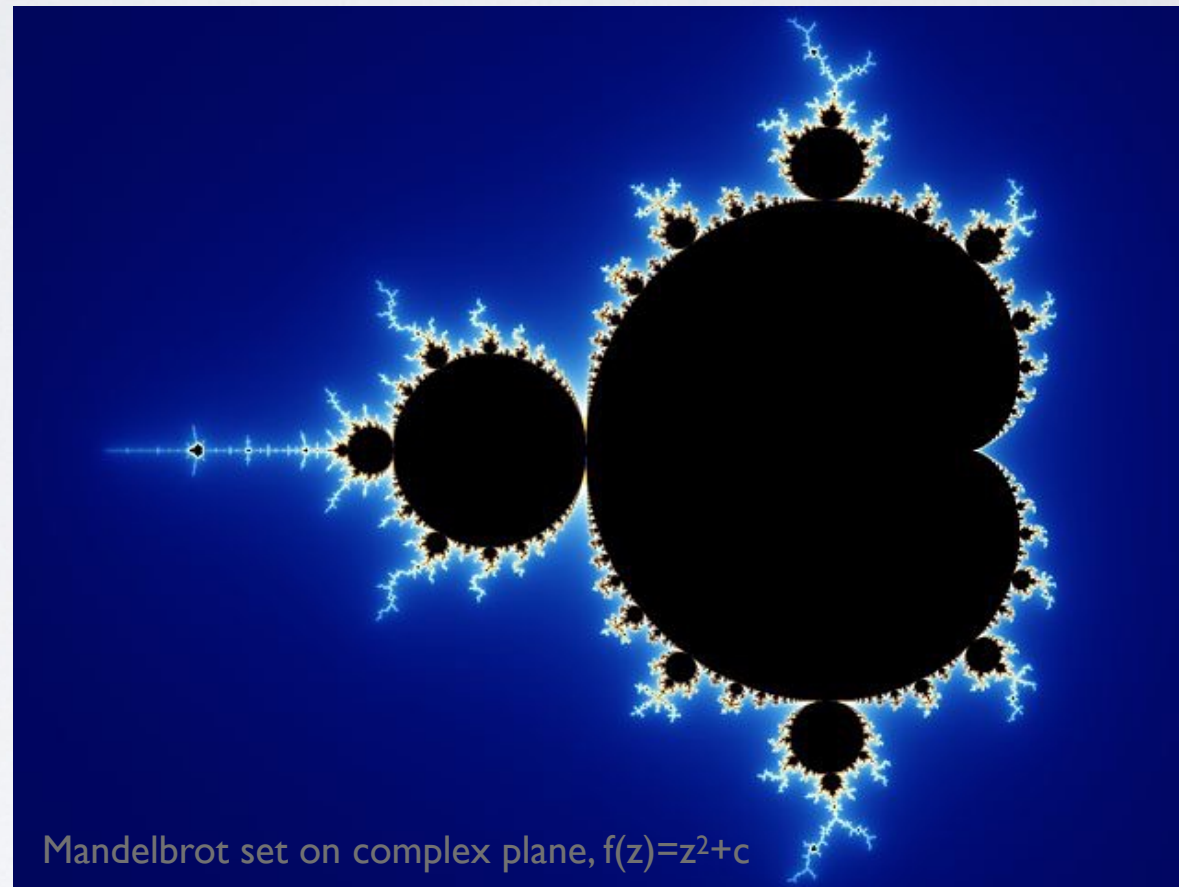
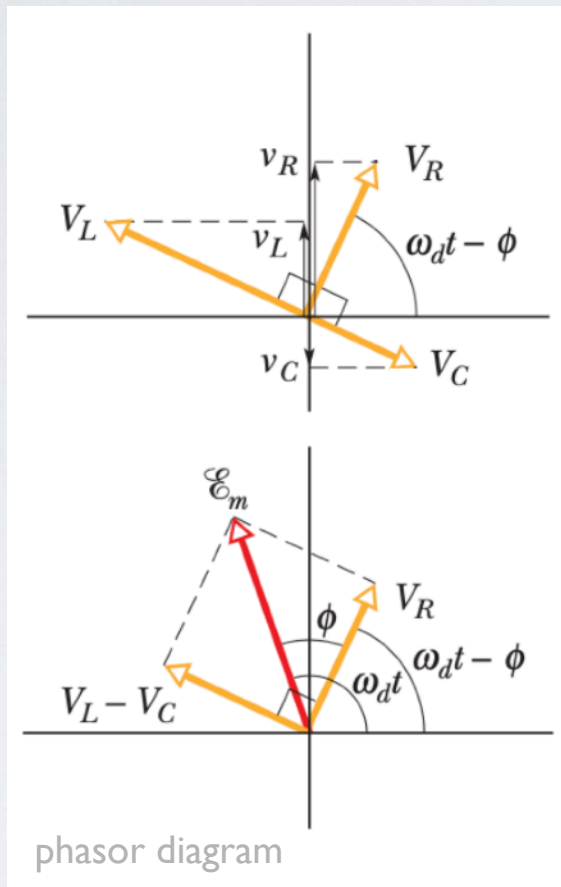


