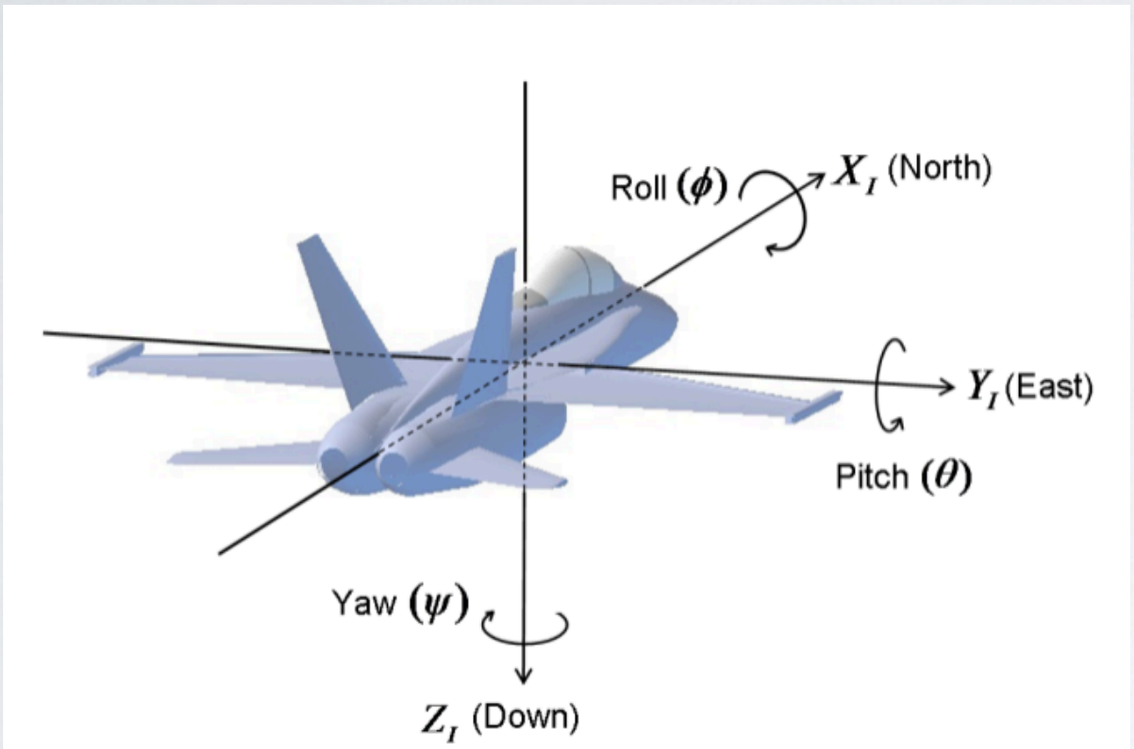
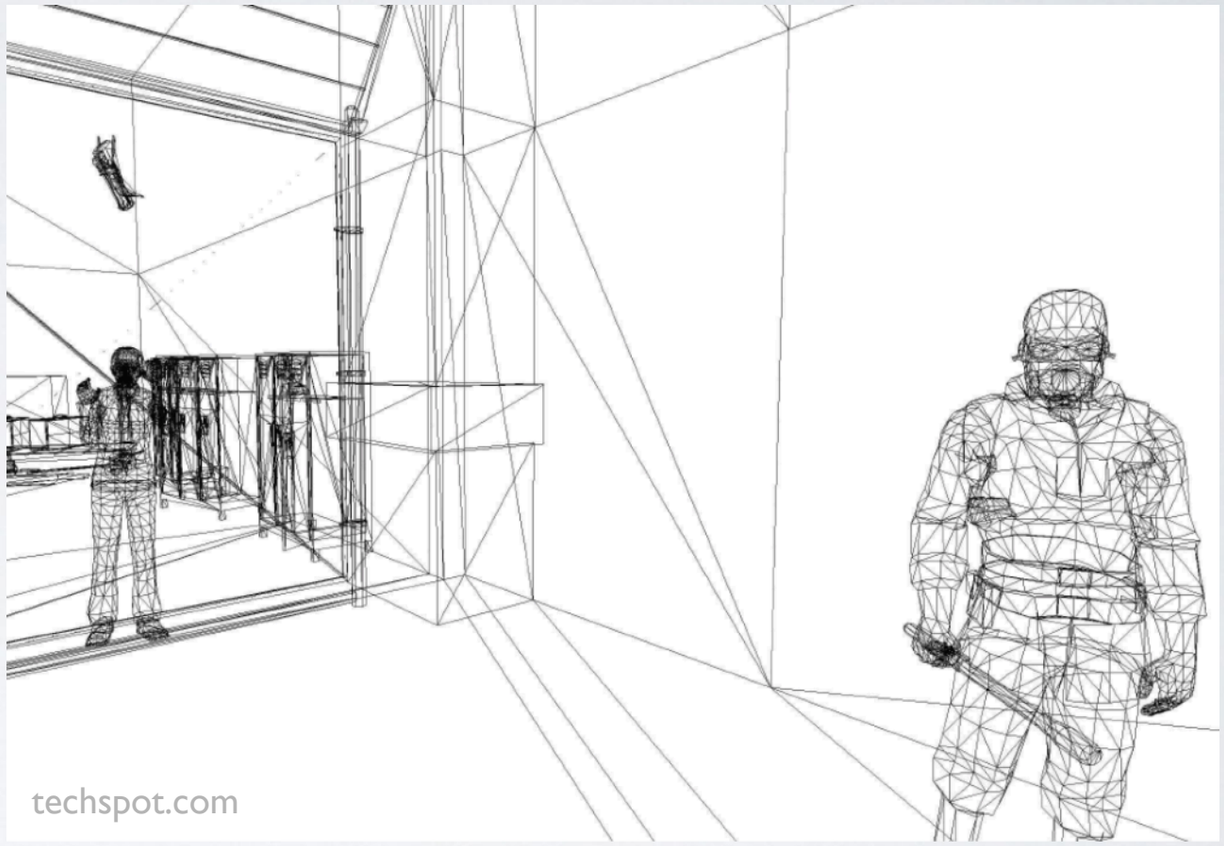
Week 4 - #2

