Classical Mechanics I (Spring 2026): Homework #2

Due Apr. 13, 2026 (Mon, 23:00pm)

[0.5 pt each, total 6 pts / turn in as a single pdf file to eTL]

• By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code, and the SNU AI Guideline!)

• Exhibit all intermediate steps to receive full credits. Only handwritten answers are accepted except for numerical problems or verifications — for which you print out and turn in not just the end results (e.g., plots) but also the source codes in your favorite tools such as MATLAB or MATHEMATICA.

• For some problems you may want to use formulae in Appendices D and E of Marion, and/or more extensive references such as Zwillinger. Do *not* use the Lagrangian method until we cover it in the class.

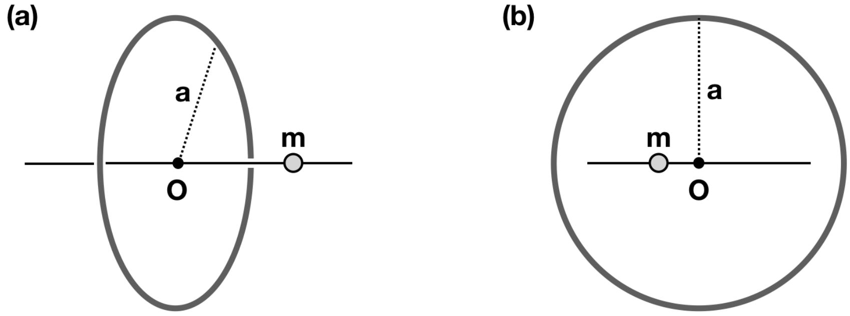
1.-8. Thornton & Marion, Problems 4-5(a)(c), 4-8, 4-13, 4-17, 4-22, 5-5, 5-14, 5-16

(Note: For Problem 4-5, you do not need to code up anything. A calculator on WOLFRAMALPHA or on any web search engine is all you need. Start with a hand-drawn "crude" graphs of $y = x^2 + x + 1$ and $y = \tan x$. Plug your first guess from the graph, e.g., $x_1 = 3\pi/8$, into the RHS of $x = \tan^{-1}(x^2 + x + 1)$, and acquire your second guess, x_2. Repeat the procedure until x_n is within 10^{-4} of x_{n-1}. You may benefit from visualizing how each x_n is acquired on your "crude" graph. For Problem 4-22, try several different values for B to draw the Poincaré sections, and guesstimate the transition value(s) that delineate chaotic and periodic behavior. For Problem 5-5, you will need to assume that the particle starts at a large — but not infinitely large — distance R_i, and the Earth is a point mass (i.e., its radius is 0). You may need to utilize an integral table; see the third bullet point above. Alternatively, you could try a change of variable, $x = R_i \sin^2\theta$, to make $\sqrt{\frac{1}{x} - \frac{1}{R_i}} = \frac{1}{\sqrt{R_i}} \frac{\cos\theta}{\sin\theta}$. For Problem 5-16, determine both the magnitude and the direction of the force. Confirm your result by applying Gauss's law to the gravitational field $\mathbf{g}$ produced by the infinite sheet. You may want to review the analogous derivation for the electric field with planar symmetry in Halliday & Resnick (Chapter 23.5) or in Griffiths (Chapter 2.2).)

9. Consider a thin, uniform circular ring of mass M and radius a. A small bead of mass m ($\ll M$), with a hole through it, is threaded on a frictionless, massless rod. The size of the bead is negligible. The ring attracts the bead gravitationally, and no other gravitational source exists.

(a) The rod is placed perpendicular to the plane of the ring and through the center of the ring, O. Find a position of equilibrium. If a stable equilibrium point exists, find the period of small oscillation about the point.

(b) The rod is now placed in the plane of the ring and through the center of the ring. Discuss the stability of the bead when it is at O.



10. The Laplacian operator appears frequently in the equations that describe a wide range of physical systems. One example is Poisson's equation (or Laplace's equation) for the gravitational potential, Eq.(5.38). And in Problem 5-16, we already observed the close analogy between the gravitational potential and the electrostatic potential. Explore three to four additional well-known physical equations in which the Laplacian operator plays a central role. Explain qualitatively why the Laplacian appears so often in physical equations.

(Note: In addition to Chapter 13 of Thornton & Marion, you may want to consult standard textbooks in electrodynamics (e.g., Griffiths), quantum mechanics (e.g., Griffiths & Schroeter), and/or mathematical physics (e.g., Boas). If needed, you must reference your sources appropriately with a proper citation convention, but your answer must still be your own work in your own words. To access the electronic resources — e.g., academic journals — off-campus via SNU library's proxy service, see http://library.snu.ac.kr/using/proxy.)

11. In this problem, let us make the bifurcation diagram of the logistic map, Eq.(4.46).

(a) First, reproduce Figure 4-23. Start testing your script with a small number of points, e.g., α going from 2.8 to 4.0 in steps of 0.2. Once you are ready to make a final diagram, you may increase the number of points, e.g., α going from 2.8 to 4.0 in steps of 0.01 or less.

(b) Make a zoomed-in version of the bifurcation diagram from (a) in a small rectangular region of $\alpha \in [3.840, 3.857]$ and $x_n \in [0.44, 0.55]$. You may want to use much finer steps in α, of course. Notice the emerging self-similarity, though the new diagram is upside down in a smaller scale.

(c) Thornton & Marion, Problem 4-11

12. In the class we discussed the method of Green's function. Starting from a response to a Heaviside step function, Eq.(3.100), follow step by step the logical procedure that eventually leads to Eq.(3.118). In particular, carefully extract Eq.(3.110) by combining Eq.(3.108) and the limiting case assumptions – which was briefly discussed in the class and left for your exercise.