Week 2 - #1

Complex Numbers (II)



Today: Ch 1-2

Next Class: Ch 2

Ji-hoon Kim (Seoul National University)

Rudimentary Mathematical Methods of Physics (Fall 2025): Quiz #2

— [open book and open note, “and” cellphone or laptop, drop it off as you leave the class] —

Please write down your name and student ID in the top right corner. (0.0 pt: no paper found with your name / 0.5 pt: paper found with your name and some answers / 1.0 pt: good answers)

1. Define “complex plane” from your textbook; then, Boas, Chapter 2, Section 5, Problem 59
2. Briefly explain the so-called Mandelbrot set demonstrated in the following movie — in particular, how this one of the most famous images in the fractal/chaos theory is plotted on a “complex plane”.



youtube.com/watch?v=G_GBwuYuOOs,
Mandelbrot set on complex plane

Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 2, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following.
The class homework assignments will mainly be from this list.

- Section 04: Problems 4, 13
- Section 05: Problems 6, 7, 27, 28, 47, 50, 57
- Section 06: Problems 11, 14
- Section 07: Problems 12, 13, 17
- Section 08: Problems 3
- Section 09: Problems 9, 24, 25, 27, 28, 37
- Section 10: Problems 23, 32, 33
- Section 11: Problems 18
- Section 12: Problems 1, 10, 14, 15, 27, 38
- Section 14: Problems 2, 7, 9, 24, 25
- Section 15: Problems 6, 7, 18
- Section 16: Problems 1, 2, 5, 8, 11
- Section 17: Problems 19, 21, 25, 30

48 problems / 337 total

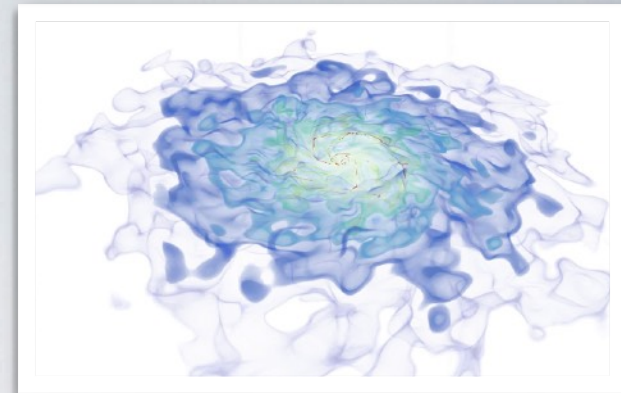
Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 3, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following.
The class homework assignments will mainly be from this list.

- Section 02: Problems 8, 13, 14, 17, 18
- Section 03: Problems 2, 6, 9, 10, 13, 15
- Section 04: Problems 5, 6, 9, 21, 23
- Section 05: Problems 17, 21, 32, 36, 42
- Section 06: Problems 6, 7, 16, 18, 21, 29, 30
- Section 07: Problems 12, 27, 31, 34, 35
- Section 08: Problems 2, 10, 15, 16, 17, 24
- Section 09: Problems 3, 5, 10, 17, 19(c), 24, 25(a)(b)
- Section 10: Problems 4(c), 5(a), 7
- Section 11: Problems 9, 10, 19, 21, 31, 33, 42, 44, 50, 57, 60, 61
- Section 12: Problems 9 (for Problems 4, 7), 16, 21
- Section 13: Problems 1, 4, 7
- Section 14: Problems 13