Linear Algebra (V), Tensor Analysis (I)



Sean Morrison



techspot.com

Today: Ch 3, 10.1-10.4

Next Class: Ch 4

Ji-hoon Kim (Seoul National University)

Rudimentary Mathematical Methods of Physics (Fall 2025): Quiz #6

— [open book and open note, "and" cellphone or laptop, drop it off as you leave the class] —

Please write down your name and student ID in the top right corner. (0.0 pt: no paper found with your name / 0.5 pt: paper found with your name and some answers / 1.0 pt: good answers)

1. Boas, Chapter 3, Section 11, Problem 13

2. Study and briefly explain how matrix operations are used in 3-dimensional rendering in various fields such as game design, immersive visualization, augmented reality, and others.

www.techspot.com/article/1857-how-to-3d-rendering-vertex-processing

Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 3, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following. The class homework assignments will mainly be from this list.

- Section 02: Problems 8, 13, 14, 17, 18
- Section 03: Problems 2, 6, 9, 10, 13, 15
- Section 04: Problems 5, 6, 9, 21, 23
- Section 05: Problems 17, 21, 32, 36, 42
- Section 06: Problems 6, 7, 16, 18, 21, 29, 30
- Section 07: Problems 12, 27, 31, 34, 35
- Section 08: Problems 2, 10, 15, 16, 17, 24
- Section 09: Problems 3, 5, 10, 17, 19(c), 24, 25(a)(b)
- Section 10: Problems 4(c), 5(a), 7
- Section 11: Problems 9, 10, 19, 21, 31, 33, 42, 44, 50, 57, 60, 61
- Section 12: Problems 9 (for Problems 4, 7), 16, 21
- Section 13: Problems 1, 4, 7
- Section 14: Problems 13

<u>Chapter 10, Sections 1-4</u>

- Section 02: Problems 6
- Section 03: Problems 1, 2, 8
- Section 04: Problems 2, 6, 8

64 problems / 399 total

Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 4, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following. The class homework assignments will mainly be from this list.

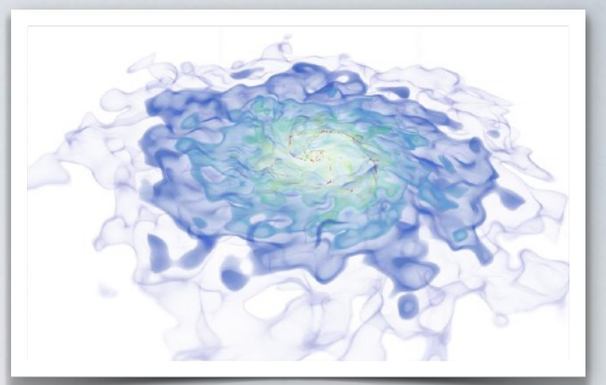
- Section 01: Problems 7, 8, 9, 15, 16
- Section 02: Problems 1, 2, 3
- Section 04: Problems 2, 3, 7, 15
- Section 05: Problems 1, 2, 6, 8
- Section 06: Problems 1, 4, 9
- Section 07: Problems 2, 4, 7, 10, 16, 25, 28
- Section 08: Problems 2, 8, 11, 13
- Section 09: Problems 2, 5, 9, 11
- Section 10: Problems 2, 5, 10, 14
- Section 11: Problems 1, 4, 6, 7, 8
- Section 12: Problems 2, 5, 10, 13, 16, 19
- Section 13: Problems 6, 9, 16, 27, 29

54 problems / 218 total

Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 5, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following. The class homework assignments will mainly be from this list.

