Classical Mechanics I (Spring 2019): Homework #2

Due Apr. 18, 2019

[0.5 pt each, total 6 pts, turn in your homework *in the class* before the class starts]

• By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code!)

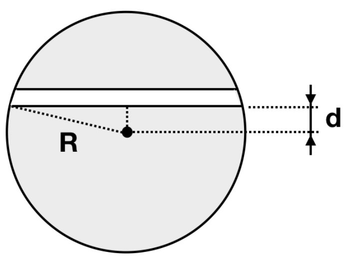
• Only handwritten answers are accepted except for numerical problems – for which you print out and turn in not just the end results (e.g., plots) but also the source codes.

• Do *not* use the Lagrangian method until we cover it in the class.

1.-8. Thornton & Marion, Problems 4-5(a), 4-17, 4-19, 4-20, 4-22, 5-4, 5-7, 5-14

(Note: For Problem 4-5(a), you don't need to code up anything. A calculator or web search engine is more than enough. Start with a hand-drawn "crude" graphs of $y = x^2 + x + 1$ and $y = \tan x$. Plug your first guess from the graph, e.g., $x_1 = 3\pi/8$, into the RHS of $x = \tan^{-1}(x^2 + x + 1)$, and acquire your second guess, x_2. Repeat the procedure until x_n is within 10^{-4} of x_{n-1}. You may benefit from visualizing how each x_n is acquired on your "crude" graph. For Problem 4-22, try several different values for B to draw the Poincaré sections, and guesstimate the transition value(s) that delineate chaotic and periodic behavior. For some problems, you may want to use the integral table in Appendix E.)

9. Thornton & Marion, Problem 5-15. In addition, consider the case where the hole is not through the center of Earth, but obliquely through Earth as in the figure below. Explain how the period of the oscillation changes assuming the particle slides in the frictionless hole.



10. Reproduce the bifurcation diagram of the logistic map, Figure 4-23. Read the relevant chapters to see how the figure is made. Start by using a small number of points, e.g., α going from 2.8 to 4.0 in steps of 0.2. Once you are ready to make the final diagram, you may increase the number of points, e.g., α going from 2.8 to 4.0 in steps of 0.01 or less.

11. Make a zoomed-in version of the bifurcation diagram from Problem 10 above in a small rectangular region of $\alpha \in [3.840, 3.857]$ and $x_n \in [0.44, 0.55]$. You may want to use much finer steps in α, of course. Notice the emerging self-similarity, though the new diagram is upside down at a smaller scale.

12. In the class we discussed the method of Green's function. Starting from a response to a Heaviside step function, Eq.(3.100), follow step by step the logical procedure that eventually leads to Eq.(3.118). In particular, carefully extract Eq.(3.110) by combining Eq.(3.108) and the limiting case assumptions – which was briefly discussed in the class and left for your exercise.