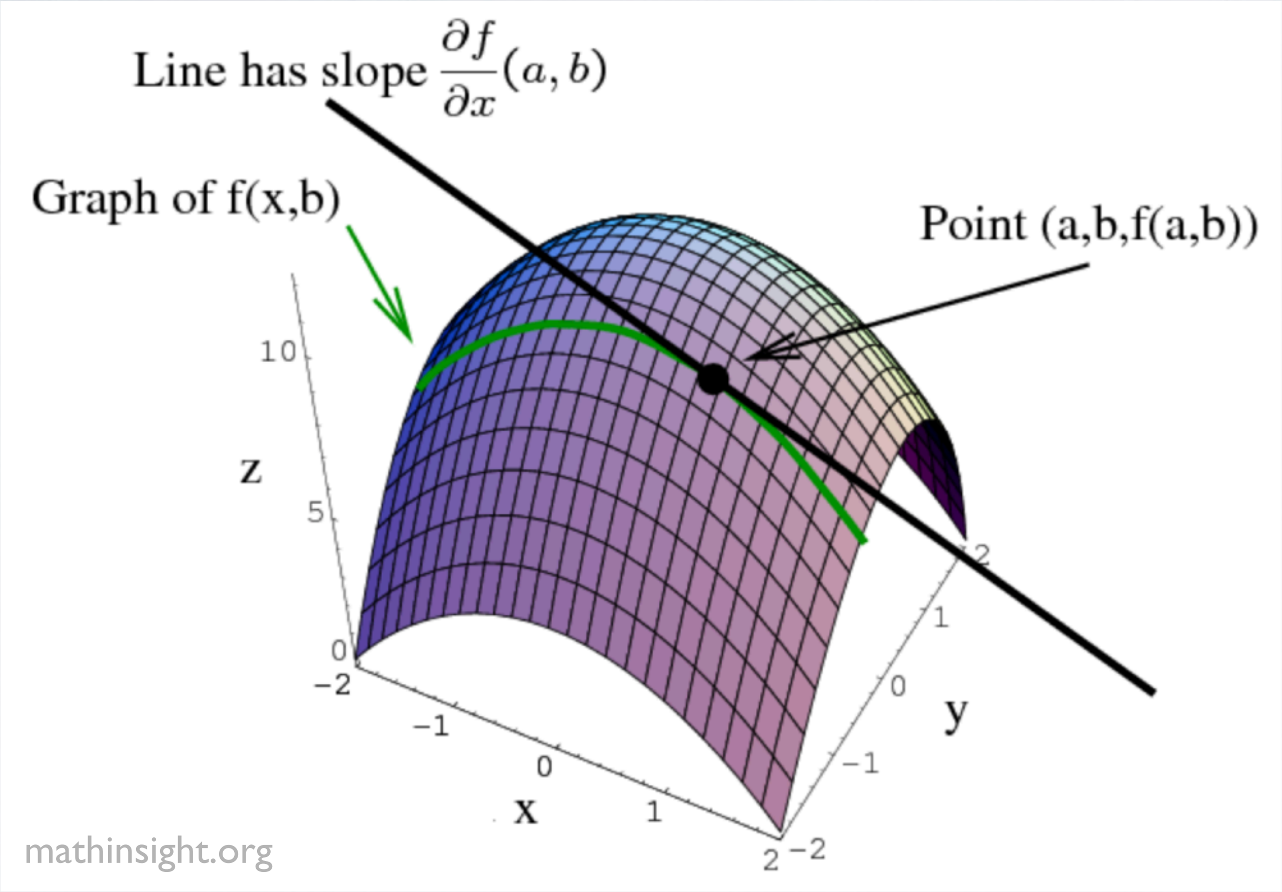
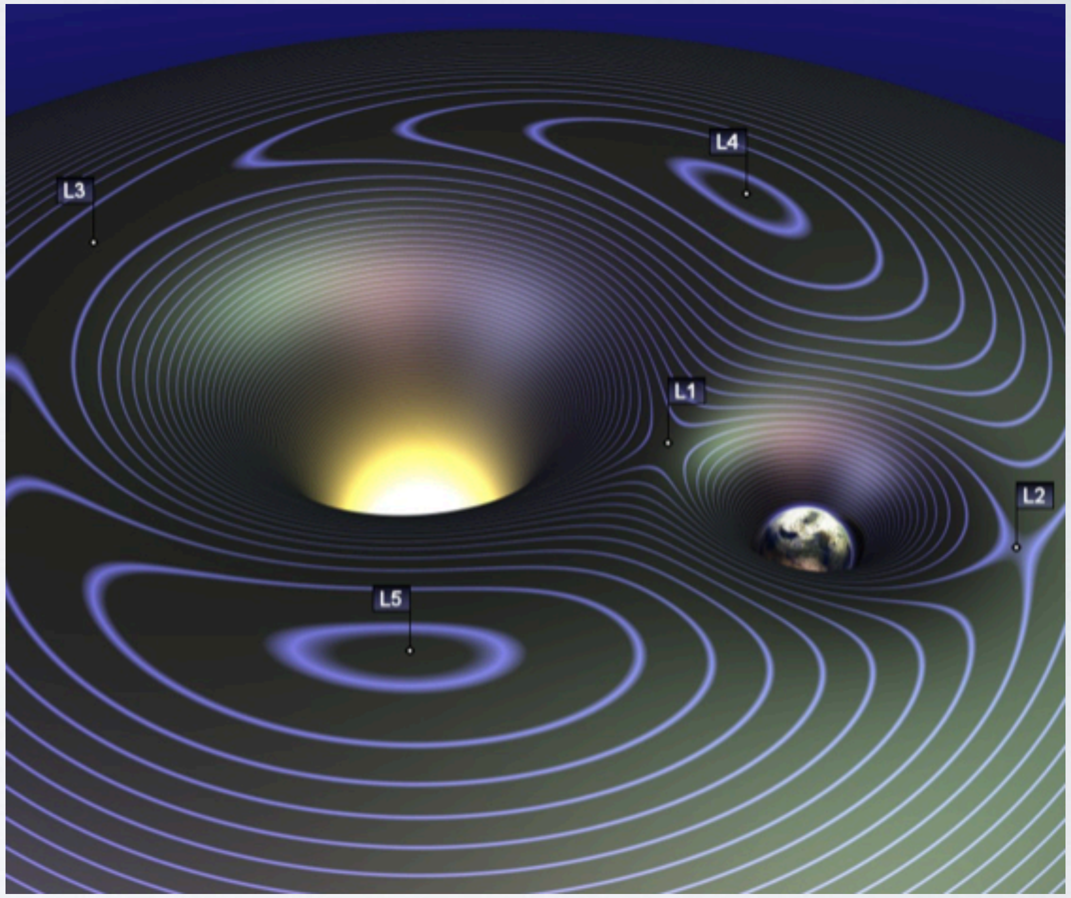
Week 5 - #1

