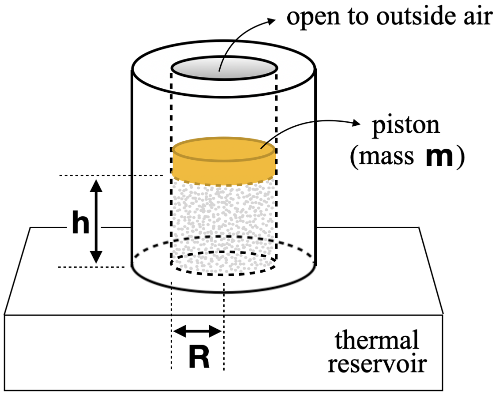




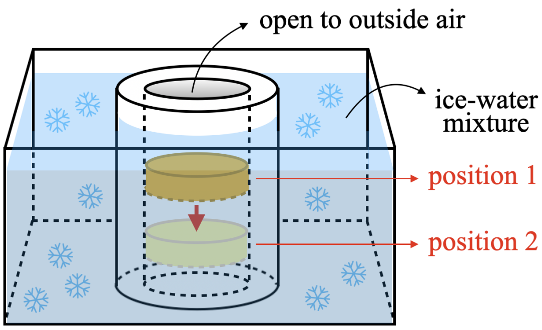
4. (a) [1 pt] An air bubble of volume $V_1 = 10\,\text{cm}^3$ is at the bottom of a lake $d = 40\,\text{m}$ deep, where the temperature is $T_1 = 4.0°\text{C}$. The bubble slowly rises to the surface, which is at a temperature of $T_2 = 27°\text{C}$. Assuming that the temperature of the bubble's air is always the same as that of the surrounding water, what is the bubble's volume just as it reaches the surface? (Note: Use the

water density $\rho = 1.0\,\mathrm{g/cm^3}$, and the gas constant $R = 8.3$ J/mol · K. $1\,\mathrm{atm} = 1.013 \times 10^5\,\mathrm{Pa}$ at the lake surface. $1\,\mathrm{Pa} = 1\,\mathrm{N/m^2}$.)

(b) [2 pt] A monatomic ideal gas is confined to a cylinder of inner radius R fitted with a movable piston of mass m. The piston moves smoothly while experiencing no friction with the wall. The entire cylinder is placed in a constant-temperature reservoir. The upper side of the piston is exposed to the outside air at atmospheric pressure p_0 (see figure). In equilibrium, the piston is at a height h above the bottom of the cylinder. *(i)* Find the pressure p of the gas trapped below the piston. *(ii)* The piston is pulled up by a small distance z ($\ll h$). Find the magnitude and direction of the net force F acting on the piston, using $(1 + x)^{-1} \simeq 1 - x$ when $x \ll 1$. *(iii)* After the piston is displaced from equilibrium by z and released, it oscillates up and down. Find the period of these small oscillations. (Note: For *(iii)*, you are asked to first derive the piston's equation of motion explicitly. But then, you may utilize the fact that the equation of motion in the form of $a(t) = -\omega^2 x(t)$ depicts an oscillatory motion of angular frequency ω.)



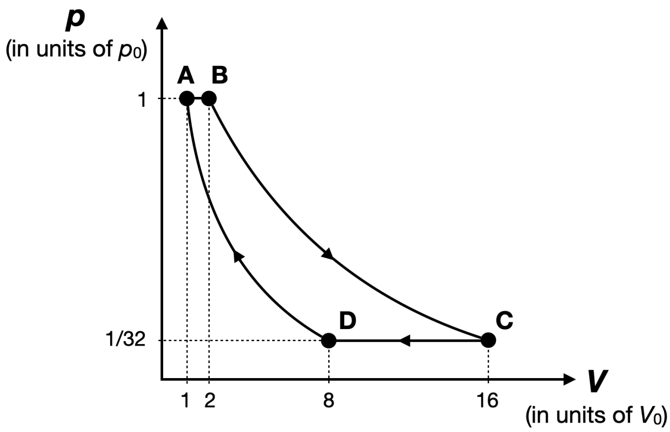
(c) [2 pt] A cylinder contains a monatomic ideal gas — initially at pressure p_0 and volume V_0 — and is closed by a movable piston. The cylinder is kept submerged in an ice-water mixture (see figure). The piston is *quickly* pushed down from position 1 to position 2 (from V_0 to $0.4\,V_0$; *process* "*A*"), and then held at position 2 until the gas is again in equilibrium with the ice-water mixture (*process* "*B*"). It is then *slowly* raised back to position 1 (*process* "*C*"). *(i)* Sketch the full cycle on a p-V diagram. Indicate which process corresponds to which curve. Write down the pressure and volume of the gas in units of (p_0, V_0) at each vertex.[3] Identify each process as one of the followings: isochoric/isobaric/isothermal/adiabatic. *(ii)* During which process(es) does the energy leave the gas as heat? During which process(es) does the gas absorb heat? During which process(es) does the gas do positive work? *(iii)* If 50 g of ice is melted during one full cycle, how much net work has been done *on* the gas? (Note: Use the heat of fusion $L_F = 333\,\mathrm{kJ/kg}$ for water.)