Chapter 10, Sections 1-4

- Section 02: Problems 6
- Section 03: Problems 1, 2, 8
- Section 04: Problems 2, 6, 8



Pitfalls of Blind Computation

Pitfalls of Blind Computation

- Inaccurate estimation on Google:

The screenshot shows a Google search for the expression $\ln(\sqrt{(1+0.0015)/(1-0.0015)}) - \tan(0.0015)$. The search bar contains the expression, and the results show "About 602 results (0.47 seconds)". Below the results, a calculator interface displays the same expression as $\ln(\sqrt{(1 + 0.0015) / (1 - 0.0015)}) - \tan(0.0015 \text{ radians}) =$ followed by the result $\underline{6.0606901e-16}$. The calculator interface includes buttons for Rad, Deg, x!, (,), %, AC, Inv, sin, ln, 7, 8, 9, ÷, π, cos, log, 4, 5, 6, ×, e, tan, √, 1, 2, 3, −, Ans, EXP, x^y, 0, ., =, and +.

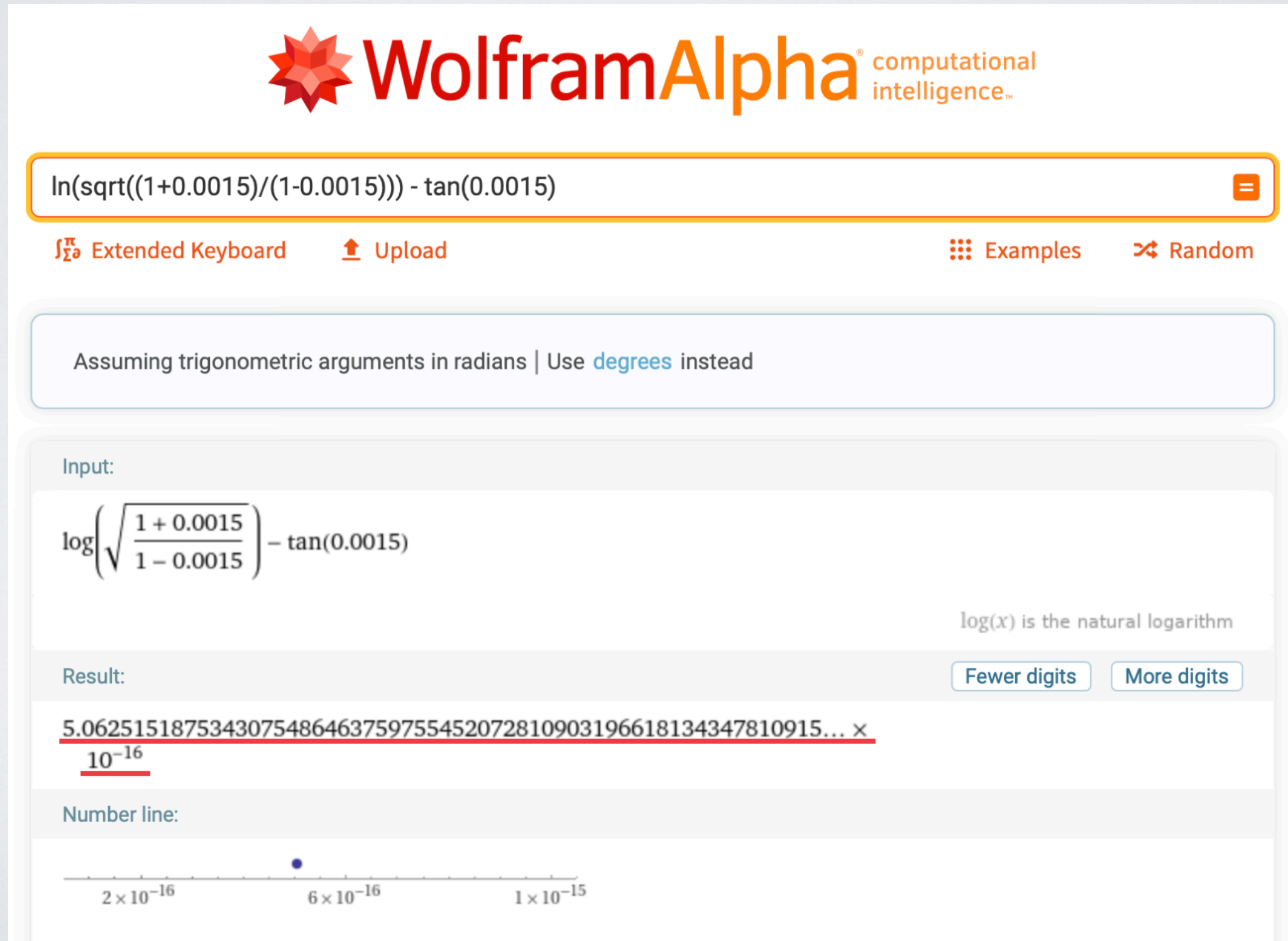
Pitfalls of Blind Computation

- Inaccurate estimation on Python:

```
bash-3.2$ python
Python 2.7.9 (default, Sep  9 2015, 22:07:15)
[GCC 4.2.1 Compatible Apple LLVM 6.1.0 (clang-602.0.53)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import math
>>> math.log(math.sqrt((1+0.0015)/(1-0.0015))) - math.tan(0.0015)
6.06069014419397e-16
```

Pitfalls of Blind Computation

- Accurate estimation on WolframAlpha:



The screenshot shows the WolframAlpha interface. At the top is the logo with the text "WolframAlpha computational intelligence". Below it is a search bar containing the expression $\ln(\sqrt{(1+0.0015)/(1-0.0015)}) - \tan(0.0015)$. Below the search bar are links for "Extended Keyboard", "Upload", "Examples", and "Random". A message states "Assuming trigonometric arguments in radians | Use [degrees](#) instead". The "Input:" section shows the expression in a more formal mathematical notation: $\log\left(\sqrt{\frac{1+0.0015}{1-0.0015}}\right) - \tan(0.0015)$. A note says "log(x) is the natural logarithm". The "Result:" section shows the value $5.0625151875343075486463759755452072810903196618134347810915... \times 10^{-16}$. To the right of the result are buttons for "Fewer digits" and "More digits". The "Number line:" section shows a horizontal axis with labels 2×10^{-16} , 6×10^{-16} , and 1×10^{-15} , with a blue dot indicating the position of the result.

WolframAlpha[®] computational intelligence[™]

$\ln(\sqrt{(1+0.0015)/(1-0.0015)}) - \tan(0.0015)$

$\int \frac{\pi}{2}$ Extended Keyboard Upload Examples Random

Assuming trigonometric arguments in radians | Use [degrees](#) instead

Input:

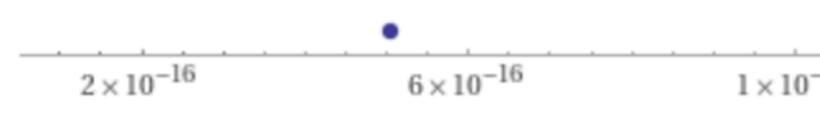
$$\log\left(\sqrt{\frac{1+0.0015}{1-0.0015}}\right) - \tan(0.0015)$$

log(x) is the natural logarithm

Result: [Fewer digits](#) [More digits](#)

$5.0625151875343075486463759755452072810903196618134347810915... \times 10^{-16}$

Number line:



A horizontal number line with tick marks. Labels are 2×10^{-16} , 6×10^{-16} , and 1×10^{-15} . A blue dot is placed on the line between 2×10^{-16} and 6×10^{-16} , closer to 2×10^{-16} .

Pitfalls of Blind Computation

- Accurate estimation on Full Precision Calculator:

Full Precision Calculator

This calculator calculates answers to full* accuracy.
If you want more functions (but not full precision) try the [Scientific Calculator](#)

$$\ln(\sqrt{(1+0.0015)/(1-0.0015)}) - \tan(0.0015 \cdot 180/\pi)$$

0.0000000000000000506251518753430754864637597554
5207281090319661813434781091560762983344871629
59393151147072175618546084275046494927908515814
2080150864641203183784359975199564279750604123
6612968728336675572119291634

7	8	9	×	÷	deg
4	5	6	+	-	
1	2	3	(	)	
0	.	^	←	C	
!	e	pi			

History:

$\ln(\sqrt{(1+0.0015)/(1-0.0015)}) - \tan(0.0015 \cdot 180/\pi) =$
0.000000000000000050625151875343075486463759755452072810903
1966181343478109156076298334487162959393151147072175618546
0842750464949279085158142080150864641203183784359975199564
2797506041236612968728336675572119291634
 $\ln(\sqrt{(1+0.0015)/(1-0.0015)}) =$
0.001500001125001518752440852485777952715102659838705448085
5820599653054716307555148275394381298635589258005404247102

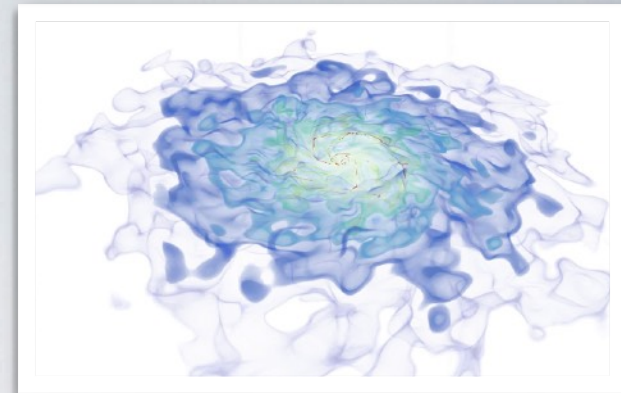
Decimals: 200

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Pitfalls of Blind Computation

- Inaccurate estimation on Python:

```
bash-3.2$ python
Python 2.7.9 (default, Sep  9 2015, 22:07:15)
[GCC 4.2.1 Compatible Apple LLVM 6.1.0 (clang-602.0.53)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import math
>>> math.log(math.sqrt((1+0.0015)/(1-0.0015))) - math.tan(0.0015)
6.06069014419397e-16
>>> math.log(math.sqrt((1+0.0015)/(1-0.0015)))
0.0015000011250016186
>>> math.tan(0.0015)
0.0015000011250010125
>>>
>>>
```

Pitfalls of Blind Computation (Another Example: Problem 15.2)

Pitfalls of Blind Computation

- Inaccurate estimation on Google:

The screenshot shows a Google search for the expression $1/\sqrt{1+0.012^4}-\cos(0.012^2)$. The search bar contains the expression, and the results show "About 226,000 results (0.55 seconds)". Below the results, a calculator interface is displayed. The calculator shows the expression $(1 / \sqrt{1 + (0.012^4)}) - \cos(0.012^2 \text{ radians}) =$ and the result 0 . The calculator interface includes buttons for Rad, Deg, x!, (,), %, AC, Inv, sin, ln, 7, 8, 9, ÷, π, cos, log, 4, 5, 6, ×, e, tan, √, 1, 2, 3, −, Ans, EXP, x^y, 0, ., =, and +.

