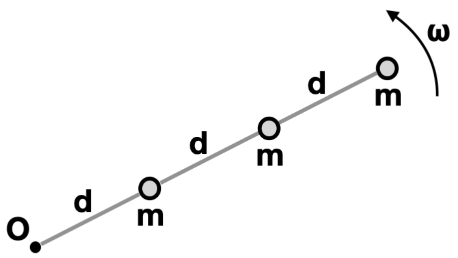
Physics I (Spring 2022): Final Exam Solution

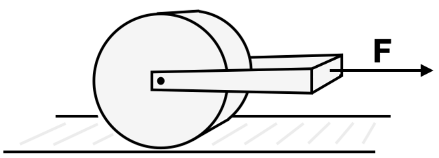
June 3, 2022

[total 25 pts, closed book, 90 minutes]

1. (a) [2 pt] Three particles, each with mass $m = 0.30$ kg, are fastened to each other and to a rotation axis at point O, by three thin rods of length $d = 10$ cm and negligible mass (see figure). The particles and rods are joined in a straight line at all times. The system rotates around O with an angular speed $\omega = 0.80$ rad/s. Measured about O, what are the system's *(i)* rotational inertia and *(ii)* angular momentum (magnitude only)?



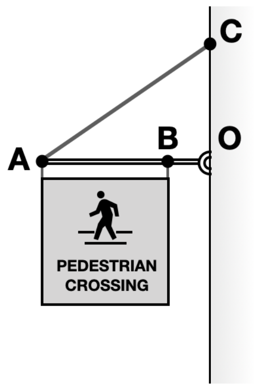
(b) [2 pt] A constant rightward horizontal force $\vec{F}$ of magnitude 120 N is applied to a steam roller which, as a result, rolls smoothly on the horizontal surface with an acceleration of $a = 0.60$ m/s^2 to the right. The steam roller can be modeled as *an azimuthally symmetric, but nonuniform cylinder* of radius $R = 0.50$ m and mass $M = 120$ kg (see figure). All other parts of the steam roller such as the axle or the frame have negligible masses. Find *(i)* the frictional force on the cylinder from the ground (both magnitude and direction), and *(ii)* the rotational inertia of the cylinder about the rotation axis through its center of mass.



- (a) $I = md^2 + 4md^2 + 9md^2$ and $L = I\omega = 14md^2\omega$.
- (b) $F - f_s = Ma$ and $\tau = Rf_s = I\alpha = I\left(\frac{a}{R}\right)$.

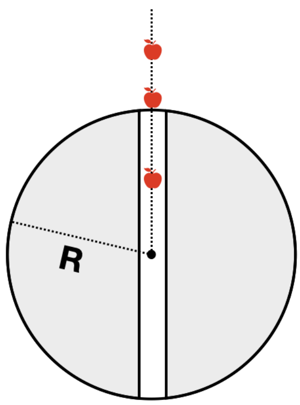
2. (a) [2 pt] When we walk, our legs alternately swing forward about the hip joint as a pivot. By treating the leg as a physical pendulum of a thin uniform rod of length $L = 0.80$ m, find the time it takes for the leg to swing forward. (Note: The table of various rotational inertias is in the last page of this exam. The time it takes for the leg to swing forward is one-half the period of this physical pendulum.)

(b) [2 pt] A uniform, thin, square steel sign of mass $M = 20$ kg and edge $\overline{\mathrm{AB}} = 3.0$ m is hanging from a massless rod of length $\overline{\mathrm{AO}} = 4.0$ m hinged at point O (see figure). A cable is attached to the one end of the rod (point A) and to the wall (point C), 3.0 m above O (i.e., $\overline{\mathrm{CO}} = 3.0$ m). *(i)* What is the tension T in the cable? *(ii)* What are the magnitude and direction — left or right — of the horizontal component of the force F on the rod from the wall? *(iii)* What are the magnitude and direction — up or down — of the vertical component of this force F?

