

Classical Mechanics I (Spring 2020): Homework #3

Due May 26, 2020

[0.5 pt each, total 7 pts / turn in as a single pdf file to eTL before the class starts]

- By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code!)
- Only handwritten answers are accepted except for numerical problems – for which you print out and turn in not just the end results (e.g., plots) but also the source codes.
- For some problems you may want to use formulae in Appendices D and E, and/or more extensive references such as Zwillinger.

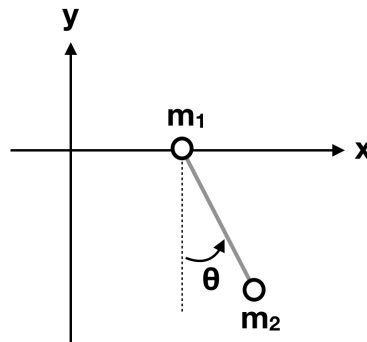
1.-8. Thornton & Marion, Problems 6-8, 6-16, 7-4, 7-5, 7-11, 7-12, 7-13, 7-15

(Note: For Problem 6-8, you will need to utilize the technique developed in Section 6.6 of Thornton & Marion. For Problem 7-12, you will need to find the displacement of m as a function of time. Try both Lagrangian and Newtonian approaches. For Problem 7-13, you will need to find the line (or angle) about which the small oscillation takes place. Explain why the sign of the acceleration a cannot affect the period, and why the signs of a^2 and g^2 in the period are the same. For Problem 7-15, also find the equations of motion, and the period of small oscillations. You will need to consider small oscillations in all generalized coordinates of your choice. See how your answer changes if the pendulum's point of support rises vertically with constant acceleration a .)

9. Two particles of masses m_1 and m_2 are connected by a massless rod of length ℓ and constrained to move on a vertical plane. Assume further that m_1 moves on a frictionless rail along the x -axis.

(a) Determine the Lagrangian and Lagrange's equation(s) of motion describing the system. Define your choice of the generalized coordinates clearly.

(b) Discuss the behavior of the system when the angle θ undergoes small oscillations about the vertical. What is the period for small oscillations?



10. [1 pt] Consider a simple pendulum whose point of support moves on a rotating massless rim, as depicted in Example 7.5.

(a) Work out Example 7.5.

(b) Solve the resulting nonlinear equation of motion numerically. Check if $\theta(t)$ resembles the sinusoidal oscillations of a linear simple pendulum if the rim is sufficiently small and rotates slowly. Assume $\theta(0) = 0.2$ and $\dot{\theta}(0) = 0$. For convenience, plot the solution for $0 < t < 100$ with $a = 0.2$, $b = 1$, $g = 1$, and $\omega = 0.1$. Experiment other values and ranges if you want.

11. [1 pt] Consider a case of the calculus of variations where the equation of constraint is written in an integral form.

(a) First, let us study this case of the calculus of variations (p.222 in Section 6.6 of Thornton & Marion, which was briefly discussed in the class and left for your exercise) in order to tackle the problems that follow. Prove Eq.(6.78) if you desire — no penalty even if you don't.

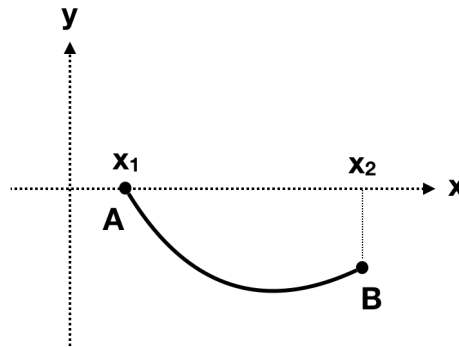
(b) Then, work out Example 6.6, the so-called *Dido problem* of isoperimetry. Note that there is a typo in the textbook; check out the collection of typos posted on the course webpage.



(c) Now solve the problem differently by finding the functional in the form of $\int f\{y, y'\} ds$ where s is the distance along the curve y .

(d) Finally, consider the so-called *catenary problem*. Consider a chain of uniform line density λ and length ℓ , hanging under its own weight from two fixed points, A and B (the word *catenary*

came from the Latin word *catena* meaning chain). Find the shape of this chain by using the fact that in stable equilibrium the chain will assume the shape that minimizes its potential energy.



12. Choose a topic in any scientific research in which the calculus of variations we learned in Chapter 6 of Thornton & Marion is utilized. While the areas of research that seem obvious to you — such as geometry, mechanics, optics, etc. — may surely be interesting, you are highly encouraged to venture outside your comfort zone — to structural engineering, control theory, game theory, programming, economics, etc. in all of which a certain optimization strategy is desired to find the best possible solution.

(Note: 2-3 paragraphs or more per topic are expected to clearly unfold your story. You must reference your sources appropriately with a proper citation convention, and your answer must be your own work in your own words. To access the electronic resources — e.g., academic journals — off-campus via SNU library's proxy service, see <http://library.snu.ac.kr/using/proxy>.)