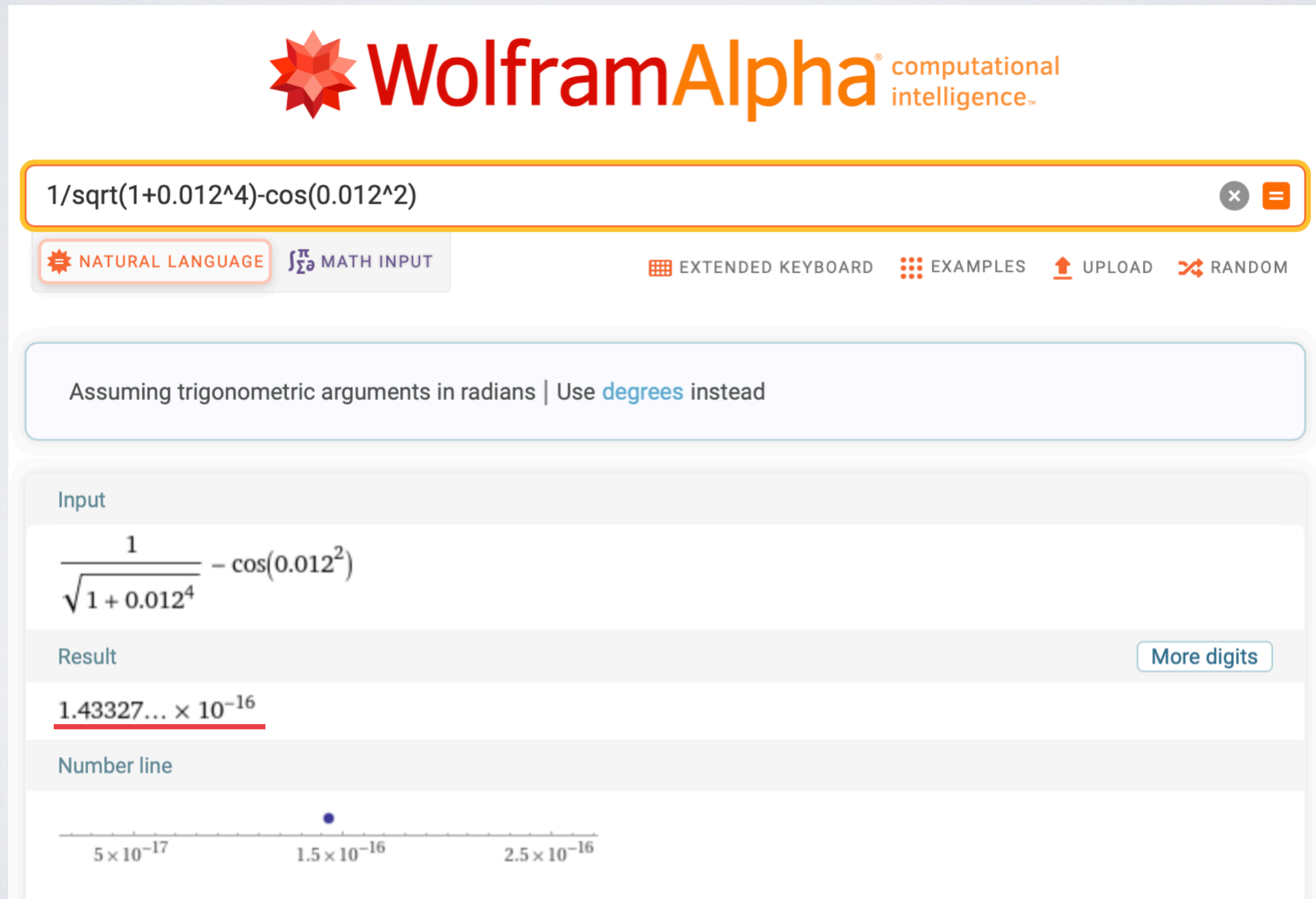
Pitfalls of Blind Computation

- Inaccurate estimation on Python:

```
bash-3.2$ python
Python 2.7.9 (default, Sep  9 2015, 22:07:15)
[GCC 4.2.1 Compatible Apple LLVM 6.1.0 (clang-602.0.53)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import math
>>> 1/math.sqrt(1+0.012**4) - math.cos(0.012**2)
1.1102230246251565e-16
>>>
```

Pitfalls of Blind Computation

- Accurate estimation on WolframAlpha:



The screenshot shows the WolframAlpha interface. At the top is the logo "WolframAlpha" with the tagline "computational intelligence". Below the logo is a search bar containing the expression $1/\sqrt{1+0.012^4}-\cos(0.012^2)$. To the right of the search bar are icons for clearing and submitting. Below the search bar are two tabs: "NATURAL LANGUAGE" and "MATH INPUT", with "MATH INPUT" being the active tab. To the right of these tabs are links for "EXTENDED KEYBOARD", "EXAMPLES", "UPLOAD", and "RANDOM". Below the search bar is a message: "Assuming trigonometric arguments in radians | Use [degrees](#) instead". Below this is the "Input" section, which shows the expression $\frac{1}{\sqrt{1+0.012^4}} - \cos(0.012^2)$. Below the input is the "Result" section, which shows the value $1.43327... \times 10^{-16}$. To the right of the result is a button labeled "More digits". Below the result is the "Number line" section, which shows a horizontal axis with tick marks at 5×10^{-17} , 1.5×10^{-16} , and 2.5×10^{-16} . A blue dot is placed on the number line at the position corresponding to the result.