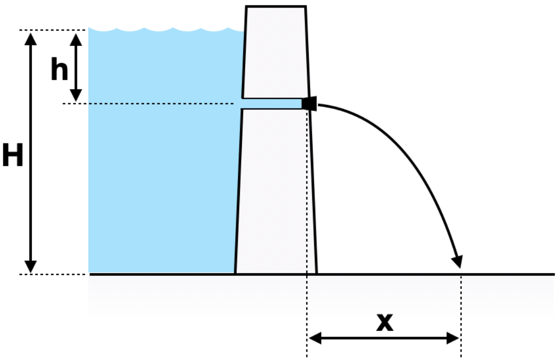


- (a) $\frac{T}{2} = \pi\sqrt{\frac{I}{mgh}} = \pi\sqrt{\frac{\frac{1}{3}ML^2}{Mg(\frac{L}{2})}} = \pi\sqrt{\frac{2L}{3g}}$.
- (b) $T_x + F_x = \frac{4}{5}T + F_x = 0, \quad T_y - Mg + F_y = \frac{3}{5}T - Mg + F_y = 0, \quad -\frac{3}{5}T \cdot \overline{\mathrm{AO}} + Mg \cdot \frac{\overline{\mathrm{AO}} + \overline{\mathrm{BO}}}{2} = 0.$

3. (a) [2 pt] Consider a planet modeled as a uniform sphere of radius R that has a narrow radial tunnel through its center (see figure). Let F_R be the magnitude of the gravitational force on an apple when it is located at the planet's surface. How far from the surface does the magnitude of the force become $\frac{1}{2}F_R$ if you move the apple *(i)* away from the planet and *(ii)* into the tunnel? (Note: For *(ii)*, you are asked to derive a law describing the gravitation inside the planet.)

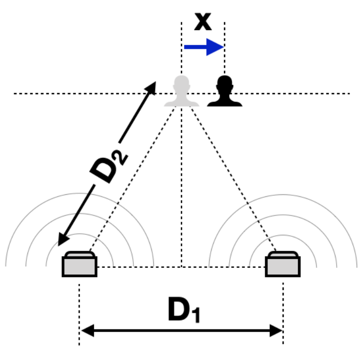


(b) [2 pt] A reservoir dam holds fresh water to height $H = 20$ m. A horizontal pipe of diameter $D = 4.0$ cm passes through the dam at depth $h = 6.0$ m, but a plug secures the pipe's opening on the right (see figure). *(i)* What is the magnitude of the frictional force F between the plug and the pipe wall? *(ii)* When the plug is removed, at what distance x from the pipe's opening (see figure) does the water stream strike the ground? (Note: Use $\rho = 1.0 \times 10^3$ kg/m^3 for the water density. For *(ii)*, you are asked to prove from Bernoulli's principle that the speed v of the water exiting the pipe is $\sqrt{2gh}$.)

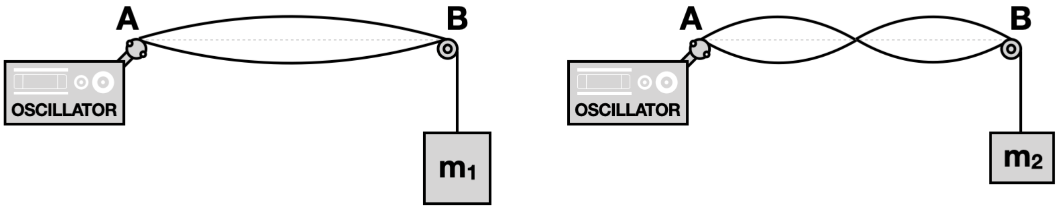


- (a) $\frac{GmM}{r^2} = \frac{1}{2}F_R = \frac{1}{2}\frac{GmM}{R^2}$ and $\frac{GmMr}{R^3} = \frac{1}{2}F_R = \frac{1}{2}\frac{GmM}{R^2}$.
- (b-1) $F = pA = (\rho g h)\,\pi\left(\frac{D}{2}\right)^2$.
- (b-2) From Sample Problem 14.07 of Halliday & Resnick we discussed in one of the class quizzes, $v = \sqrt{2gh}$. Therefore, $x = vt = \sqrt{2gh}\sqrt{\frac{2(H-h)}{g}} = 2\sqrt{h(H-h)}$.

4. (a) [2 pt] A person is standing in front of two speakers ($D_1 = 6.0$ m apart) that are producing sound of the same frequency and amplitude, but vibrating *out of phase*. Initially, the distance between the person and each speaker is the same ($D_2 = 5.0$ m; see figure). Assume that each speaker and the person can be treated as a point source and observer, respectively, and that the speakers and the person's ears are located at the same height above the ground. As the person slowly moves sideways, he finds that the sound intensity gradually increases. When the distance he moved becomes $x = 0.75$ m, he finds that the intensity reaches the maximum for the first time. Determine the frequency of the sound coming from the speakers. (Note: Use $v_s = 340$ m/s for the speed of sound.)