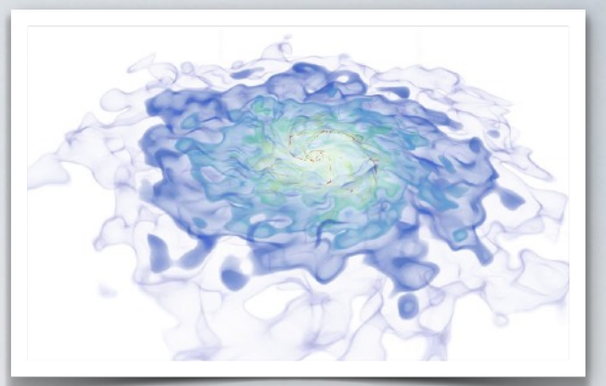
- Section 01: Problems 2, 4, 5
- Section 02: Problems 3, 5, 9, 11, 13, 16, 20, 22, 26, 39, 41
- Section 03: Problems 6, 12, 13, 17, 18, 19, 20, 21, 25
- Section 04: Problems 2, 4, 7, 14, 16, 19, 20
- Section 05: Problems 1, 5
- Section 06: Problems 4, 6, 11, 18, 23, 25, 27



HW #2 will be posted!

(To be posted on jihoonkim.org, Due: Oct. 3 (Fri), 23:00pm,

Grader TA this time: 서현우, hyeonu@snu.ac.kr)



Matrix Diagonalization On WolframAlpha

Matrix Diagonalization (Ch 3.11, Ex 1)

WolframAlpha computational intelligence

determinant of {{1-a, -4, 2}, {-4, 1-a, -2}, {2, -2, -2-a}}

Extended Keyboard Upload Examples Random

Input interpretation:

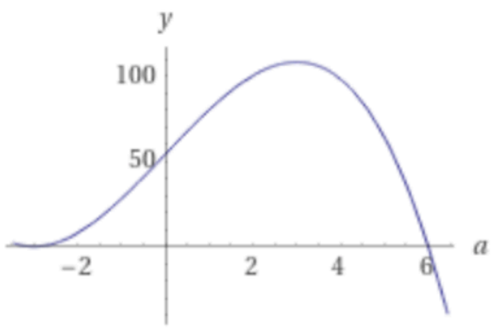
$$\begin{vmatrix} 1-a & -4 & 2 \\ -4 & 1-a & -2 \\ 2 & -2 & -2-a \end{vmatrix}$$

$|m|$ is the determinant

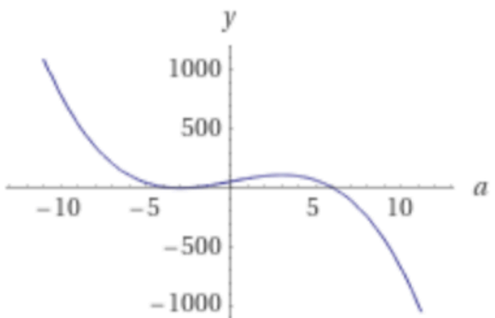
Result: Step-by-step solution

$$-a^3 + 27a + 54$$

Plots:



(a from −3.4 to 6.4)



(a from −12.7 to 12.7)

Matrix Diagonalization (Ch 3.11, Ex 5)

WolframAlpha computational intelligence

characteristic equation (1/7){{2, 6, 3}, {6, -3, 2}, {3, 2, -6}}

Extended Keyboard | Upload | Examples | Random

Assuming "characteristic equation" is a math function | Use as referring to linear algebra instead

Input:

eigenvectors	$\frac{1}{7}\begin{pmatrix} 2 & 6 & 3 \\ 6 & -3 & 2 \\ 3 & 2 & -6 \end{pmatrix}$

Results: Approximate forms | Step-by-step solution

$v_1 = \left(-\frac{1}{3}, 0, 1\right)$

$v_2 = \left(-\frac{2}{3}, 1, 0\right)$

$v_3 = (3, 2, 1)$

Corresponding eigenvalues: Step-by-step solution

$\lambda_1 = -1$

$\lambda_2 = -1$

$\lambda_3 = 1$

Plot of eigenvectors: Enlarge | Data | Customize | Plain Text

Matrix Diagonalization (Ch 3.11, Ex 3)

WolframAlpha computational intelligence

eigenvector {{1, sqrt(2), 1}, {-sqrt(2), 0, sqrt(2)}, {1, -sqrt(2), 1}}

Extended Keyboard | Upload | Examples | Random

Input:

eigenvectors $\begin{pmatrix} 1 & \sqrt{2} & 1 \\ -\sqrt{2} & 0 & \sqrt{2} \\ 1 & -\sqrt{2} & 1 \end{pmatrix}$

