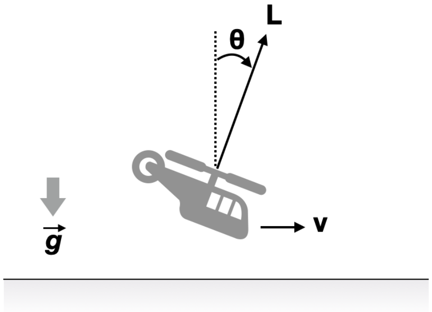
# Physics I (Spring 2022): Midterm Exam Solution

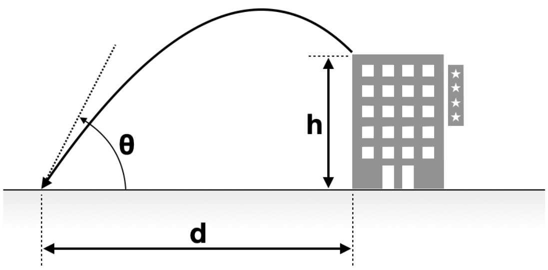
Apr. 15, 2022

[total 25 pts, closed book, 90 minutes]

1. (a) [2 pt] A helicopter of mass $m = 2000$ kg is moving horizontally to the right at a constant velocity, $\vec{v}$ (see figure). The lift force $\vec{L}$ generated by the rotating blade makes an angle $\theta = 20°$ with respect to the vertical. Determine the magnitude of the lift force $\vec{L}$, and the drag force $\vec{D}$ (air resistance) that opposes the helicopter's motion.



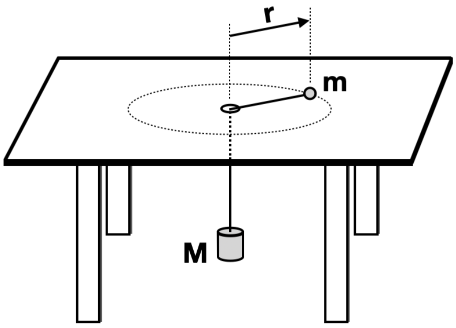

(b) [2 pt] A boy throws a ball from the left edge of the roof of a building at height $h$ above the ground (see figure). The ball hits the ground $t = 1.5$ second later, at distance $d = 25$ m from the building and at angle $\theta = 60°$ with the horizontal. Assuming negligible air resistance, find $h$. (Note: You may want to *time-reverse* the motion, as if on video.)



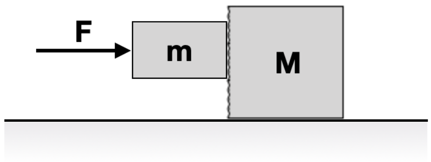

- (a) $mg = L\cos\theta$ and $D = L\sin\theta$.
- (b) $d = v_0\cos\theta \cdot t$ and $h = v_0\sin\theta \cdot t - \frac{1}{2}gt^2$, where $v_0$ is the ball's speed hitting the ground.

2. (a) [2 pt] A particle of mass $m = 1.5$ kg is connected to a cylinder of mass $M = 2.5$ kg via a massless string that passes through a small hole in a table. The particle $m$ slides in a circle

of radius $r = 0.20$ m on the frictionless table, while the cylinder $M$ hangs below the table (see figure). The string always remains tight. What is the speed of the particle, $v_{\text{circ}}$, that keeps the cylinder at rest? What would happen if you started the circular motion of $m$ with the speed $v_{\text{circ}}$ but on a frictional tabletop?




(b) [2 pt] The two blocks in the figure, $m = 2.0$ kg and $M = 8.0$ kg, are not attached to each other. The surface beneath the larger block is frictionless, but the coefficient of static friction between the blocks is $\mu_s = 0.25$ (see figure). Find the *minimum* magnitude of the horizontal force $\vec{F}$ required to keep the smaller block from slipping down the larger block. (Note: If the two blocks move together as a single system, they have the same acceleration.)