Partial Differentiation





Today: Ch 4

Next Class: Ch 5

Ji-hoon Kim (Seoul National University)

Rudimentary Mathematical Methods of Physics (Fall 2025): Quiz #7

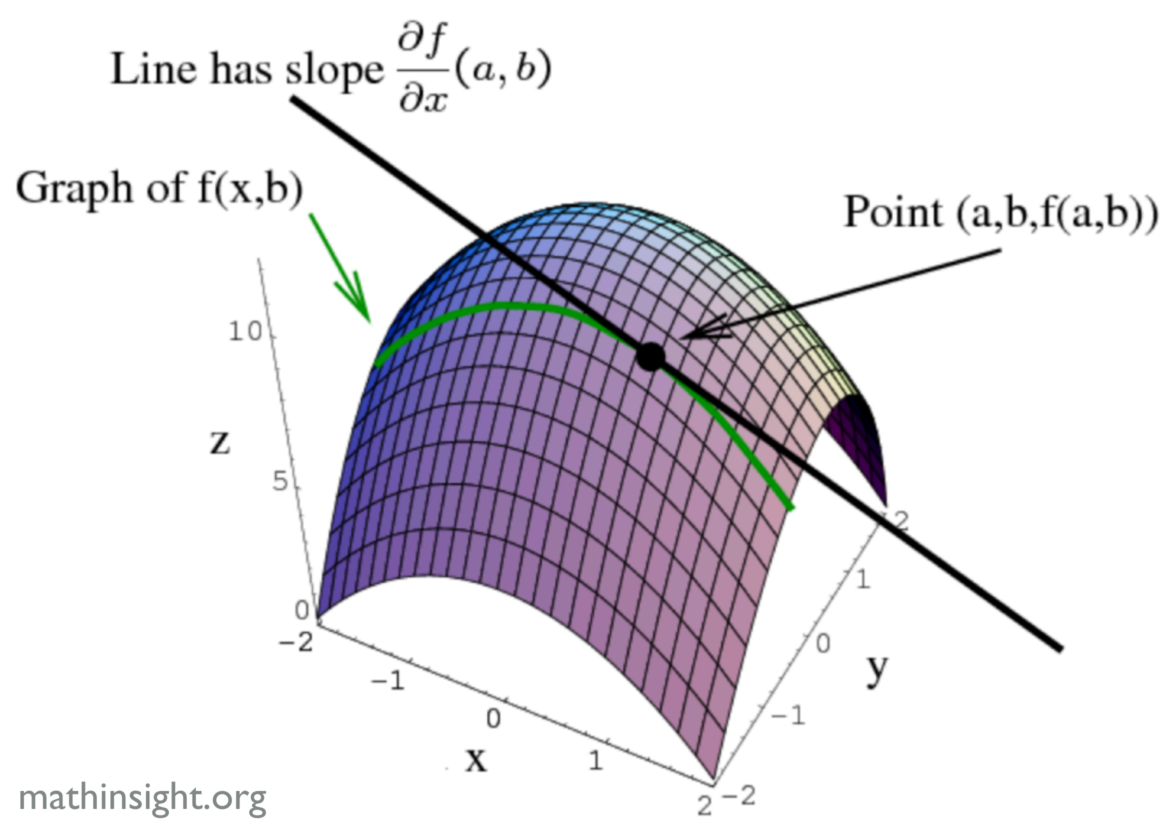
— [open book and open note, “and” cellphone or laptop, drop it off as you leave the class] —