Results: Approximate forms | Step-by-step solution

$v_1 = \left(-1, -i\sqrt{2}, 1\right)$

$v_2 = \left(-1, i\sqrt{2}, 1\right)$

$v_3 = (1, 0, 1)$

Corresponding eigenvalues: Step-by-step solution

$\lambda_1 = 2i$

$\lambda_2 = -2i$

$\lambda_3 = 2$

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POWERED BY THE **WOLFRAM LANGUAGE**

Matrix Diagonalization (Ch 10.4, Ex 5)

WolframAlpha computational intelligence

diagonalize{{pi,-1,-1},{-1,pi,-1},{-1,-1,pi}}

Extended Keyboard | Upload | Examples | Random

Input interpretation:

diagonalize	$\begin{pmatrix} \pi & -1 & -1 \\ -1 & \pi & -1 \\ -1 & -1 & \pi \end{pmatrix}$

Result: Approximate forms

$$M = S.J.S^{-1}$$

where

$$M = \begin{pmatrix} \pi & -1 & -1 \\ -1 & \pi & -1 \\ -1 & -1 & \pi \end{pmatrix}$$

$$S = \begin{pmatrix} 1 & -1 & -1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{pmatrix}$$

$$J = \begin{pmatrix} \pi - 2 & 0 & 0 \\ 0 & 1 + \pi & 0 \\ 0 & 0 & 1 + \pi \end{pmatrix}$$

$$S^{-1} = \begin{pmatrix} \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \end{pmatrix}$$

Matrix Diagonalization (Ch 3.11, Ex 4)

WolframAlpha computational intelligence

inverse {{1/sqrt(2), -1/sqrt(6), 1/sqrt(3)}, {1/sqrt(2), 1/sqrt(6), -1/sqrt(3)}, {0, 2/sqrt(6), 1/sqrt(3)}}

Extended Keyboard | Upload | Examples | Random

Input:

$$\begin{pmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{3}} \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{6}} & -\frac{1}{\sqrt{3}} \\ 0 & \frac{2}{\sqrt{6}} & \frac{1}{\sqrt{3}} \end{pmatrix}^{-1}$$ (matrix inverse)

Result: Approximate form | Step-by-step solution

$$\frac{1}{6}\begin{pmatrix} 3\sqrt{2} & 3\sqrt{2} & 0 \\ -\sqrt{6} & \sqrt{6} & 2\sqrt{6} \\ 2\sqrt{3} & -2\sqrt{3} & 2\sqrt{3} \end{pmatrix}$$

Expanded form:

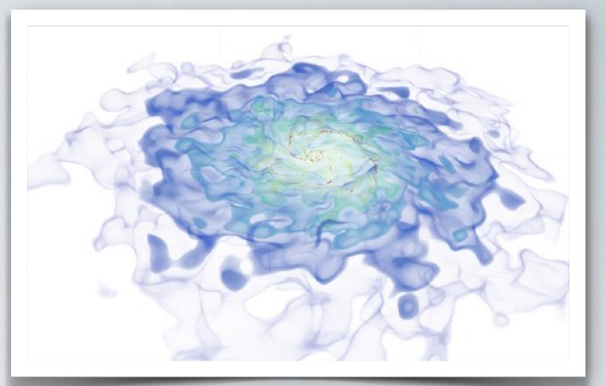
$$\begin{pmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{6}} & \frac{1}{\sqrt{6}} & \sqrt{\frac{2}{3}} \\ \frac{1}{\sqrt{3}} & -\frac{1}{\sqrt{3}} & \frac{1}{\sqrt{3}} \end{pmatrix}$$

Dimensions: Enlarge | Data | Customize | Plain Text

3 (rows) × 3 (columns)

Matrix plot:

1 2 3



Matrix Diagonalization: Applications

Matrix Diagonalization (Ch 3.12, Ex 1)

WolframAlpha computational intelligence

plot 5x^2-4xy+2y^2=30

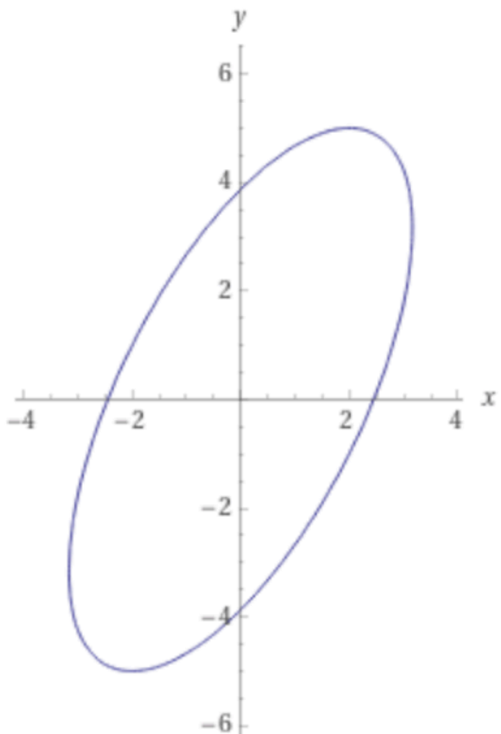
NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES | UPLOAD | RANDOM

