Mathematical Physics I (Fall 2021): Homework #3

Due Oct. 15, 2021 (Fri, 23:00pm)

[0.5 pt each, total 5 pts, turn in as a single pdf file to eTL]

• By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code!)

• Textbook problem numbering convention (Boas, 3rd ed.): [Section].[Problem No.]. Many problems in this assignment are from the list of suggested problems announced in the class.

• Exhibit all intermediate steps to receive full credits. Only handwritten answers are accepted except for numerical problems or verifications — for which you print out and turn in not just the end results (e.g., plots) but also the source codes in your favorite tools such as Matlab or Mathematica.

1.-3. Boas Chapter 4, Problems 7.28, 8.8, 10.10

(Note: For Problem 7.28, you may want to prove and utilize the findings in Problem 7.27. Note the meaning of the subscripts next to the partial derivatives from Boas Chapter 4, Section 1 in case you did not read it. Prove further that the resulting formula is the familiar $c_p - c_v = nR$ found in e.g., Schroeder or Halliday & Resnick, where n is the number of moles of gas present and R is the gas constant. For Problem 8.8, tackle the stated problem in two ways, one with the method in Section 8 and the other with the Lagrange multiplier described in Section 9.)

4.-5. Boas Chapter 5, Problems 5.5, 6.27

(Note: For Problem 6.27, you will first have to show that the intervals of integration for u and v are $[0, 1]$ and $[0, 1 + u]$, respectively. See the hint in Problem 4.20 for more information.)

6.-8. Boas Chapter 6, Problems 3.12, 6.16, 9.8

(Note: For Problem 9.8, you will need to prove and utilize the findings in Problem 9.6.)

9. The ground state energy of a quantum particle of mass m in a *right circular cylinder* of

radius R and height H is given by

$$E = \frac{\hbar^2}{2m}\left(\frac{k_{10}^2}{R^2} + \frac{\pi^2}{H^2}\right), \tag{1}$$

where k_{mn} is the mth positive zero of the Bessel function $J_n(x)$ (k_{mn} defined right before Eq.(5.17) in Boas Chapter 13; their numerical values such as $k_{10} = 2.4048$ can be found in e.g., http://mathworld.wolfram.com/BesselFunctionZeros.html). After briefly reviewing what each term of Eq.(1) means, find the ratio of R to H that minimizes the energy for a fixed cylinder volume.

(Note: The meaning of the second term in Eq.(1) should be obvious from the *particle in a box* problem in rudimentary quantum mechanics (i.e., quantum particle in an infinite potential well; see e.g., Halliday & Resnick). For the first term, you may want to briefly look through the textbooks in quantum mechanics, or the description of its classical analog in Boas Chapter 13.6. Once again, you are simply asked to come up with 2-3 sentences about what Eq.(1) means, without laboriously deriving the equation in great detail. If needed, you must reference your sources appropriately with a proper citation convention, but your answer must still be your own work in your own words. To access the electronic resources — e.g., academic journals — off-campus via SNU library's proxy service, see http://library.snu.ac.kr/using/proxy.)

10. (a) Review Boas Chapter 6, Example 7.2, thus prove Eq.(7.6) or Eq.(f) in p.339.

(b) Review how Gauss's law $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ is acquired from Coulomb's law in Boas Chapter 6, Section 10, where $\mathbf{E}$ is the electric field, ρ is the volume charge density, and ϵ_0 is the permittivity of free space.

(c) Prove that the energy of a continuous charge distribution is given by

$$\frac{1}{2}\int \rho V d\tau = \frac{\epsilon_0}{2}\int |\mathbf{E}|^2 d\tau$$

for integration over all space, where V is the electrostatic potential. You are first asked to briefly discuss how one came up with the term $\frac{1}{2}\int \rho V d\tau$. To prove the equality, you may assume that V vanishes at large distance r at least as fast as r^{-1}.

(Note: The exercises here should sound familiar to most of you as you have begun to study and explore electromagnetism. If not, you may want to briefly review the classic textbooks in electromagnetism such as Griffiths or Jackson.)