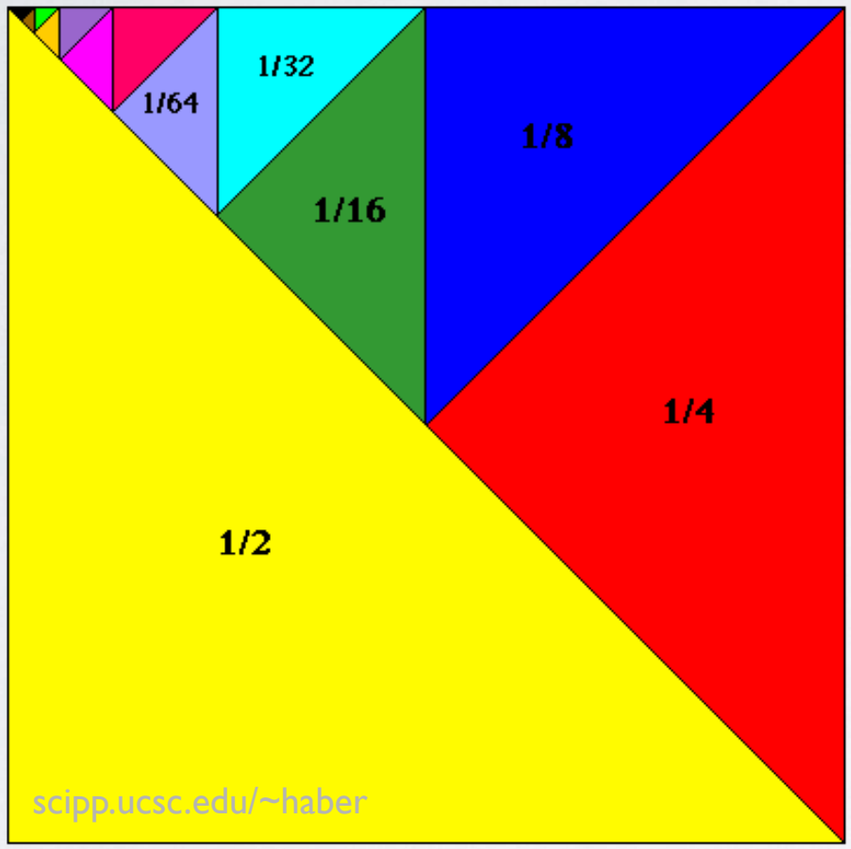
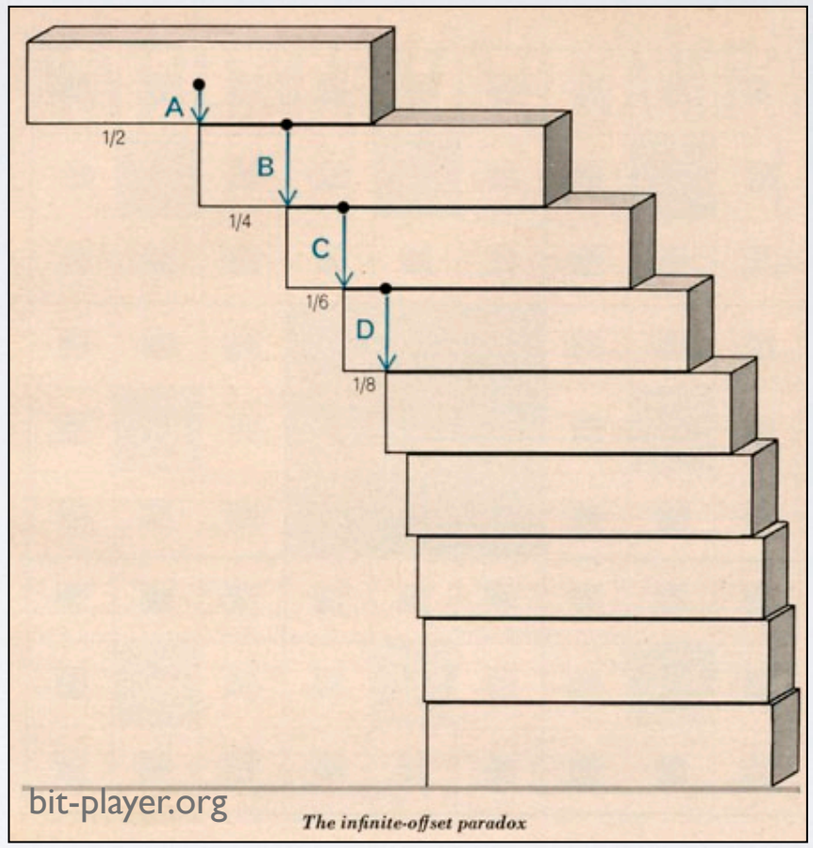
Week 1 - #2

Infinite & Power Series, Complex Numbers (I)





Today: Ch 1-2

Next Class: Ch 2

Ji-hoon Kim (Seoul National University)