Assuming "plot" is a plotting function | Use as referring to geometry instead

Input interpretation

plot	$5x^2 - 4xy + 2y^2 = 30$

Implicit plot



Matrix Diagonalization (Ch 3.12, Ex 1)

WolframAlpha computational intelligence

plot 5x^2-4xy+2y^2=30

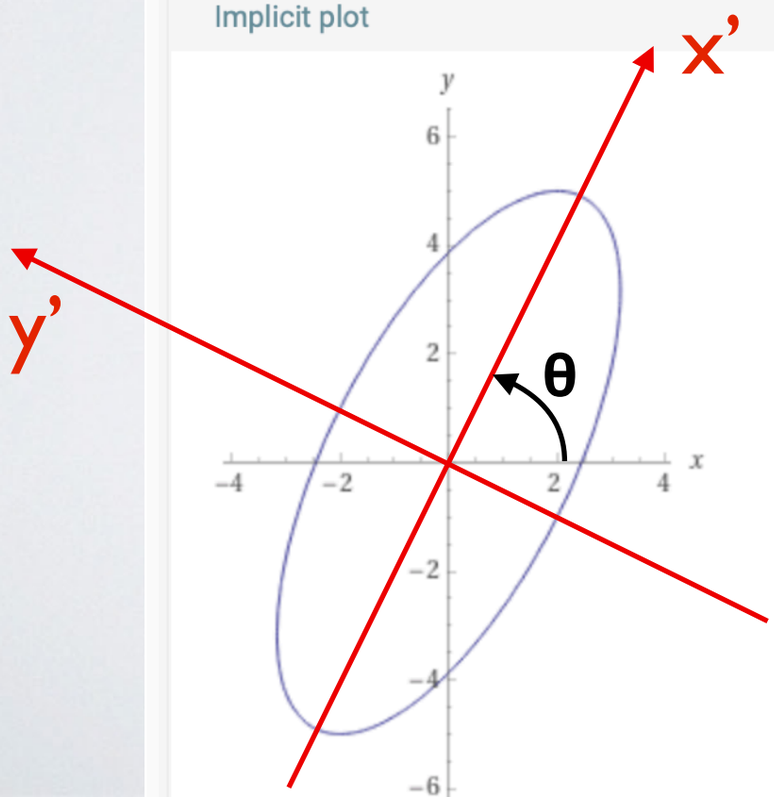
NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES | UPLOAD | RANDOM

Assuming "plot" is a plotting function | Use as referring to geometry instead

Input interpretation

plot	$5x^2 - 4xy + 2y^2 = 30$

Implicit plot



Matrix Diagonalization (Ch 3.12, Ex 1)

WolframAlpha computational intelligence

plot x^2+6xy-2y^2-2yz+z^2=24

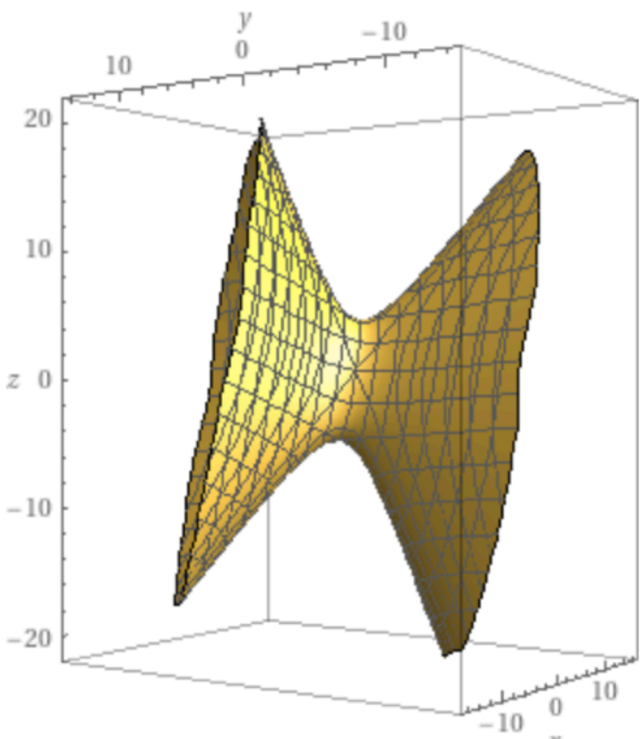
NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES | UPLOAD | RANDOM

