Classical Mechanics I (Spring 2020): Homework #2

Due Apr. 28, 2020

[0.5 pt each, total 7 pts / turn in as a single pdf file to eTL before the class starts]

• By turning in your homework, you acknowledge that you have not received any unpermitted aid, nor have compromised your academic integrity during its preparation. (Remember the SNU College of Natural Sciences Honor Code!)

• Only handwritten answers are accepted except for numerical problems – for which you print out and turn in not just the end results (e.g., plots) but also the source codes.

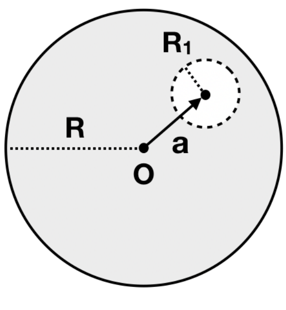
• For some problems you may want to use formulae in Appendices D and E, and/or more extensive references such as Zwillinger.

• Do *not* use the Lagrangian method until we cover it in the class.

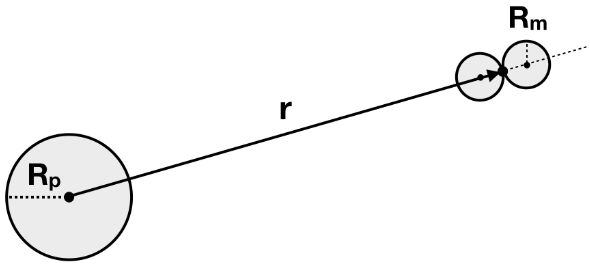
1.-8. Thornton & Marion, Problems 4-4, 4-10, 4-14, 4-17, 4-19, 5-4, 5-10, 5-16

(Note: For Problem 4-10, drawing a phase diagram and a Poincaré section for each ω will help you appreciate if the system exhibits chaotic behavior or not. You do not have to include all the plots from your investigations in your answer, but include only a few representative one.)

9. A sphere of uniform density ρ and radius R has within it a spherical cavity of radius R_1 whose center is at $\mathbf{a}$ as seen in the figure below ($\mathbf{a}$ originates from the center of the sphere O and points to the center of the cavity O_1). Show that the gravitational field within the cavity is uniform. Determine its magnitude and direction.



10. An oddly-shaped moon orbiting a massive exoplanet at radius r can be modeled as two identical spheres of uniform density ρ_{m} and radius R_{m} ($\ll r$) just touching each other. Their centers are always in line with the center of the planet whose mean density is ρ_{p} and radius R_{p}. The only force between the three objects is gravitational. Show that the two spheres will be pulled apart by the planet's tidal force if the moon is less than $r_{\mathrm{lim}} = 2\left(\frac{\rho_{\mathrm{p}}}{\rho_{\mathrm{m}}}\right)^{\frac{1}{3}} R_{\mathrm{p}}$ away from the planet's center. (Note: Assume that the two spheres themselves cannot be torn apart.)



11. [1 pt] In this problem, let us make the bifurcation diagram of the logistic map, Eq.(4.46).

(a) First, reproduce Figure 4-23. Start testing your script with a small number of points, e.g., α going from 2.8 to 4.0 in steps of 0.2. Once you are ready to make a final diagram, you may increase the number of points, e.g., α going from 2.8 to 4.0 in steps of 0.01 or less.

(b) Make a zoomed-in version of the bifurcation diagram from (a) in a small rectangular region of $\alpha \in [3.840, 3.857]$ and $x_n \in [0.44, 0.55]$. You may want to use much finer steps in α, of course. Notice the emerging self-similarity, though the new diagram is upside down in a smaller scale.

(c) Thornton & Marion, Problem 4-11

12. Choose a topic in any scientific research — physics, biology, epidemiology, economics, human behavioral science, etc. — in which the concepts we newly learned in Chapter 4 of Thornton & Marion are used. Describe how a complicated and seemingly random phenomenon can be modeled with the concepts in nonlinear dynamics or in chaos theory. Use diagrams if desired.

(Note: 2-3 paragraphs or more per phenomenon are expected to clearly unfold your story. You must reference your sources appropriately with a proper citation convention, and your answer must be your own work in your own words. To access the electronic resources — e.g., academic journals — off-campus via SNU library's proxy service, see http://library.snu.ac.kr/using/proxy.)

13. In the class we discussed the method of Green's function. Starting from a response to a Heaviside step function, Eq.(3.100), follow step by step the logical procedure that eventually leads to Eq.(3.118). In particular, carefully extract Eq.(3.110) by combining Eq.(3.108) and the limiting case assumptions – which was briefly discussed in the class and left for your exercise.