Rudimentary Mathematical Methods of Physics (Fall 2025): Quiz #1

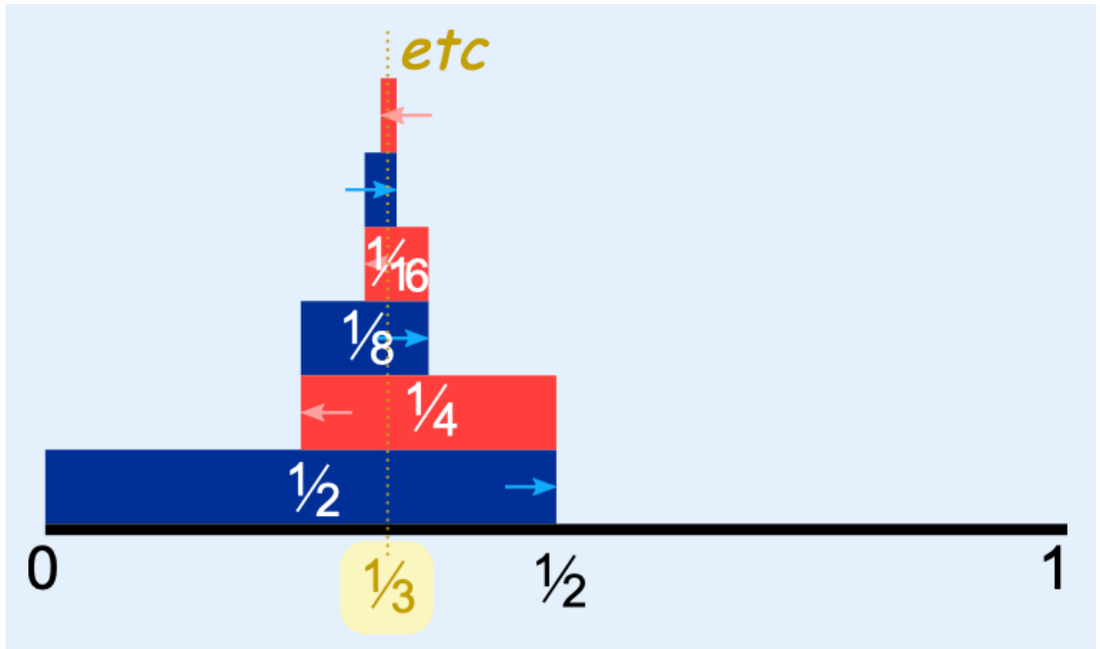
— [open book and open note, but no cellphone or laptop, drop it off as you leave the class] —

Please write down your name and student ID in the top right corner. (0.0 pt: no paper found with your name / 0.5 pt: paper found with your name and some answers / 1.0 pt: good answers)

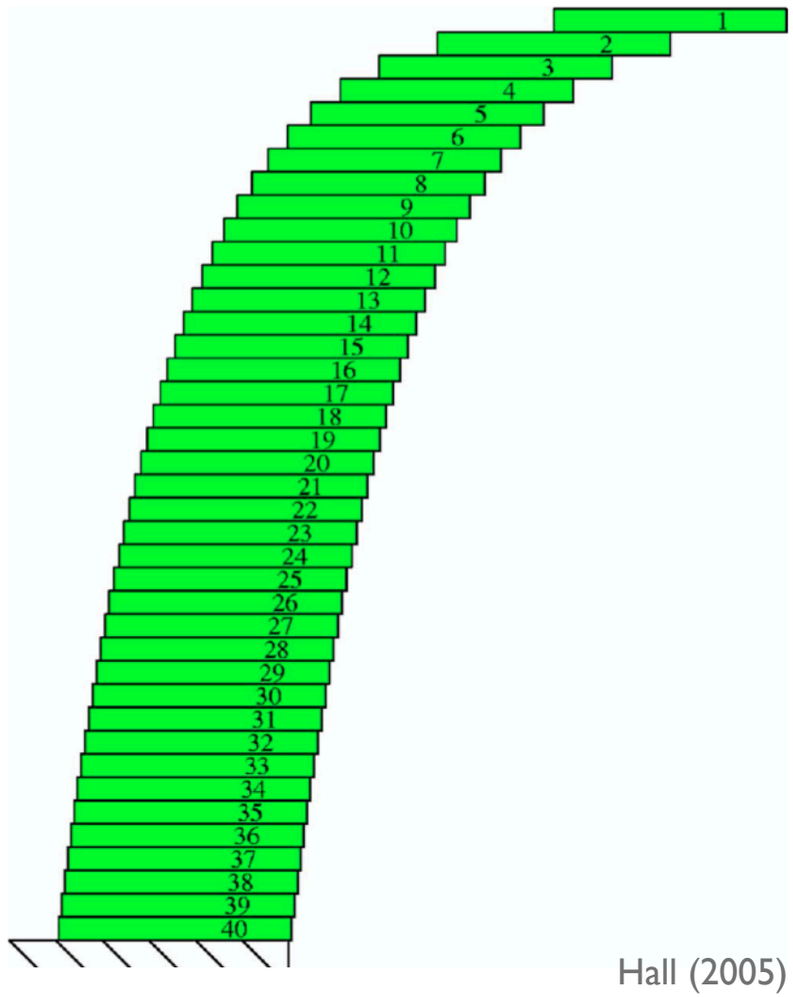
1. Boas, Chapter 1, Section 6, Problem 19

2. Boas, Chapter 1, Section 6, Problem 35

3. Imagine you are a tutor for high school math. A student complains that she sees no use of "series" in any aspect of her future life. How would you convince her that it is useful?



mathisfun.com



Hall (2005)

Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 1, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following. The class homework assignments will mainly be from this list.

