Classical Mechanics II (Fall 2020): Homework #1

Due Sep. 29, 2020

[0.5 pt each, total 6 pts, turn in as a single pdf file to eTL before the class starts]

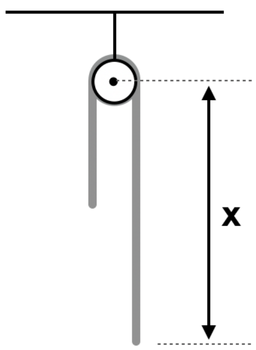
• By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code!)

• Only handwritten answers are accepted except for numerical problems – for which you print out and turn in not just the end results (e.g., plots) but also the source codes.

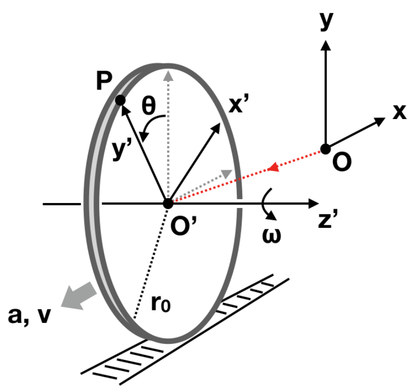
• For some problems you may want to use formulae in Appendices D and E, and/or more extensive references such as Zwillinger.

1.-7. Thornton & Marion, Problems 9-15, 9-20, 9-43, 9-45, 9-62, 9-64, 10-2

(Note: For Problem 9-15, discuss the difference between its set-up and the one in Problem 9-21. Once you have the equation of motion, you may find it useful to assume that $\dot{x}^2$ can be written as $\dot{x}^2 = \sum_n a_n x^n$. This is a so-called power series solution — similar in philosophy to the Frobenius' power series method covered in e.g., Chapter 7.5 of Arfken, Weber & Harris, 7th ed., 2013. For Problem 9-62, note that we do not assume a constant burn rate of the fuel. For Problem 9-64(b), use Eq.(2.21) with parameters given in Problem 9-63(b). For Problem 9-64(c), prove and use $g(y) = \frac{9.8}{(1+y/R_E)^2}\ \mathrm{m\,s^{-2}}$ where y is the altitude above Earth and R_E is Earth's radius. For Problem 10-2, investigate the problem in two ways by using (a) an inertial reference frame $x-y$ centered on the initial position of the tire's center, and (b) a rotating noninertial reference frame $x'-y'$, as illustrated below.)

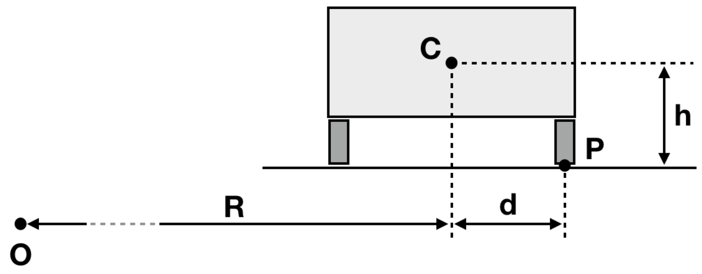


Thornton & Marion, Problem 9-20



Thornton & Marion, Problem 10-2

8. A car of mass m travels with speed v on a horizontal, circular track of radius R. h is the height of the center of mass C above the ground, and $2d$ ($\ll R$) is the separation between the inner and outer wheels. The track is sufficiently rough that the wheels are not skidding. Show that the car will overturn if v is larger than $\sqrt{\frac{gRd}{h}}$. You are asked to consider the problem using (a) an inertial reference frame fixed on the ground and (b) a noninertial reference frame rotating at the same rate as the car.

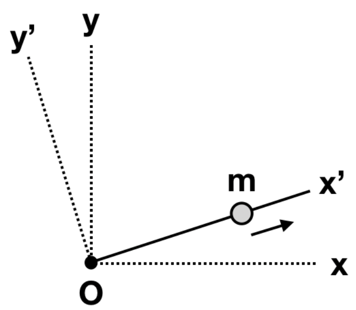


9. A rod of length b rotates with a constant angular speed ω about the z-axis through one end of the rod (point O) and perpendicular to the plane of rotation. A small bead of mass m, with a hole through it, is threaded on this frictionless rod.

(a) Write down the equation of motion of the bead in the $x' - y'$ frame rotating with the rod. Express the force that the rod exerts on the bead.

(b) The bead is now placed at O then pushed down the rod with an initial speed of $b\omega$ with respect to the rod. Calculate the time and velocity when the bead leaves the rod. In your answer you may choose to leave the trigonometric or hyperbolic functional form, or the inverse function thereof — e.g., $\sin(\square)$, $\sin^{-1}(\square)$, $\sinh(\square)$, $\sinh^{-1}(\square)$.

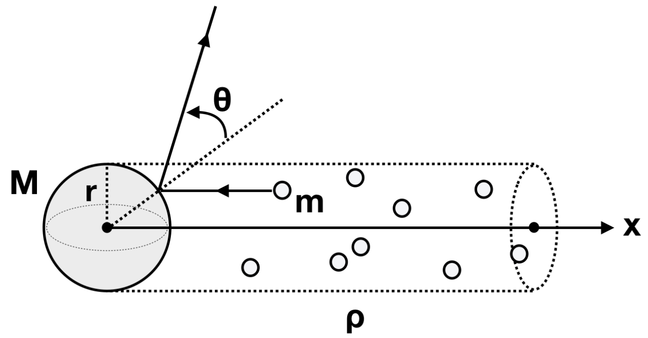
(c) The bead is now placed at the midpoint of the rod, and released from rest with respect to the rod. Calculate the time and velocity when the bead leaves the rod.



10. A spherical satellite of radius r and mass $M(t)$ is moving with velocity $v(t)$ through an atmosphere of uniform density ρ. The atmospheric particles are of identical sizes and masses, all initially at rest. Show that the retarding force on the satellite is written as $F_{\mathrm{r}} = -\rho(\pi r^2)v^2(t)$ in both of the following cases (resembling Eq.(2.21) in Thornton & Marion).

(a) Each atmospheric particle strikes the satellite and adheres to its surface.

(b) Each atmospheric particle strikes the satellite and bounces off from it elastically (see the figure below).



11. Work out Example 9.2. In particular, prove explicitly the statements made in the last paragraph (the bottom of p.335) about the continuity — or discontinuity — of the tension on either side of the bottom bend, for both free fall and energy-conserving cases.

12. In the class we discussed the kinematics of elastic collisions. Starting from the initial energy of the system in the LAB and CM systems, Eqs.(9.78) and (9.79), follow step by step the logical procedure that leads to Eq.(9.87a), the LAB energy of the particle m_1 written with the CM scattering angle θ. Continue to derive Eqs.(9.88), (9.90), (9.91) and (9.92) — which were briefly discussed in the class but left for your exercise. An ambitious student seeking an additional +0.5 point may venture to derive Eq.(9.87b).