Please write down your name and student ID in the top right corner. (0.0 pt: no paper found with your name / 0.5 pt: paper found with your name and some answers / 1.0 pt: good answers)

1. Boas, Chapter 4, Section 4, Problem 1

2. What is the difference between the two formulae, Eq.(3.6) and (5.3) of Boas, Chapter 4?

3. List any research area you can think of where tensors are used — e.g., in physics textbooks, or in describing daily phenomena of any kind.



Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 4, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following. The class homework assignments will mainly be from this list.

- Section 01: Problems 7, 8, 9, 15, 16
- Section 02: Problems 1, 2, 3
- Section 04: Problems 2, 3, 7, 15
- Section 05: Problems 1, 2, 6, 8
- Section 06: Problems 1, 4, 9
- Section 07: Problems 2, 4, 7, 10, 16, 25, 28
- Section 08: Problems 2, 8, 11, 13
- Section 09: Problems 2, 5, 9, 11
- Section 10: Problems 2, 5, 10, 14
- Section 11: Problems 1, 4, 6, 7, 8
- Section 12: Problems 2, 5, 10, 13, 16, 19
- Section 13: Problems 6, 9, 16, 27, 29

54 problems / 218 total

Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 5, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following. The class homework assignments will mainly be from this list.

- Section 01: Problems 2, 4, 5
- Section 02: Problems 3, 5, 9, 11, 13, 16, 20, 22, 26, 39, 41
- Section 03: Problems 6, 12, 13, 17, 18, 19, 20, 21, 25
- Section 04: Problems 2, 4, 7, 14, 16, 19, 20
- Section 05: Problems 1, 5
- Section 06: Problems 4, 6, 11, 18, 23, 25, 27

39 problems / 160 total

Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 6, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following. The class homework assignments will mainly be from this list.

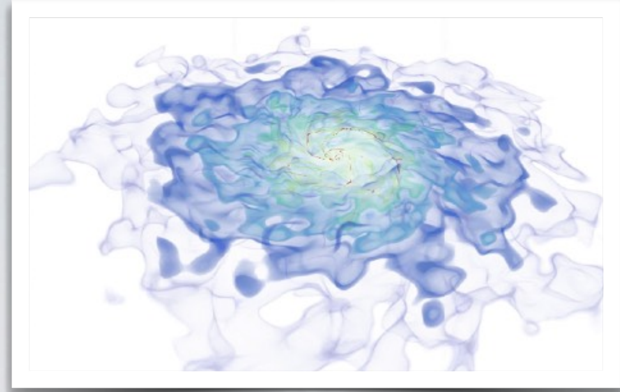
- Section 03: Problems 12, 13, 14, 16, 17
- Section 04: Problems 6, 9
- Section 06: Problems 8, 13, 14, 16
- Section 07: Problems 6, 7, 15, 16, 17
- Section 08: Problems 6, 14, 18, 19, 20
- Section 09: Problems 3, 4, 7, 8
- Section 10: Problems 6, 10, 12, 15, 16
- Section 11: Problems 8, 10, 12, 16, 17(b)(d)(e), 18
- Section 12: Problems 3, 16, 19