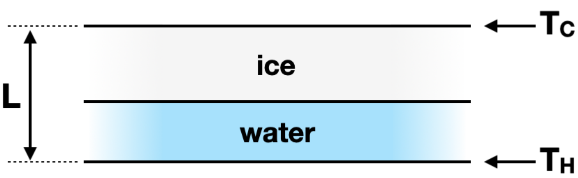


(b) [2 pt] Two strings have the same length L and linear density μ. The left end of each string is tied to a sinusoidal oscillator of the same frequency f (as seen in the movies in our classes; point A in the figure below), while the right end passes over a small pulley (point B) and is connected to blocks of different weights, m_1 and m_2. The amplitude of the oscillator's motion at A is small enough for that point to be considered a node. Another node also exists at B. Different standing waves are set up on each string as seen below. If $m_1 = 4.8$ kg, what is m_2? (Note: You may simply utilize the fact that the wave speed on a stretched string is $v = \sqrt{\frac{\tau}{\mu}}$ where τ is the tension in the string and μ is the linear density of the string.)



- (a) Path length difference $\Delta L = \frac{\lambda}{2} = \frac{v_s}{2f} \;\rightarrow\; f = \frac{v_s}{2\Delta L}$.
- (b) $f_A = f_B \;\rightarrow\; \frac{v_A}{\lambda_A} = \frac{v_B}{\lambda_B} \;\rightarrow\; \frac{\sqrt{m_1 g/\mu}}{2L} = \frac{\sqrt{m_2 g/\mu}}{L}$.

5. (a) [2 pt] An ice slab has formed in a shallow pond, and a steady state has been reached (see figure). Assume that the air right above the ice — and the top surface of the ice — is at $T_C = -5.0°$C, while the soil right below the pond — and the bottom of the pond — is at $T_H = 4.0°$C. Let us also presume that the total depth of ice+water is $L = 1.4$ m from observing the pond for many years, and heat transfer occurs only by conduction. How thick is the ice slab? (Note: Use the thermal conductivitiy of water, $k_{\rm w} = 0.12$ cal/s · m · K, and that of ice, $k_{\rm i} = 0.40$ cal/s · m · K.)



(b) [2 pt] Suppose that 1.0 mol of a monatomic ideal gas is taken from a volume of $V_1 = 1.5$ m^3 to $V_2 = 3.0$ m^3 via an isothermal expansion at $T_0 = 27°$C. *(i)* How much work is done by the gas during the expansion? How much energy is transferred as heat to the gas during the expansion? *(ii)* Now consider the reverse process — that is, 1.0 mol of an ideal gas is taken from V_2 to V_1 via an isothermal compression at T_0. How much energy is transferred as heat during the compression? Is the energy transfer *to* or *from* the gas? (Note: Use the gas constant $R = 8.3$ J/mol · K.)

- (a) From Eq.(18-34) of Halliday & Resnick, $\frac{k_{\rm w}(T_H - T_X)}{L - L_{\rm i}} = \frac{k_{\rm i}(T_X - T_C)}{L_{\rm i}}$ with the temperature at the interface of ice and water $T_X = 0.0°$C.

• (b-1) From Eq.(19-14) of Halliday & Resnick, during the isothermal expansion, $W_e = nRT_0 \ln\frac{V_2}{V_1}$ $= Q_e$ since $\Delta E_{\rm int} = 0$.

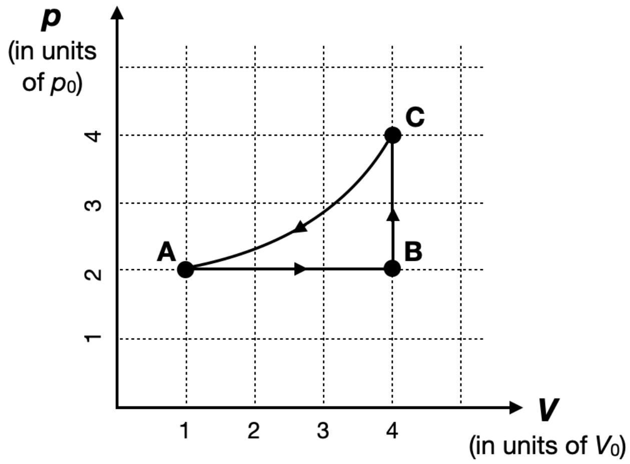
• (b-2) During the isothermal compression, $W_c = nRT_0 \ln\frac{V_1}{V_2} = Q_c = -Q_e = -W_e$.