[3]vertex: 꼭짓점

5. (a) [2 pt] An object of mass m_1, specific heat c_1, and temperature T_L comes in contact with another object of mass m_2, specific heat c_2, and temperature T_H $(> T_L)$. As a result, the temperature of the first object increases to T_L' and that of the other decreases to T_H'. *(i)* Show that the energy conservation requires $m_1 c_1 (T_L' - T_L) = m_2 c_2 (T_H - T_H')$ and that the entropy increase of the system is $\Delta S = m_1 c_1 \ln\left(\frac{T_L'}{T_L}\right) + m_2 c_2 \ln\left(\frac{T_H'}{T_H}\right)$. *(ii)* Writing ΔS as a function of T_L' only, show that ΔS is maximized if $T_L' = T_H' = \frac{m_1 c_1 T_L + m_2 c_2 T_H}{m_1 c_1 + m_2 c_2}$. Discuss its implication using the following keywords: the 2nd law of thermodynamics, thermodynamic equilibrium.

(b) [2 pt] 2.0 mol of an ideal gas acts as the working substance in an engine that operates on the cycle shown in the p-V diagram below. Processes BC and DA are adiabatic and reversible. *(i)* Is the gas monatomic, diatomic, or polyatomic? *(ii)* What is the engine efficiency? (Note: Use the gas constant $R = 8.3$ J/mol $\cdot$ K if needed.)

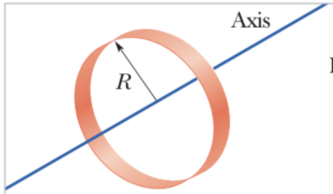
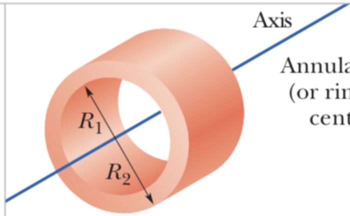
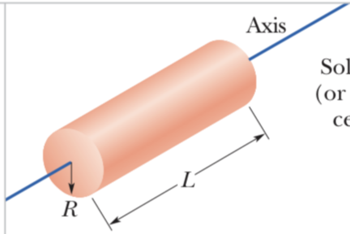
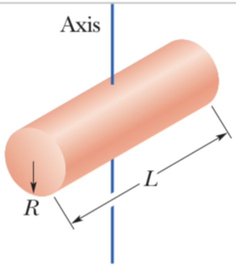
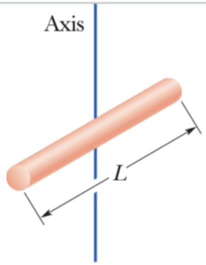
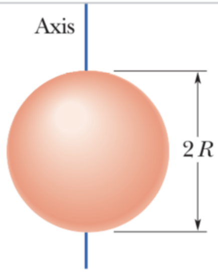
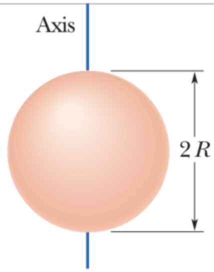
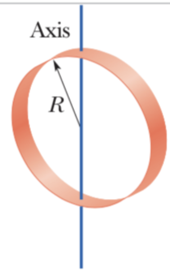
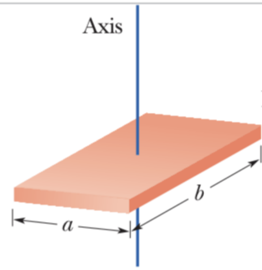


6. (a) [1 pt] As we cover the topic of waves, we discussed many examples where the classical wave theory is utilized to understand seemingly very different domains within modern physics. Among the following sets of keywords, select *two* and craft a concise yet physically meaningful narrative for each set that incorporates all the keywords in it. For each set, a paragraph of at least 3-4 sentences is expected to explain how the wave theory is employed in these (relatively new) fields of study, or how it facilitates analogies to be drawn as new ideas emerge in these fields. To aid the grader, circle the keywords in your answers. Use diagrams if desired.

- Gravitational wave detection: *interference*, spacetime distortion, Michelson and Morley
- Exoplanet detection: *Doppler effect*, binary system, invisible object, redshift and blueshift
- Neutrino detection: *sonic boom*, supersonic motion, charged particle, Cherenkov radiation
- Quantum mechanical wave function: *standing wave*, boundary condition, quantized states

(b) [1 pt] Throughout the semester we discussed many examples in which concepts in classical physics are utilized in contemporary research or in explaining daily phenomena. In this regard, four of your peers presented their term projects at the end of the semester. Describe the key idea of one of the presentations you found interesting. A paragraph of at least 3-4 sentences is expected to clearly convey the core physics idea of his/her term project. Use diagrams if desired. If you were one of the presenters, please choose someone else's.

Table 10-2 Some Rotational Inertias

 Hoop about central axis $I = MR^2$ (*a*)	 Annular cylinder (or ring) about central axis $I = \frac{1}{2}M(R_1^2 + R_2^2)$ (*b*)	 Solid cylinder (or disk) about central axis $I = \frac{1}{2}MR^2$ (*c*)
 Solid cylinder (or disk) about central diameter $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (*d*)	 Thin rod about axis through center perpendicular to length $I = \frac{1}{12}ML^2$ (*e*)	 Solid sphere about any diameter $I = \frac{2}{5}MR^2$ (*f*)
 Thin spherical shell about any diameter $I = \frac{2}{3}MR^2$ (*g*)	 Hoop about any diameter $I = \frac{1}{2}MR^2$ (*h*)	 Slab about perpendicular axis through center $I = \frac{1}{12}M(a^2 + b^2)$ (*i*)