<u>Chapter 10, Sections 5 & 7-9</u>

- Section 05: Problems 5, 7, 10
- Section 08: Problems 6
- Section 09: Problems 2, 5

39 problems / 173 total

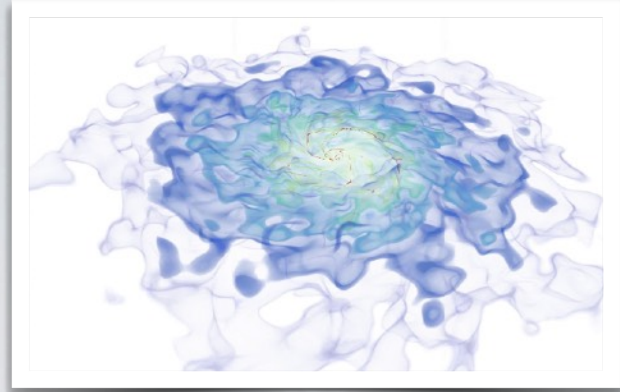
(Ch.10) 13 problems / 138 total



HW #2 has been posted!

(Posted on jihoonkim.org, Due: Oct. 3 (Fri), 23:00pm,

Grader TA this time: 서현우, hyeonu@snu.ac.kr)



Tensors In Physics

Tensors In Physics: Examples

- (1) Coordinate transformation tensor (Cartesian ↔ Cartesian)

$$\begin{pmatrix} x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

Boas Ch 10.2, Eq.(2.7)

- (2) Coordinate transformation tensor (Cartesian ↔ curvilinear)

$$\begin{pmatrix} dx \\ dy \\ dz \end{pmatrix} = \begin{pmatrix} \sin\theta\cos\phi & r\cos\theta\cos\phi & -r\sin\theta\sin\phi \\ \sin\theta\sin\phi & r\cos\theta\sin\phi & r\sin\theta\cos\phi \\ \cos\theta & -r\sin\theta & 0 \end{pmatrix} \begin{pmatrix} dr \\ d\theta \\ d\phi \end{pmatrix}$$

Boas Ch 10.10, Eq.(10.2)

- (3) Metric tensor

$$ds^2 = \begin{pmatrix} dx_1 & dx_2 & dx_3 \end{pmatrix} \begin{pmatrix} h_1^2 & 0 & 0 \\ 0 & h_2^2 & 0 \\ 0 & 0 & h_3^2 \end{pmatrix} \begin{pmatrix} dx_1 \\ dx_2 \\ dx_3 \end{pmatrix}$$

Boas Ch 10.2, Eq.(2.7)

Tensors In Physics: Examples

- (4) Stress tensor, Strain tensor, Elastic constant tensor

$$P_{ij} = \begin{pmatrix} P_{xx} & P_{xy} & P_{xz} \\ P_{yx} & P_{yy} & P_{yz} \\ P_{zx} & P_{zy} & P_{zz} \end{pmatrix} = C_{ijkm} S_{km}$$

Boas Ch 10.7, Eq.(7.5)

- (5) Lorentz spacetime transformation tensor

$$\Lambda = \begin{pmatrix} \gamma & -\gamma\beta & 0 & 0 \\ -\gamma\beta & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

$$x^0 \equiv ct,$$
$$x^1 = x, \quad x^2 = y, \quad x^3 = z$$

Griffiths Eq.(12.117, 12.21-22)

$$\boldsymbol{\lambda} = \begin{pmatrix} \gamma & 0 & 0 & i\beta\gamma \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -i\beta\gamma & 0 & 0 & \gamma \end{pmatrix}$$

$$\mathbb{X} = (x_1, x_2, x_3, ict)$$

Marion Eq.(14.77, 14.79a)

- (6) Electromagnetic field tensor

$$F^{\mu\nu} = \begin{Bmatrix} 0 & E_x/c & E_y/c & E_z/c \\ -E_x/c & 0 & B_z & -B_y \\ -E_y/c & -B_z & 0 & B_x \\ -E_z/c & B_y & -B_x & 0 \end{Bmatrix}$$

Griffiths Eq.(12.122)

Tensors In Physics: Examples

- (7) Ricci curvature tensor, Stress-Energy tensor

$$R_{\alpha\beta} - \frac{1}{2} g_{\alpha\beta} R = 8\pi G T_{\alpha\beta}$$

Hartle Eq.(22.51)