Assuming "plot" is a plotting function | Use as referring to geometry instead

Input interpretation

plot	$x^2 + 6\,x\,y - 2\,y^2 - 2\,y\,z + z^2 = 24$

Surface plot



Matrix Diagonalization (Ch 3.12, Ex 1)

WolframAlpha computational intelligence

plot x^2+6xy-2y^2-2yz+z^2=24

NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES | UPLOAD | RANDOM

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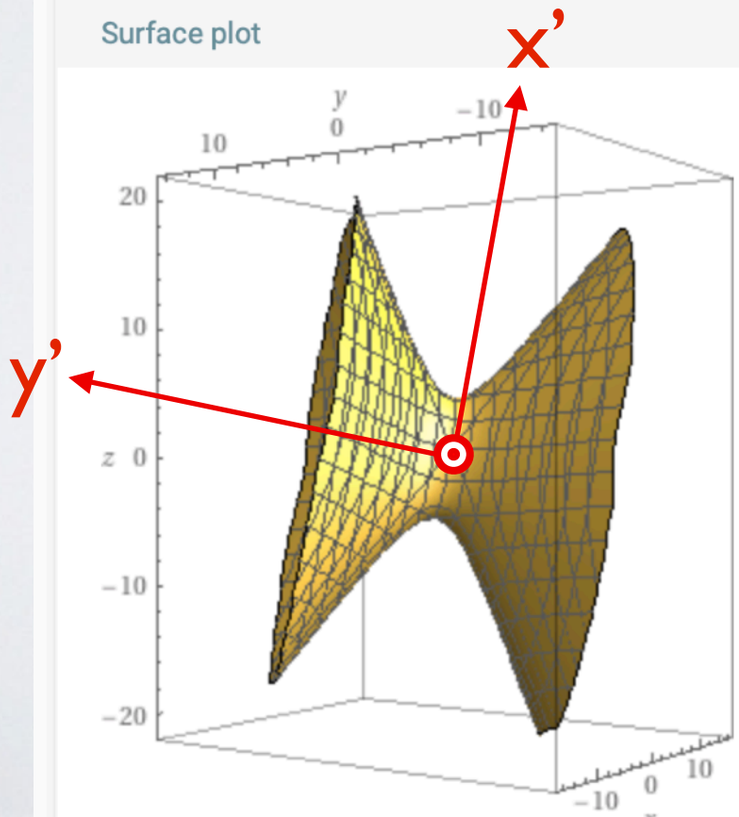

Matrix Diagonalization (Ch 3.12, Ex 1)

WolframAlpha computational intelligence

plot x^2+6xy-2y^2-2yz+z^2=24

NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES | UPLOAD | RANDOM

Assuming "plot" is a plotting function | Use as referring to geometry instead

Input interpretation

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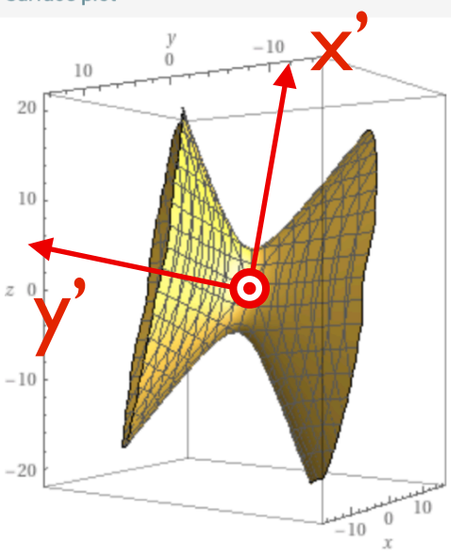
Surface plot