6. In the p-V diagram below, 1.0 mol of a monatomic ideal gas is taken through the cycle A→B→C→A. (Note: Use the gas constant $R = 8.3$ J/mol $\cdot$ K and the Boltzmann constant $k = 1.4 \times 10^{-23}$ J/K.)

(a) [1 pt] What is $\frac{W}{p_0 V_0}$ for path A→B→C, where W is the work done by the gas during the process?

(b) [2 pt] What is $\frac{\Delta E_{\rm int}}{p_0 V_0}$ for *(i)* path B→C, and for *(ii)* one full cycle, where $\Delta E_{\rm int}$ is the change in the internal energy?

(c) [2 pt] What is the entropy change ΔS for *(i)* path B→C, and for *(ii)* one full cycle?

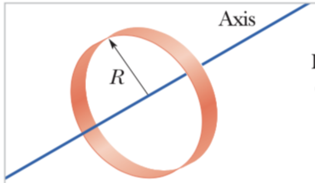
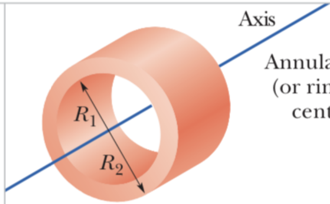
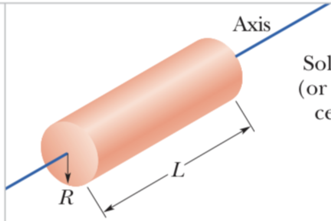
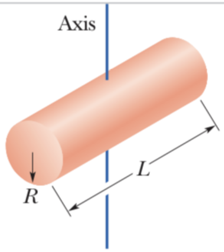
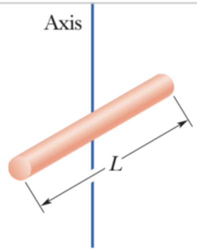
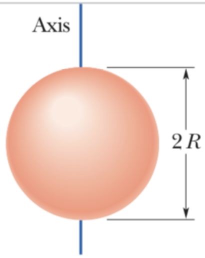
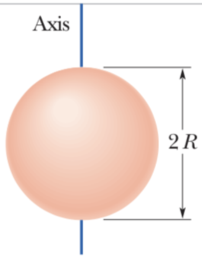
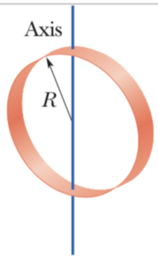
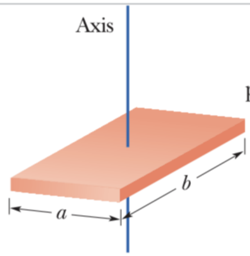


• (a) From Eq.(19-16) of Halliday & Resnick, during the isobaric process A→B, $W = \int_{\rm A}^{\rm B} 2p_0 dV =$ $2p_0 \Delta V = 6p_0 V_0$.

• (b) From Eq.(19-42) of Halliday & Resnick, during the isochoric process B→C, $\Delta E_{\rm int} = Q =$ $nC_V \Delta T = \frac{3}{2} nR\Delta T = \frac{3}{2} nR(T_C - T_B) = \frac{3}{2} nR(\frac{4p_0 \cdot 4V_0}{nR} - \frac{2p_0 \cdot 4V_0}{nR}) = 12p_0 V_0$. $\Delta E_{\rm int} = 0$ for a full cycle, obviously.

• (c) From Eq.(20-4) of Halliday & Resnick, during the isochoric process B→C, $\Delta S = nC_V \ln\frac{T_C}{T_B} =$ $\frac{3}{2} nR \ln 2 = 8.6$ J/K. Again, $\Delta S = 0$ for a full cycle, obviously.

Table 10-2 Some Rotational Inertias

 Hoop about central axis $I = MR^2$ (a)	 Annular cylinder (or ring) about central axis $I = \frac{1}{2}M(R_1^2 + R_2^2)$ (b)	 Solid cylinder (or disk) about central axis $I = \frac{1}{2}MR^2$ (c)
 Solid cylinder (or disk) about central diameter $I = \frac{1}{4}MR^2 + \frac{1}{12}ML^2$ (d)	 Thin rod about axis through center perpendicular to length $I = \frac{1}{12}ML^2$ (e)	 Solid sphere about any diameter $I = \frac{2}{5}MR^2$ (f)
 Thin spherical shell about any diameter $I = \frac{2}{3}MR^2$ (g)	 Hoop about any diameter $I = \frac{1}{2}MR^2$ (h)	 Slab about perpendicular axis through center $I = \frac{1}{12}M(a^2 + b^2)$ (i)