Classical Mechanics II (Fall 2019): Homework #1

Due Sep. 25, 2019

[0.5 pt each, total 6 pts, turn in your homework *in the class* before the class starts]

• By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code!)

• Only handwritten answers are accepted except for numerical problems – for which you print out and turn in not just the end results (e.g., plots) but also the source codes.

• For some problems you may want to use formulae in Appendices D and E, and/or more extensive references such as Zwillinger.

1.-7. Thornton & Marion, Problems 9-9, 9-15, 9-24, 9-35, 9-50, 9-64, 10-6

(Note: For Problem 9-15, discuss the difference between its set-up and the one in Problem 9-21. Once you have the equation of motion, you may find it useful to assume that $\dot{x}^2$ can be written as $\dot{x}^2 = \sum_n a_n x^n$. This is a so-called power series solution — similar in philosophy to the Frobenius' power series method covered in e.g., Chapter 7.5 of Arfken, Weber & Harris, 7th ed., 2013. For Problem 9-24, note that the force on m is always perpendicular to its velocity. For Problem 9-64, you will very likely need to use your favorite numerical tool such as MATLAB or MATHEMATICA to solve the equation of motion. For Problem 9-64(b), use Eq.(2.21) with parameters given in Problem 9-63(b). For Problem 9-64(c), prove and use $g(y) = \frac{9.8}{1+(y/R_E)^2}\ \mathrm{m\,s^{-2}}$ where y is the altitude above Earth and R_E is Earth's radius. For Problem 10-6, consider the fact that the surface of the water will be an equipotential once the water has settled in equilibrium.)

8. Work out Example 9.2. In particular, prove explicitly the statements made in the last paragraph (the bottom of p.335) about the continuity — or discontinuity — of the tension on either side of the bottom bend, for both free fall and energy-conserving cases.

9. A water droplet falls in the atmosphere through a mist cloud. Assume that as the droplet passes through the cloud, it remains spherical and acquires mass at a rate proportional to its cross-sectional area πr^2 multiplied by the speed of the fall.

(a) Find the differential equation of motion of the droplet, assuming it starts with an infinitely small size ($r_0 = 0$).

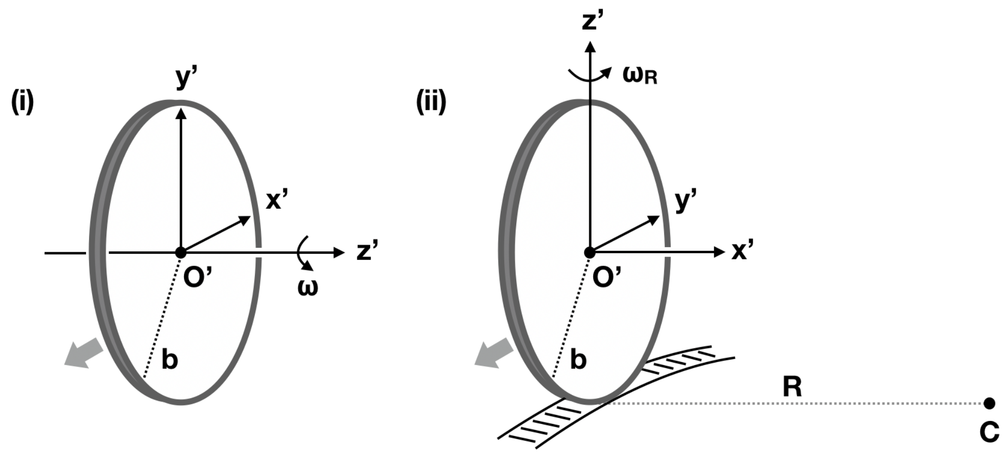
(b) Show that if the droplet starts from rest, the acceleration of the fall is equal to $g/7$. You may find it useful to utilize the power series solution mentioned above for Problem 9-15.

10. A car with wheels of radius b travels in the following ways. Find the acceleration, relative to the ground, of the highest point on one of its wheels. Depending on the problem setup, you may consider different types of rotating coordinate systems illustrated below.

(a) With constant forward velocity $\mathbf{v}_0$.

(b) With constant forward acceleration $\mathbf{a}_0$ and instantaneous velocity $\mathbf{v}$.

(c) With constant forward speed v_0 around a circular track of radius R.

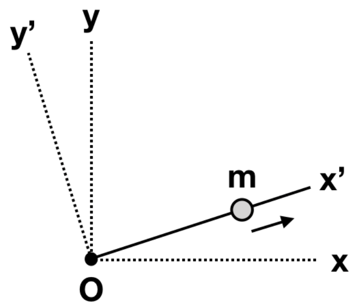


11. A rod of length b rotates with a constant angular speed ω about the z-axis through one end of the rod (point O) and perpendicular to the plane of rotation. A small bead of mass m, with a hole through it, is threaded on this frictionless rod.

(a) Write down the equation of motion in the $x' - y'$ frame rotating with the rod. Express the force that the rod exerts on the bead.

(b) The bead is now placed at O then pushed down the rod with an initial speed of $b\omega$ with respect to the rod. Calculate the time and velocity when the bead leaves the rod. In your answer you may choose to leave the trigonometric or hyperbolic functional form, or the inverse function thereof — e.g., $\sin(\square)$, $\sin^{-1}(\square)$, $\sinh(\square)$, $\sinh^{-1}(\square)$.

(c) The bead is now placed at the midpoint of the rod, and released from rest with respect to the rod. Calculate the time and velocity when the bead leaves the rod.



12. In the class we discussed the kinematics of elastic collisions. Starting from the initial energy of the system in the LAB and CM systems, Eqs.(9.78) and (9.79), follow step by step the logical procedure that leads to Eq.(9.87a), the LAB energy of the particle m_1 written with the CM scattering angle θ. Continue to derive Eqs.(9.88), (9.90), (9.91) and (9.92) — which were briefly discussed in the class but left for your exercise. An ambitious student seeking an additional +0.5 point may venture to derive Eq.(9.87b).