Classical Mechanics II (Fall 2020): Homework #4

Due Dec. 3, 2020

[0.5 pt each, total 6 pts, turn in as a single pdf file to eTL before the class starts]

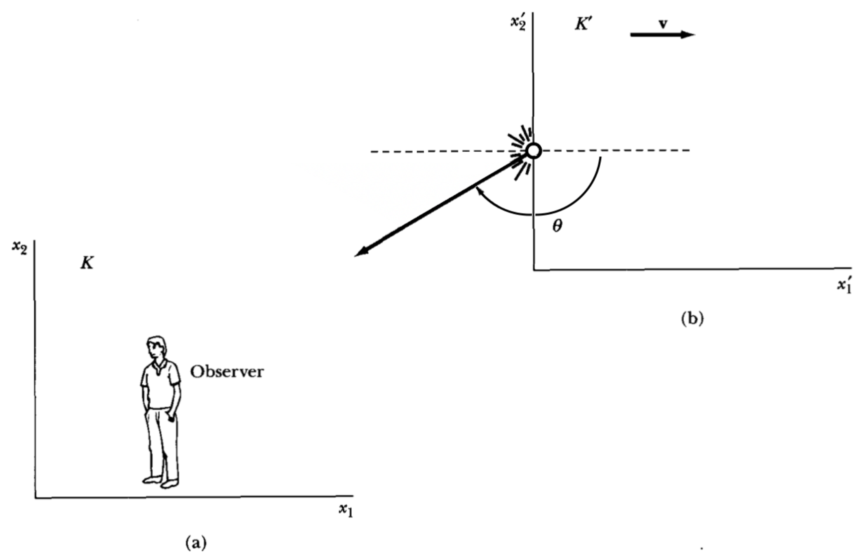
• By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code!)

• Only handwritten answers are accepted except for numerical problems – for which you print out and turn in not just the end results (e.g., plots) but also the source codes.

• For some problems you may want to use formulae in Appendices D and E, and/or more extensive references such as Zwillinger.

1.-8. Thornton & Marion, Problems 13-9, 13-19, 13-22, 14-6, 14-18, 14-33, 14-37, 14-42

(Note: For Problem 13-9, first review Chapter 13.5 and work out Example 13.2, which was briefly discussed in the class, but left for your exercise. In Problem 14-6, as in other examples and problems, unprimed quantities such as Δx is measured in the unprimed system K. For Problem 14-18, consider the choice of systems below, in which the angle between the light source and the direction of the relative motion is again θ as in Example 14.11.)



Thornton & Marion, Problem 14-18 (compare with Figure 14-8)

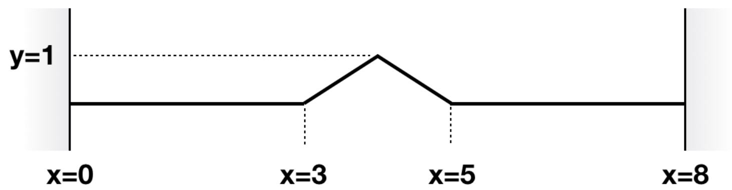
9. Consider a triangular pulse on a finite, dissipationless string discussed in the class. Here we will investigate the general traveling wave solution to the wave equation.

(a) First, let us re-examine the plucked string described in Figure 13-1 and Example 13.1. Using a trigonometric identity $\sin A \cos B = \frac{1}{2}[\sin(A+B) + \sin(A-B)]$, show that Eq.(13.13) can be written in the form of $\frac{1}{2}[f(x+vt) + f(x-vt)]$, or Eq.(13.62b). Find $f(X)$.

(b) By a numerical calculation using e.g., the first 5 nonzero terms of the series to get a moderate approximation, draw the shapes of *(i)* $\frac{1}{2}f(x+vt)$, *(ii)* $\frac{1}{2}f(x-vt)$, and *(iii)* the combined actual string shape. You may plot the position of the string at e.g., 8 equally spaced times from $t = 0$ to $t = T$, where T is the period of the string's motion. You may certainly reuse what you developed for Problem 13-3 in Homework #3.

(c) Now consider a slightly different initial configuration. A string of length $L = 8$ is fixed at both ends, and is initially given a small triangular displacement shown in the figure below. The string is then released from rest at $t = 0$. Describe the vibration of the string in terms of normal modes. Find the Fourier coefficients, Eq.(13.8).

(d) By a numerical calculation using e.g., the first 5 nonzero terms of the series to get a moderate approximation, see how the wave propagates in time. You may plot the position of the string at e.g., 20 equally spaced times from $t = 0$ to $t = T$, where T is the period of the string's motion. What is T? In your first few snapshots, can you reproduce the behavior seen in Figure 13-3?