- (8) Inertia tensor

$$I_{xx} = \sum_i m_i (y_i^2 + z_i^2)$$
$$I_{xy} = -\sum_i m_i x_i y_i$$

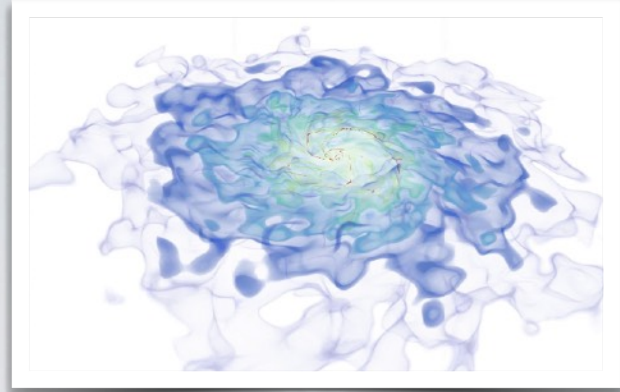
$$\Rightarrow$$

$$I_{xx} = \int (y^2 + z^2)\, dm$$
$$I_{xy} = -\int xy\, dm$$

Boas Ch 10.4, Eq.(4.5)

$$\{\mathbf{I}\} = \begin{Bmatrix} \sum_\alpha m_\alpha (x_{\alpha,2}^2 + x_{\alpha,3}^2) & -\sum_\alpha m_\alpha x_{\alpha,1} x_{\alpha,2} & -\sum_\alpha m_\alpha x_{\alpha,1} x_{\alpha,3} \\ -\sum_\alpha m_\alpha x_{\alpha,2} x_{\alpha,1} & \sum_\alpha m_\alpha (x_{\alpha,1}^2 + x_{\alpha,3}^2) & -\sum_\alpha m_\alpha x_{\alpha,2} x_{\alpha,3} \\ -\sum_\alpha m_\alpha x_{\alpha,3} x_{\alpha,1} & -\sum_\alpha m_\alpha x_{\alpha,3} x_{\alpha,2} & \sum_\alpha m_\alpha (x_{\alpha,1}^2 + x_{\alpha,2}^2) \end{Bmatrix}$$

Marion Eq.(11.13a)



Inertia Tensors

Matrix Diagonalization (Ch 10.4, Ex 4)

WolframAlpha computational intelligence.

eigenvector {{4,2,0}, {2,3,-1}, {0,-1,5}}

NATURAL LANGUAGE   MATH INPUT   EXTENDED KEYBOARD   EXAMPLES   UPLOAD   RANDOM

Input

eigenvectors	$\begin{pmatrix} 4 & 2 & 0 \\ 2 & 3 & -1 \\ 0 & -1 & 5 \end{pmatrix}$

Results   Approximate forms   Step-by-step solution

$v_1 = (-1, -1, 1)$

$v_2 = \left(-1 + \sqrt{3}, 2 - \sqrt{3}, 1\right)$

$v_3 = \left(-1 - \sqrt{3}, 2 + \sqrt{3}, 1\right)$

Enlarge   Data   Customize   Plain Text

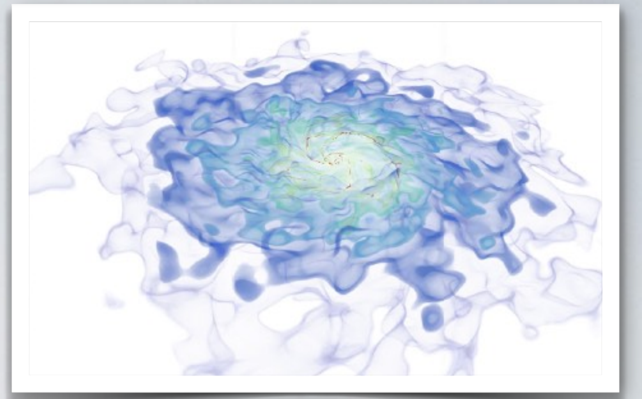
Corresponding eigenvalues

$\lambda_1 = 6$

$\lambda_2 = 3 + \sqrt{3}$

$\lambda_3 = 3 - \sqrt{3}$

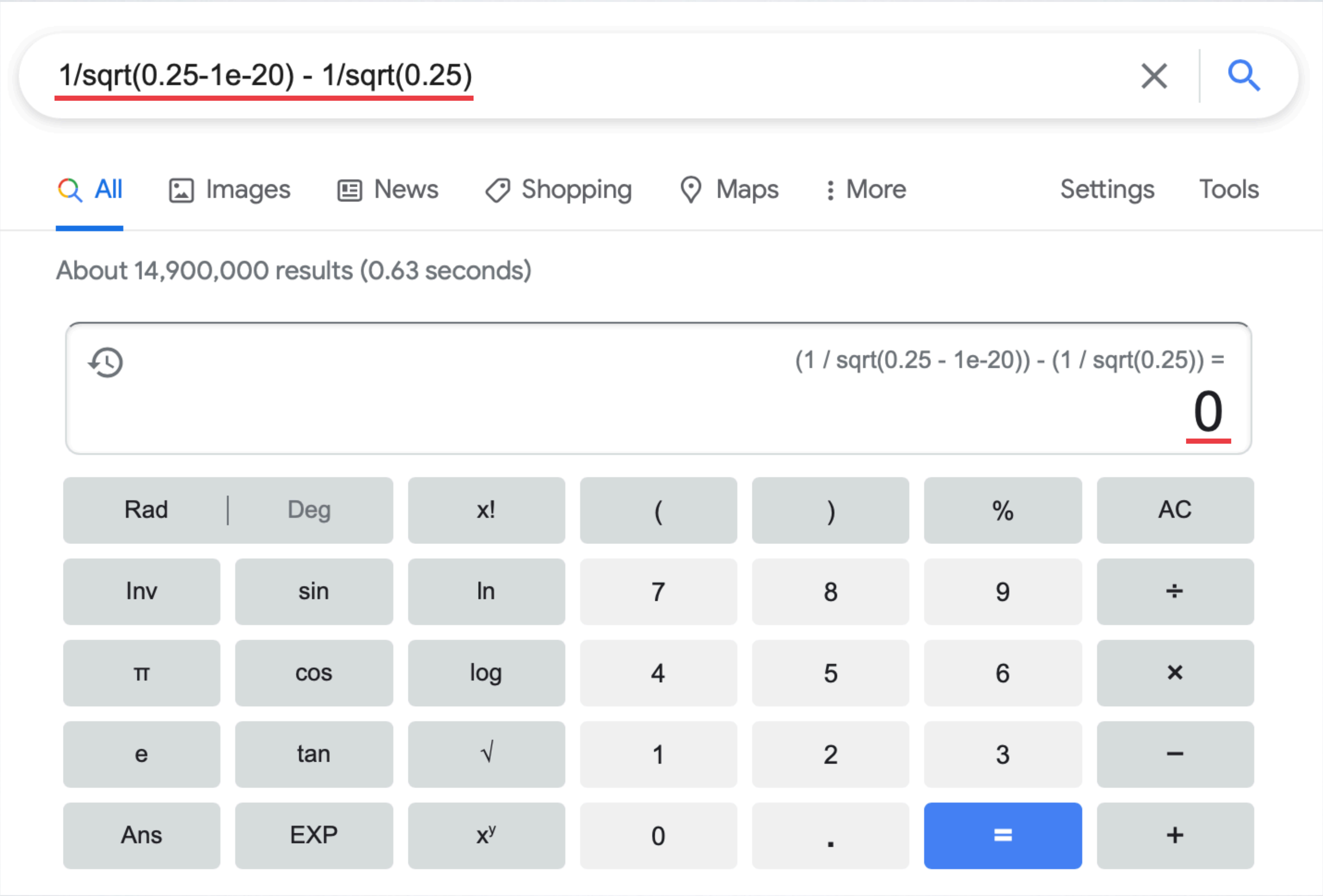
Plot of eigenvectors



Pitfalls of Blind Computation
(Again)

Pitfalls of Blind Computation

- Inaccurate estimation on Google:



Pitfalls of Blind Computation

- Inaccurate estimation on Python:

```
bash-3.2$ python
Python 2.7.9 (default, Sep  9 2015, 22:07:15)
[GCC 4.2.1 Compatible Apple LLVM 6.1.0 (clang-602.0.53)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import math
>>> 1/math.sqrt(0.25-1e-20) - 1/math.sqrt(0.25)
0.0
>>> 1/math.sqrt(0.25-1e-10) - 1/math.sqrt(0.25)
4.000000330961484e-10
>>>
```

Pitfalls of Blind Computation

- Accurate estimation on WolframAlpha: