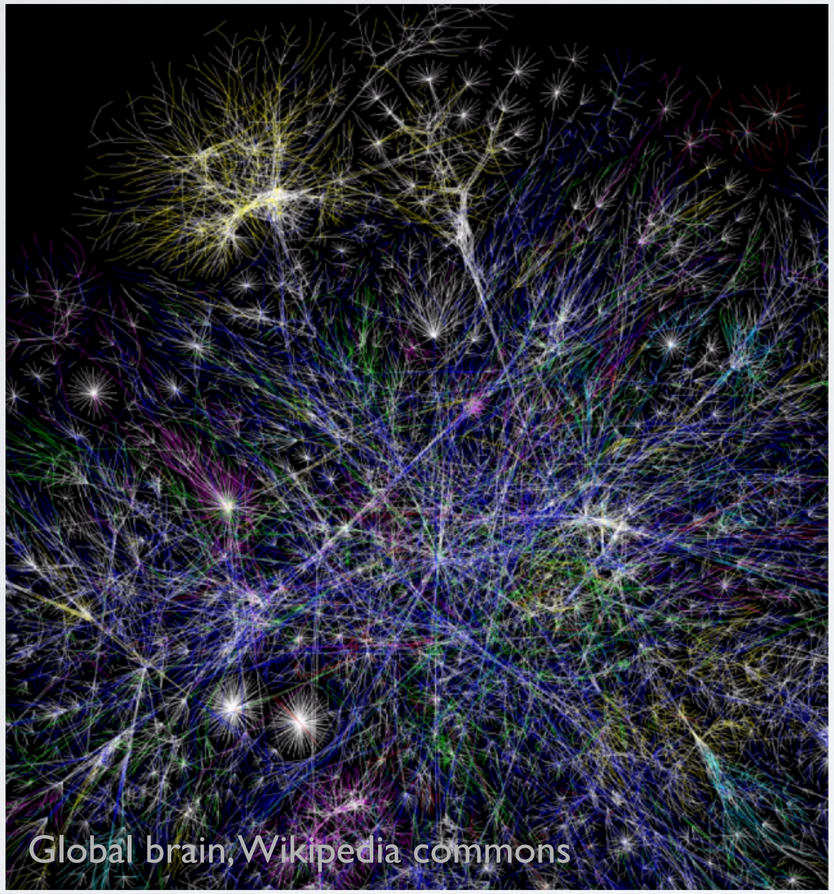
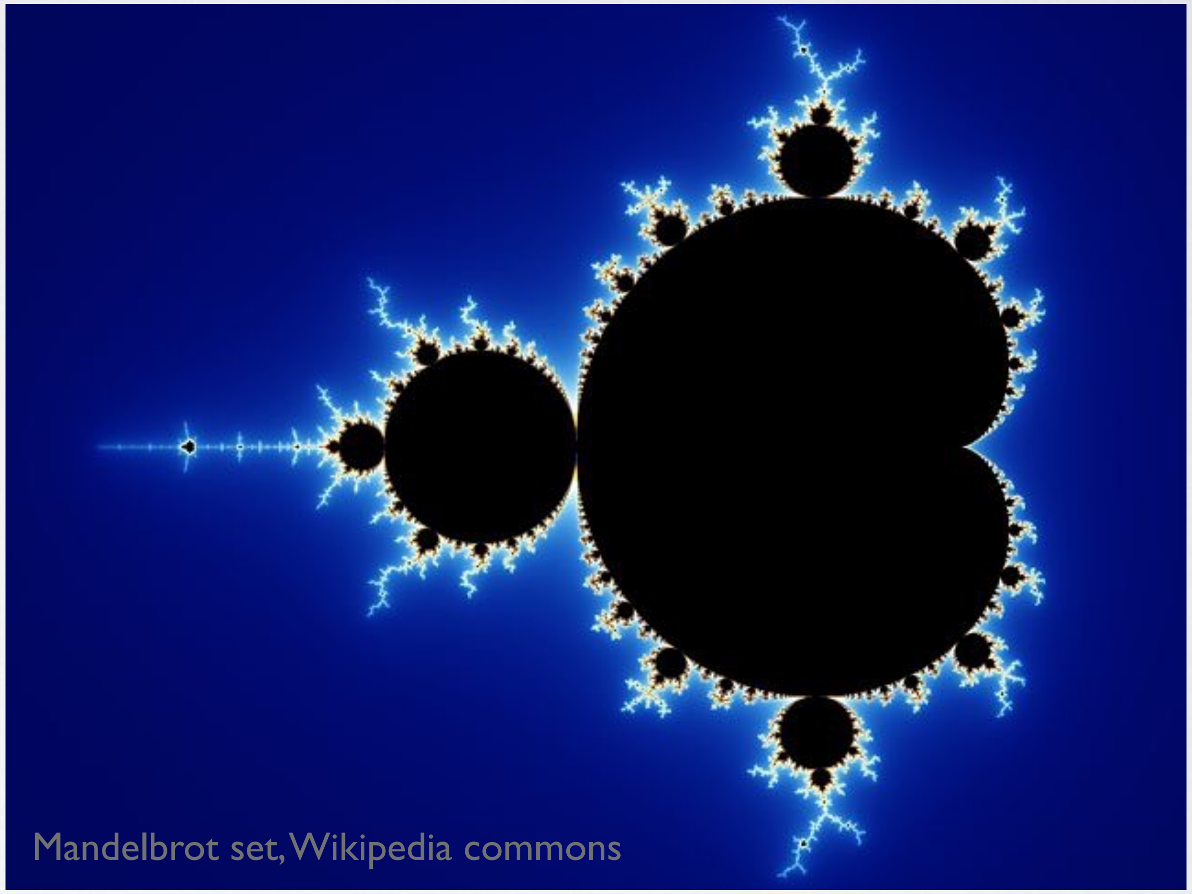
Week 5 - #1

Nonlinear Oscillations and Chaos (III)





Today: Ch 4.7-4.8

Next Class: Ch 5.1-5.4

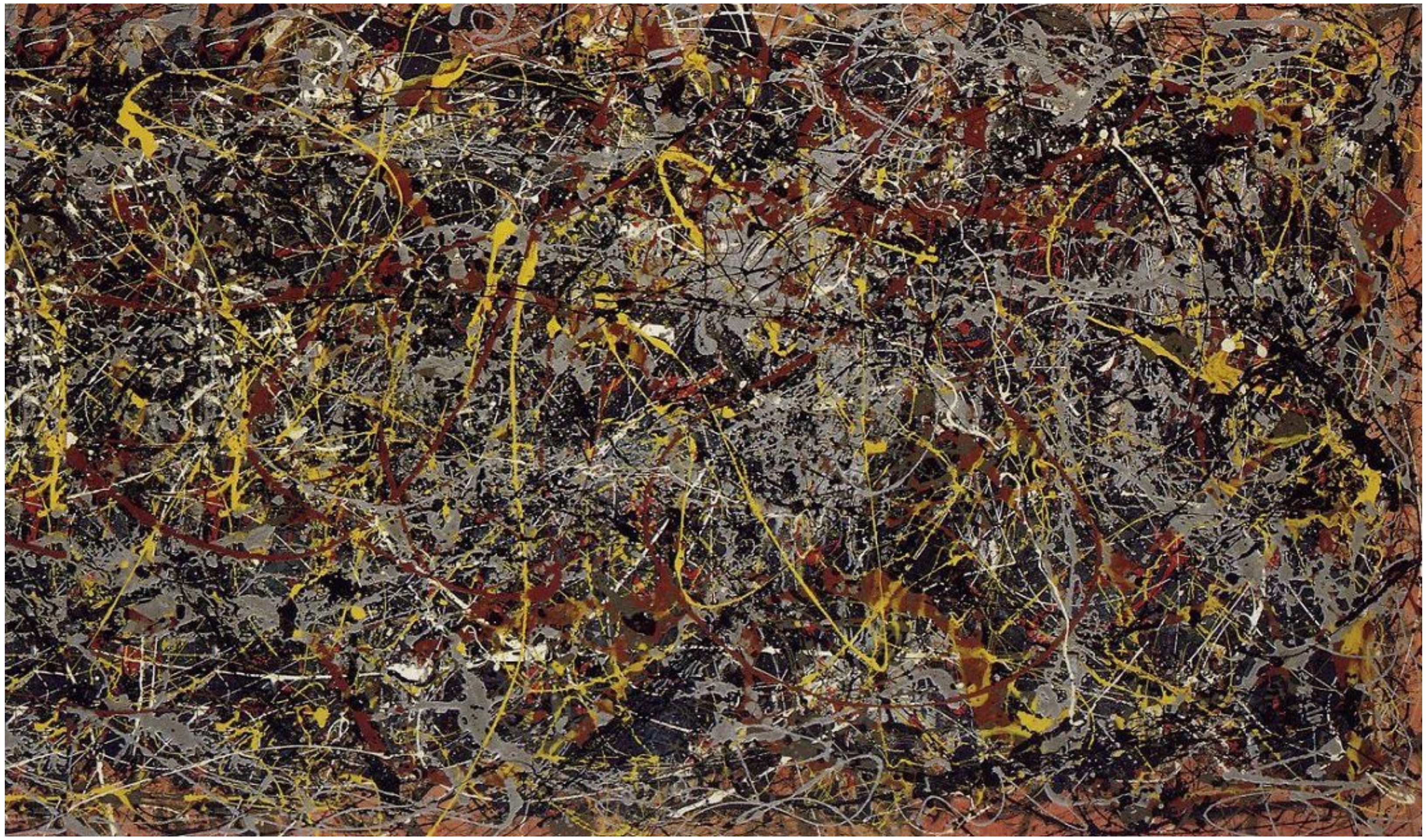
Ji-hoon Kim (Seoul National University)

Classical Mechanics I (Spring 2026): Quiz #8

— [open book and open note, but no cellphone or laptop, drop it off as you leave the class] —

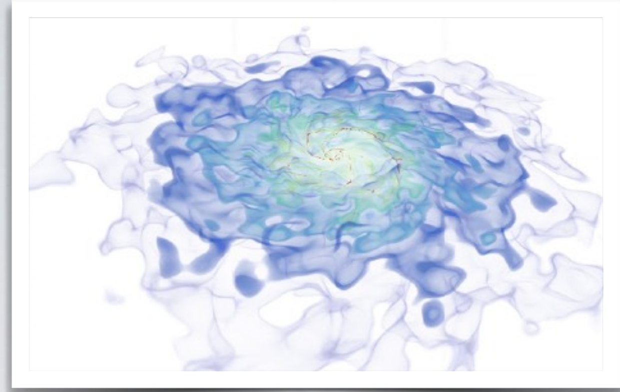
Please write down your name and student ID in the top right corner. (0.0 pt: no paper found with your name / 0.5 pt: paper found with your name and some answers / 1.0 pt: good answers)

1. Thornton & Marion, Problem 4-8.
2. What is the difference between "chaos" and "randomness", as defined in your textbook?



No. 5 (1948; Jackson Pollack, sold at $140 million in 2006)

michael-chen-mg2n.squarespace.com

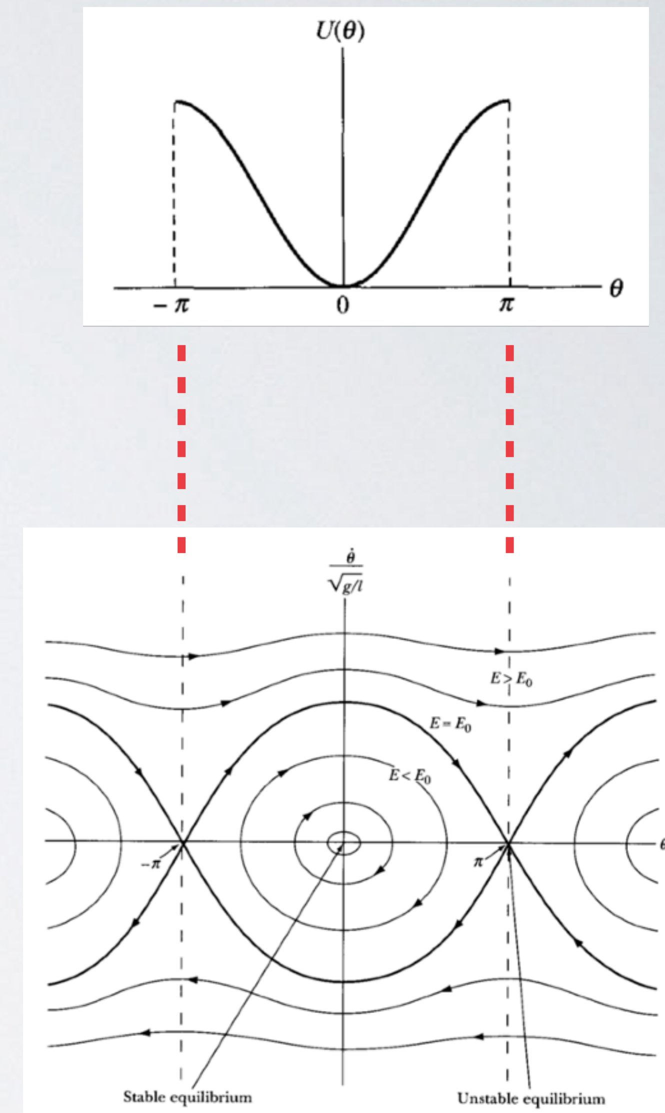


Nonlinear Plane Pendulum: Phase Diagram & Poincaré Section

Damped Drive Plane Pendulum

- Pendulum supported by a massless, extension less rod.

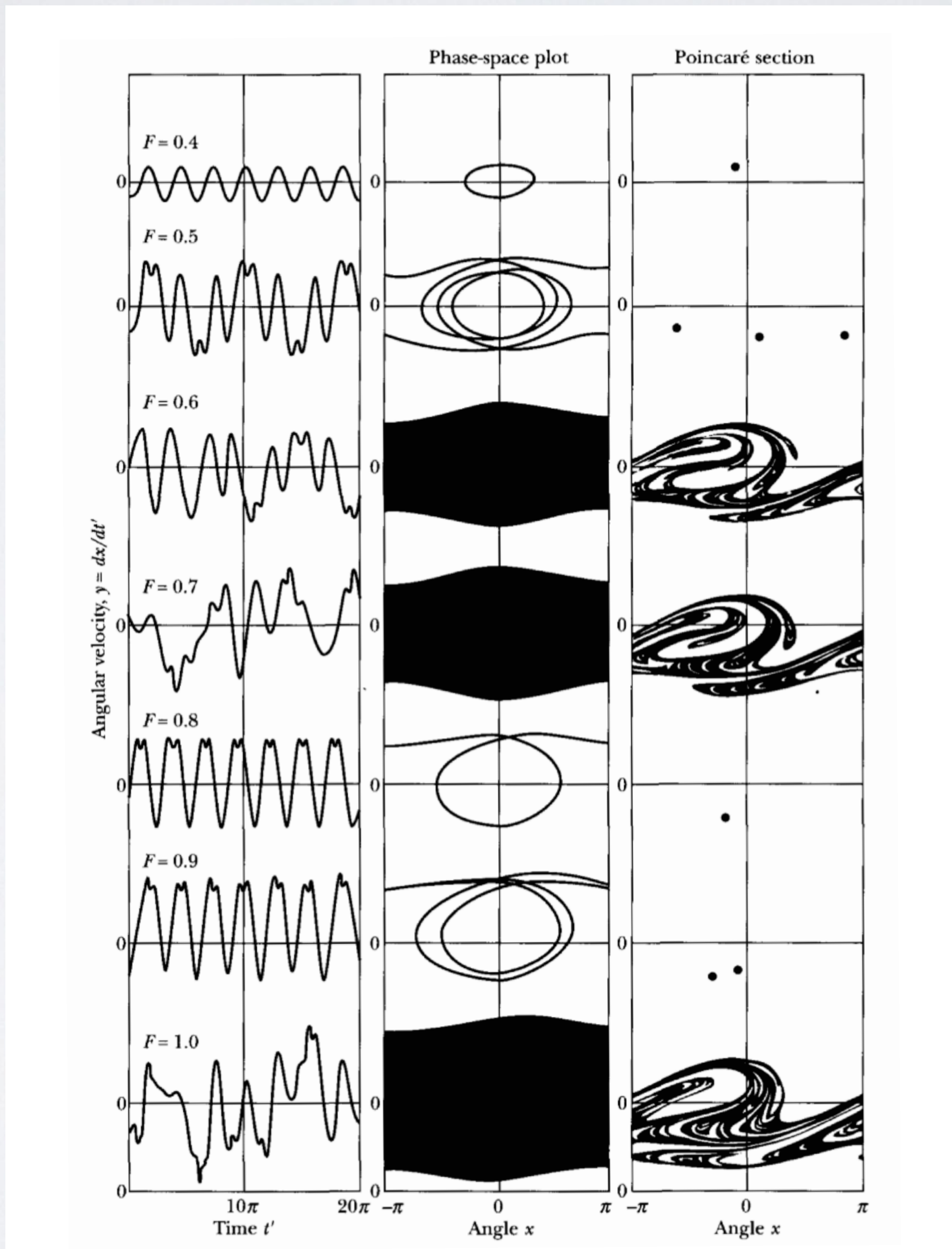
youtube.com/watch?v=TwbUEcVsuwI



Thornton & Marion, Fig 4-10/4-11

Phase Diagram & Poincaré Section

- Damped, driven oscillator



Thornton & Marion Fig. 4-19

- Damped, driven oscillator

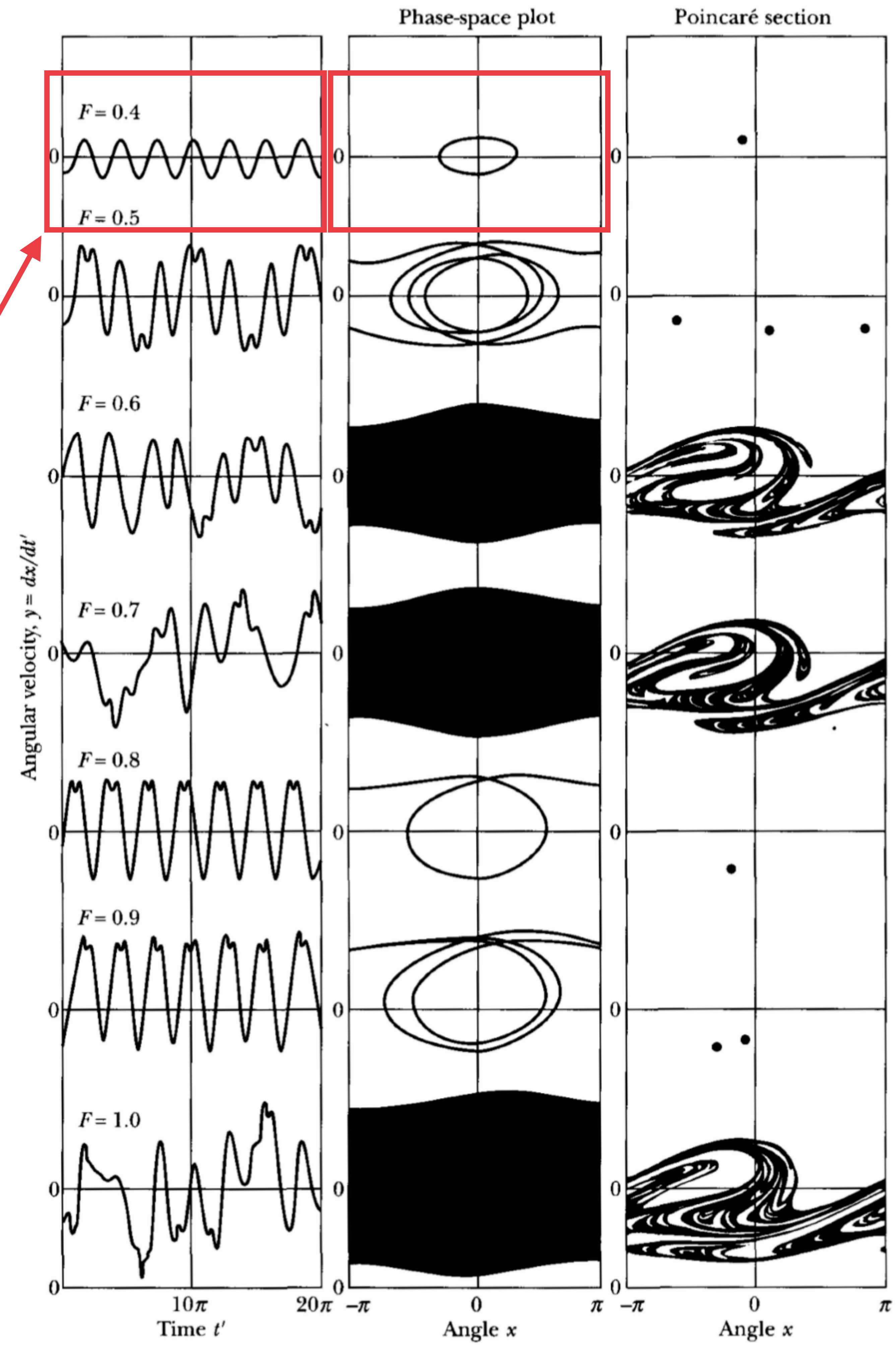
$$\ddot{x} = -c\dot{x} - \sin x + F\cos\omega t'$$

Steady-state solution after the transient effect dies out:

The state x(t) at t≫1 is predictable (insensitive to ICs).

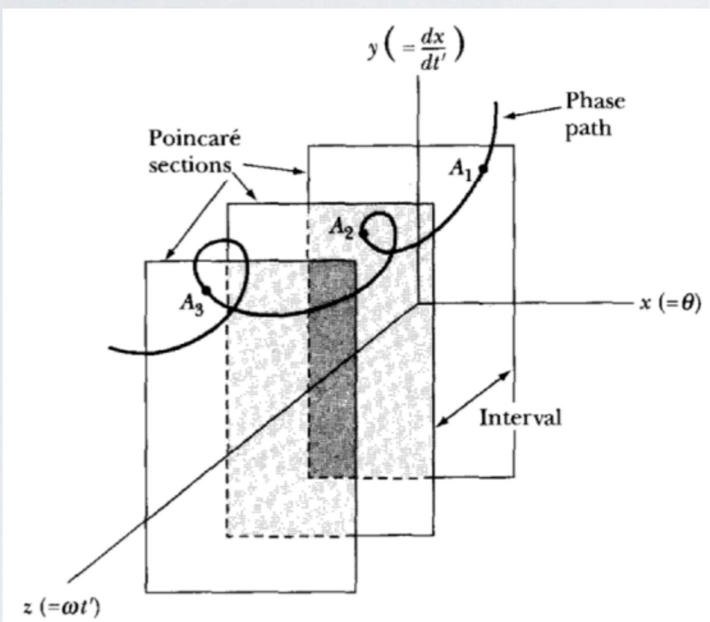
→ "nearly linear" regime

Nonlinearity of the EoM is not a sufficient condition for chaos.

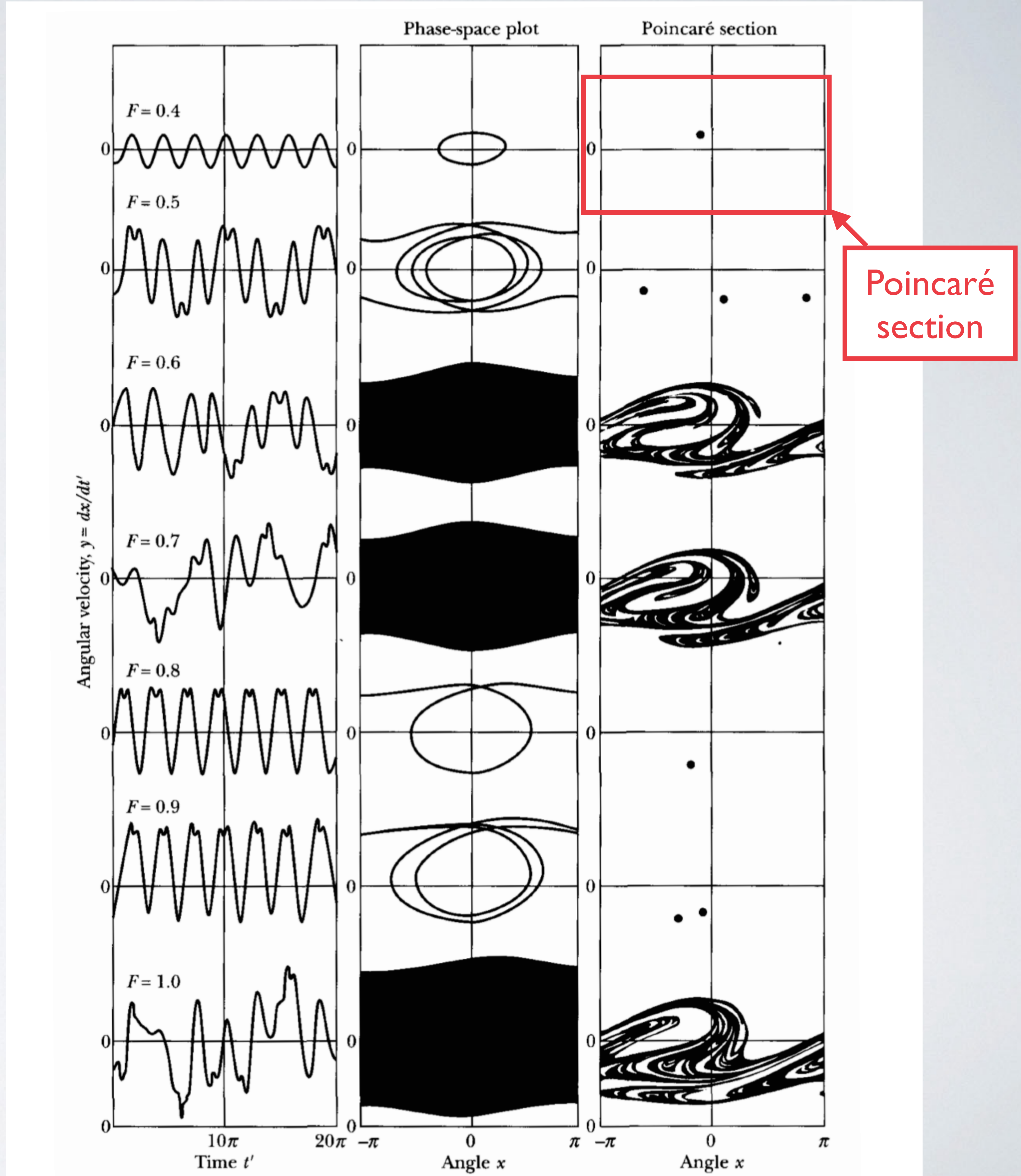


- Damped, driven oscillator

$$\ddot{x} = -c\dot{x} - \sin x + F\cos\omega t'$$



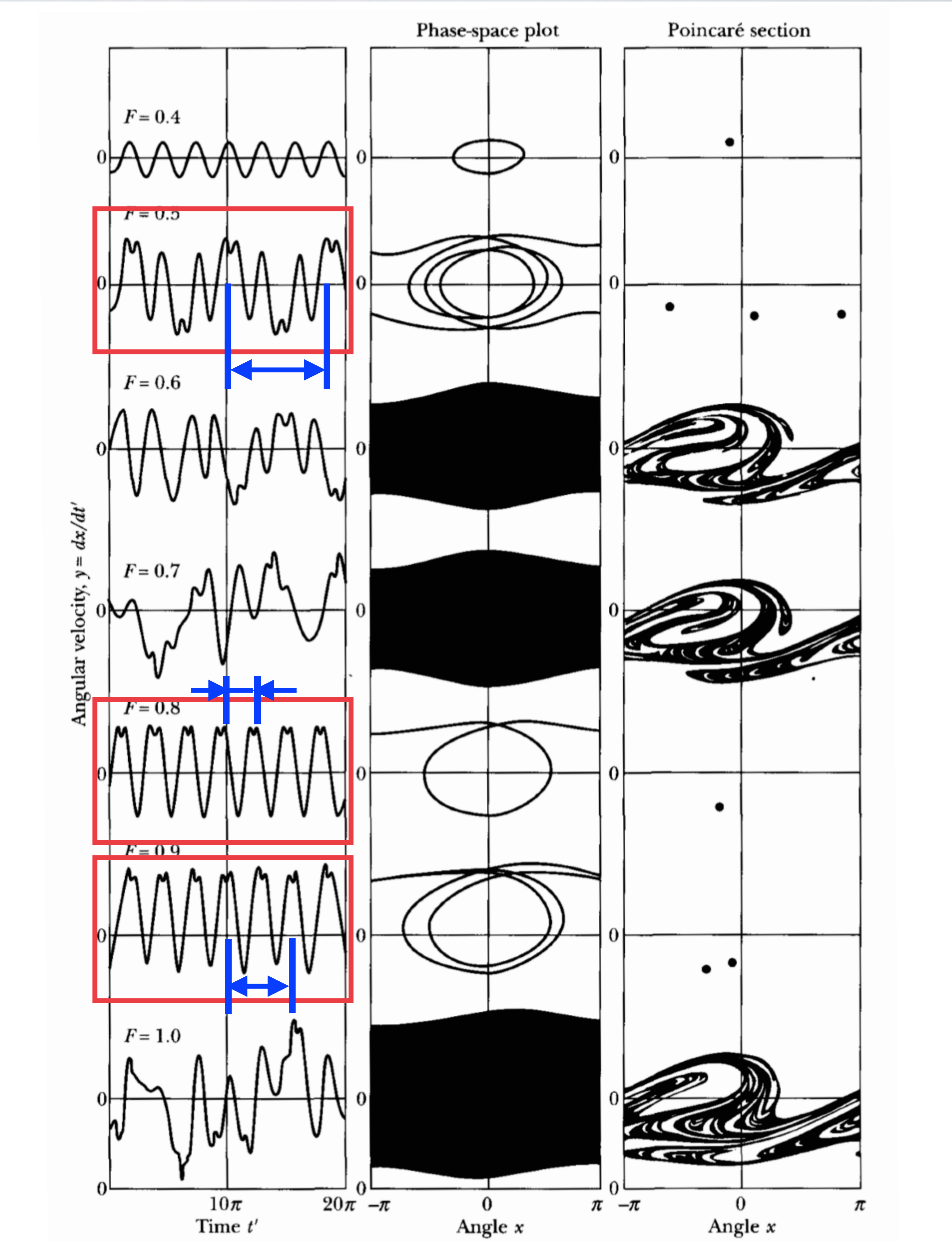
Thornton & Marion Fig. 4-20(a)



Thornton & Marion Fig. 4-19

- Damped, driven oscillator

$$\ddot{x} = -c\dot{x} - \sin x + F \cos \omega t'$$



Thornton & Marion Fig. 4-19

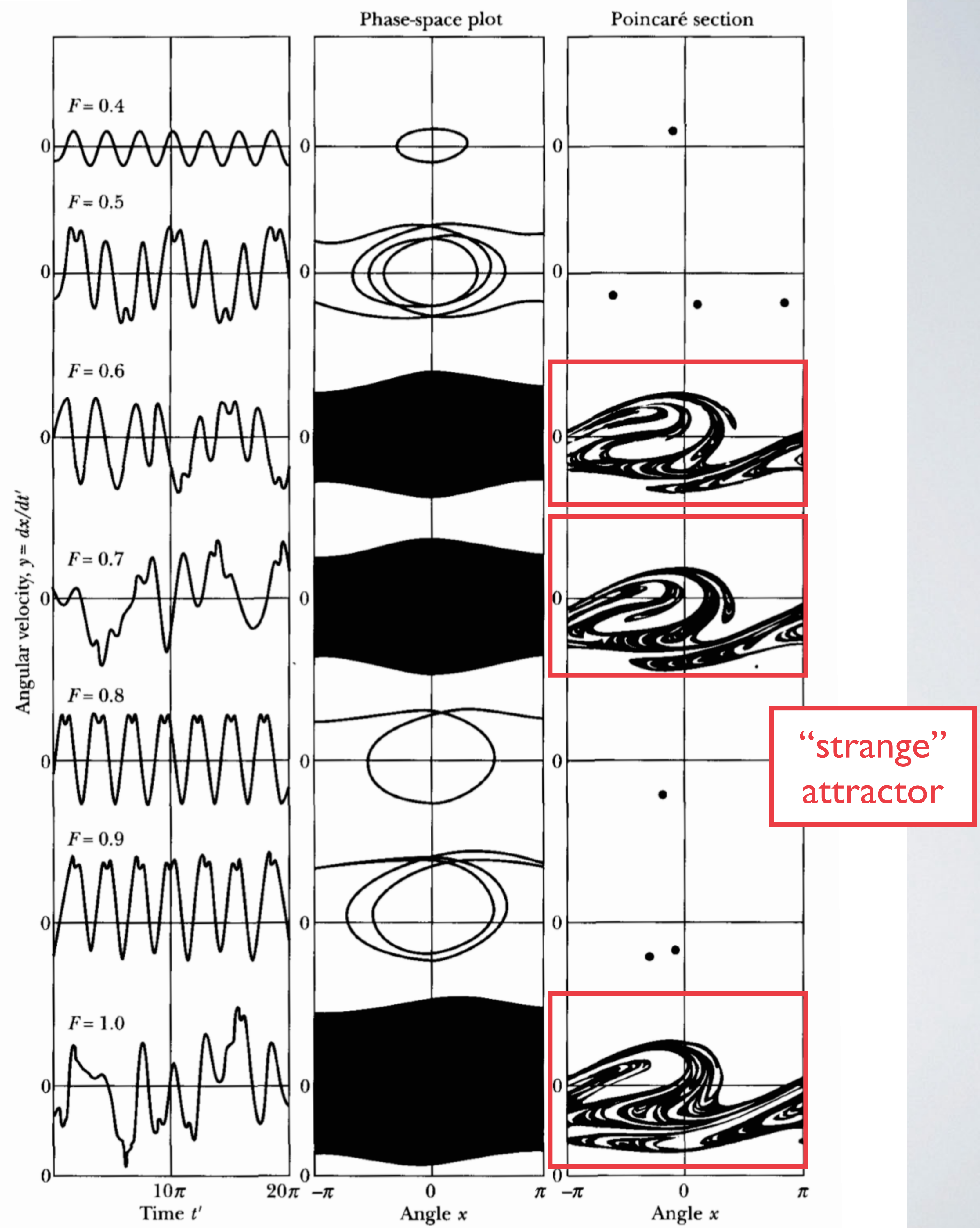
• Damped, driven oscillator

$$\ddot{x} = -c\dot{x} - \sin x + F\cos\omega t'$$

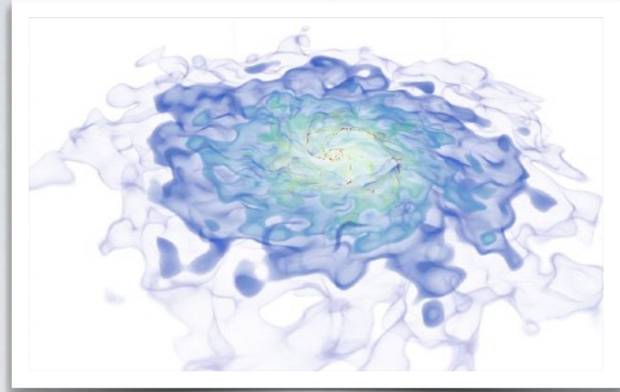
Never converges to a steady-state solution:

The state x(t) at t≫1 is practically unpredictable (very sensitive to ICs).

→ "chaotic" regime



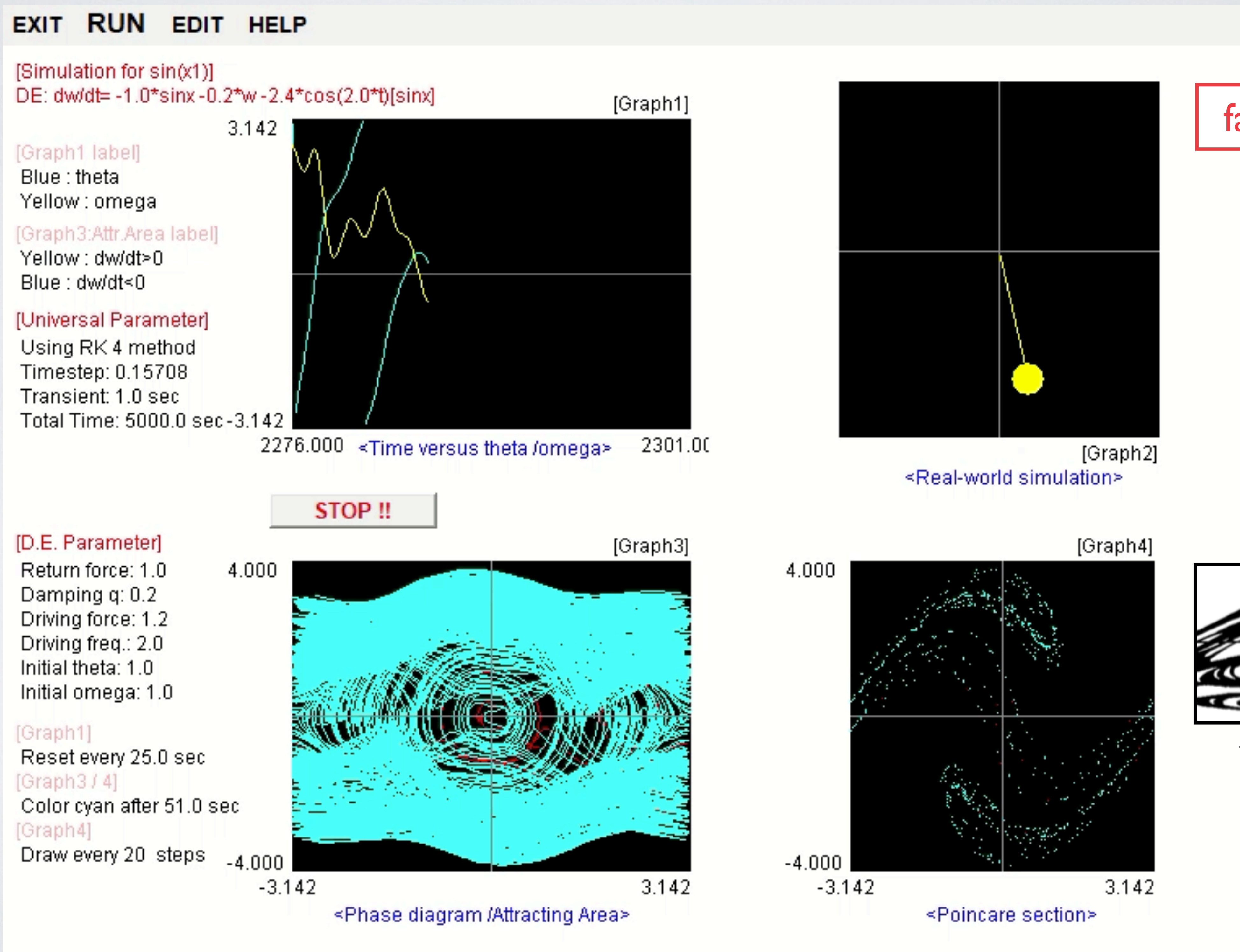
Thornton & Marion Fig. 4-19



Nonlinear Plane Pendulum: Simulator

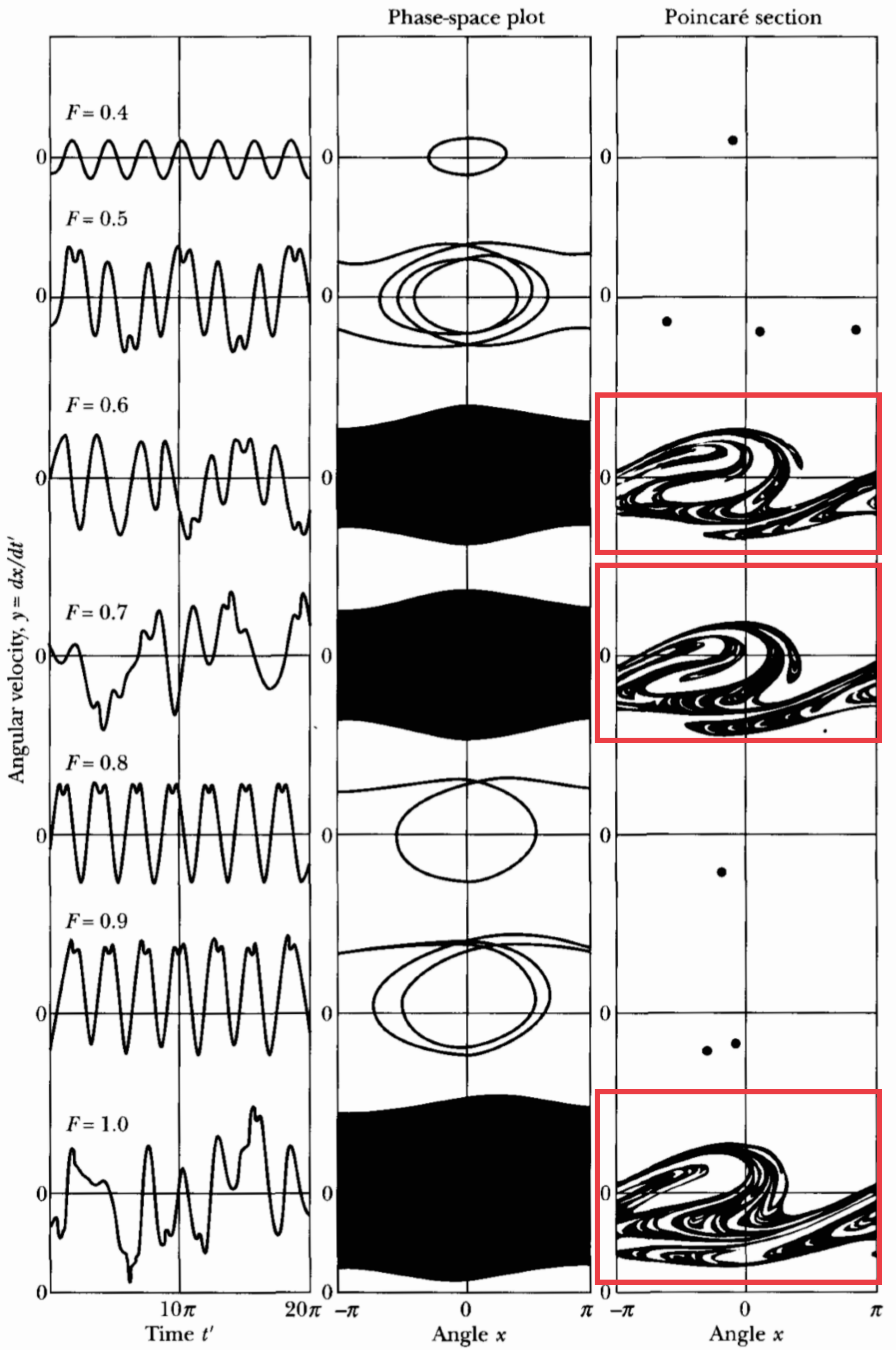
Pendulum: Numerical Simulation

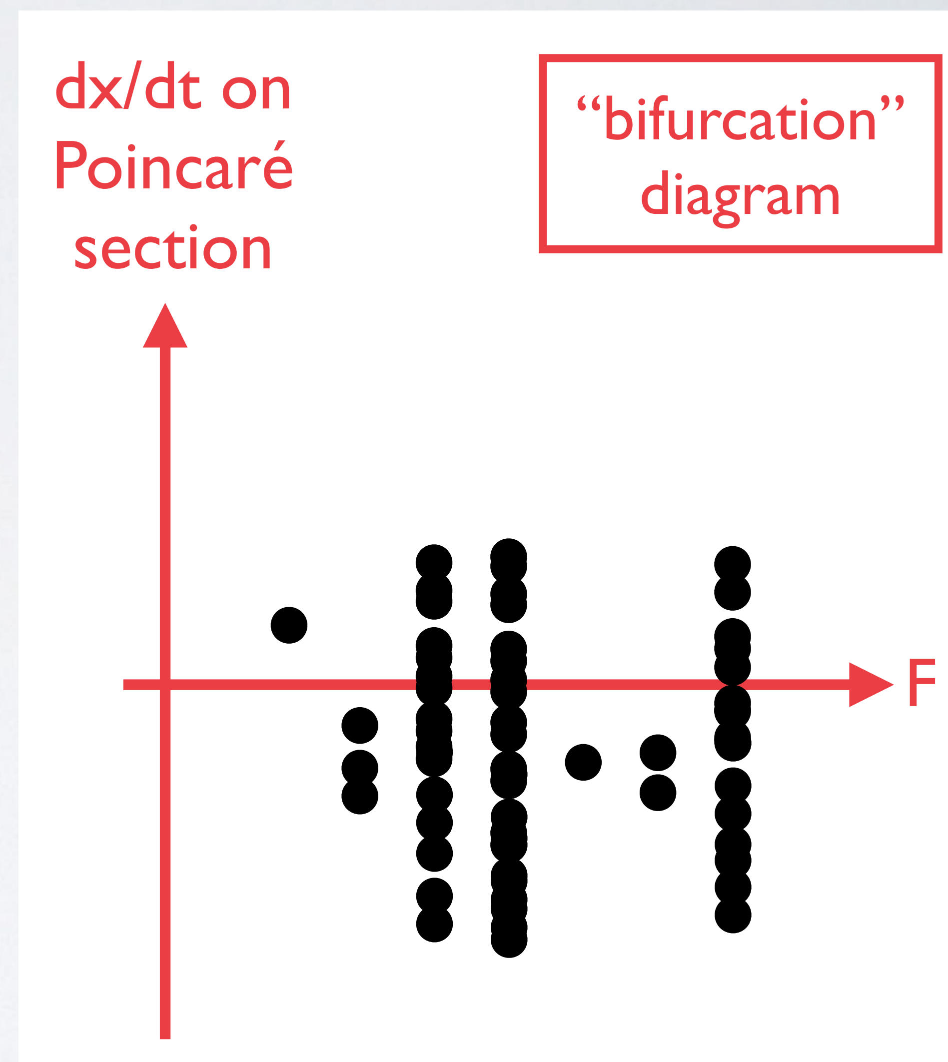
- Damped, driven oscillator (chaotic behavior, filling phase space)



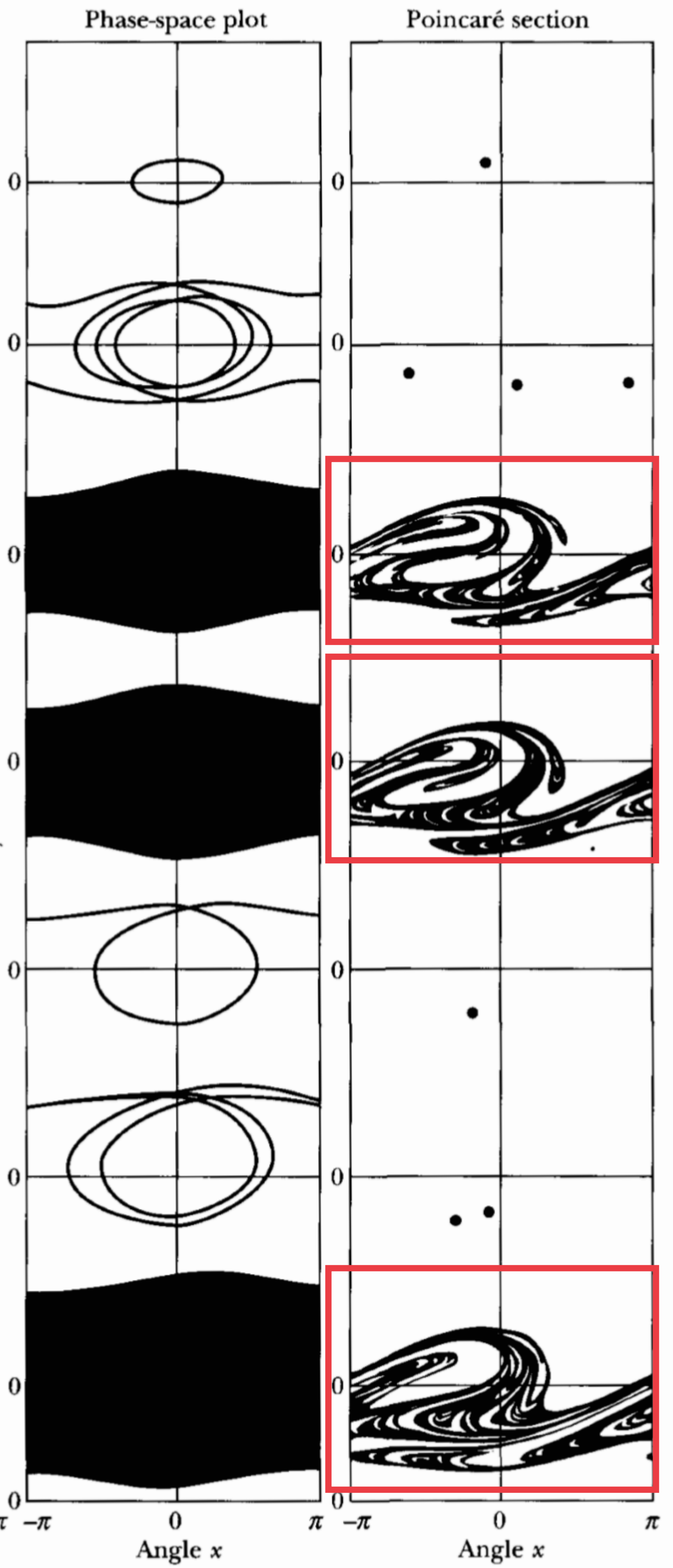
fast forward!

Thornton & Marion
Fig. 4-19

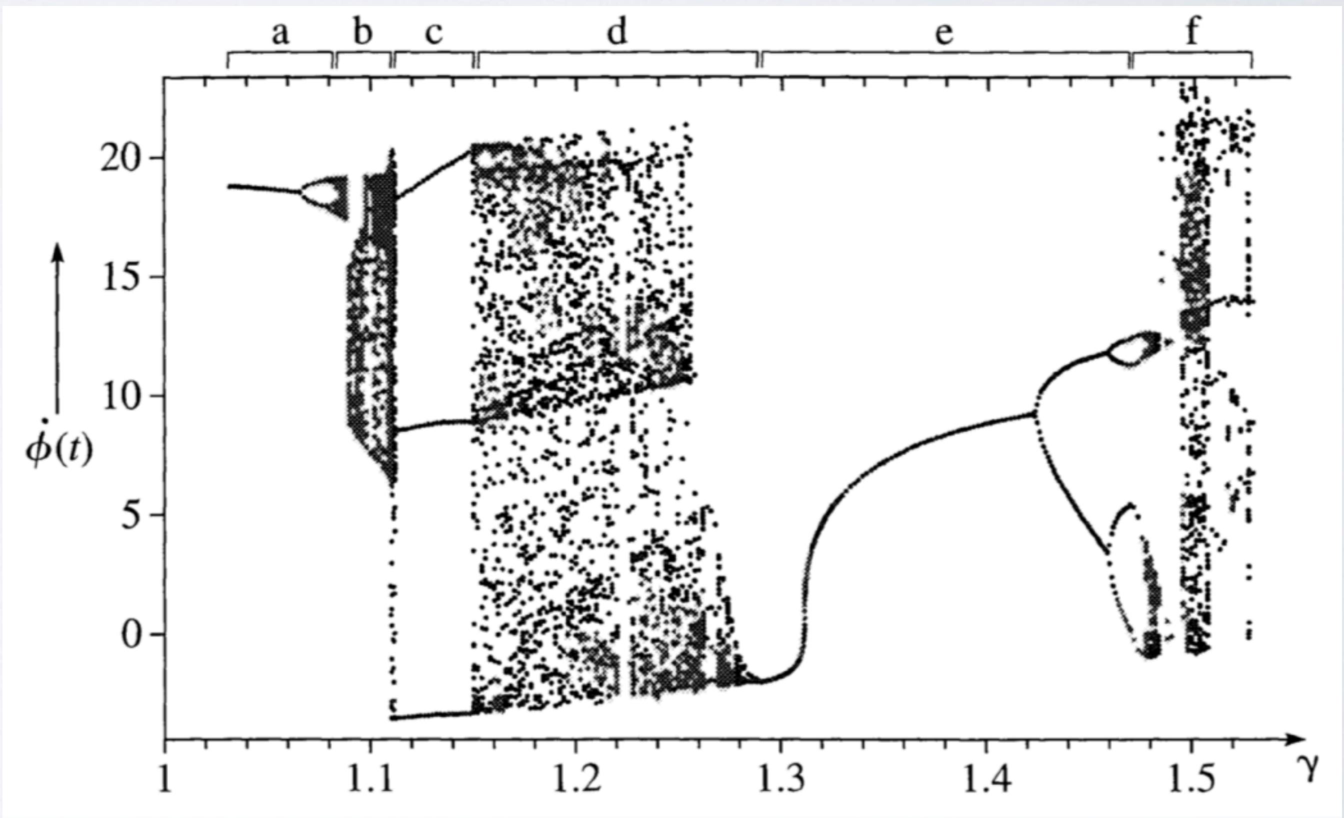

Phase-space plot
Poincaré section
F = 0.4
F = 0.5
F = 0.6
F = 0.7
F = 0.8
F = 0.9
F = 1.0
Angular velocity, y = dx/dt'
10π
20π
Time t'
−π
0
π
Angle x
−π
0
π
Angle x


dx/dt on Poincaré section
"bifurcation" diagram
F

“bifurcation” diagram



dx/dt on Poincaré section



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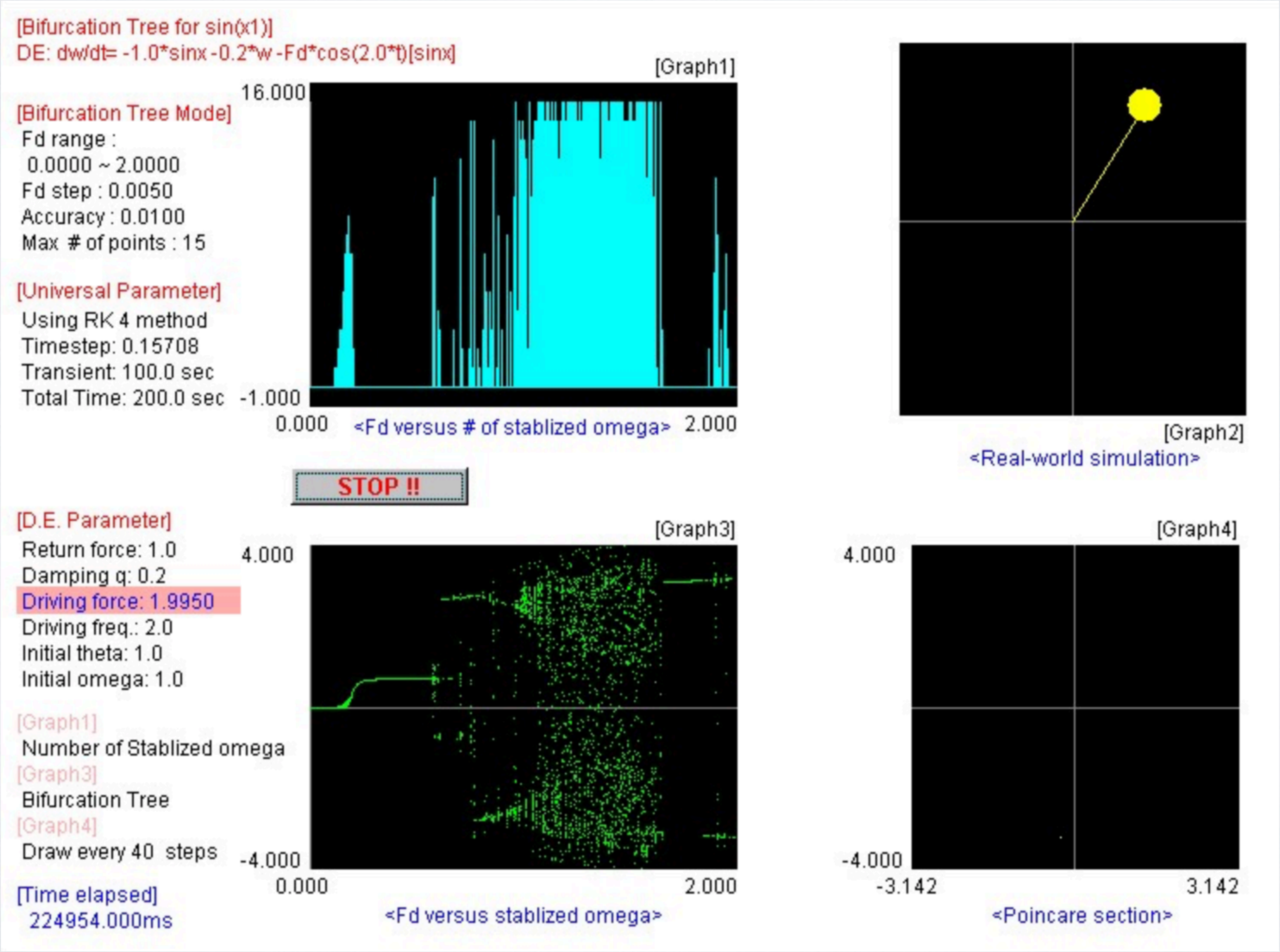
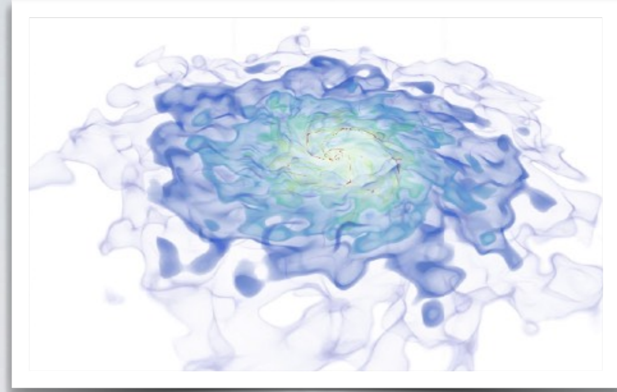
Pendulum: Numerical Simulation

- Damped, driven oscillator ("bifurcation diagram")





Linearity vs. Chaos vs. Randomness: Lorenz System

Linearity vs. Chaos vs. Randomness

www.youtube.com/watch?v=aAJkLh76QnM

Linearity vs. Chaos vs. Randomness

[1] The research reported in this work has been sponsored by the Geophysics Research Directorate of the Air Force Cambridge Research Center, under Contract No. AF 19(604)-4969.

values of the continuous variables at a chosen grid of points, or the coefficients in the expansions of these variables in series of orthogonal functions. The governing laws then become a finite set of ordinary differential

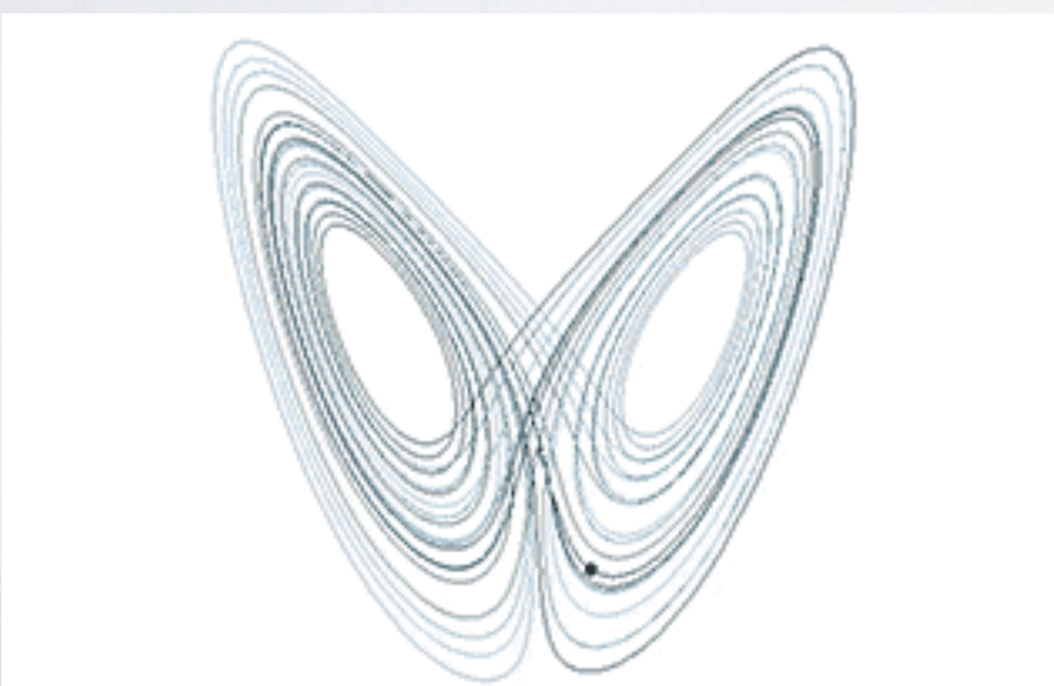
www.youtube.com/watch?v=aAJkLh76QnM

Lorenz System, Lorenz Attractor

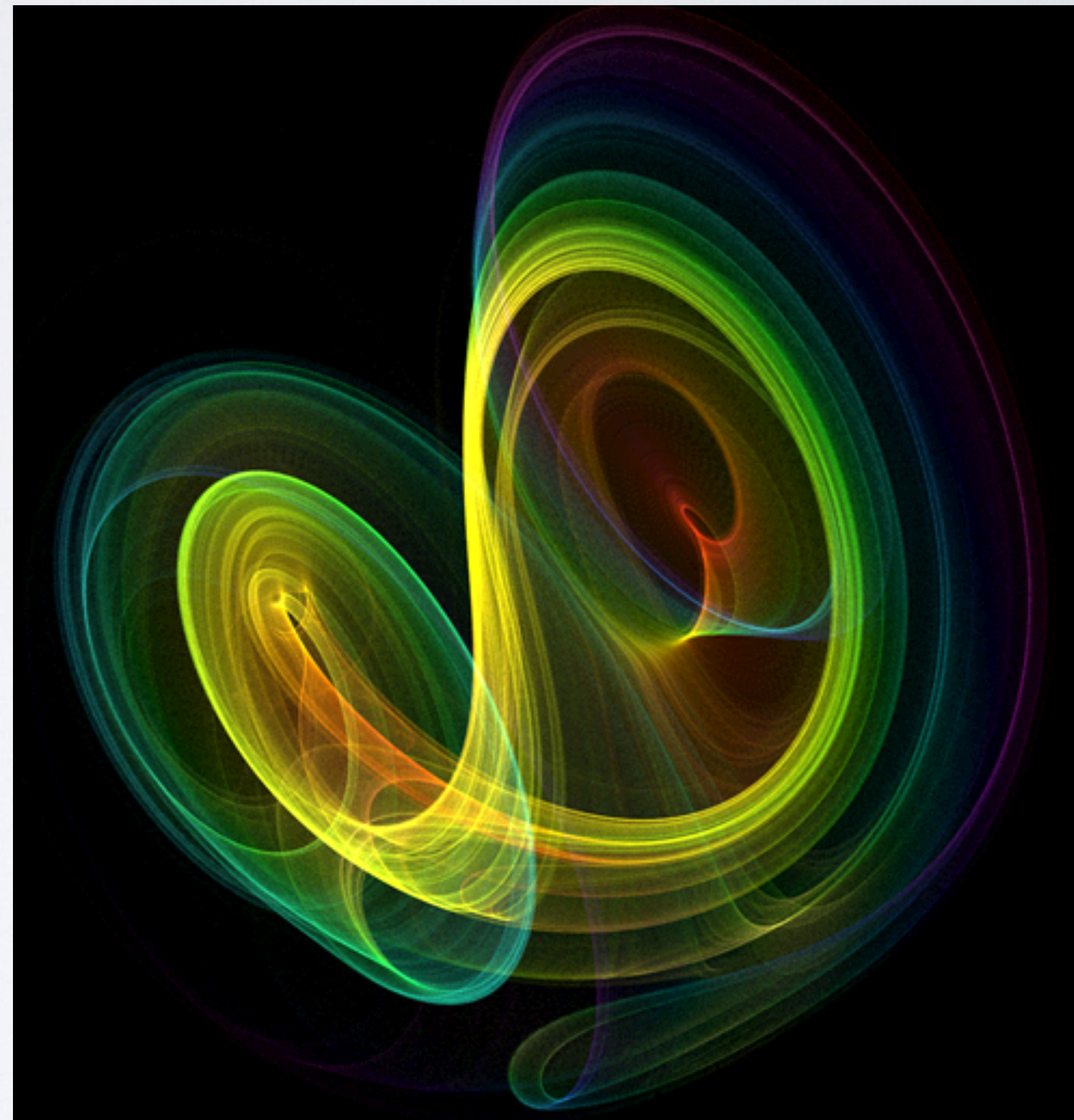
- Linearity vs. Chaos vs. Randomness

$$\frac{dx}{dt} = a(y - x)$$
$$\frac{dy}{dt} = x(b - z) - y$$
$$\frac{dz}{dt} = xy - cz$$

Lorenz (1963)



Wikipedia commons



www.metabluedb.com/lorentz-attractor.html