10. A particle (particle 1) of mass m and relativistic total energy E_1 collides with an identical particle (particle 2) initially at rest. The particles collide to reach a final state containing N particles of mass m each (e.g., a collection of particles and anti-particles, all with mass m). Find the *threshold* energy, or the minimum value of E_1, for which this process can occur. Show that more energy would be needed to produce more particles.

11. A spaceship is initially at rest in the LAB frame. At a given instant (LAB clock $t' = 0$ and the spaceship's clock $t = 0$), it starts to accelerate with the constant *proper* acceleration a along the x_1'- and x_1-axes. Here, the *proper* acceleration is defined as follows: Let t be the time coordinate in the spaceship's frame. If the proper acceleration is a, then at time $t + dt$, the spaceship is moving at speed adt relative to the frame it was in at time t.

(a) Show that the relative speed of the spaceship and the LAB frame at the spaceship's time t is written as $v(t) = c \tanh\left(\frac{at}{c}\right)$. (Note: You may start by setting $v_1 = v(t)$ and $v_2 = adt$ in the velocity addition formula Eq.(14.98) of Thornton & Marion, and expand $v(t + dt)$ to first order in dt.)

(b) Later on, an observer in the LAB frame measures t' and t. Find the relation between t' and t? Check if your formula yields $t' = t$ in the nonrelativistic limit.

(c) Find the four-vector velocities $\mathbb{V}'$ and $\mathbb{V}$ of the spaceship in the LAB frame and the spaceship's frame, respectively. Show that they transform like four-vectors between the two frames.

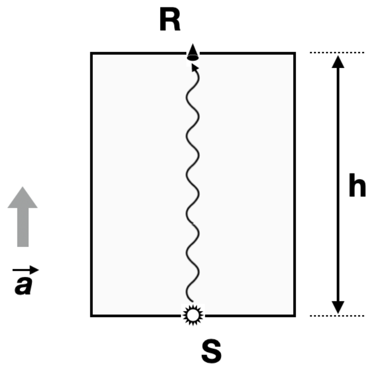
12. To understand the gravitational shift of spectral lines, let us consider an elevator of height h starting to move upward from rest at $t = 0$ with an acceleration $a = g = 9.8\,\mathrm{m/s^2}$ in free space (no gravity) with a photon source S on its floor. A photon of frequency ν_0 is emitted at $t = 0$ and travels upward as seen in the figure below.

(a) An observer R fixed to the elevator's ceiling — thus accelerating with the elevator — receives the photon with Doppler-shifted frequency ν. Show that the frequency difference is given by $\Delta\nu = \nu - \nu_0 \simeq -\frac{gh}{c^2}\nu_0$ when we can assume $\sqrt{gh} \ll c$. (Note: You may want to consider two inertial reference frames — K' moving at the same velocity as the elevator when the photon is emitted, and K moving at the same velocity as the elevator when the photon is received — and the transformation of a four-vector momentum between K' and K.)

(b) Combining the Equivalence Principle (what is this?) and the result of the above thought experiment, explain the *gravitational* redshift (and the *gravitational* time dilation). Notice that, in your gravitational redshift formula, gh is the change in the Newtonian gravitational potential experienced by the photon.

(c) What do you expect if a photon falls from the ceiling rather than moving upward from the floor?

(d) Two people stand a distance h apart. They simultaneously start accelerating in the same direction (along the line connecting the two) with the same *proper* acceleration a. At the instant they start to move, how fast does each person's clock tick when observed by the other person?



13. [For additional +0.5pt] Einstein's Speed of Light Postulate states that the speed of light c is a universal constant independent of any relative motion of the source and the observer, implying that no information can be transmitted faster than c (i.e., *signal velocity* $\leq c$). Write a short essay that includes your thoughts and research about the following questions: *(i)* Can the *phase velocity* be faster than c, and if so, why does it not contradict the Speed of Light Postulate? *(ii)* Can the *group velocity* be faster than c, and if so, why does it not contradict the Postulate? *(iii)* Can the *velocity on a screen* described in p.576 of Thornton & Marion be faster than c, and if so, why does it not contradict the Postulate?

(Note: 2-3 paragraphs or more are expected to clearly demonstrate what you learned from various scientific articles. You must reference your sources appropriately with a proper citation convention, and your answer must be your own work in your own words. Sources like Wikipedia or YouTube are *not* scientific literatures. To access the electronic resources — e.g., academic journals — off-campus via SNU library's proxy service, see http://library.snu.ac.kr/using/proxy. You may start by reviewing p.541-542 and p.576 of Thornton & Marion. Other articles that may be a good starting point for your research include: *(i)* Rothman, M. A., 1960, "Things that go faster than light", Scientific American, 203/1, 142-152, *(ii)* Feinberg, G., 1970, "Particles that go faster than light", Scientific American, 222/2, 68-75. Obviously, your literature search should not stop there.)