WolframAlpha[®] computational intelligence™

$1/\sqrt{1+0.012^4}-\cos(0.012^2)$

NATURAL LANGUAGE MATH INPUT

EXTENDED KEYBOARD EXAMPLES UPLOAD RANDOM

Assuming trigonometric arguments in radians | Use [degrees](#) instead

Input

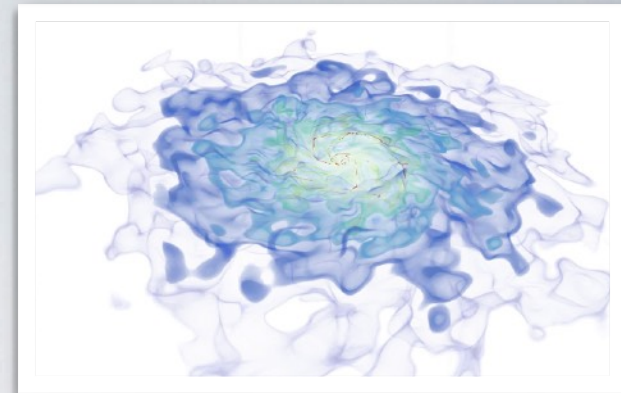
$$\frac{1}{\sqrt{1+0.012^4}} - \cos(0.012^2)$$

Result [More digits](#)

$1.43327... \times 10^{-16}$

Number line

5×10^{-17} 1.5×10^{-16} 2.5×10^{-16}



Roots of Complex Numbers

Roots of Complex Numbers



$(-8i)^{(1/6)}$



NATURAL LANGUAGE

MATH INPUT

EXTENDED KEYBOARD

EXAMPLES

UPLOAD

RANDOM

Assuming i is the imaginary unit | Use i as [a variable](#) instead

Input

$\sqrt[6]{-8i}$

i is the imaginary unit

Exact result

$-(-1)^{11/12} \sqrt{2}$

Decimal approximation

Enlarge | Data | Customize | Plain Text

1.36602540378443864676372317075293618347140262690519031402790348...

-

0.366025403784438646763723170752936183471402626905190314027903489...

i

Polar coordinates

Radians ▾

Approximate forms

$r = \sqrt{2}$ (radius), $\theta = -\frac{\pi}{12}$ (angle)

Roots of Complex Numbers



$(-8i)^{(1/6)}$

NATURAL LANGUAGE

MATH INPUT

EXTENDED KEYBOARD

EXAMPLES

UPLOAD

RANDOM

All 6th roots of $-8i$

Fewer roots

More digits

Polar form

$$\sqrt{2} e^{-i\pi/12} \approx 1.36603 - 0.36603 i$$

$$\sqrt{2} e^{i\pi/4} = 1 + i \text{ (principal root)}$$

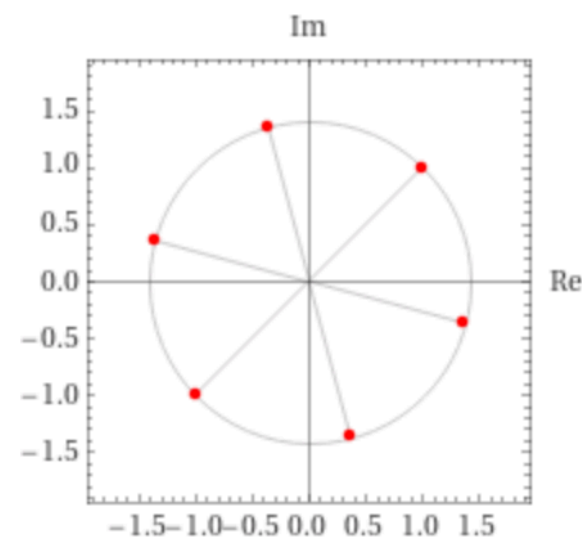
$$\sqrt{2} e^{7i\pi/12} \approx -0.3660 + 1.3660 i$$

$$\sqrt{2} e^{11i\pi/12} \approx -1.3660 + 0.3660 i$$

$$\sqrt{2} e^{-3i\pi/4} = -1 - i$$

$$\sqrt{2} e^{-5i\pi/12} \approx 0.3660 - 1.3660 i$$

Plot of all roots in the complex plane



→ Boas Ch2 Fig 10.4