Mathematical Physics I (Fall 2025): Homework #1

Due Sep. 19, 2025 (Fri, 23:00pm)

[0.5 pt each, total 5 pts, turn in as a single pdf file to eTL]

• By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code!)

• Textbook problem numbering convention (Boas, 3rd ed.): [Section].[Problem No.]. Many problems in this assignment are from the list of suggested problems announced in the class.

• Exhibit all intermediate steps to receive full credits. Only handwritten answers are accepted except for numerical problems or verifications — for which you print out and turn in not just the end results (e.g., plots) but also the source codes in your favorite tools such as MATLAB or MATHEMATICA.

1.-3. Boas Chapter 1, Problems 10.21, 13.36, 16.1

(Note: For Problem 13.36, read the instruction in the textbook carefully; among others, you have been asked to compare your result with a computer solution as well. For Problem 16.1, it is helpful to draw a diagram that considers the balance between forces and torques, a so-called *extended free-body diagram.*)

4.-7. Boas Chapter 2, Problems 10.23, 14.24, 15.6, 17.21

(Note: For Problem 14.24, compare your result with a computer solution. Resolve or explain any disagreement in the two results. For Problem 15.6, read the instruction in the textbook carefully; that is, you have been asked to compare your result with a computer solution as well. Resolve or explain any disagreement in the two results. For Problem 17.21, verify only the first equality, $\cosh^{-1} z = \ln(z \pm \sqrt{z^2 - 1})$.)

8. Boas Chapter 3, Problem 2.8

(Note: For Problem 2.8, again, read the instruction in the textbook carefully.)

9. In the quantum statistical distribution of bosons with unspecified total number of particles (so-called Bose-Einstein statistics of quantized oscillators), the average energy of the system is found to be

$$\bar{\epsilon} = \frac{\sum_{n=1}^{\infty} n\epsilon_0 e^{-\frac{n\epsilon_0}{k_{\mathrm{B}}T}}}{\sum_{n=0}^{\infty} e^{-\frac{n\epsilon_0}{k_{\mathrm{B}}T}}}, \tag{1}$$

where ϵ_0 is a fixed energy, k_B is the Boltzmann constant, and T is the temperature. (a) After briefly discussing how this formula is acquired, show that the ratio becomes

$$\bar{\epsilon} = \frac{\epsilon_0}{e^{\frac{\epsilon_0}{k_{\mathrm{B}}T}} - 1}$$

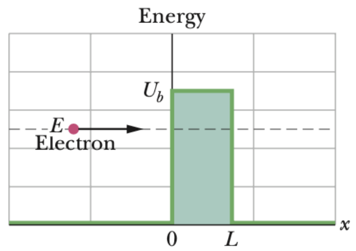
by first identifying Eq.(1)'s denominator as a binomial expansion of $\frac{1}{1-x}$ with $x = e^{-\frac{\epsilon_0}{k_{\mathrm{B}}T}}$, and its numerator as a constant times $\frac{x}{(1-x)^2}$. (b) Using a power series expansion, show that $\bar{\epsilon}$ reduces to $k_{\mathrm{B}}T$ in the classical limit of $k_{\mathrm{B}}T \gg \epsilon_0$.

(Note: You may want to briefly review the classic textbooks in statistical mechanics such as Schroeder or Reif. To receive full credit, you are asked to come up with a short paragraph about what Eq.(1) means, without having to laboriously derive the equation in great detail. If needed, you must reference your sources appropriately with a proper citation convention, but your answer must still be your own work in your own words. To access the electronic resources — e.g., academic journals — off-campus via SNU library's proxy service, see http://library.snu.ac.kr/using/proxy.)

10. In quantum mechanics, consider the transmission coefficient T of an electron matter wave (of mass m and energy E) incident on a rectangular potential barrier of height U_b (see figure). Briefly explain why the following expression must be evaluated to determine the probability of the electron being transmitted through the barrier and detected on the other side:

$$T^{-1} = \left| \frac{(k+i\kappa)^2 - (k-i\kappa)^2 e^{-2\kappa L}}{4ik\kappa e^{-\kappa L} e^{-ikL}} \right|^2 = 1 + \left(\frac{k^2+\kappa^2}{2k\kappa} \right)^2 \sinh^2(\kappa L), \tag{2}$$

where $k \equiv \frac{\sqrt{2mE}}{\hbar}$ and $\kappa \equiv \frac{\sqrt{2m(U_b-E)}}{\hbar}$ are real numbers. Prove the second equality using complex algebra.



[Adapted from Halliday & Resnick, Chapter 38.9]

(Note: You may want to review the freshman physics textbooks such as Halliday & Resnick (Chapters 38.8 and 38.9), or the quantum mechanics textbooks such as Griffiths & Schroeter (Chapter 2.6 and problems therein) or Landau & Lifshitz. To receive full credit, you should include a brief explanation — without a detailed derivation — of how the absolute square of a complex number like Eq.(2) arises in calculating the transmission probability.)