Classical Mechanics I (Spring 2020): Homework #4

Due Jun. 11, 2020

[0.5 pt each, total 7 pts / turn in as a single pdf file to eTL before the class starts]

• By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code!)

• Only handwritten answers are accepted except for numerical problems – for which you print out and turn in not just the end results (e.g., plots) but also the source codes.

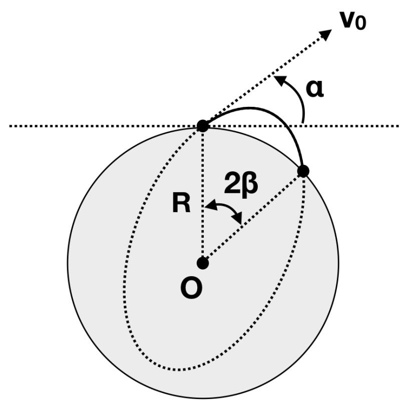
• For some problems you may want to use formulae in Appendices D and E, and/or more extensive references such as Zwillinger.

1.-8. Thornton & Marion, Problems 7-17, 7-30, 7-34, 7-40, 8-13, 8-17, 8-22, 8-33

(Note: For Problem 7-30, note that there is a typo in the textbook; check out the collection of typos posted on the course webpage. For Problem 7-34(b), acquire the reaction using both Lagrangian and Newtonian methods. The "reaction" refers to the normal force of constraint exerted by the wedge on the particle, which of course depends on the initial position of the particle. For Problem 8-13, show also that for $\lambda < 0$ the particle's orbit has the form similar to Eq.(8.41) but slightly different: $\frac{\alpha}{r} = 1 + \epsilon \cos \beta\theta$. Here, find α and β in terms of the variables given. Describe the particle's orbit for $0 < \epsilon < 1$, and determine for what values of β the orbit is closed. For Problem 8-22, sketch the particle's motion in the effective potential diagrams, $V(r)$, of various ℓ and E values – i.e., equivalents of Figure 8-6. For Problem 8-33, you may want to use an approximation technique similar to what we did in Eq.(8.86) or (8.99).)

9. Consider a projectile launched on the surface of the Earth of radius R, with an initial velocity v_0 at an angle α with respect to the horizontal plane. The gravitational acceleration at radius R is g. Show that the angular range of the projectile, 2β, can be found by $\tan\beta = \frac{(v_0^2/gR)\sin 2\alpha}{2\left[1-(v_0^2/gR)\cos^2\alpha\right]}$.

Verify that this result reduces to the familiar formula, $D = \frac{v_0^2 \sin 2\alpha}{g}$, if we assume the flat Earth and constant gravity. (Note: Ignore the Earth's rotation while the projectile is in motion.)



10. [1 pt] A *spherical pendulum* is a simple pendulum that is free to move in any sideways direction – as opposed to a *plane pendulum*. It consists of a bob of mass m attached to a weightless, extensionless rod of length b.

(a) Determine Lagrange's equation(s) of motion.

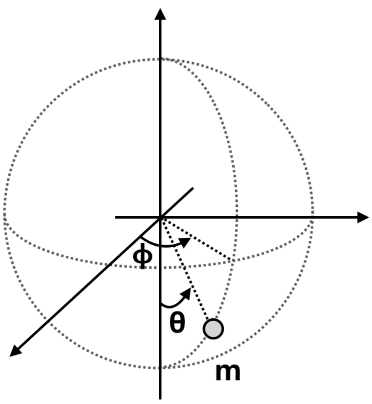
(b) Find the generalized momenta, p_ϕ and p_θ, and Hamilton's equation(s) of motion.

(c) Discuss the special case of $\phi = \phi_0 =$ constant.

(d) Discuss the existence of an angle θ_0 at which θ remains unchanged. This special case is called a *conical pendulum*. It will be helpful to define an effective potential $V(\theta)$ and (numerically) sketch it for several p_ϕ values. Find θ_0 and discuss the Newtonian force equilibrium.

(e) Determine if θ_0 is a stable equilibrium position. If so, find the period of small oscillations in the variable θ about the point (a *near-conical pendulum*). Discuss the pendulum's motion in the effective potential diagram.

(f) Now drop the small oscillation assumption in (e), and solve the nonlinear equation of motion numerically. Plot your solution $\theta(t)$ and compare it with the approximate solution from (e). For simplicity, one may use the following initial conditions: $\theta(0) = 0.785$ rad, $\dot{\theta}(0) = 0$ rad/s, $\phi(0) = 0$ rad, $\dot{\phi}(0) = 10.57$ rad/s, and $b = 0.284$ m. Estimate the limits of θ and the period of the θ-motion.



11. A particle of mass m is constrained to move on the inside surface of a smooth torus shown in the figure below. The symmetry axis of the torus (z-axis) is vertical, and the particle is subject to the gravitational field g. The location of the particle is then described by $(\rho, \phi, z) = (a + b\cos\theta, \phi, -b\sin\theta)$, where ϕ is the azimuthal angle in the cylindrical coordinate system and θ is the angle measured in the vertical plane containing the particle with respect to the $z = 0$ plane (see the figure).

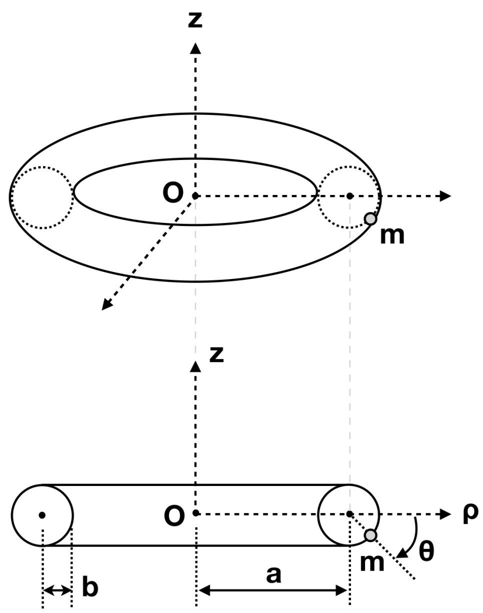
(a) Determine Lagrange's equation(s) of motion.

(b) Find the generalized momenta, p_ϕ and p_θ, and Hamilton's equation(s) of motion.

(c) Discuss the existence of an angle θ_0 at which θ remains unchanged – i.e., the particle is in the state of uniform circular motion. It will be helpful to define an effective potential $V(\theta)$ and (numerically) sketch it for several p_ϕ values. Find θ_0.

(d) Find the orbital period $T_{\rm orb}(\theta_0)$ of the circular motion described in (c).

(e) Determine if θ_0 is a stable equilibrium position. If so, find the period of small oscillations in the variable θ about the point. Discuss the particle's motion in the effective potential diagram.



12. [1 pt] Here we review the sections of Thornton & Marion we briefly discussed in the class and left for your exercise.

(a) In the class we discussed the connection between invariance or symmetry properties and conserved quantities (Noether's theorem). Starting from three characteristics of an inertial reference frame, follow step by step the logical procedures that eventually lead to Eqs.(7.130), (7.137) and (7.149). During the process, you will need to carefully derive Eq.(7.122), and also need to show that an expression like Eq.(1.106) is possible for infinitesimal rotation.

(b) Study Liouville's theorem, Eq.(7.198), and the virial theorem, Eq.(7.204). For example, you may follow step by step the procedures in your textbook that eventually proves each theorem. These two theorems have important applications in statistical mechanics and in astrophysics. Explain briefly.