

# Classical Mechanics I (Spring 2026): Homework #3

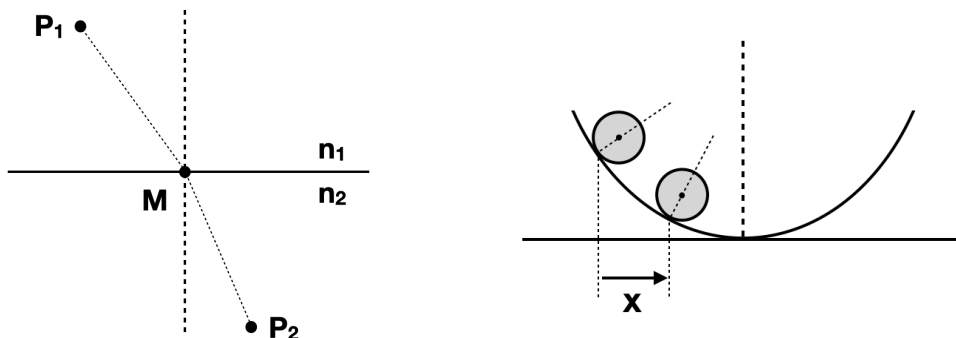
Due May 11, 2026 (Mon, 23:00pm)

[0.5 pt each, total 6 pts / turn in as a single pdf file to eTL]

- By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code, and the SNU AI Guideline!)
- Exhibit all intermediate steps to receive full credits. Only handwritten answers are accepted except for numerical problems or verifications — for which you print out and turn in not just the end results (e.g., plots) but also the source codes in your favorite tools such as MATLAB or MATHEMATICA.
- For some problems you may want to use formulae in Appendices D and E of Marion, and/or more extensive references such as Zwillinger.

1.-8. Thornton & Marion, Problems 6-7, 6-11, 6-16, 7-2, 7-6, 7-9, 7-10, 7-15

(Note: For Problem 6-7, despite your urge to immediately reduce the problem into 2-dimension, you need to explicitly demonstrate that the mid-point along the light path on the planar interface between the two media,  $M$ , lies in the same plane as the starting and ending points of the light ray,  $P_1$  and  $P_2$ . For Problem 6-11, first show that the *radius of curvature* for the parabola is  $R_c = \left| \frac{ds}{d\phi} \right| = \left| \frac{(1+y'^2)^{3/2}}{y''} \right|$  where  $d\phi$  is the angle turned by a line tangential to the curve. For Problems 7-6 and 7-9, you will first need to clearly define your choice of the generalized coordinates. Use diagrams if needed. For Problem 7-10, show that your answer in (b) asymptotes to your answer in (a) when  $m \rightarrow 0$ . For Problem 7-15, also find the equations of motion, and the period of small oscillations. You will need to consider small oscillations in all generalized coordinates of your choice. See how your answer changes if the pendulum's point of support rises vertically with constant acceleration  $a$ .)



9. Consider a case of the calculus of variations where the equation of constraint is written in an integral form.

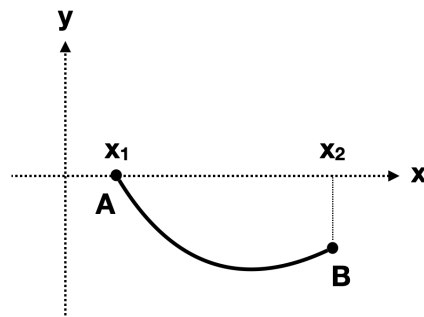
(a) First, let us study this case of the calculus of variations (p.222 in Section 6.6 of Thornton & Marion, which was briefly discussed in the class and left for your exercise) in order to tackle the problems that follow. Prove Eq.(6.78) if you desire — no penalty even if you don't.

(b) Then, work out Example 6.6, the so-called *Dido problem* of isoperimetry. Note that there is a typo in the textbook; check out the collection of typos posted on the course webpage.



(c) Now solve the problem differently by finding the functional in the form of  $\int f\{y, y'\} ds$  where  $s$  is the distance along the curve  $y$ .