- Section 01: Problems 12, 14, 15
- Section 02: Problems 5, 7, 8, 9
- Section 04: Problems 3, 5, 7
- Section 05: Problems 2
- Section 06: Problems 5, 6, 10, 13, 17, 18, 26, 33, 34
- Section 07: Problems 3, 5, 6, 10
- Section 09: Problems 4, 8, 17, 21, 22(a)(b)
- Section 10: Problems 2, 8, 14, 19, 21
- Section 13: Problems 4, 9, 13, 14, 19, 24, 36
- Section 14: Problems 4, 6
- Section 15: Problems 2, 15, 16, 18, 23, 24(e)(f), 28, 29, 32, 33
- Section 16: Problems 1, 3, 18, 22, 24, 25

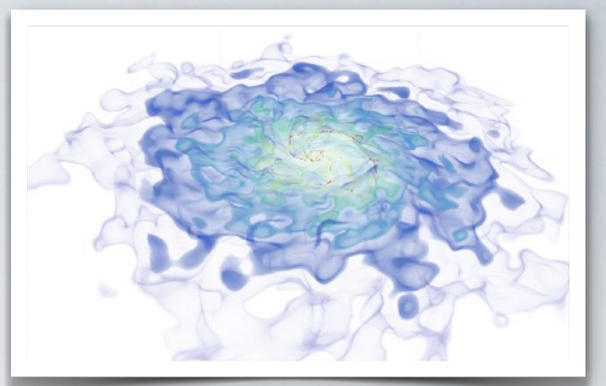
59 problems / 259 total

Rudimentary Mathematical Methods of Physics (Fall 2025): Suggested Problems in Chapter 2, Boas, 3rd ed.

The problems I suggest you to take a deeper look into include, but are not limited to, the following. The class homework assignments will mainly be from this list.

- Section 04: Problems 4, 13
- Section 05: Problems 6, 7, 27, 28, 47, 50, 57
- Section 06: Problems 11, 14
- Section 07: Problems 12, 13, 17
- Section 08: Problems 3
- Section 09: Problems 9, 24, 25, 27, 28, 37
- Section 10: Problems 23, 32, 33
- Section 11: Problems 18
- Section 12: Problems 1, 10, 14, 15, 27, 38
- Section 14: Problems 2, 7, 9, 24, 25
- Section 15: Problems 6, 7, 18
- Section 16: Problems 1, 2, 5, 8, 11
- Section 17: Problems 19, 21, 25, 30

48 problems / 337 total



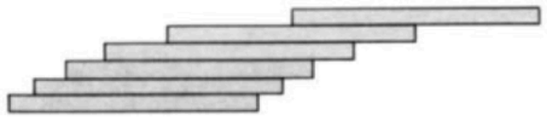
Fun with Stacking Blocks (Example of Diverging Infinite Series)

Fun with Stacking Blocks

- See: Problem 16.1 + "Fun with Stacking Blocks" (Hall, 2005), etc.

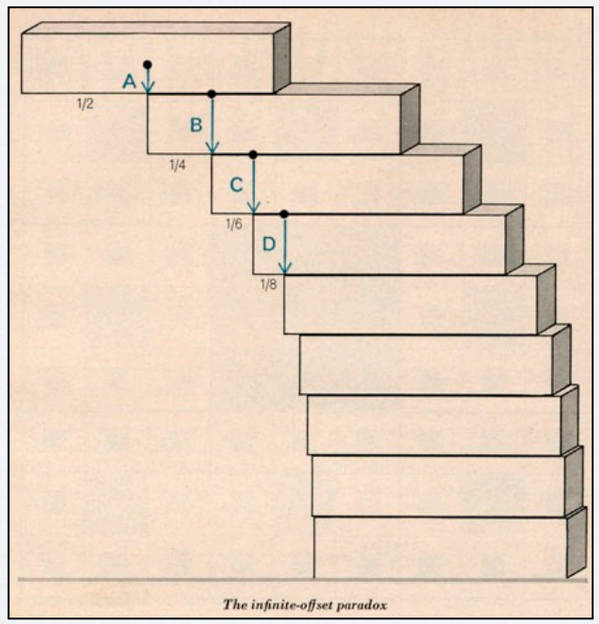
16. MISCELLANEOUS PROBLEMS

1. (a) Show that it is possible to stack a pile of identical books so that the top book is as far as you like to the right of the bottom book. Start at the top and each time place the pile already completed on top of another book so that the pile is just at the point of tipping. (In practice, of course, you can't let them overhang quite this much without having the stack topple. Try it with