Geometric figure

one-sheeted hyperboloid

WolframAlpha computational intelligence

plot x^2-4y^2+3z^2=24

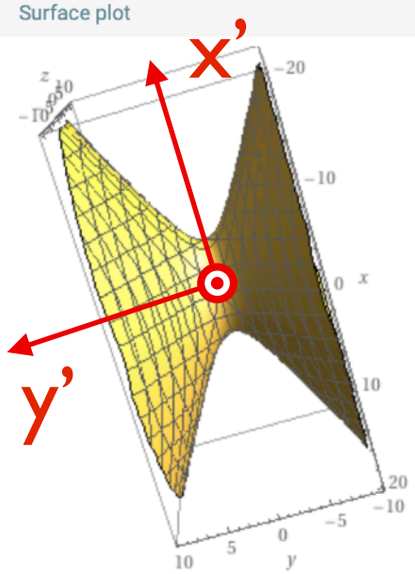
NATURAL LANGUAGE | MATH INPUT | EXTENDED KEYBOARD | EXAMPLES | UPLOAD | RANDOM

Assuming "plot" is a plotting function | Use as referring to geometry instead

Input interpretation

plot $x^2 - 4\,y^2 + 3\,z^2 = 24$

Surface plot

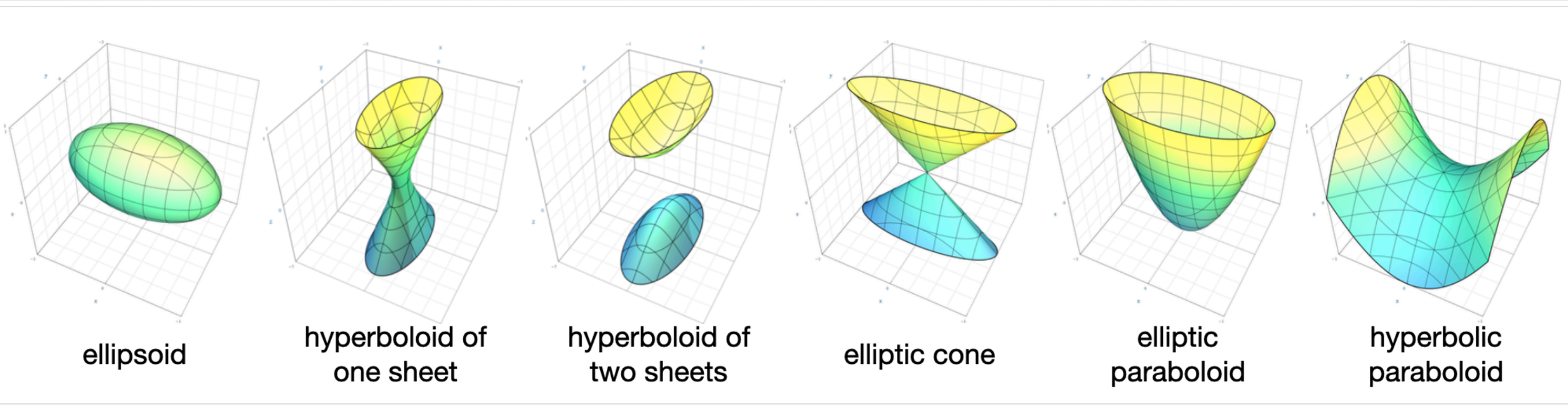


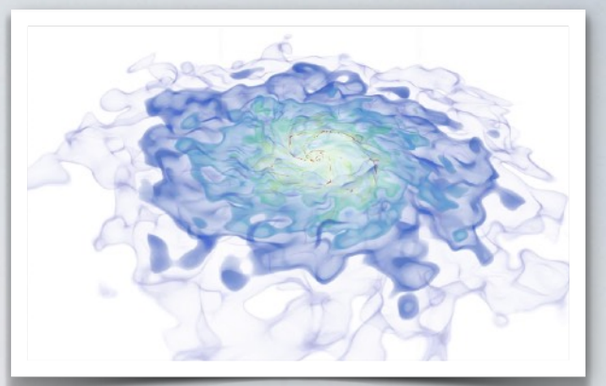
Geometric figure

one-sheeted hyperboloid

Matrix Diagonalization (Ch 3.12, Ex 2)

- Various quadric surfaces:

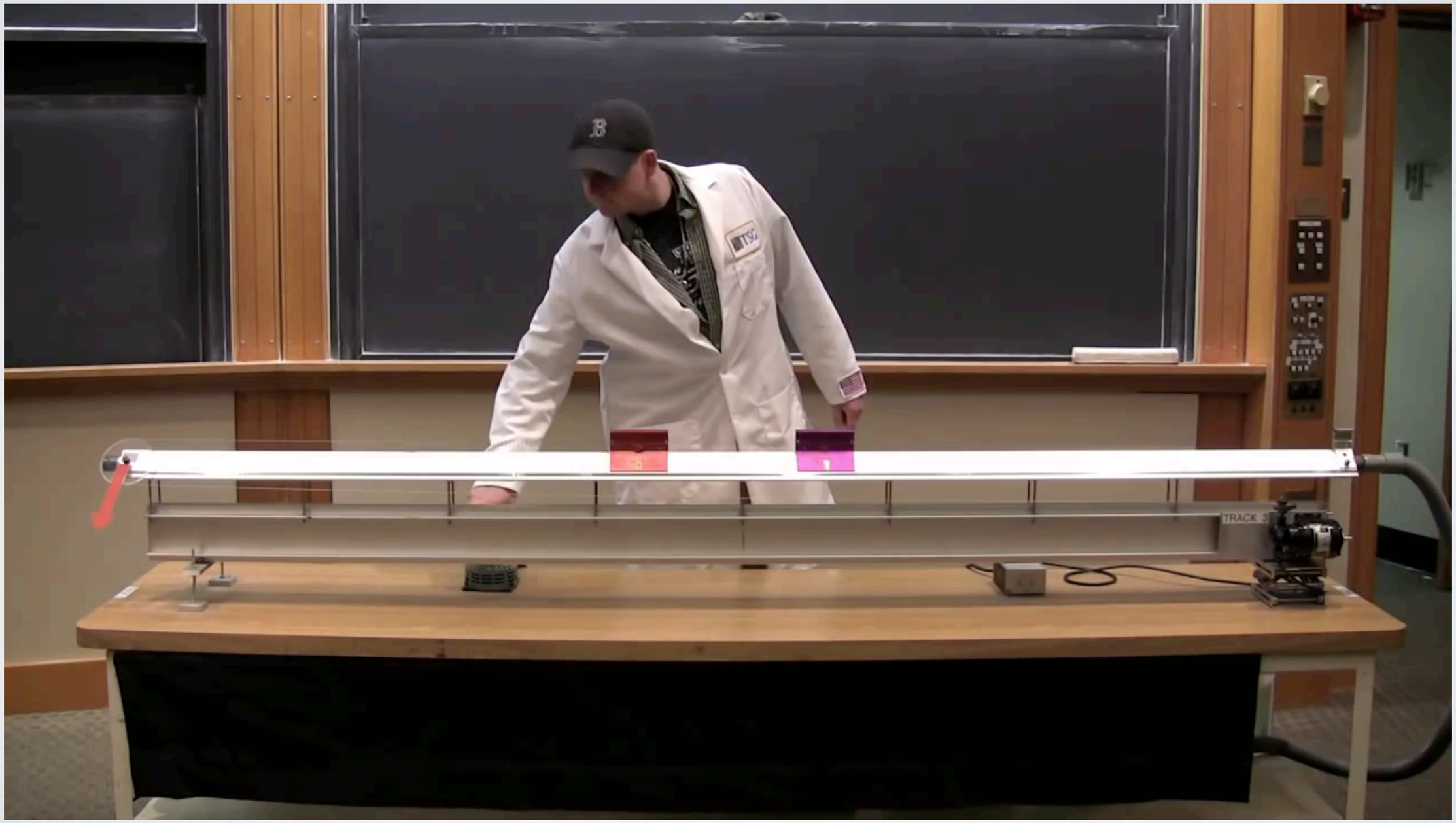




Coupled Oscillators

Coupled Oscillators

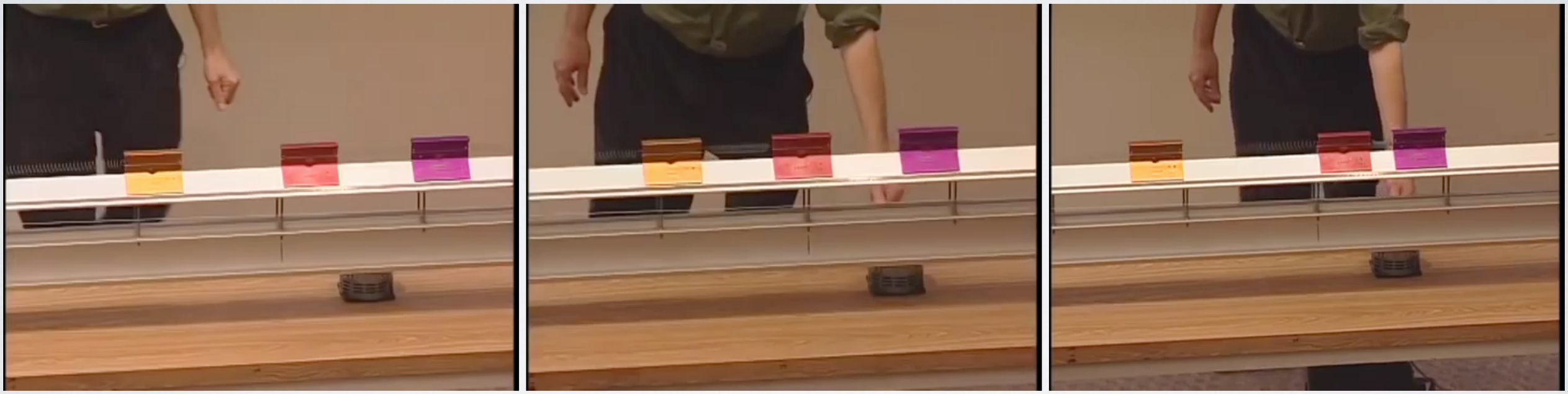
- Two carts on a frictionless air track coupled & fixed by springs.



www.youtube.com/watch?v=zlzns5PjmJ4

Coupled Oscillators

- Three carts on an air track coupled with four springs.



www.youtube.com/watch?v=Ye92jN6FrlU