(d) Finally, consider the so-called *catenary problem*. Consider a chain of uniform line density  $\lambda$  and length  $\ell$ , hanging under its own weight from two fixed points, A and B (the word *catenary* came from the Latin word *catena* meaning chain). Find the shape of this chain by using the fact that in stable equilibrium the chain will assume the shape that minimizes its potential energy.



10. Show that the shortest path between two given points on a plane is a straight line — in three different ways below.

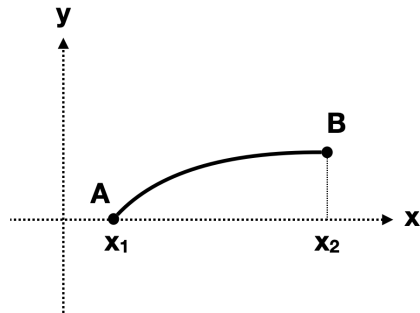
(a) By introducing a functional in the form of  $\int f_1\{y'\} dx$  in Cartesian coordinates  $[x, y]$ .

(b) By rewriting the functional in the form of  $\int f_2\{x', y'\} ds$  where  $s$  denotes a variable chosen for convenient parametrization for  $x$  and  $y$  — such as the distance along the path.

(c) By finding a functional in the form of  $\int f_3\{r, \phi'\} dr$  in plane polar coordinates  $[r, \phi]$ . One may use the following formula from an integral table (e.g., Eq.151 from Chapter 5.4.11 of Zwillinger, 33rd ed., 2018):

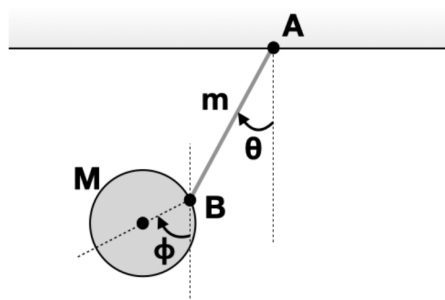
$$\int \frac{dx}{x\sqrt{x^2 - a^2}} = \frac{1}{|a|} \cos^{-1} \left| \frac{a}{x} \right|.$$

11. Consider the light path in an atmosphere where the speed of light increases in proportion to the height  $y$  (i.e., the index of refraction is proportional to  $y^{-1}$ ). Write down the transit time from A to B as an integral, and use Fermat's principle to determine the path that minimizes it. (Note: Review Example 6.3 and the subsequent discussion, and consider choosing a different independent variable to simplify the Euler equation. You will find that the resulting light path is an arc of a circle. This problem simulates a condition for a so-called *superior* mirage — as opposed to an *inferior* mirage — or a condition near a black hole's surface where one can model that the speed of light drastically changes with  $y$ , even approaching zero at the *event horizon*.)



Superior mirage (left; 위 신기루) and inferior mirage (right; 아래 신기루) [image credits: nytimes.com, epod.usra.edu]

12. A uniform rod of mass  $m$  and length  $\ell$  is free to rotate in a vertical plane about one end, A, under the influence of a uniform gravitational acceleration,  $g$ . The other end of the rod, B, is hinged to a point on the rim of a uniform circular lamina of mass  $M$  and radius  $R$ . The lamina is therefore free to rotate about point B in the same vertical plane (see figure). Determine the Lagrangian and Lagrange's equation(s) of motion describing the system. (Note: You may use the angles indicated in the figure as generalized coordinates, or define and use any other convenient set of generalized coordinates.)



[Problem 13 continues in the next page.]

13. [For additional +0.5pt] Choose a topic in any scientific research in which the calculus of variations we learned in Chapter 6 of Thornton & Marion is utilized. While the areas of research that seem obvious to you — such as geometry, mechanics, optics, etc. — may surely be interesting, you are highly encouraged to venture beyond your comfort zone — to biology, ecology, structural engineering, urban planning, control theory, game theory, economics, finance, management science, etc. in all of which optimization strategies play a crucial role in identifying the best possible solution.

(Note: See the first bullet point above. 2-3 paragraphs or more per topic are expected to clearly unfold your story. If needed, you must reference your sources appropriately with a proper citation convention, but your answer must still be your own work in your own words. To access the electronic resources — e.g., academic journals — off-campus via SNU library's proxy service, see <http://library.snu.ac.kr/using/proxy>.)