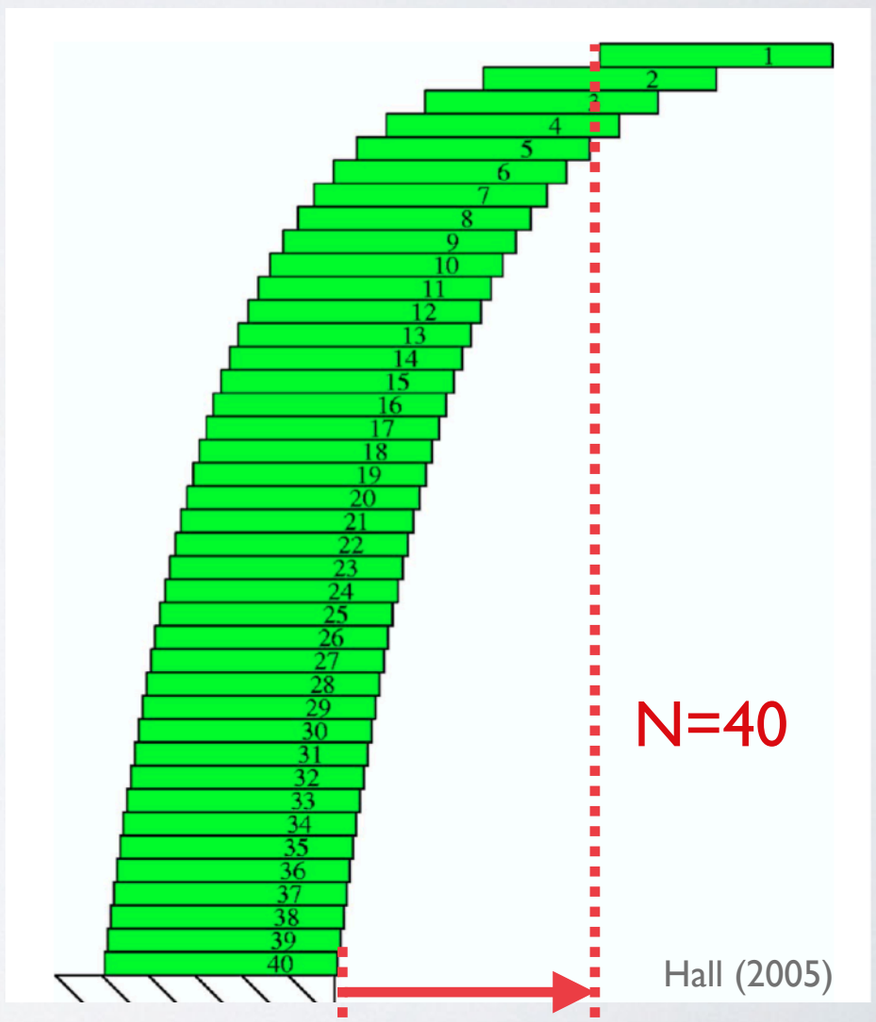


a deck of cards.) Find the distance from the right-hand end of each book to the right-hand end of the one beneath it. To find a general formula for this distance, consider the three forces acting on book n, and write the equation for the torque about its right-hand end. Show that the sum of these setbacks is a divergent series (proportional to the harmonic series). [See "Leaning Tower of *The Physical Reviews*," Am. J. Phys. **27**, 121–122 (1959).]