- (a) $Mg = \frac{mv_{\text{circ}}^2}{r}$.

- (b) For the two blocks moving together, Newton's 2nd law becomes $F - F' = ma$ for block $m$ and $F' = Ma$ for block $M$, where $F'$ is the force exerted on $M$ by $m$. From these you obviously acquire $a = \frac{F}{M+m}$. Also, the minimum value of $F'$ can be acquired from $\mu_s F'_{\text{min}} = mg$. Combining the equations, you get $F'_{\text{min}} = \frac{mg}{\mu_s} = Ma_{\text{min}} = F_{\text{min}}\frac{M}{M+m} \quad \rightarrow \quad F_{\text{min}} = \frac{m(M+m)g}{\mu_s M}$.

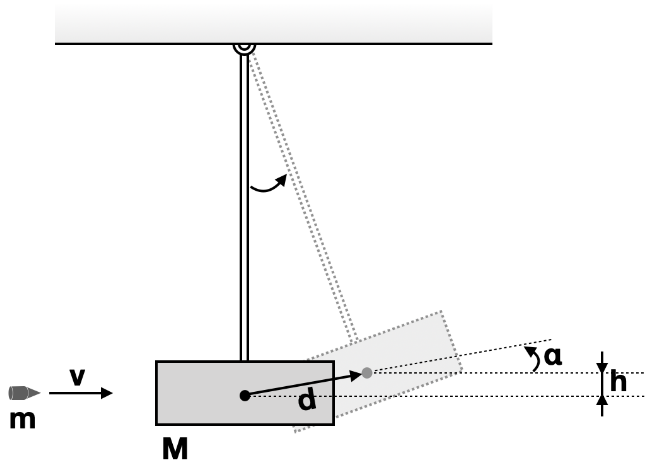
3. A ballistic pendulum consists of a large block of wood of mass $M$ (= 5.0 kg) hanging from a hinged massless rod that can swing without friction with the horizontal axle (see figure).

(a) [1 pt] A bullet of mass $m$ (= 10 g) is fired into the block with an unknown constant speed $v$, coming quickly to rest inside the block. The collision is so brief that we can assume that the *block+bullet* system is isolated during the collision, and the collision is 1-dimensional. Express the speed $V$ of the stuck-together bodies right after the collision in terms of $M$, $m$ and $v$.

(b) [1 pt] Right after the collision, the block — with the bullet at its center — begins to swing upward. Their center of mass (CoM) rises a vertical distance $h = 7.0$ cm before the pendulum comes to rest momentarily (see figure). Determine $v$ using the numerical values given herein.

(c) [1 pt] During the block's ascent to $h$ immediately after the collision, how much work is done on the *block+bullet* by *(i)* the gravitational force, and *(ii)* the force on the block from the rod, respectively? (Note: You are asked to explicitly derive each amount of work by using $W = \vec{F} \cdot \vec{d}$ where $\vec{d}$ is the displacement of the CoM noted in the figure. Here we assumed that $\alpha$ and $h$ are so small that the arc along which the CoM moves can be approximated as a straight vector $\vec{d}$.)

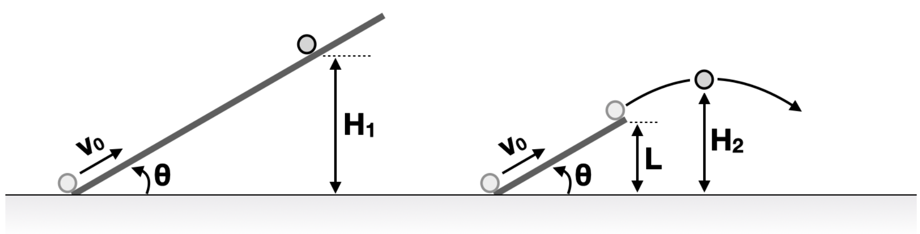
(d) [1 pt] Using your result in (c), check *(i)* that the net work done on the block is the change in its kinetic energy, $\Delta K$, and *(ii)* that the negative of the work done on the block by the gravitational force is the change in its gravitational potential energy, $\Delta U$.



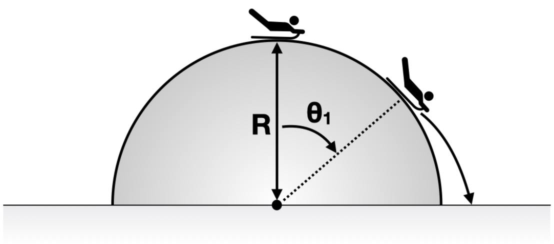

• (b) $(m+M)gh = \frac{1}{2}(m+M)V^2 = \frac{1}{2}\frac{m^2}{m+M}v^2$.

• (c) $W_g = (m+M)\vec{g} \cdot \vec{d} = (m+M)gd\cos(90° + \alpha) = -(m+M)gd\sin\alpha = -(m+M)gh$. Meanwhile, $W_r = 0$ since the force from the rod is always perpendicular to the block's velocity.

4. (a) [2 pt] Two frictionless track begin at the ground level and slope upward at the same angle $\theta = 30°$. One track is longer than the other, however. Two identical particles are projected up each track with the same initial speed $v_0 = 20$ m/s. On the longer track the particle slides upward until it reaches a maximum height $H_1$ above the ground (see figure). On the shorter track the particle slides upward, then flies off the end of the track at a height $L = 10$ m, reaches the highest point of this trajectory at a height $H_2$ above the ground. Find and compare $H_1$ and $H_2$. Why is there a difference?



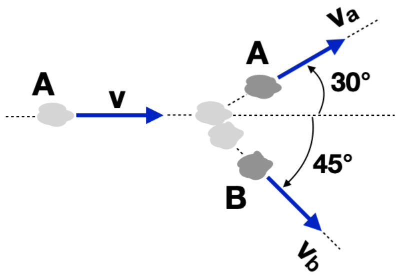

(b) [3 pt] A girl on a sled starts from rest at the top of a large hemispherical ice sculpture of radius $R = 10$ m, and begins to slide down the frictionless surface with a negligible initial speed (see figure). Find $\cos\theta_1$ where $\theta_1$ is the angle at which the girl leaves the ice surface. Find the girl's speed as she leaves the surface. (Note: When the girl loses contact with the ice, the normal force of the ice on her is zero.)



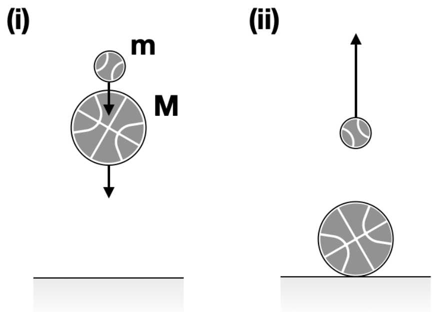

• (a) $\frac{1}{2}mv_0^2 = mgH_1$ for the longer track; $\frac{1}{2}mv_0^2 = mgL + \frac{1}{2}mv_L^2$ where $v_L$ is the particle's launch speed at height $L$ on the shorter track. With $v_L$, you can find $H_2$ using $(v_L \sin\theta)^2 = 2g(H_2 - L)$.

• (b) While in a circular motion on the surface of the sphere, $\frac{mv^2}{R} = mg\cos\theta - F_N$. Thus, when the girl leaves the surface at $\theta_1$ you find $\frac{mv_1^2}{R} = mg\cos\theta_1$, where $mv_1^2$ is acquired from the energy conservation, $mgR = \frac{1}{2}mv_1^2 + mgR\cos\theta_1$. Combining the equations, you get $\cos\theta_1 = \frac{2}{3}$.

5. (a) [2 pt] Two asteroids of equal mass in the asteroid belt between Mars and Jupiter collide. Asteroid A, which was initially traveling at $v = 50$ m/s, is deflected 30° from its original direction, while asteroid B, which was initially at rest, travels at 45° to the original direction of asteroid A (see figure). Assuming that the mass of each asteroid does not change during this 2-dimensional collision, find the speed of each asteroid after the collision. What fraction of the original kinetic energy of asteroid A dissipates during this collision?