Boas, Chapter 1



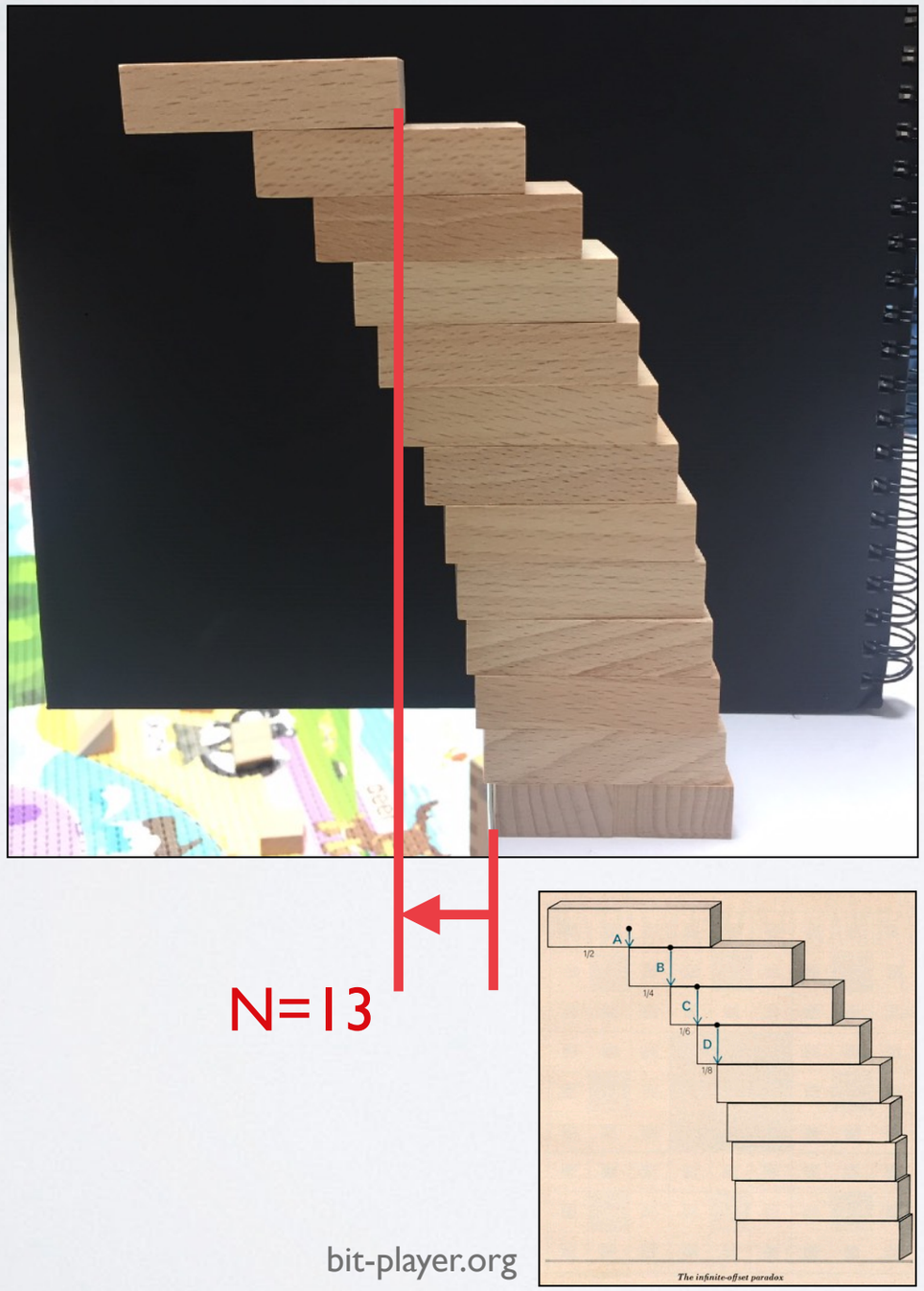
bit-player.org



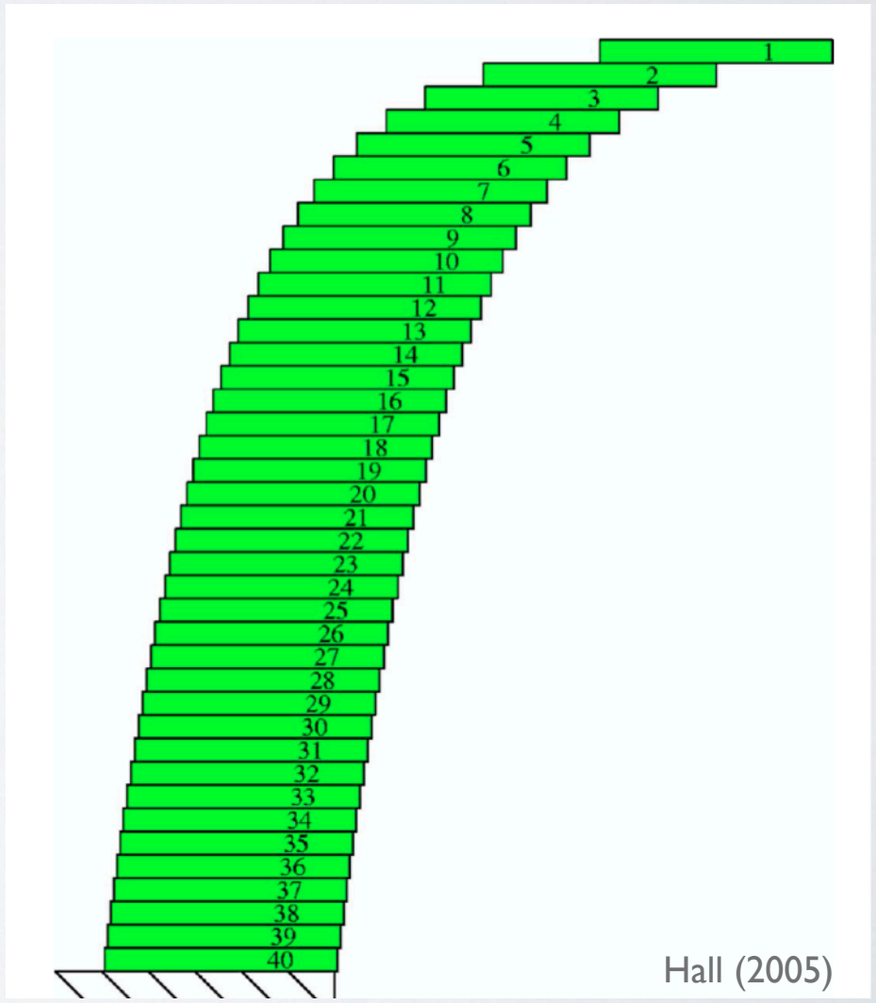
Hall (2005)

Fun with Stacking Blocks

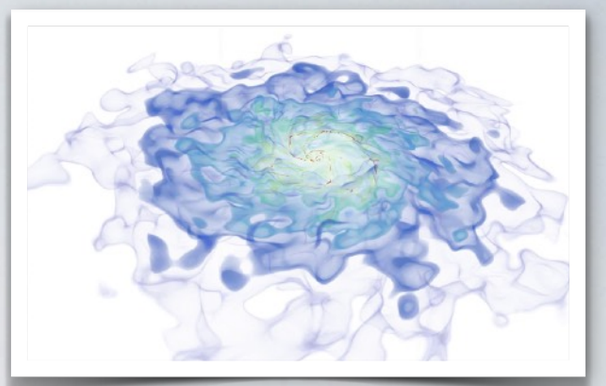
- See: Problem 16.1 + "Fun with Stacking Blocks" (Hall, 2005), etc.



bit-player.org



Hall (2005)



Taylor Series Using A Computer

Numerical Problems

- Tools you may want to try for your numerical problems:

  Matlab, Mathematica, Mathcad, Wolfram Alpha, Python, etc.

WolframAlpha computational intelligence

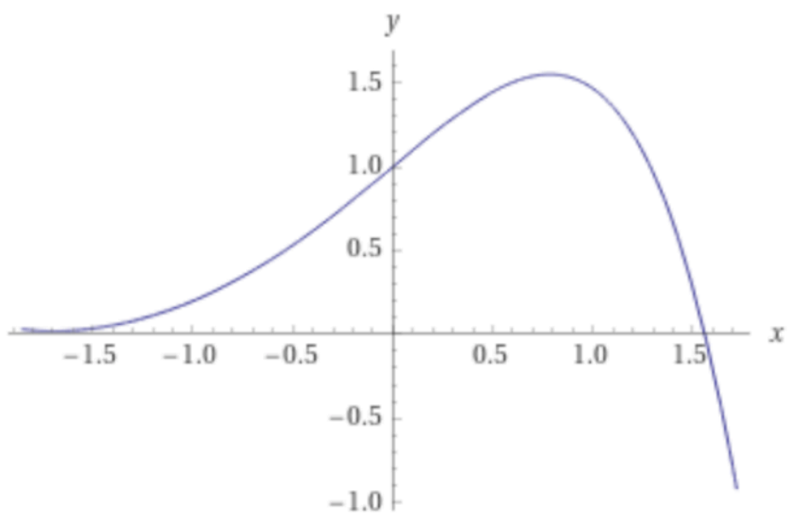
plot 1+x-x^3/3-x^4/6-x^5/30

Extended Keyboard Upload Examples Random

Input interpretation:

plot	$1 + x - \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{30}$

Plots:



(x from −1.8 to 1.7)

→ Boas Ch1 Fig13.1

Taylor Series Using A Computer

WolframAlpha computational intelligence

taylor expand e^x cosx

Extended Keyboard Upload Examples Random

Input interpretation:

series $e^x \cos(x)$

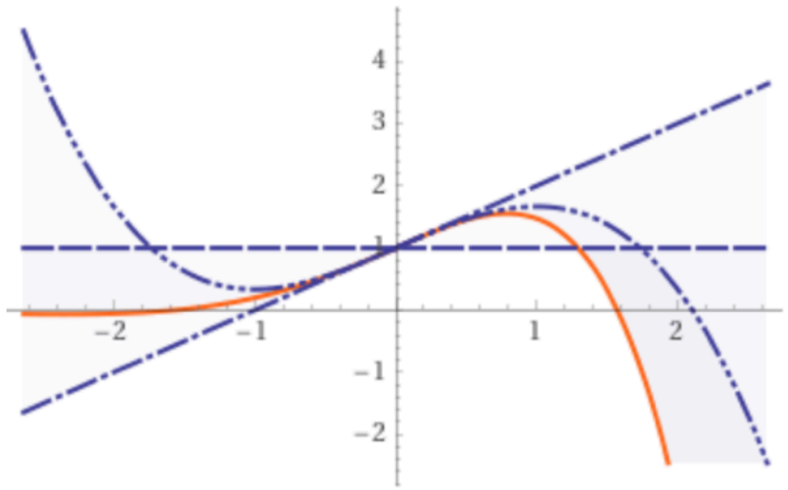
Series expansion at x = 0: Fewer terms More terms

$$1 + x - \frac{x^3}{3} - \frac{x^4}{6} - \frac{x^5}{30} + \frac{x^7}{630} + \frac{x^8}{2520} + \frac{x^9}{22\,680} - \frac{x^{11}}{1\,247\,400} + O(x^{12})$$

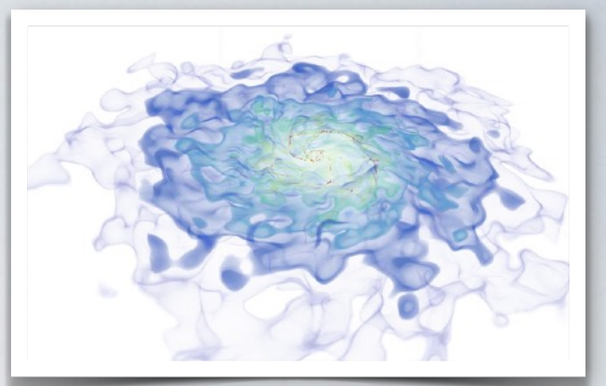
(Taylor series)

(converges everywhere)

Approximations about x = 0 up to order 3:



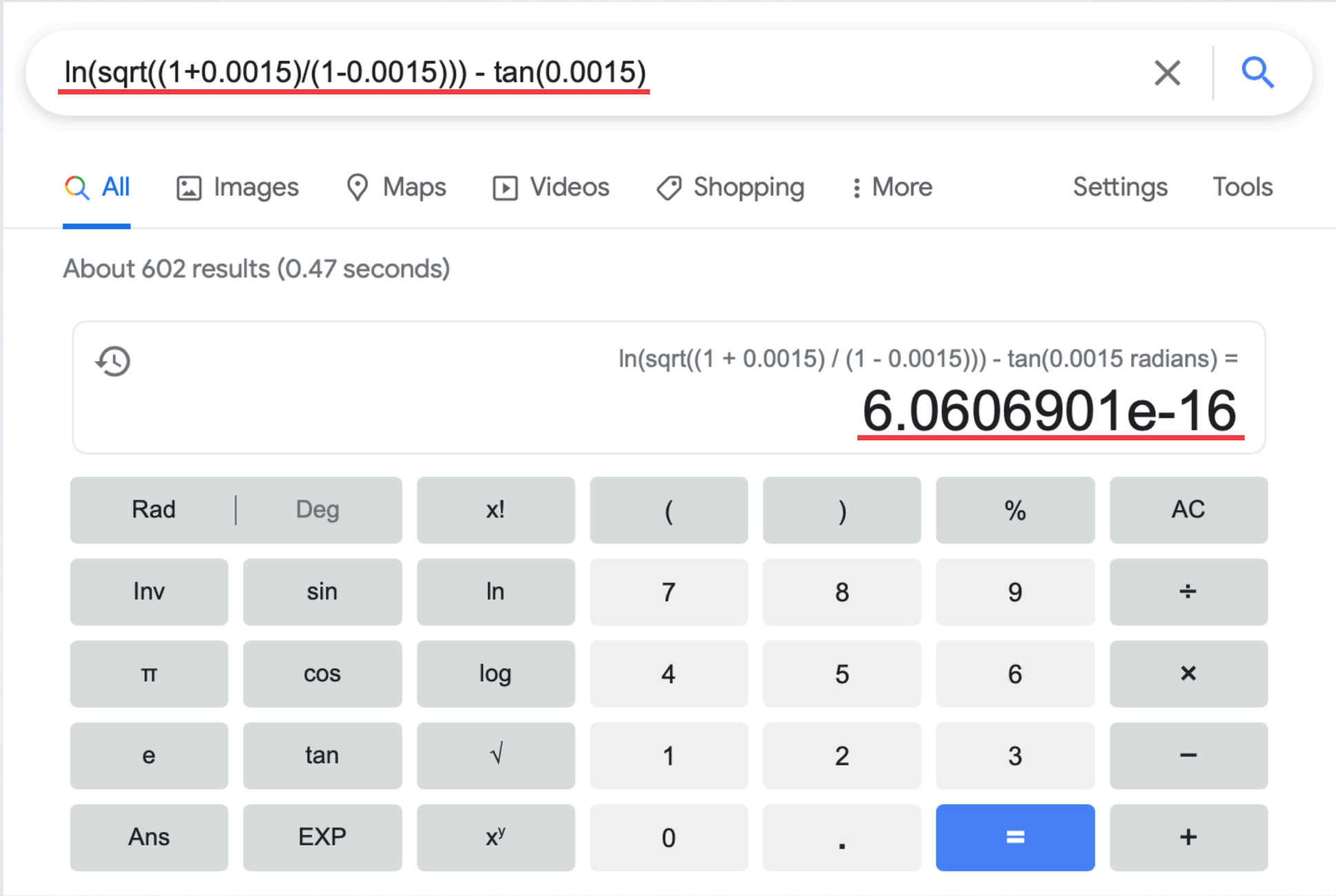
(order *n* approximation shown with *n* dots)