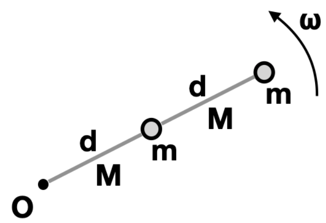

(b) [2 pt] A baseball ($m = 0.20$ kg) is aligned above a basketball ($M = 3m = 0.60$ kg) with a slight separation between them. Then the two are dropped simultaneously from a height $h = 2.0$ m (see figure *i*). Assume that the radius of each ball is negligible compared to $h$. Assume also that the basketball rebounds elastically from the floor, and then the baseball rebounds elastically from the basketball. Show that, after the collision between the baseball and the rebounding basketball, the basketball stops (see figure *ii*). What height does the baseball then reach?



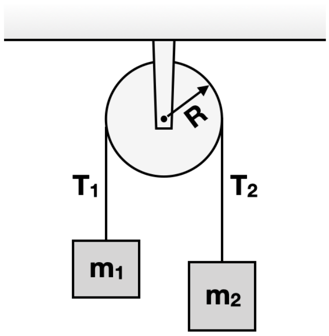
• (a) From $mv = mv_a\cos 30° + mv_b\cos 45°$ and $mv_a\sin 30° = mv_b\sin 45°$, $\frac{\Delta K}{K} = \frac{v_a^2+v_b^2-v^2}{v^2}$.

• (b) Right before the collision between the baseball and the rebounding basketball, $v_{\mathrm{m},i} = -\sqrt{2gh}$ and $v_{\mathrm{M},i} = \sqrt{2gh}$. Plugging these into Eqs.(9-75)-(9-76) of Halliday & Resnick, you get $v_{\mathrm{M},f} = \frac{2M}{m+M}v_{\mathrm{m},i} + \frac{M-m}{m+M}v_{\mathrm{M},i} = -\frac{2m}{m+3m}\sqrt{2gh} + \frac{3m-m}{m+3m}\sqrt{2gh} = 0$ and $v_{\mathrm{m},f} = \frac{m-M}{m+M}v_{\mathrm{m},i} + \frac{2M}{m+M}v_{\mathrm{M},i} = -\frac{m-3m}{m+3m}\sqrt{2gh} + \frac{6m}{m+3m}\sqrt{2gh} = 2\sqrt{2gh}$.

6. (a) [2 pt] Two particles, each with mass $m = 0.50$ kg, are fastened to each other and to a rotation axis at point $O$, by two thin rods, each with length $d = 10$ cm and mass $M = 1.0$ kg (see figure). The particles and rods are joined in a straight line at all times. The system rotates around $O$ with an angular speed $\omega = 0.30$ rad/s. Measured about $O$, what are the system's *(i)* rotational inertia and *(ii)* kinetic energy?




(b) [2 pt] Two blocks of mass $m_1 = 0.46$ kg and $m_2 = 0.50$ kg each are connected by a massless cord that passes over a pulley of radius $R = 5.0$ cm mounted on a horizontal axle with negligible friction. When released from rest, block $m_2$ falls 0.90 m in 3.0 seconds without the cord slipping on the pulley. What are the magnitudes of tensions on the cord, $T_1$ and $T_2$ (see figure)? What is the pulley's rotational inertia?




• (a) $I = I_{\mathrm{particles}} + I_{\mathrm{rods}} = md^2 + 4md^2 + \frac{1}{12}2M(2d)^2 + (2M)d^2 = 5md^2 + \frac{8}{3}Md^2$

• (b) $T_1 - m_1 g = m_1 a, \quad m_2 g - T_2 = m_2 a \quad$ and $\quad R(T_2 - T_1) = I\left(\frac{a}{R}\right)$ where $a = 0.2$ m/s$^2$.