Pitfalls of Blind Computation

Pitfalls of Blind Computation

- Inaccurate estimation on Google:



Pitfalls of Blind Computation

- Inaccurate estimation on Python:

```
bash-3.2$ python
Python 2.7.9 (default, Sep  9 2015, 22:07:15)
[GCC 4.2.1 Compatible Apple LLVM 6.1.0 (clang-602.0.53)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import math
>>> math.log(math.sqrt((1+0.0015)/(1-0.0015))) - math.tan(0.0015)
6.060690144193970e-16
>>> math.log(math.sqrt((1+0.0015)/(1-0.0015)))
0.0015000011250016186
>>> math.tan(0.0015)
0.0015000011250010125
>>>
>>>
```

Pitfalls of Blind Computation

- Accurate estimation on WolframAlpha:

WolframAlpha computational intelligence

ln(sqrt((1+0.0015)/(1-0.0015))) - tan(0.0015)

Extended Keyboard Upload Examples Random

Assuming trigonometric arguments in radians | Use degrees instead

Input:

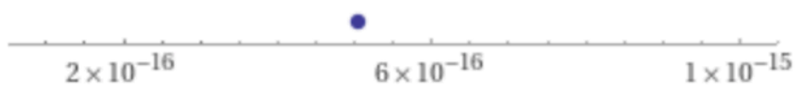
$$\log\left(\sqrt{\frac{1+0.0015}{1-0.0015}}\right) - \tan(0.0015)$$

$\log(x)$ is the natural logarithm

Result: Fewer digits More digits

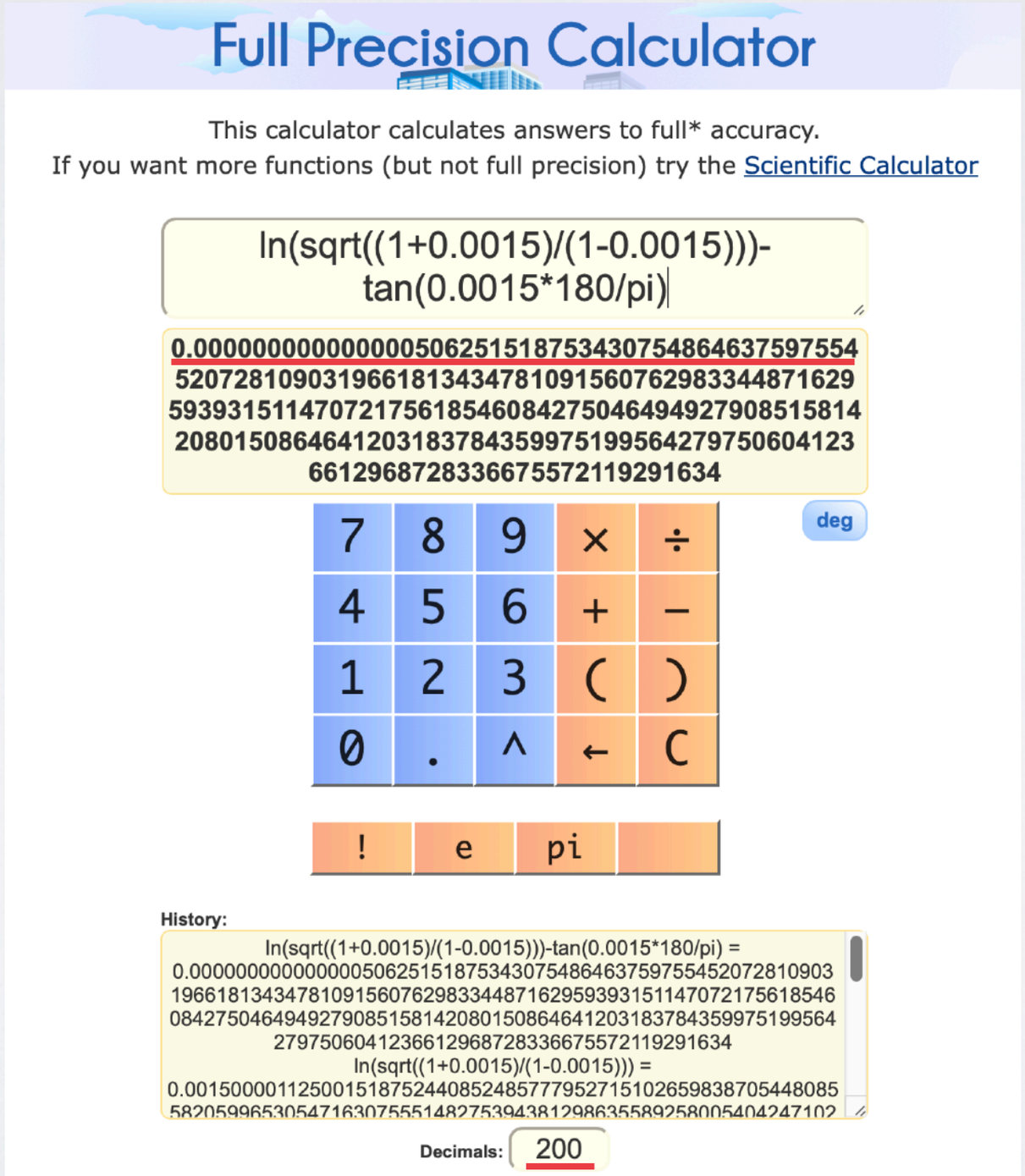
$$5.062515187534307548646375975545207281090319661813434781091 5\ldots \times 10^{-16}$$

Number line:

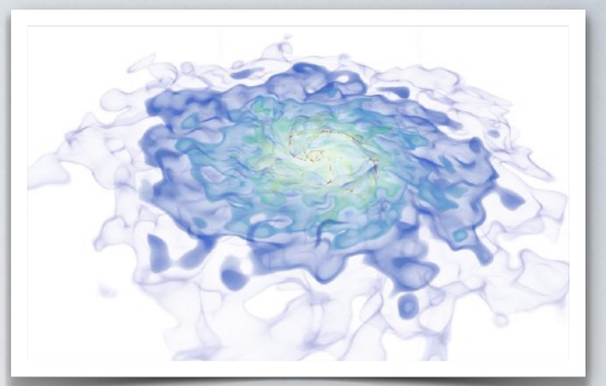


Pitfalls of Blind Computation

- Accurate estimation on Full Precision Calculator:



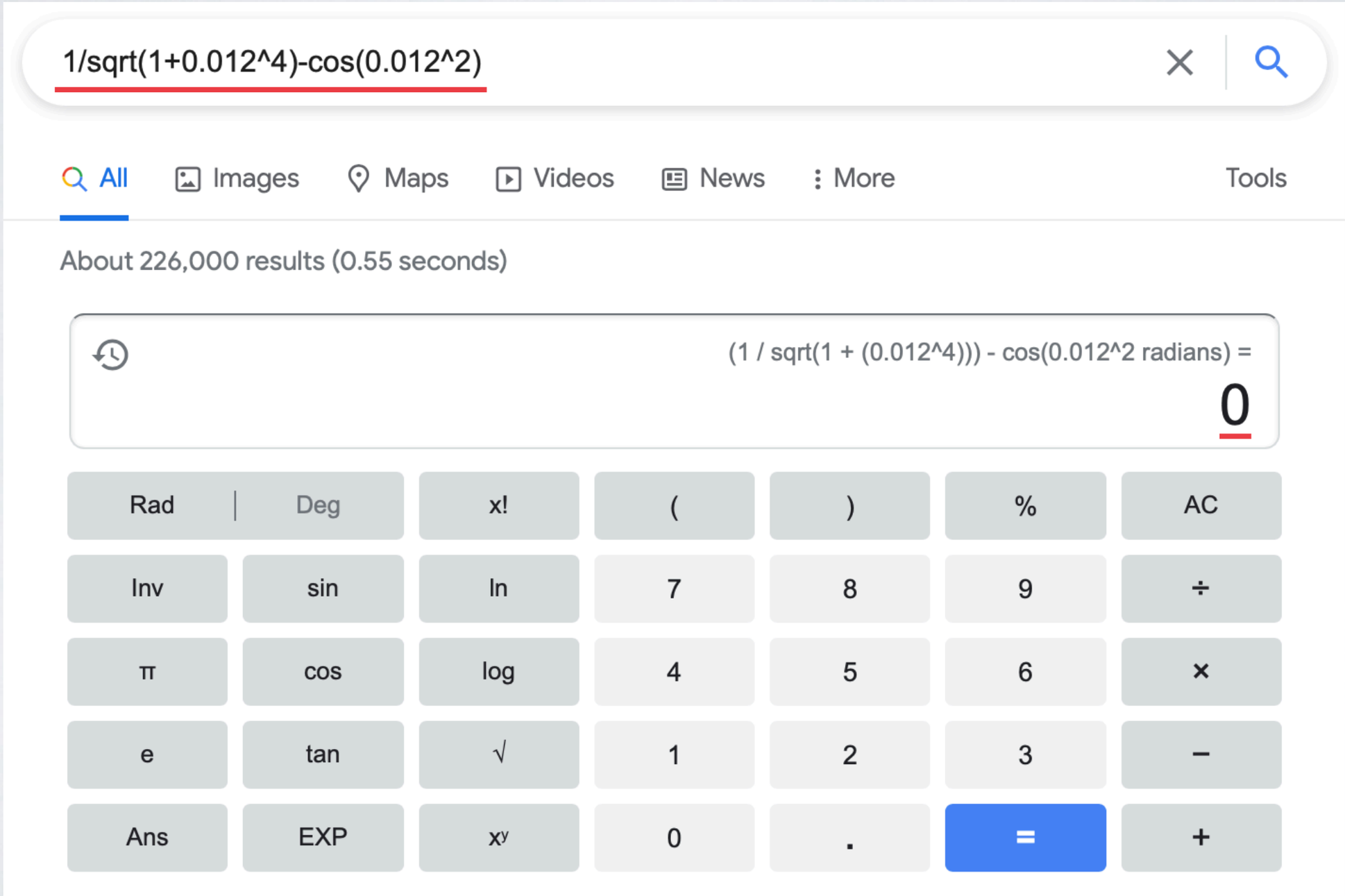
www.mathsisfun.com/calculator-precision.html



Pitfalls of Blind Computation (Another Example: Problem 15.2)

Pitfalls of Blind Computation

- Inaccurate estimation on Google:



Pitfalls of Blind Computation

- Inaccurate estimation on Python:

```
bash-3.2$ python
Python 2.7.9 (default, Sep  9 2015, 22:07:15)
[GCC 4.2.1 Compatible Apple LLVM 6.1.0 (clang-602.0.53)] on darwin
Type "help", "copyright", "credits" or "license" for more information.
>>> import math
>>> 1/math.sqrt(1+0.012**4) - math.cos(0.012**2)
1.1102230246251565e-16
>>>
```

Pitfalls of Blind Computation

- Accurate estimation on WolframAlpha: