

Quantum technology: What kind of resources do we need?

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2022. 05. 18.

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What is quantum resource?

- Resource: Something that a country, person, or organization has that they can **use** (*Cambridge Dictionary*)

Ex) Gas/Oil, Materials, Water, Food, Human resources

자원⁴ 資源

발음

표준국어대사전

고려대한국어대사전

우리말샘

예문 열기

명사

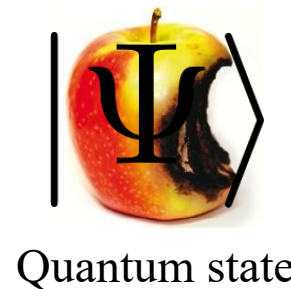
1. 경제 인간 생활 및 경제 생산에 이용되는 원료로서의 광물, 산림, 수산물 따위를 통틀어 이르는 말.

☒ 식량 자원.

2. 경제 인간 생활 및 경제 생산에 이용되는 노동력이나 기술 따위를 통틀어 이르는 말.

청소원은 우리나라의 중요한 자원이다.

- Quantum resource: Something that we can use to do a **“non-classical”** task
 - ✓ What are non-classical tasks using quantum principles? → Quantum technology
 - ✓ What kind of quantum principle make this possible?
 - ✓ How can we quantify the non-classical resources?



Quantum technologies

Quantum communication

[Sending quantum bits]

- Quantum information transfer
- Quantum cryptography (quantum key distribution)

Quantum computing

[Faster computation]

- Quantum simulation
- Quantum algorithm
- Quantum error correction

Quantum metrology

[Precision measurement]

- Precise phase/force sensing
- Accurate measurement of physical constants

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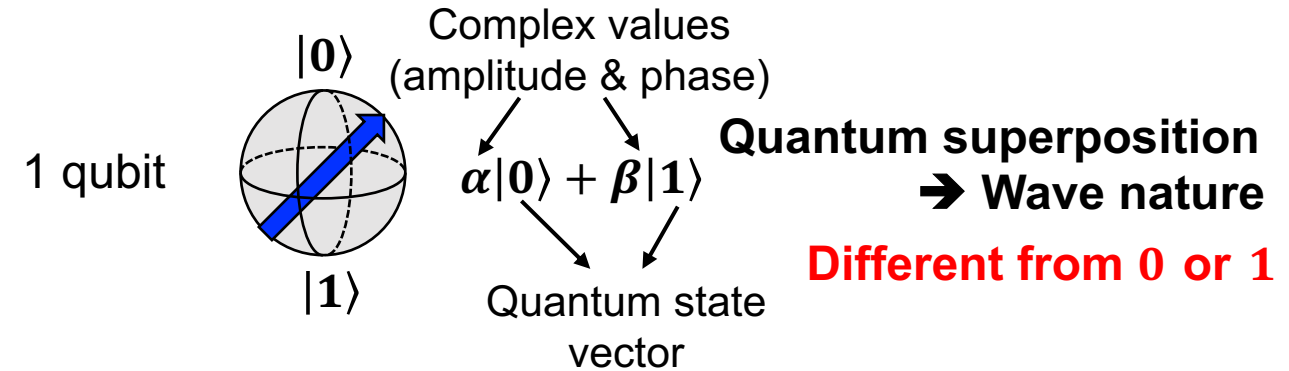
Bit and qubits

(Classical) Bit



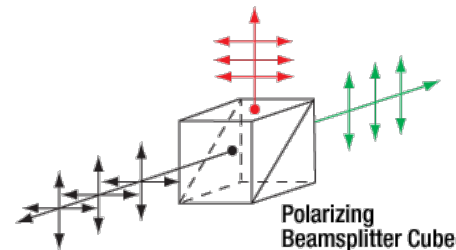
Head (0) or Tail (1)

Quantum Bit



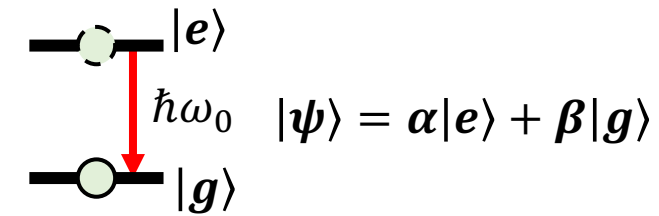
Any physical state can be in superposition

e.g., Polarization of light



$$|\psi\rangle = \alpha|H\rangle + \beta|V\rangle$$

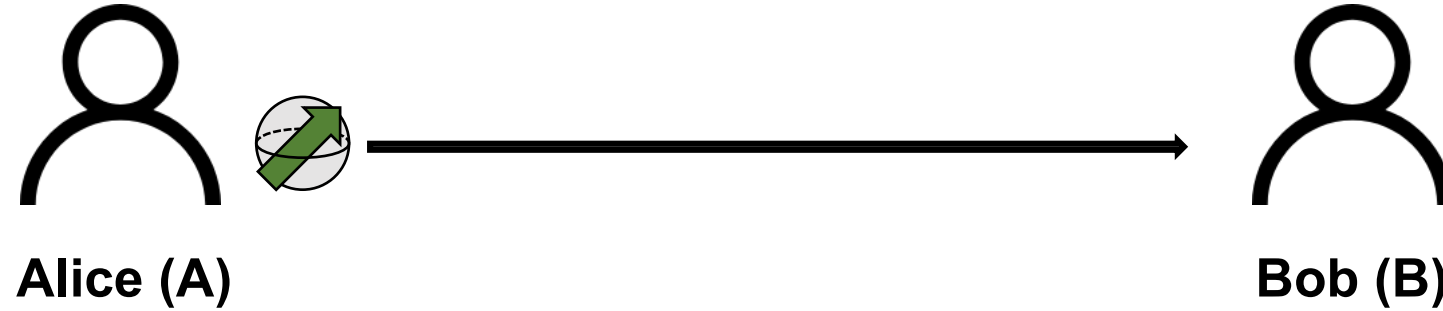
Atomic energy level



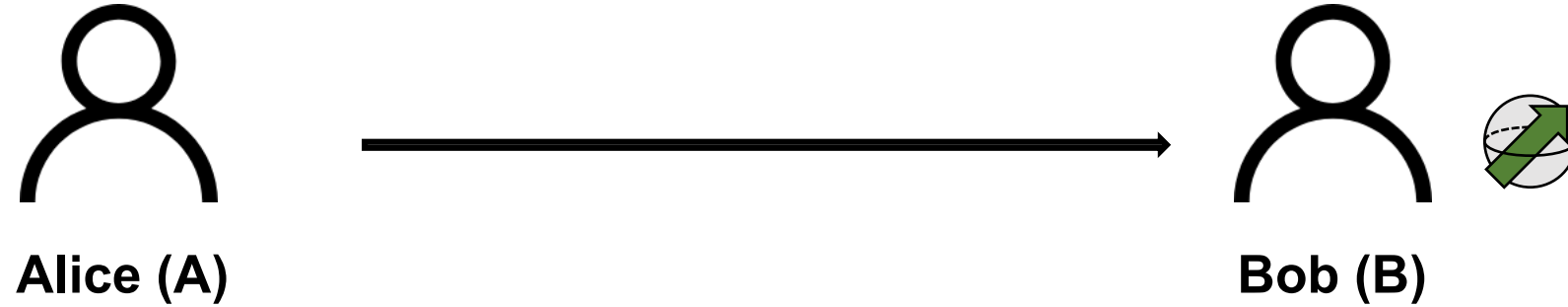
or even a cat!

$$\frac{1}{\sqrt{2}}|\text{cat sitting}\rangle + \frac{1}{\sqrt{2}}|\text{cat lying}\rangle$$

How to send a quantum state (qubit)?

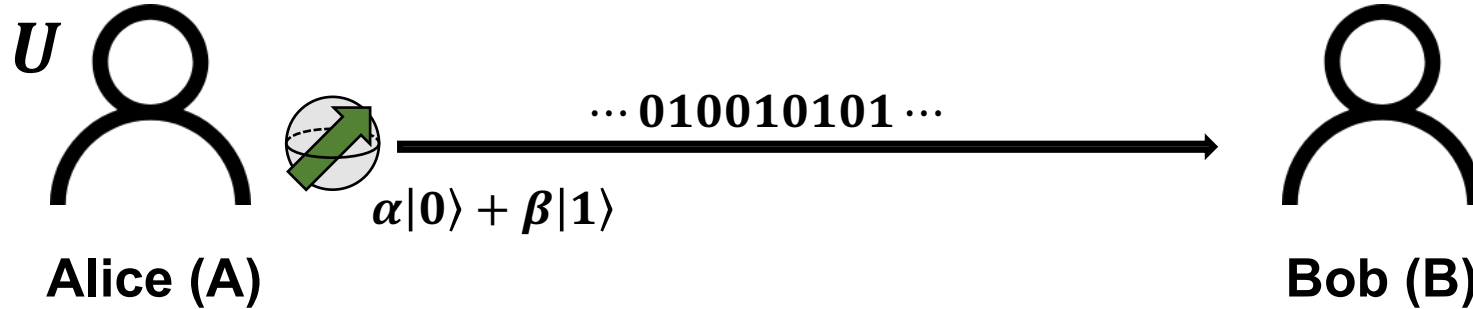


How to send a quantum state (qubit)?



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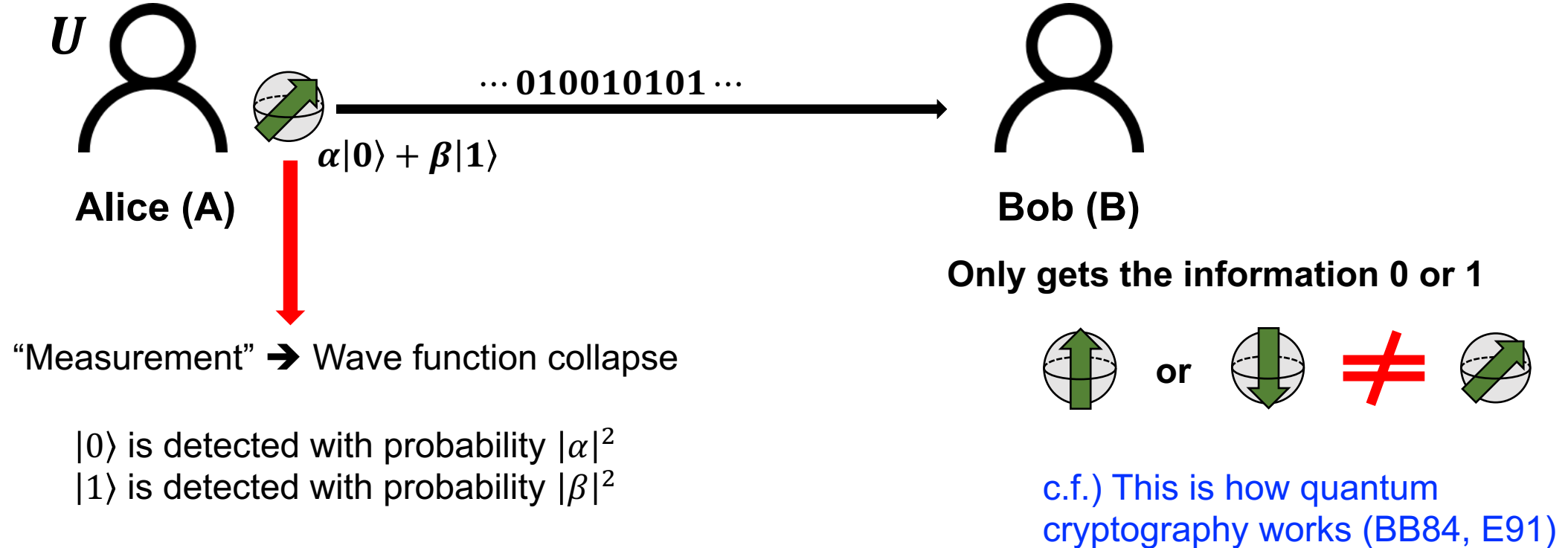
Let us suppose that only classical information (bit) can be transmitted
(but any quantum operation can be done locally)



LOCC (local operation and classical communication)

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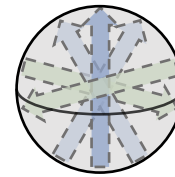
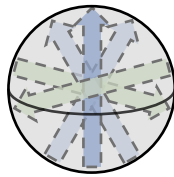


Alice may measure the state in different bases

→ To get exact values of α and β , infinitely many bits should be sent to Bob

It is impossible to send a qubit using **classical communication channels (LOCC)**

Quantum entanglement



Quantum entanglement

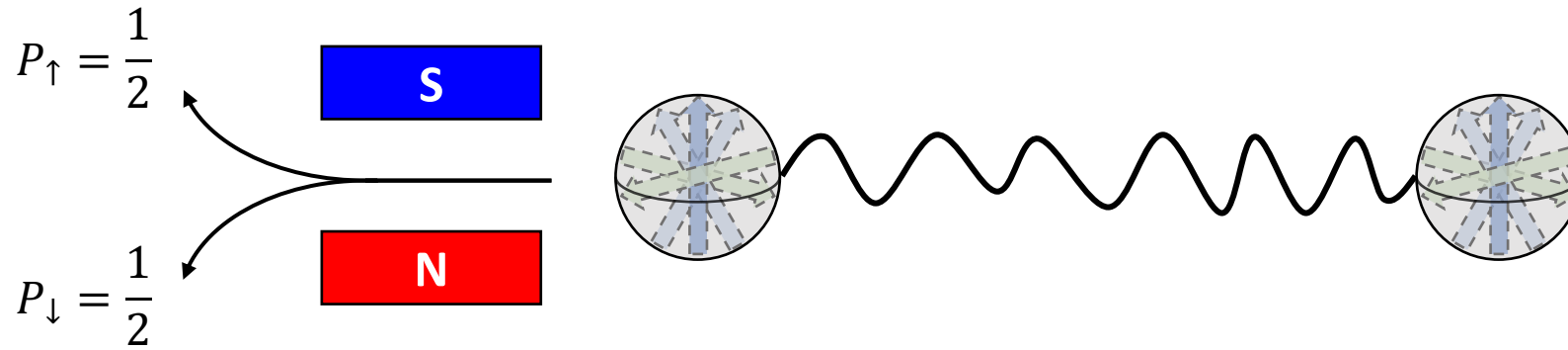
Spin=0



Quantum entanglement

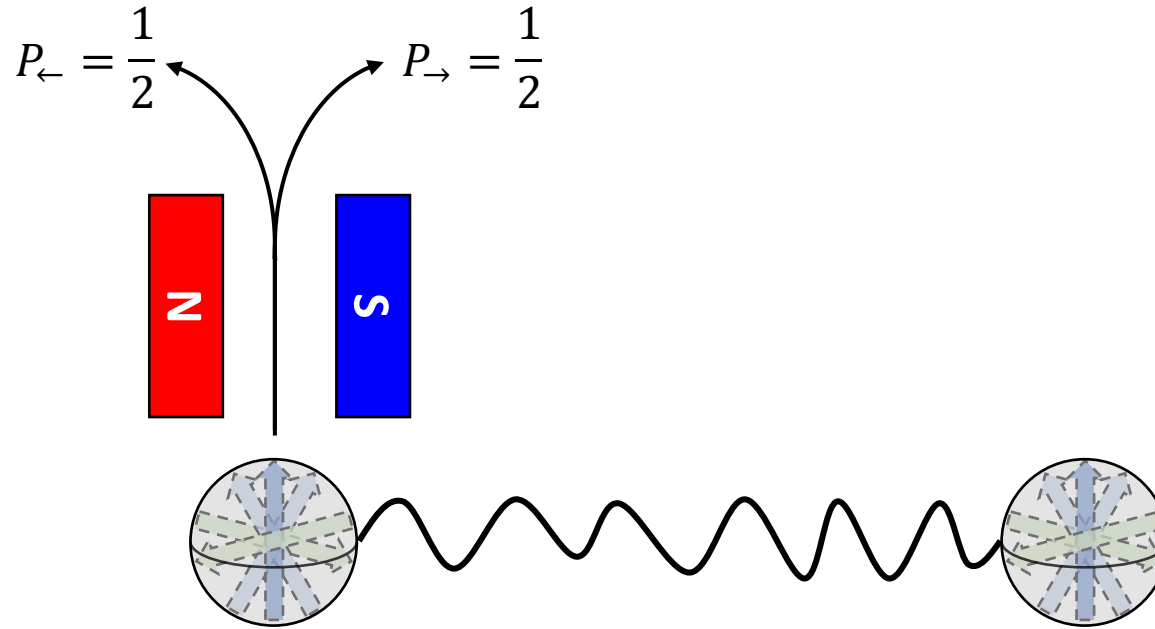


Quantum entanglement



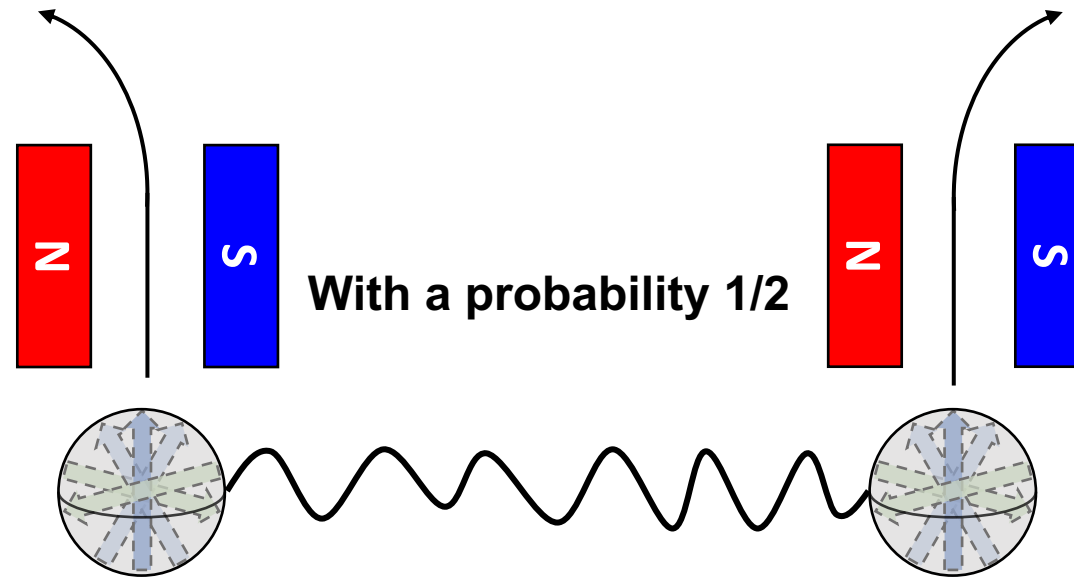
If we do only look at **the local party**, it is **perfectly random** (in any axis!)

Quantum entanglement



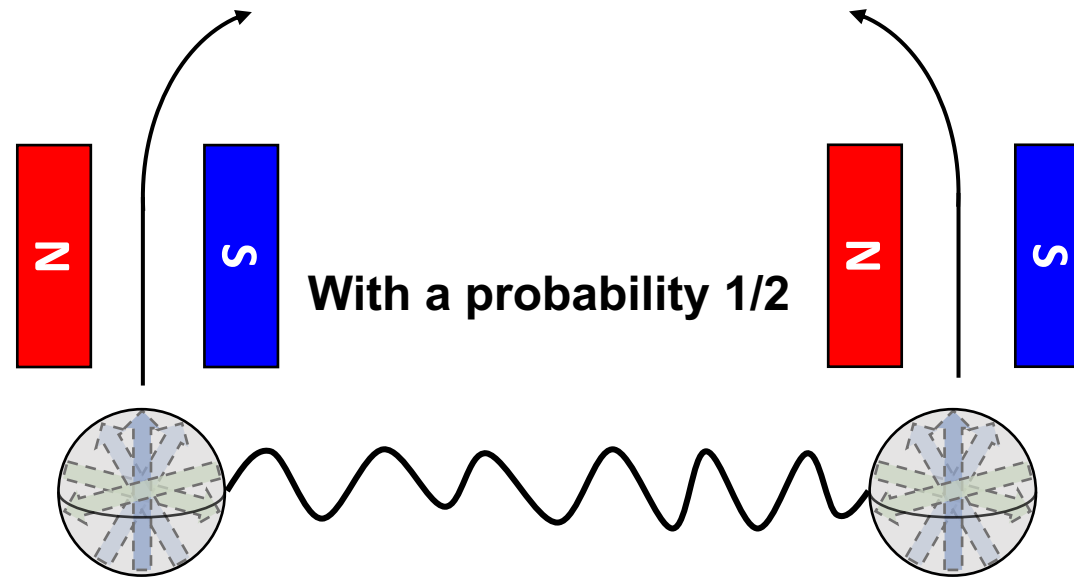
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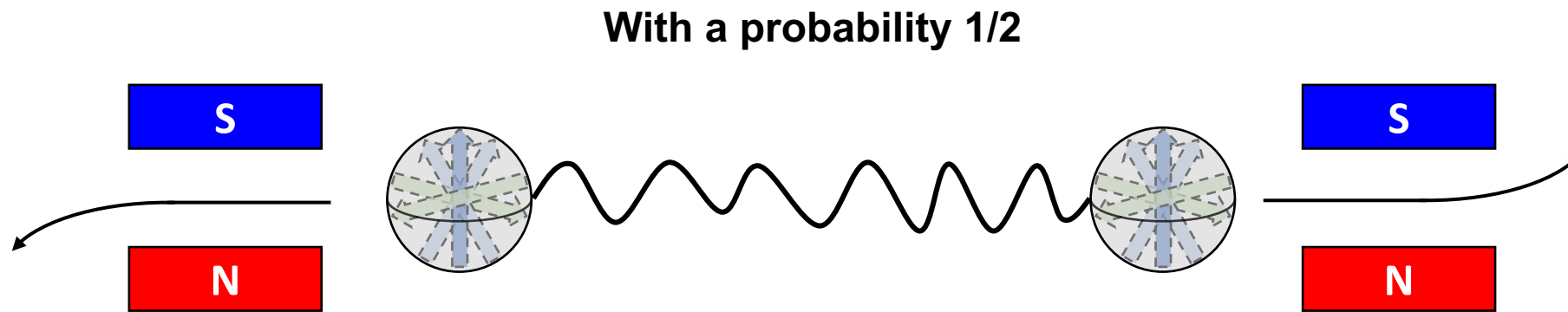
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Quantum entanglement



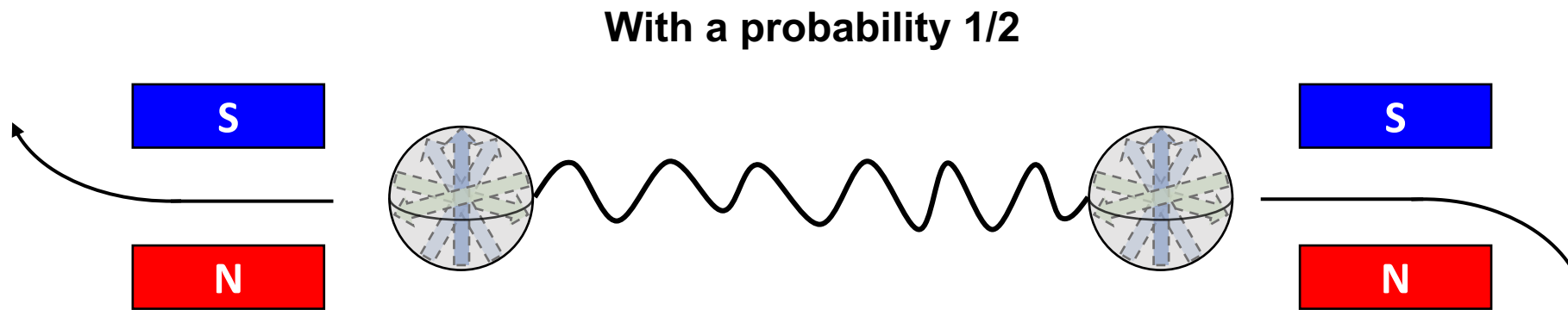
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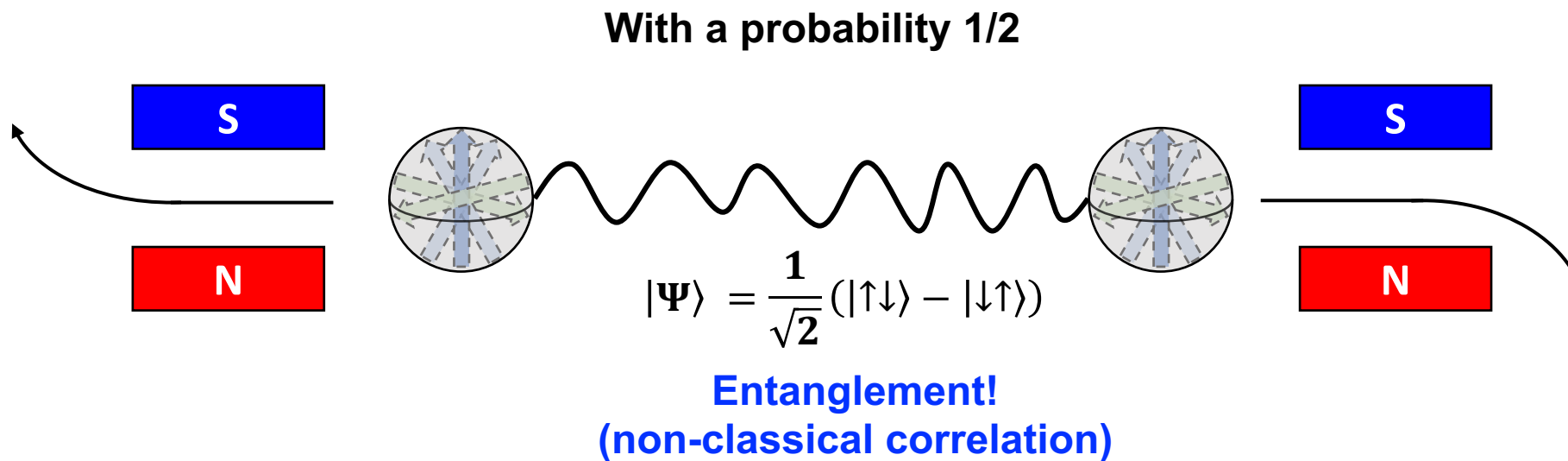
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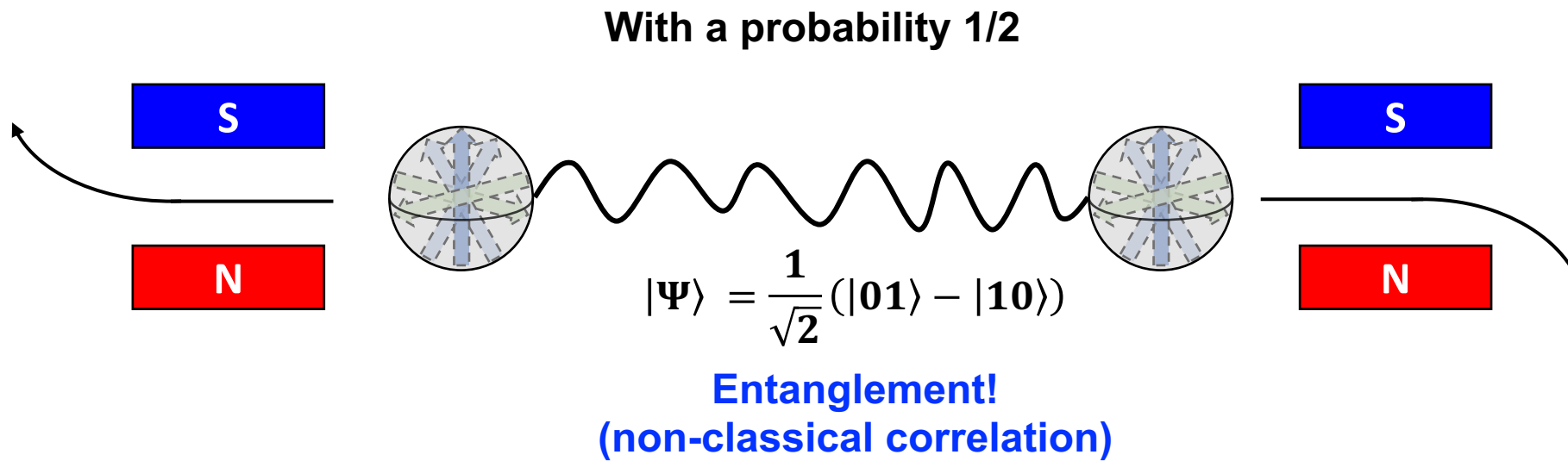
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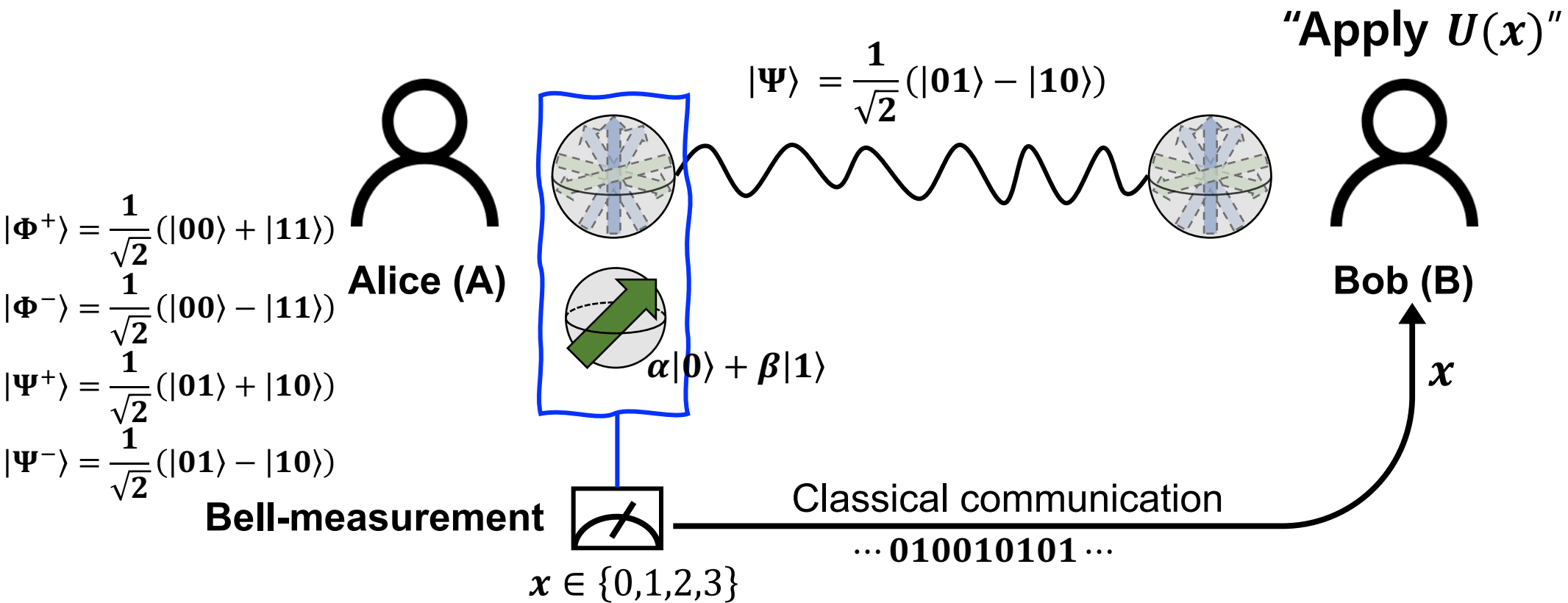
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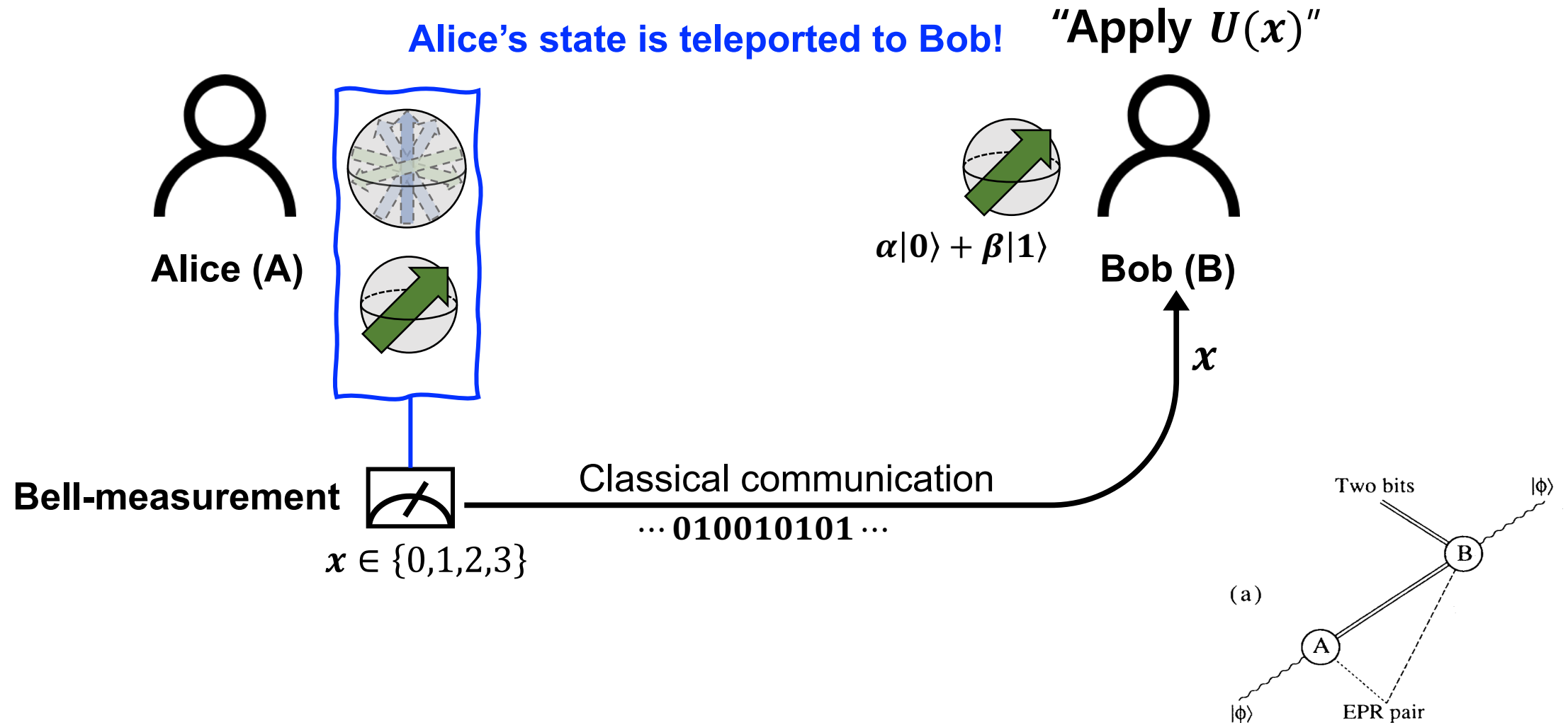
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Quantum teleportation



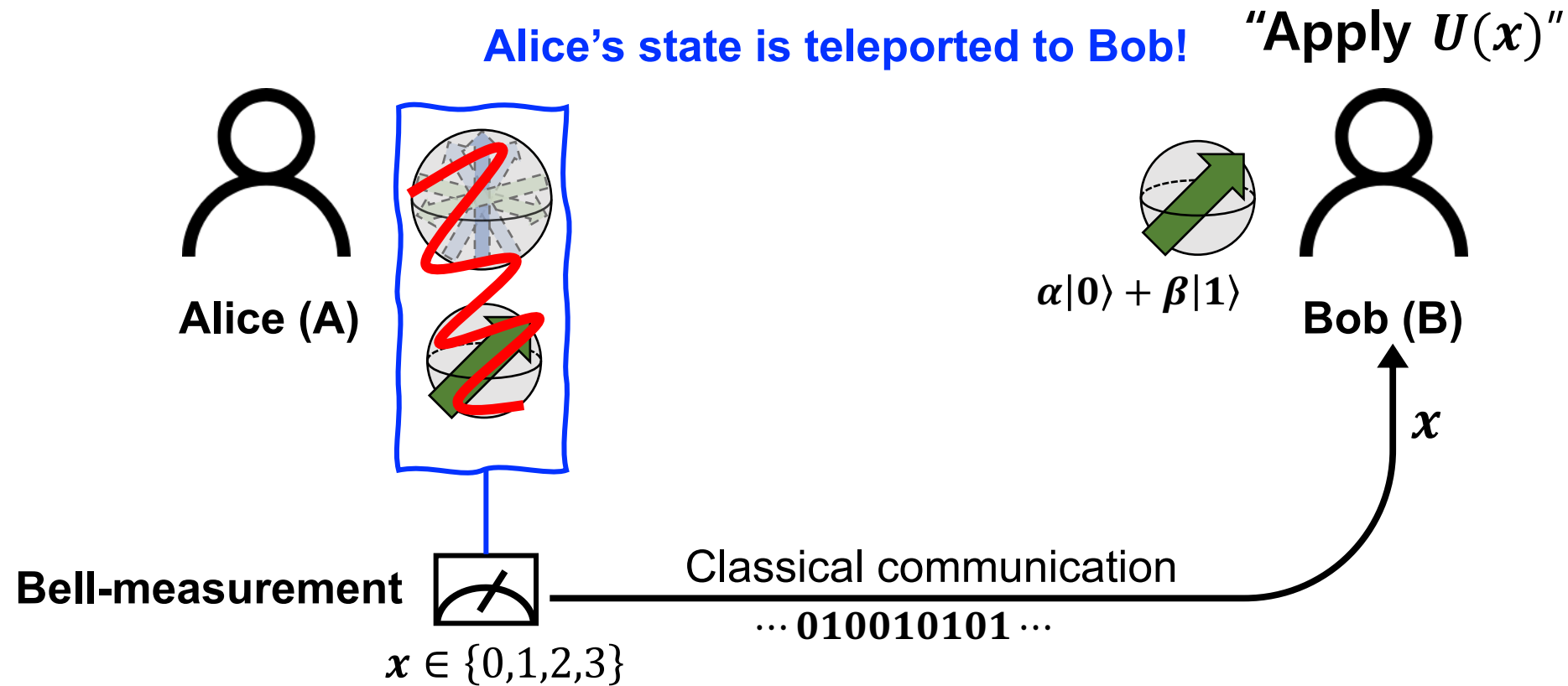
Quantum teleportation



"Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels"

C. H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W. K. Wootters, Phys. Rev. Lett. **70** 1895 (1993).

Quantum teleportation



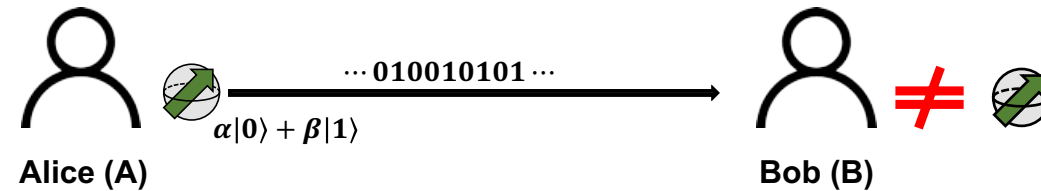
Note: 1. Quantum state is not copied (wave function collapse)
2. Not a superluminal signaling (classical communication)

"Teleporting an Unknown Quantum State via Dual Classical and Einstein-Podolsky-Rosen Channels"

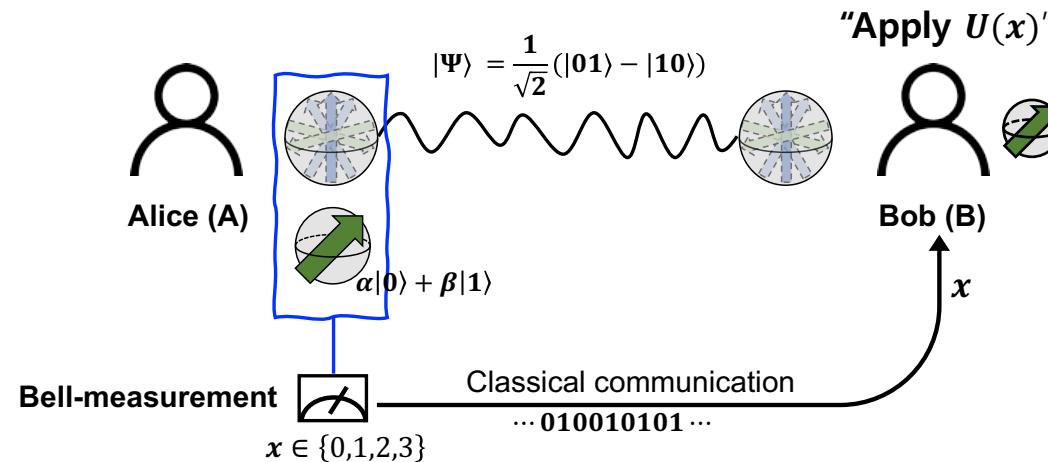
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Quantum communication

- Qubit cannot be transmitted via LOCC (local operation and classical communication)



- Entanglement makes this possible (Quantum teleportation)



→ Entanglement is a (unit) resource for quantum communication
[1-ebit (assisted by 2-bit) can transmit 1-qubit]

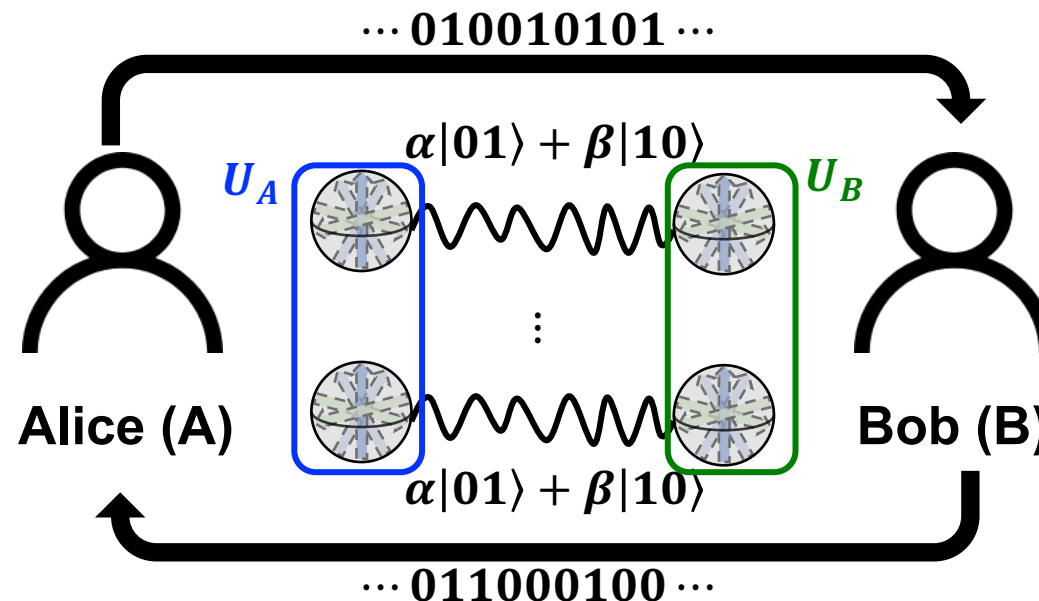
Quantifying entanglement

- A pair of perfectly entangled state (Bell-state) can transmit 1 qubit
- What happens if we have a partially entangled state? (e.g.) $|\Psi\rangle = \alpha|01\rangle + \beta|10\rangle$

If we prepare many copies of imperfect entangled states, we can **distill** the Bell-states via LOCC!

$$|\Psi\rangle^{\otimes N} \rightarrow |\Phi^+\rangle^{\otimes RN} \quad \text{for } N \rightarrow \infty \quad (|\Phi^+\rangle = \frac{|01\rangle + |10\rangle}{\sqrt{2}})$$

$$R = -|\alpha|^2 \ln |\alpha|^2 - |\beta|^2 \ln |\beta|^2 \leq 1: \text{Distillation rate}$$



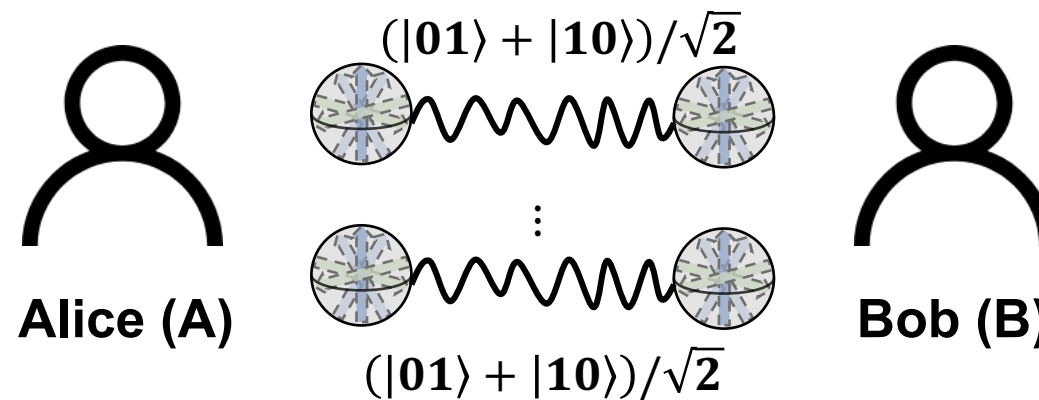
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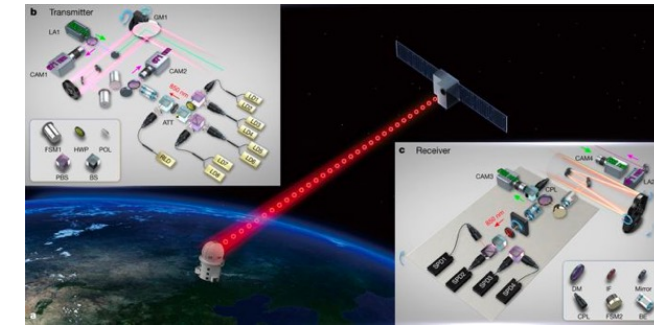
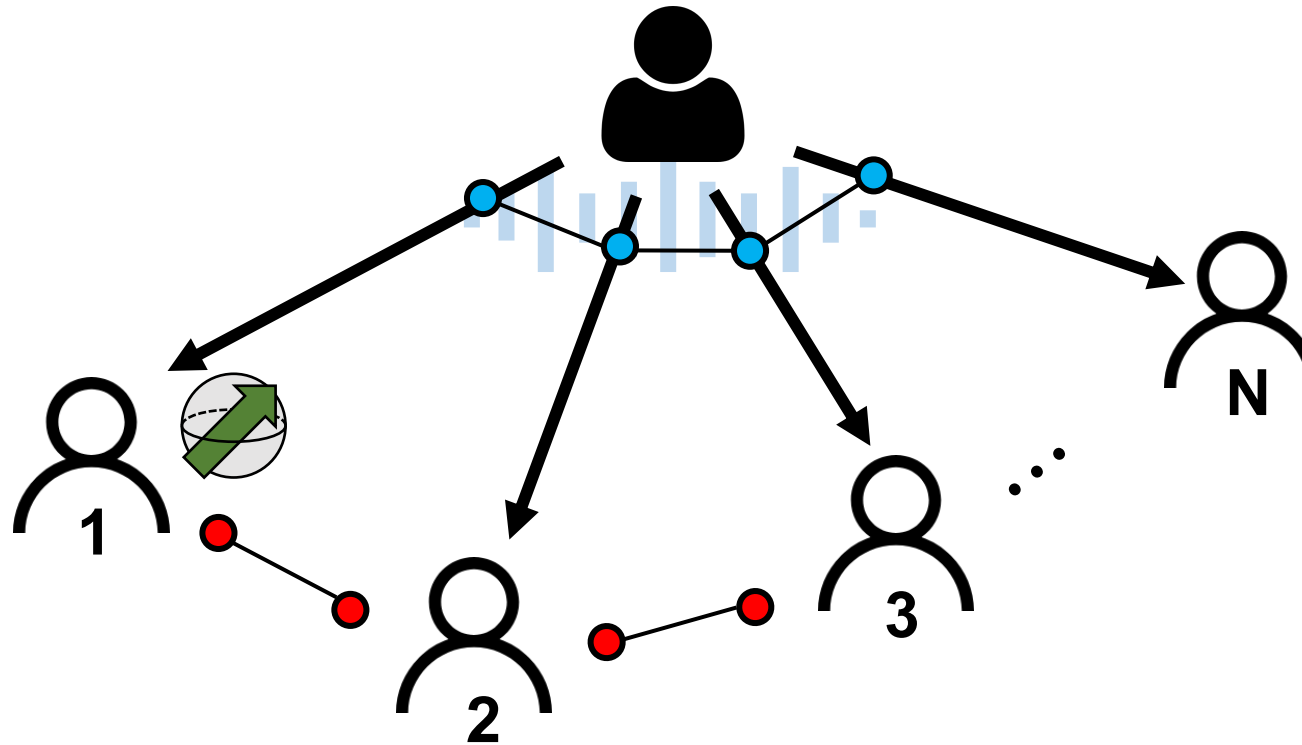


Fig. from Rev. Mod. Phys. 91, 025001 (2019)

Quantifying entanglement

- What happens if we have a noisy entangled state?
 - ✓ Decoherence can wash out **superposition**, thus the state can be no more entangled
(e.g.,) $|\Phi^+\rangle\langle\Phi^+| = \frac{1}{2}(|01\rangle\langle 01| + |01\rangle\langle 10| + |10\rangle\langle 01| + |10\rangle\langle 10|) \rightarrow \rho = \frac{1}{2}(|01\rangle\langle 01| + |10\rangle\langle 10|)$
no entanglement (classically correlated state)
 - ✓ Definition of (mixed-state) entanglement becomes more complicated ($\rho \neq \sum p_i \rho_A^{(i)} \otimes \rho_B^{(i)}$)
 - ✓ We can always distill entanglement from noisy for entangled noisy **qubit** states
but it no more holds for a higher dimension (**bound entanglement problem**)

Quantum network



S.-K Liao *et al.*, Nature **549**, 43 (2017)

Quantum network: Distributing quantum entanglement over world!

Quantum technologies

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[Sending quantum bits]

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- Quantum cryptography (quantum key distribution)

Quantum computing

[Faster computation]

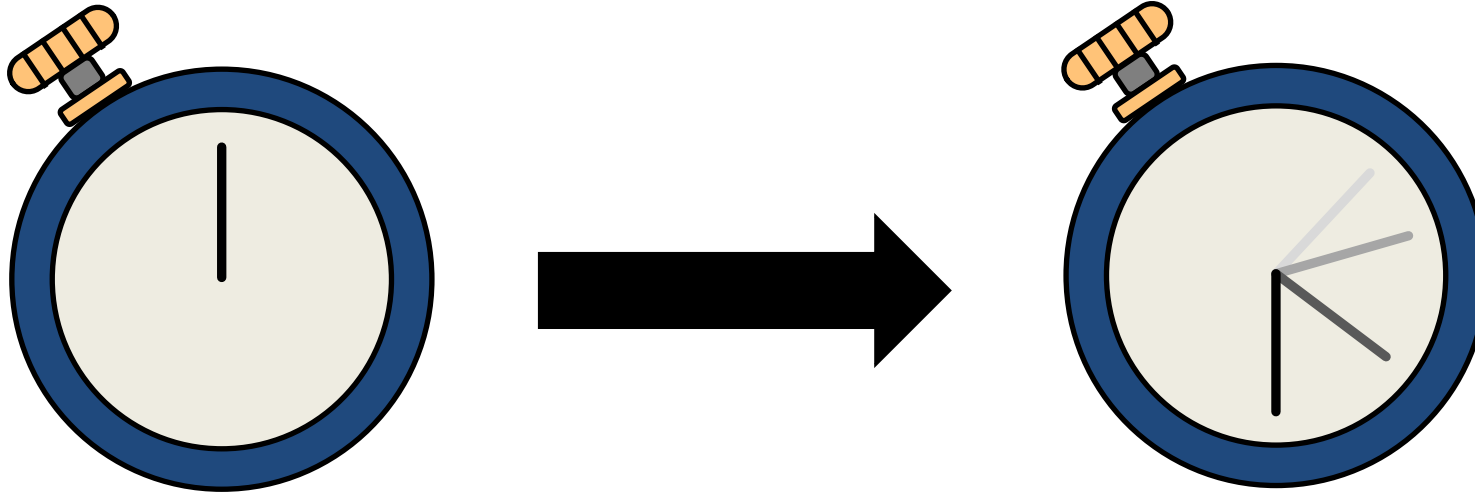
- Quantum simulation
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Quantum metrology

[Precision measurement]

- Precise phase/force sensing
- Accurate measurement of physical constants

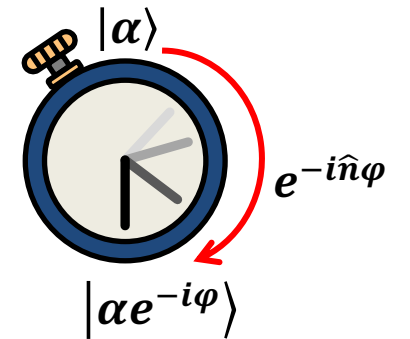
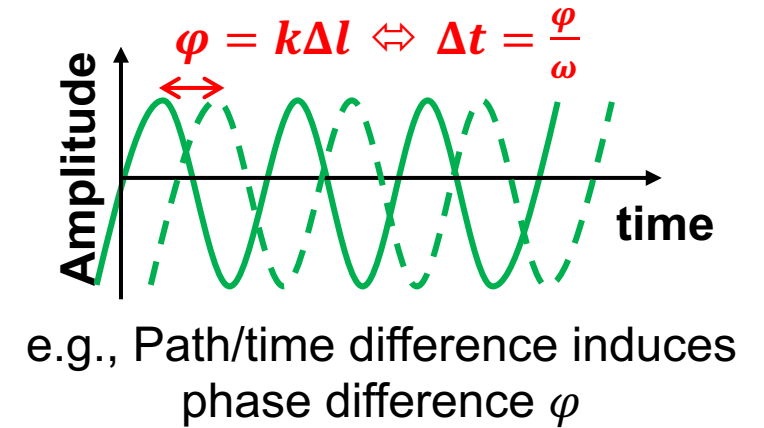
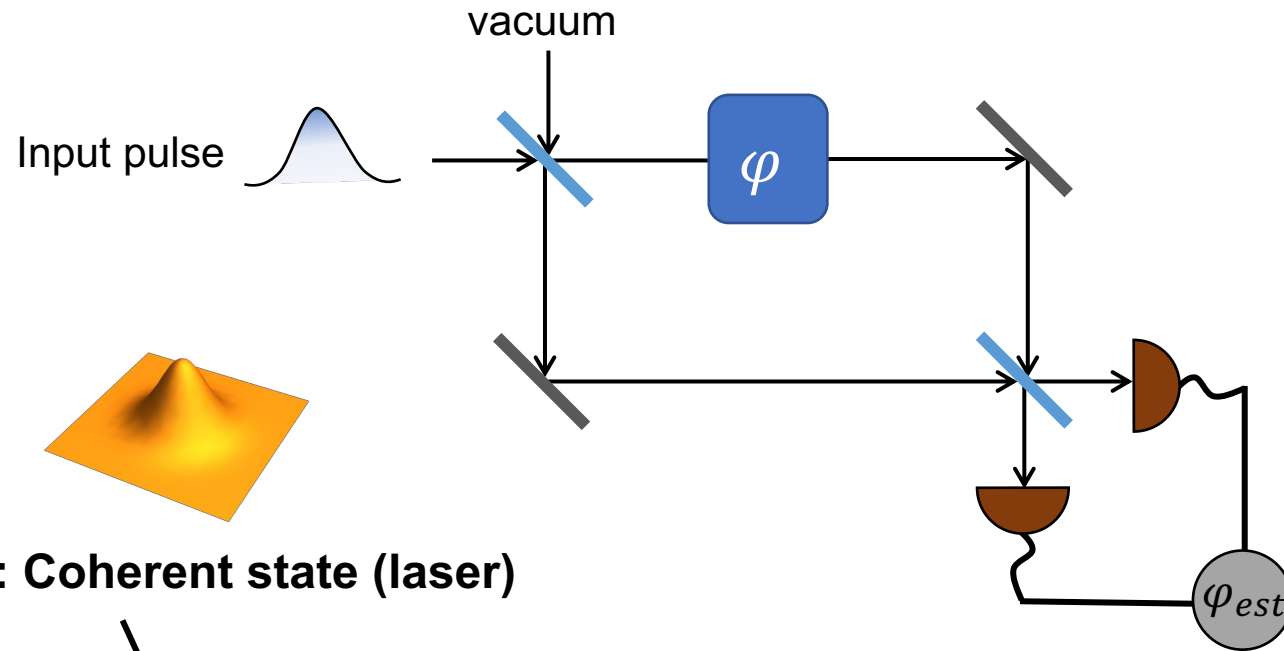
What is measurement?



Measurement is **detecting the difference**

Precise measurement → Detecting a small difference in physical quantities (time, distance)

Optical interferometer for phase sensing



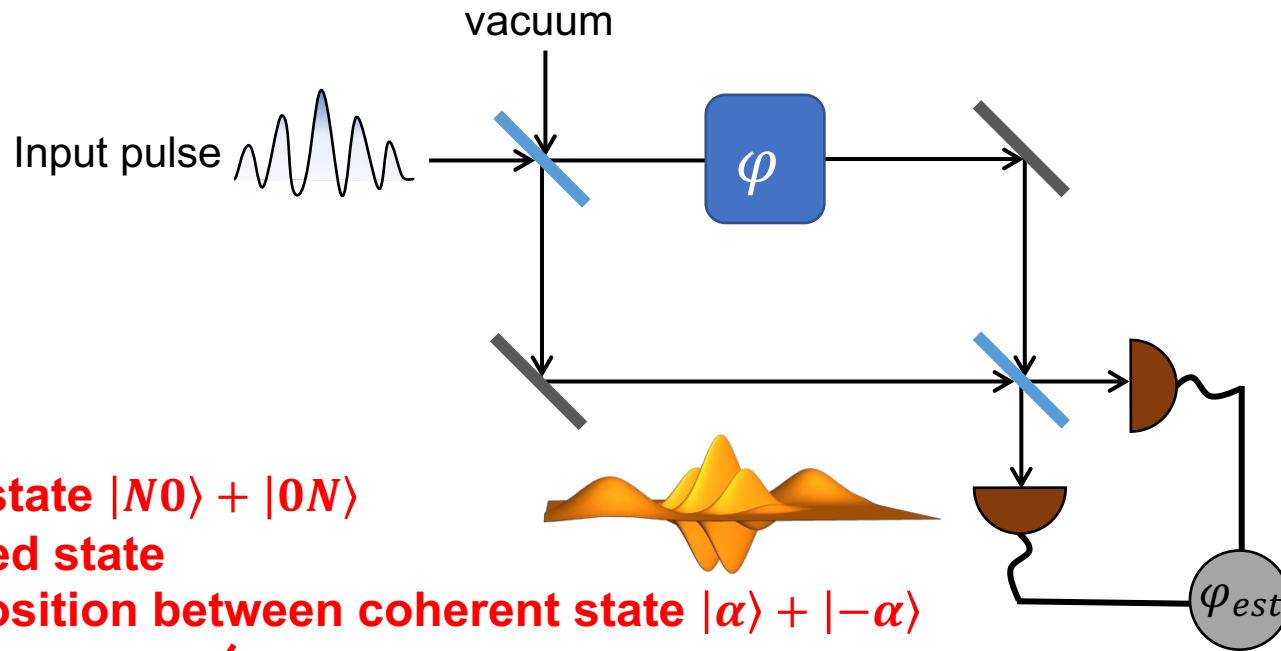
Input pulse: Coherent state (laser)

Classically, the sensitivity of phase estimation is bounded by

$$(\Delta\varphi)_{cl}^2 = \overline{|\varphi_{real} - \varphi_{est}|^2} \geq \frac{1}{\text{Energy}(\text{photon number})}$$

(standard quantum limit)

Quantum metrology

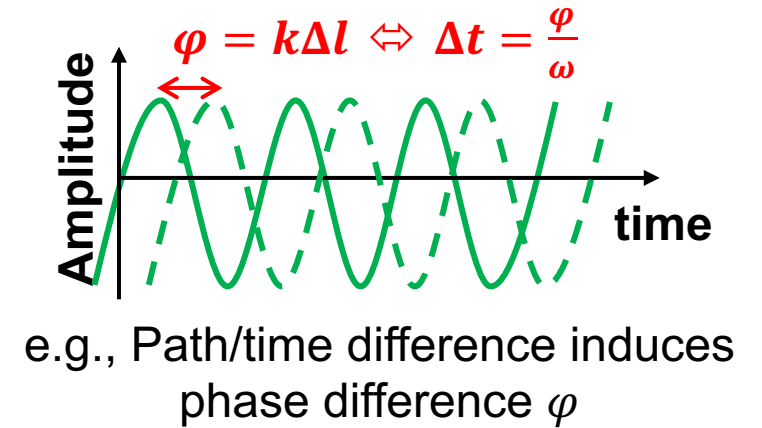


Input pulse:
 e.g.) NOON-state $|N0\rangle + |0N\rangle$
 Squeezed state
 Superposition between coherent state $|\alpha\rangle + |-\alpha\rangle$

If we use **non-classical states**, the sensitivity of phase estimation can be improved by

$$(\Delta\varphi)_{\text{qm}}^2 = \overline{|\varphi_{\text{real}} - \varphi_{\text{est}}|^2} \geq \frac{1}{(\text{Energy})^2}$$

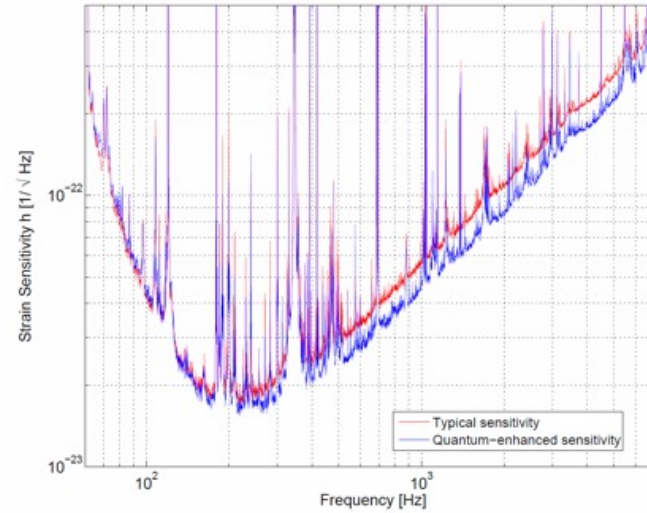
(Heisenberg limit)



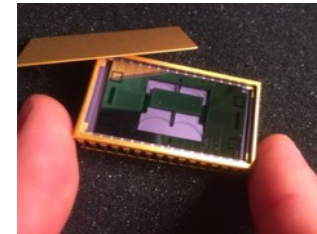
[Nature Photonics 5, 222–229 (2011)]

Applications

- LIGO interferometer + quantum metrology

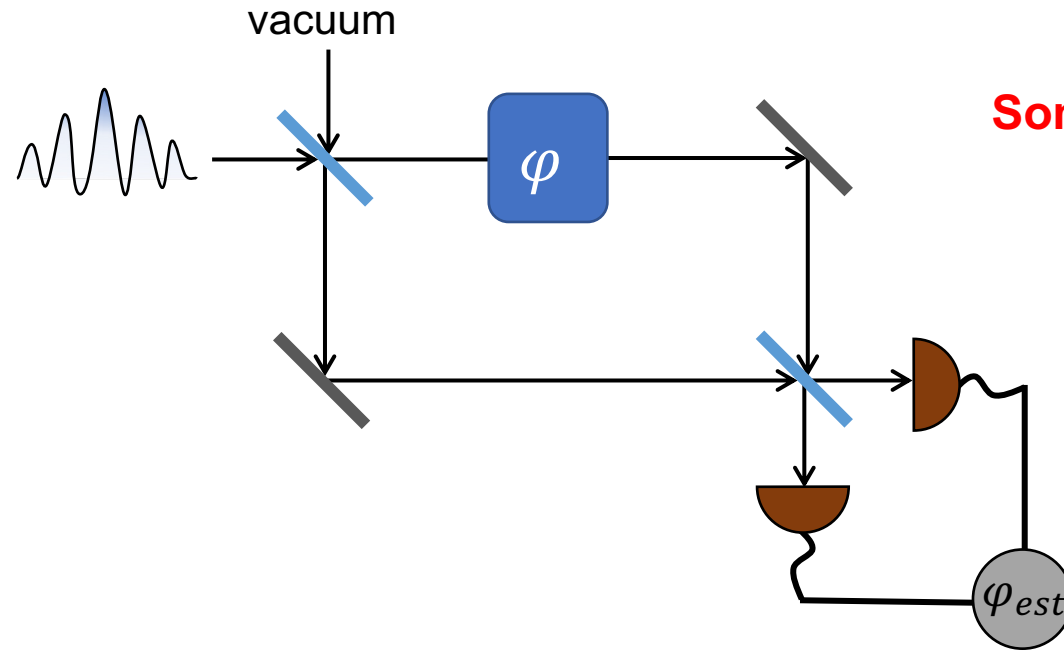


- Commercializing quantum sensing technique

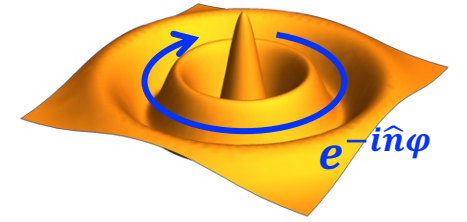


Resource for quantum metrology

Which kind of quantum states is resourceful?



Some entangled states, non-classical states
→ No quantum advantage



Fock state $|n\rangle$: phase insensitive
 $e^{-i\hat{n}\varphi}|n\rangle = |n\rangle$

What makes quantum advantage in sensing task (metrology)?

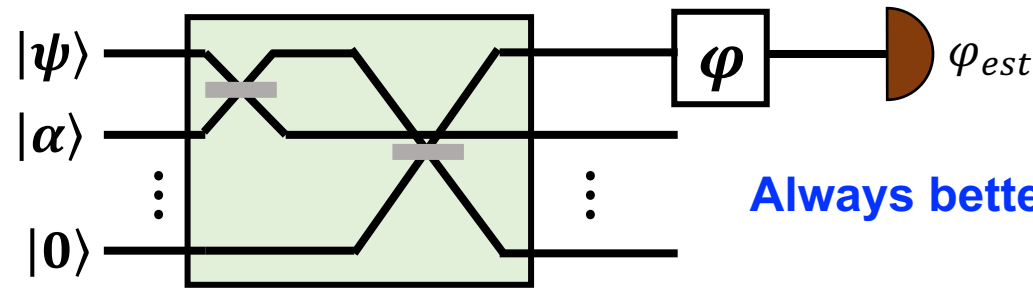
Can we quantify resource for quantum metrology?

Non-classicality and quantum metrology

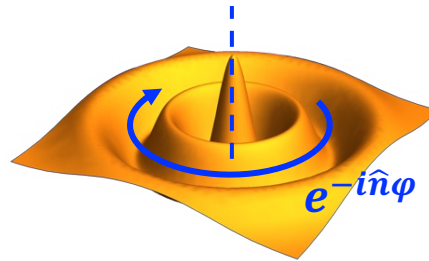
- **Non-classicality is indeed a resource!**

c.f.) Definition of non-classicality
= Negativity P-function in phase space

→ We can always convert any non-classical pure state to be useful in quantum metrology!
(using linear interferometer + additional coherent state)



Always better than the classical limit!

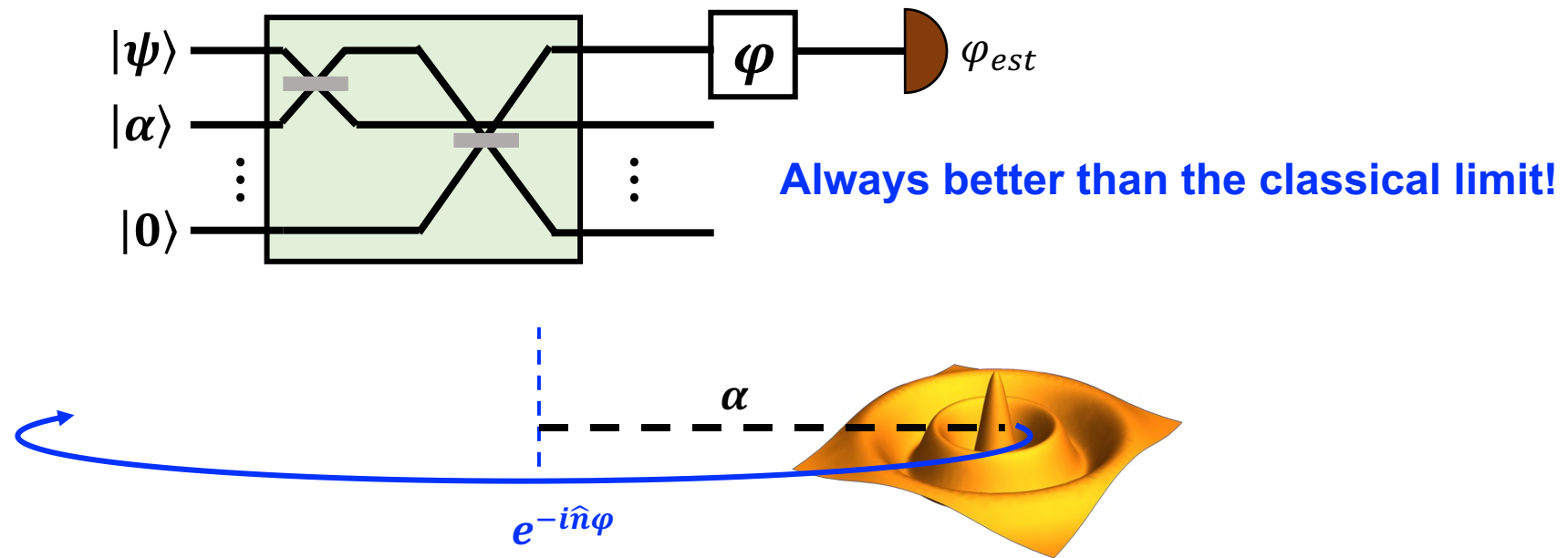


Non-classicality and quantum metrology

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Nonclassicality is necessary and sufficient condition for quantum metrology (for pure state)

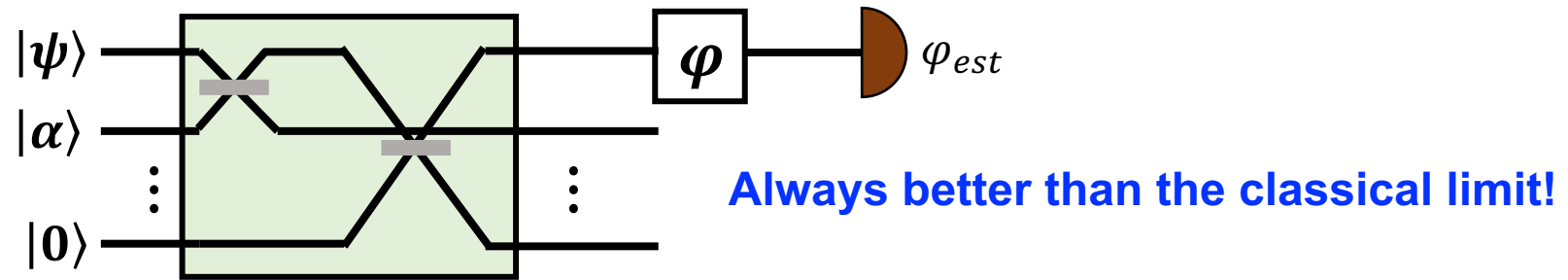
[HK, K. C. Tan, T. Volkoff, and H. Jeong, *Nonclassicality as a Quantifiable Resource for Quantum Metrology*, Phys. Rev. Lett. **122**, 040503 (2019)]

Non-classicality and quantum metrology

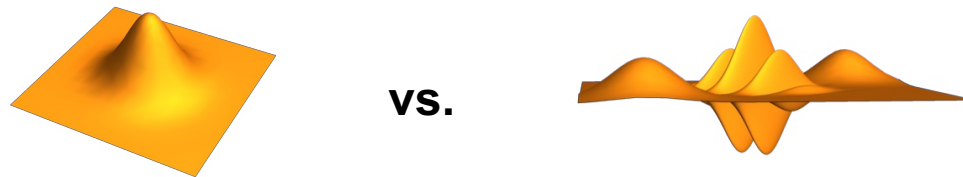
- **Non-classicality is indeed a resource!**

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- **Physical interpretation (why non-classicality?)**



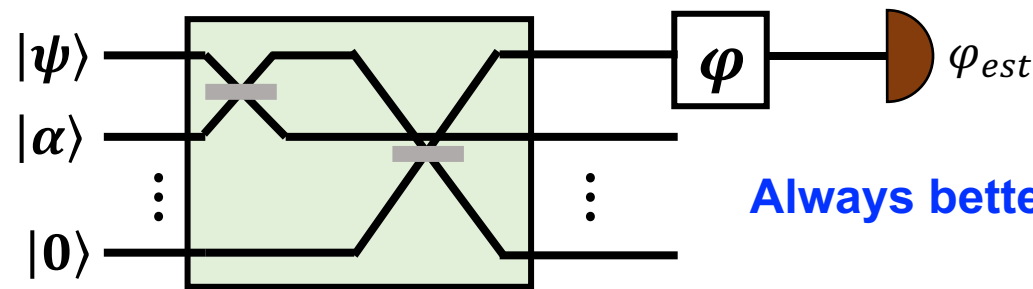
Negativity probability → Distribution can fluctuate (change) beyond classical limit → Higher sensitivity

Non-classicality and quantum metrology

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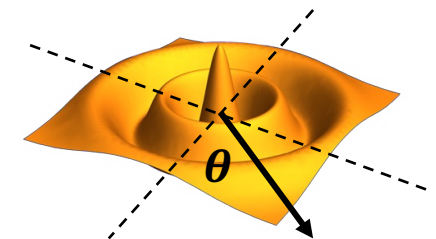


Always better than the classical limit!

- **How do we quantify negative probability?**

Simple way: Add all negative components ($N(\psi) = 1 - \int d^2\alpha |P_\psi(\alpha)|$)

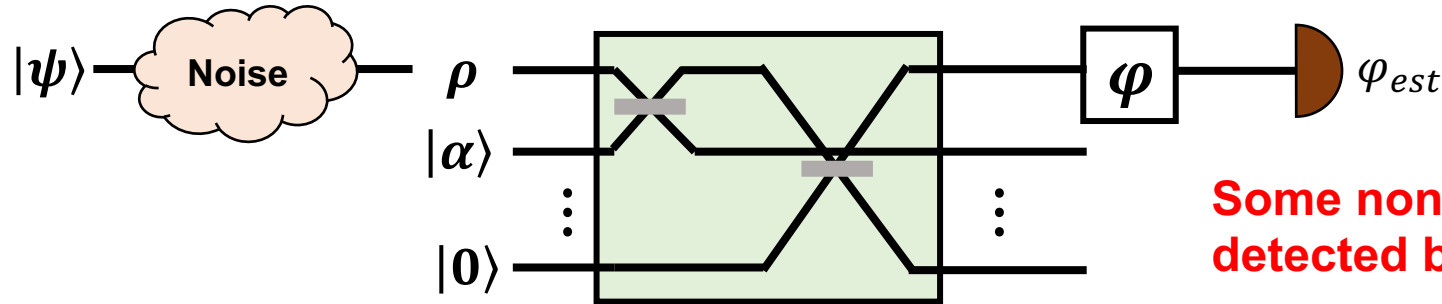
Indirect way: **Quantum Fisher information** $M(\psi) = \frac{1}{2} \max_\theta [I_F(\psi)] - 1$



Indicator of displacement sensitivity

Non-classicality and quantum metrology

- What happens when we have a noisy state (decoherence)?

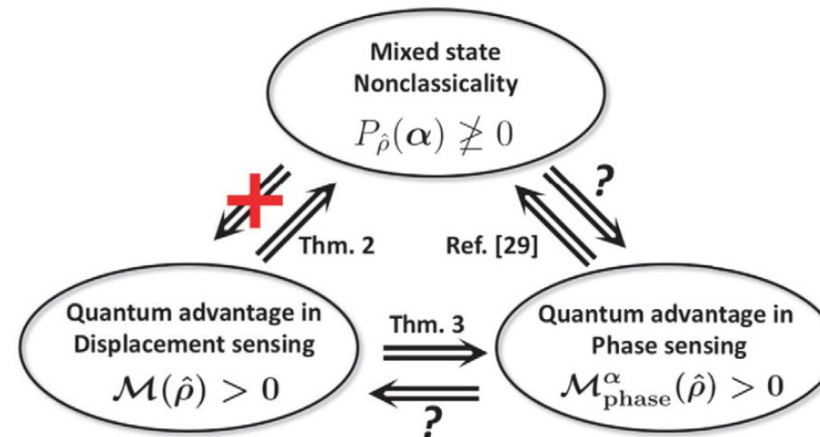


Some nonclassical states cannot be detected by quantum Fisher information $\mathcal{M}(\rho)$

(recall: bound entanglement)

→ Non-classicality may not be sufficient to have a quantum advantage in metrology

But it is still a necessary condition for mixed states



(c.f.) Detecting the non-classicality of mixed states is as complicated as detecting entanglement

Quantum technologies

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Quantum computing

[Faster computation]

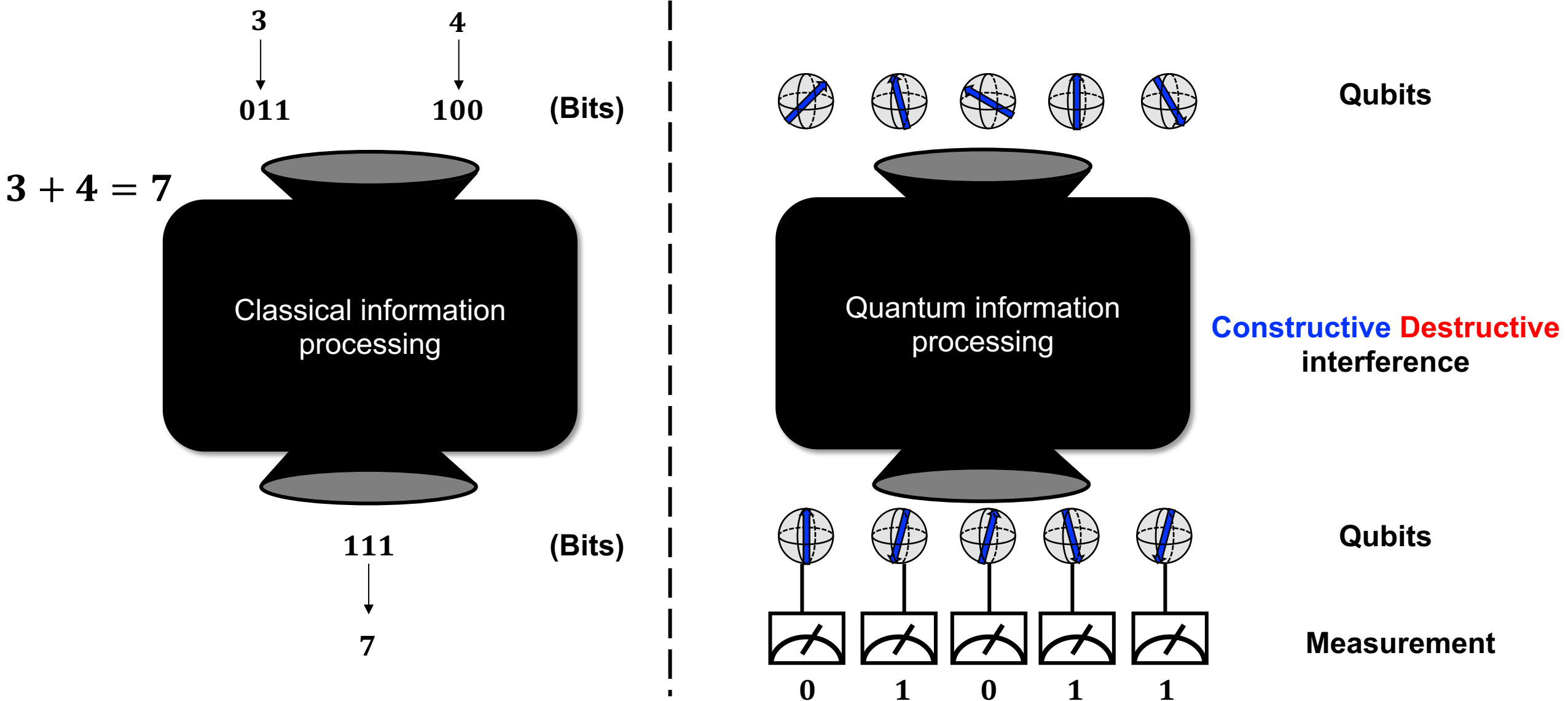
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Quantum metrology

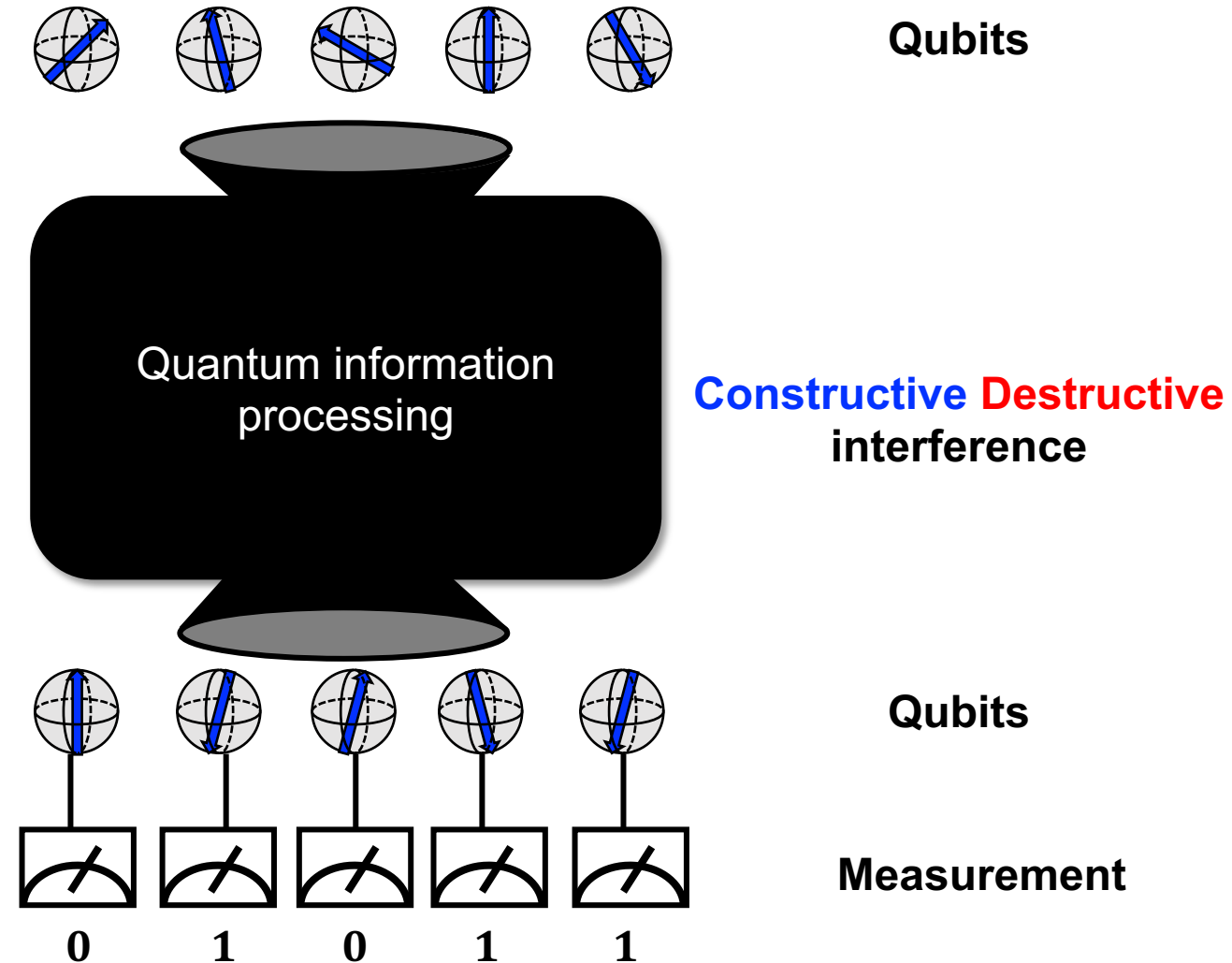
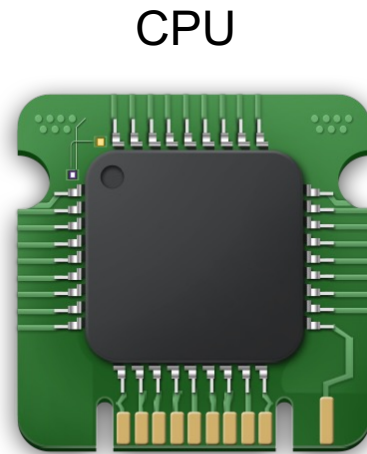
[Precision measurement]

- Precise phase/force sensing
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Quantum computing in a nutshell

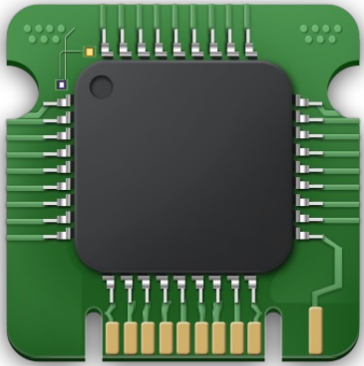


Quantum computing in a nutshell

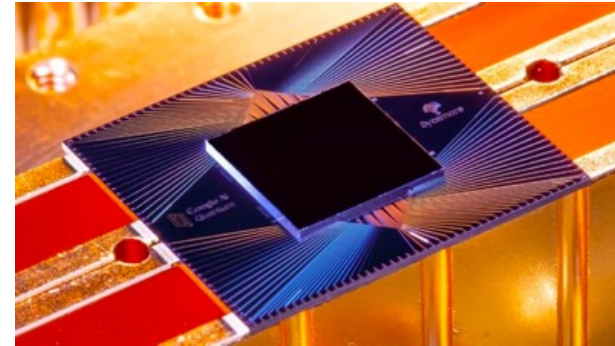


Quantum computing in a nutshell

CPU



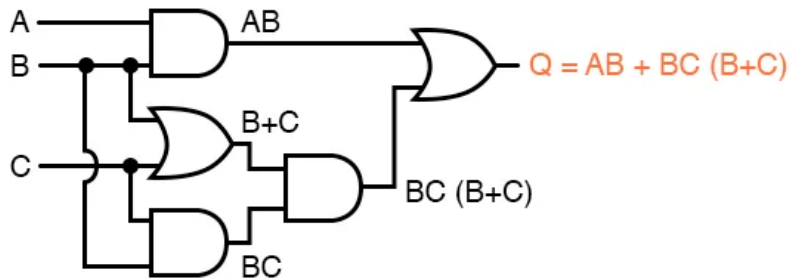
Quantum processor



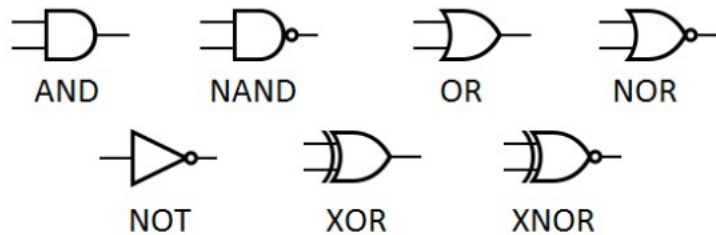
Sycamore processor (Google)

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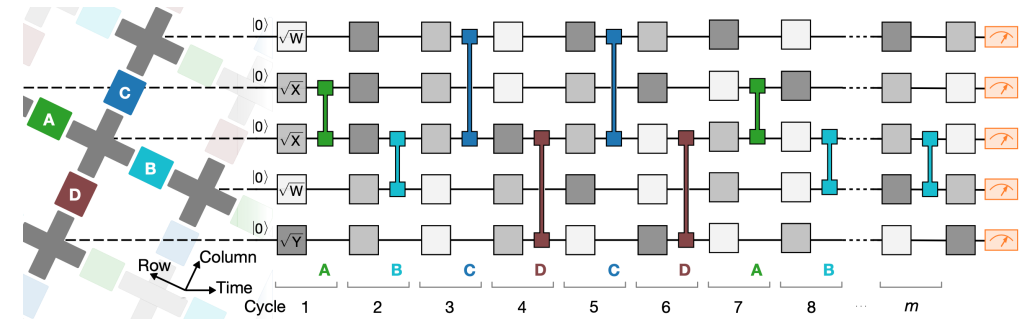
Classical circuit



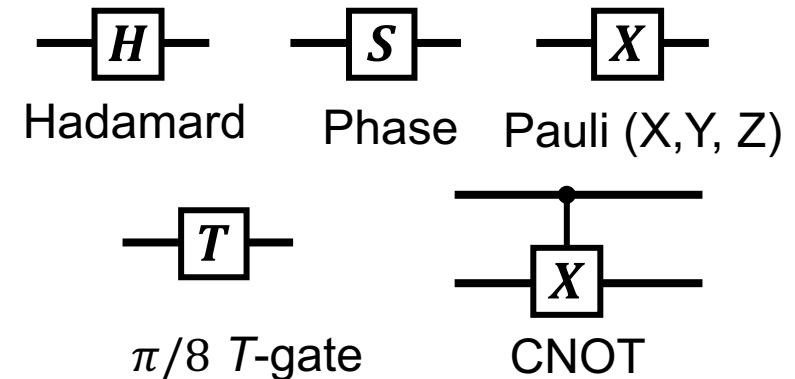
Classical (logic) gates



Quantum circuit

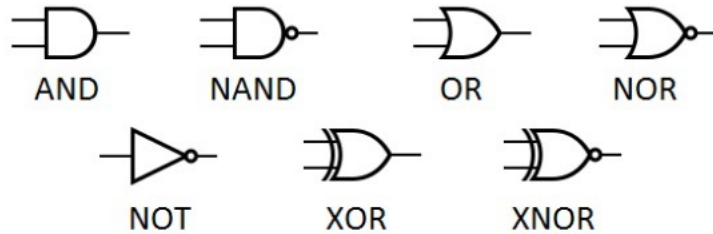


Quantum (logic) gates

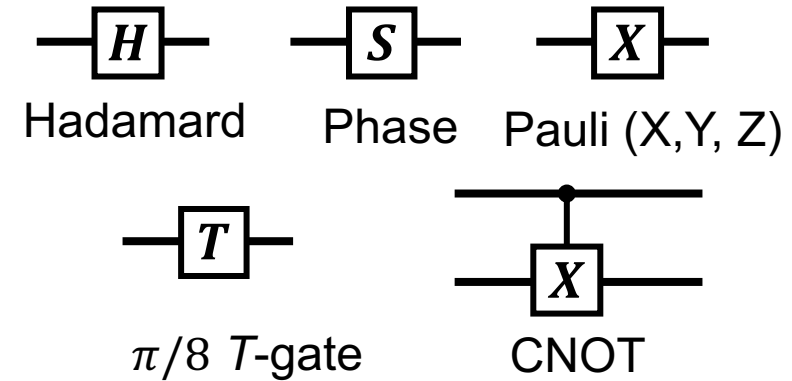


Quantum Logic Gates

Classical (logic) gates



Quantum (logic) gates



How to arrange quantum logic gates
to solve a given problem

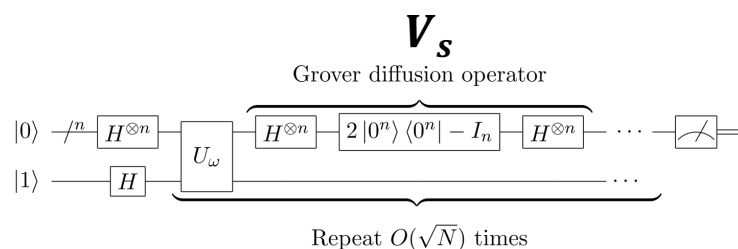


Quantum algorithm

Quantum algorithms

Grover's search algorithm

Database searching problem



Classical: $O(N)$

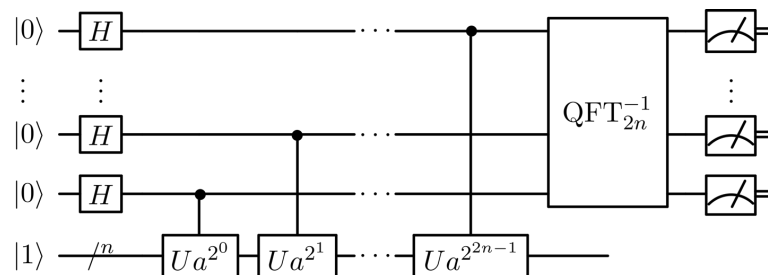
Quantum: $O(\sqrt{N})$

(polynomial speed up)

1000000 classical operations vs.
1000 quantum operations

Shor's factorization algorithm

Integer factoring problem ($15 = 3 \times 5$)
→ Can break RSA cryptosystem



Fast computation of frequency
(quantum Fourier transformation)

$$|x\rangle \rightarrow \frac{|0\rangle + e^{\left(\frac{2\pi i}{N}\right)x}|1\rangle + \dots + e^{\left(\frac{2\pi i}{N}\right)x \cdot (N-1)}|N-1\rangle}{\sqrt{N}}$$

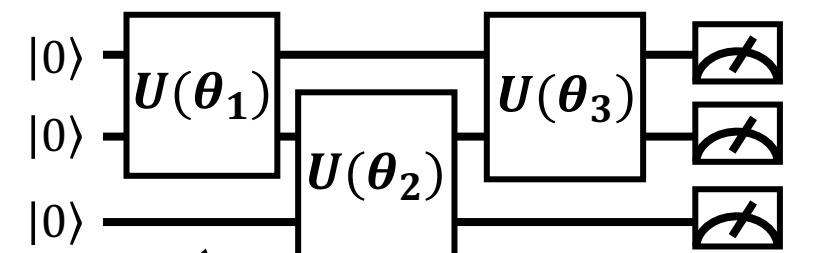
Classical: $O(\text{poly}(N))$ or $O\left(e^{\sqrt[3]{\log N}}\right)$

Quantum: $O(\text{poly}(\log N))$

(exponential speed up)

Variational quantum eigensolver

Calculate ground state energy $\langle H \rangle_{\min}$



Update
 $\theta_1, \theta_2, \theta_3$

$$C = \langle H \rangle_{\theta} = \langle \psi_{\theta} | H | \psi_{\theta} \rangle$$

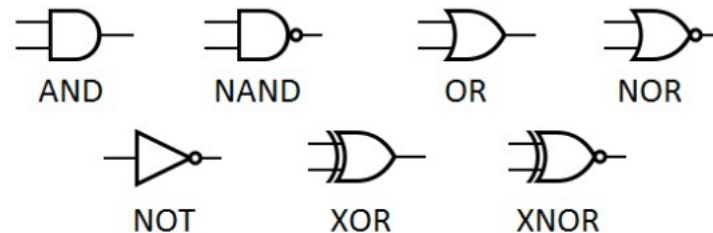
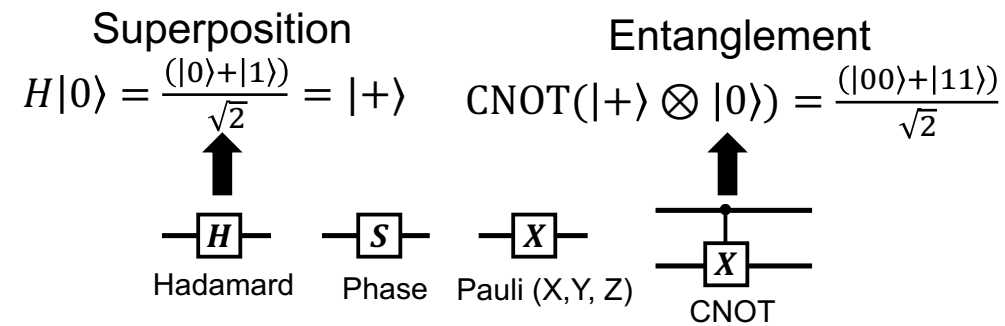
Classical
optimizer

$$\vec{\nabla} C(\theta_1, \theta_2, \theta_3)$$

Quantum-classical
hybrid algorithm

Resource for quantum computing

- Can we obtain (exponential) quantum speed up only with entanglement?

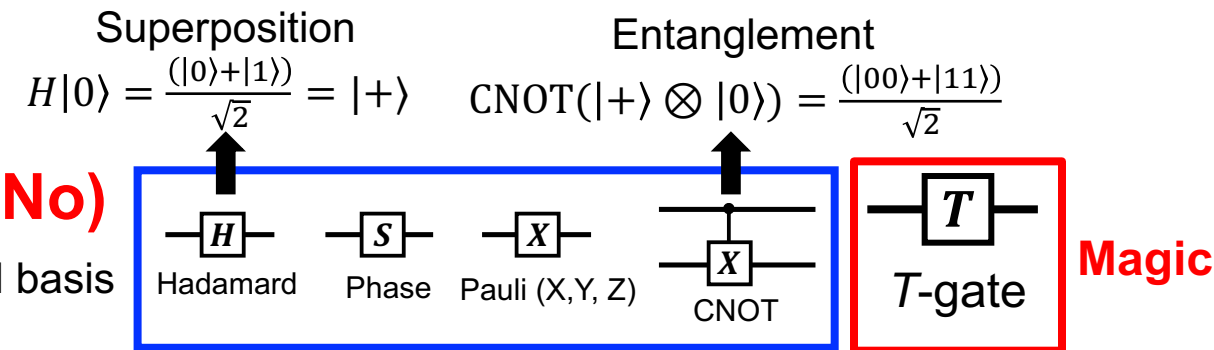


Resource for quantum computing

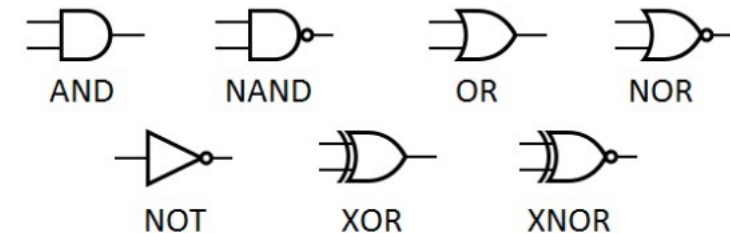
- Can we obtain (exponential) quantum speed up only with entanglement?

- **Gottesman-Knill theorem (the answer is No)**

“Quantum circuits that only involve the Clifford gates & computational basis can be **efficiently simulated on a classical computer**”



Clifford gates

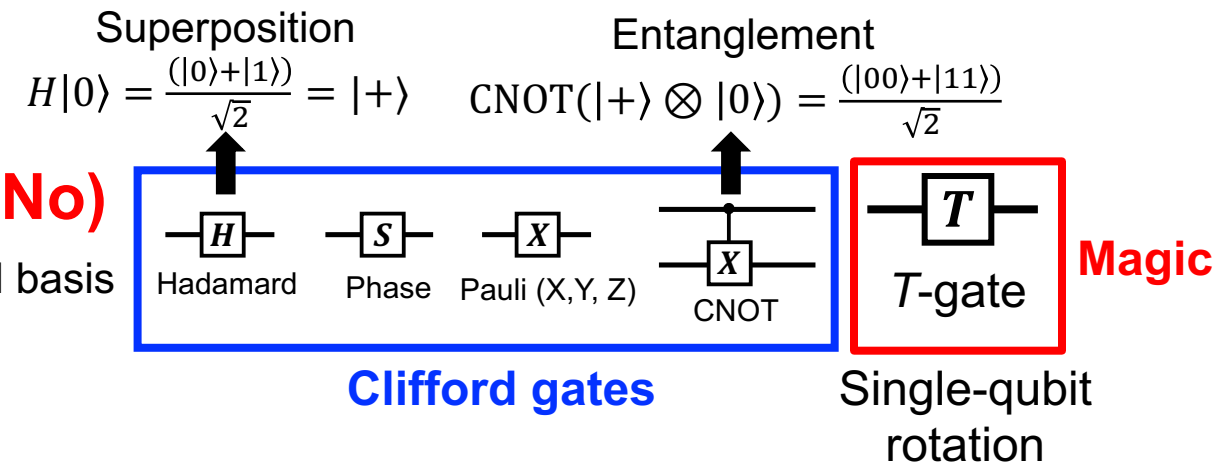


Resource for quantum computing

- Can we obtain (exponential) quantum speed up only with entanglement?

- **Gottesman-Knill theorem (the answer is No)**

“Quantum circuits that only involve the Clifford gates & computational basis can be **efficiently simulated on a classical computer**”



- Then, where does the “quantum” computing power come from?

Honest answer: We do not know yet

Resource for quantum computing

- Then, where does the “quantum” computing power come from?

Contextuality!?

ARTICLE [Nature **510**, 351 (2014)]

doi:10.1038/nature13460

Contextuality supplies the ‘magic’ for quantum computation

Mark Howard^{1,2}, Joel Wallman², Victor Veitch^{2,3} & Joseph Emerson²

Bell–Kochen–Specker theorem (1966)

Measurement of quantum observables cannot simply reveal pre-existing values
(i.e., Realistic hidden variable models fail to describe our nature)

- Example) Peres-Mermin Square

$$\begin{array}{ccc} \left[\begin{array}{ccc} A & B & C \\ a & b & c \\ \alpha & \beta & \gamma \end{array} \right] & \begin{array}{l} \rightarrow +1 \\ \rightarrow +1 \\ \rightarrow +1 \end{array} \\ \begin{array}{ccc} \downarrow & \downarrow & \downarrow \\ +1 & +1 & -1 \end{array} \end{array}$$

Can we find a configuration such that
 $\langle ABC \rangle = \dots = \langle \alpha\beta\gamma \rangle = +1 = -\langle Cc\gamma \rangle$?

Classically impossible!

$$\left[\begin{array}{ccc} \sigma_z \otimes \mathbb{1} & \mathbb{1} \otimes \sigma_z & \sigma_z \otimes \sigma_z \\ \mathbb{1} \otimes \sigma_x & \sigma_x \otimes \mathbb{1} & \sigma_x \otimes \sigma_x \\ \sigma_z \otimes \sigma_x & \sigma_x \otimes \sigma_z & \sigma_y \otimes \sigma_y \end{array} \right]$$

It is possible quantum-mechanically!

Resource for quantum computing

- Then, where does the “quantum” computing power come from?

Contextuality!?

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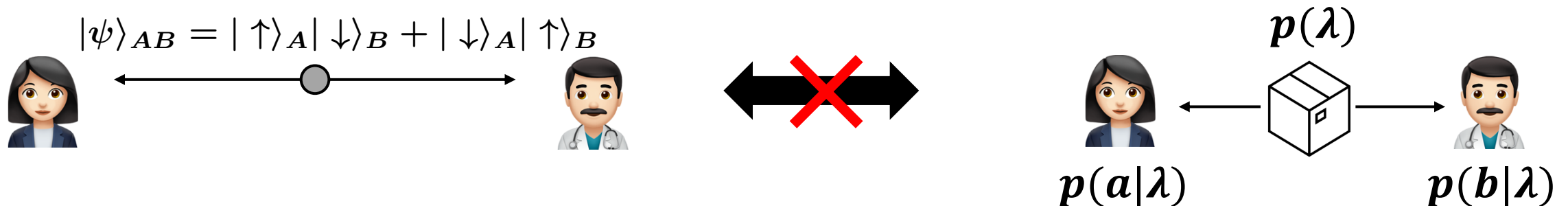
Contextuality supplies the ‘magic’ for quantum computation

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Measurement of quantum observables cannot simply reveal pre-existing values
(i.e., Realistic hidden variable models fail to describe our nature)

- Entanglement (Bell non-locality)



Resource for quantum computing

- Then, where does the “quantum” computing power come from?

Contextuality!?

[ARTICLE](#) [Nature **510**, 351 (2014)]

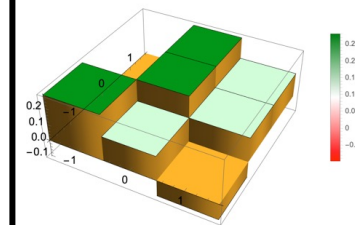
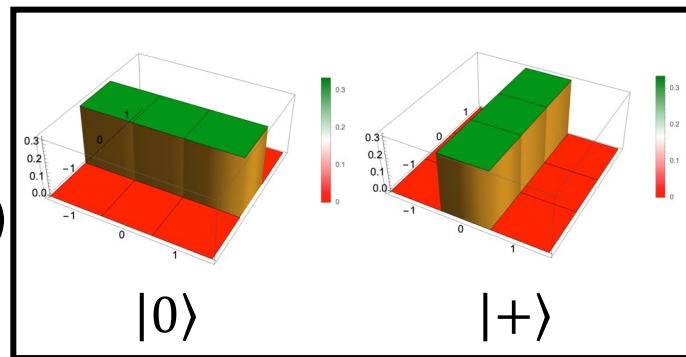
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Contextuality supplies the ‘magic’ for quantum computation

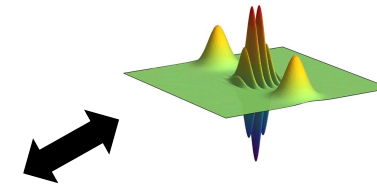
Mark Howard^{1,2}, Joel Wallman², Victor Veitch^{2,3} & Joseph Emerson²

- 0) Non-contextual state $\Leftrightarrow$ positive distribution in phase space $\Leftrightarrow$ Classically simulable
- 1) Magic gate can be converted to magic state + Clifford gates
- 2) Magic state has **negative** distribution

**Stabilizer state
(classically simulable)**



**|T>: Magic state
→ Negativity!**



But this is not a complete answer (this argument is true only for $d = 3, 5, 7, \dots$)

Recent progress

[S. Bravyi, D. Gosset, R. Koenig, Science **362**, 308 (2018)]

[S. Bravyi *et al.*, Nat. Phys. **16**, 1040 (2020)]

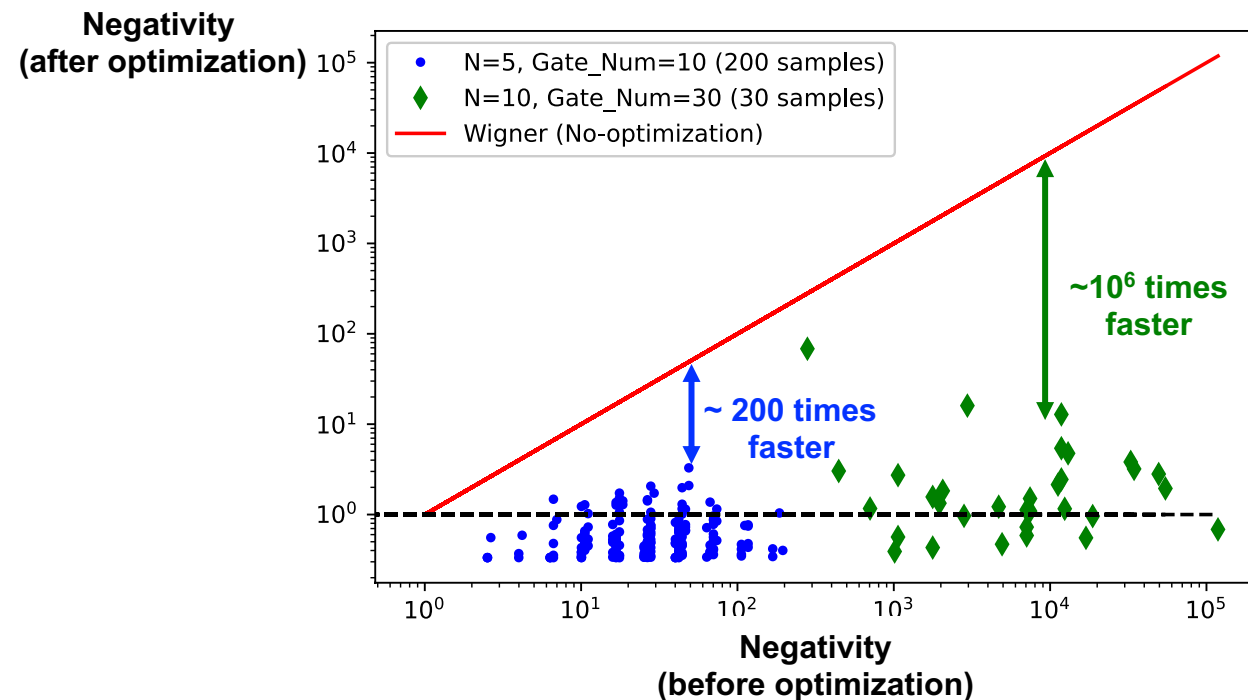
→ Quantum algorithms with a shallow circuit which has a strict quantum advantage!

Can we go beyond the Clifford framework?

[H. Pashayan, J. J. Wallman, and S. D. Bartlett, Phys. Rev. Lett. 115, 070501 (2015)]

- **Negativity $\rightarrow$ Overhead cost for classical simulation** (not exactly quantifying a speed up)
- There are many different phase-space representations (Clifford is one of them)
- Can we reduce the negativity by taking different phase space structure?

The answer is YES!



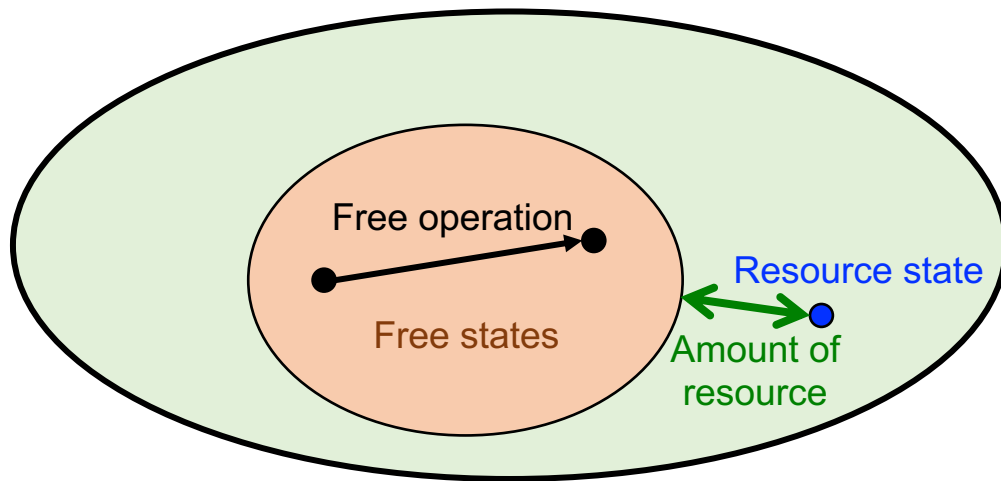
$\rightarrow$ Quantum circuits can be classically simulated faster after optimizing the negativity!

Born probability estimation via gate merging and dynamic frame optimisation

Nikolaos Koukoulekidis¹, Hyukjoon Kwon^{1,2}, Hyejung H. Jee³, David Jennings^{4,1}, and M. S. Kim^{1,2}

General framework of quantum resource theory

- In order to quantify quantum resources, we need
 - 1) Set of operations than can be done classically (Free operation)
 - 2) Set of quantum states that do not have a quantum advantage (Free state)
- Then,
 - 0) If a quantum state is a free state → No quantum advantage
 - 1) If a quantum state is not a free state → It contains non-classical resource
 - 2) How far it is from the free states quantifies the amount of resource



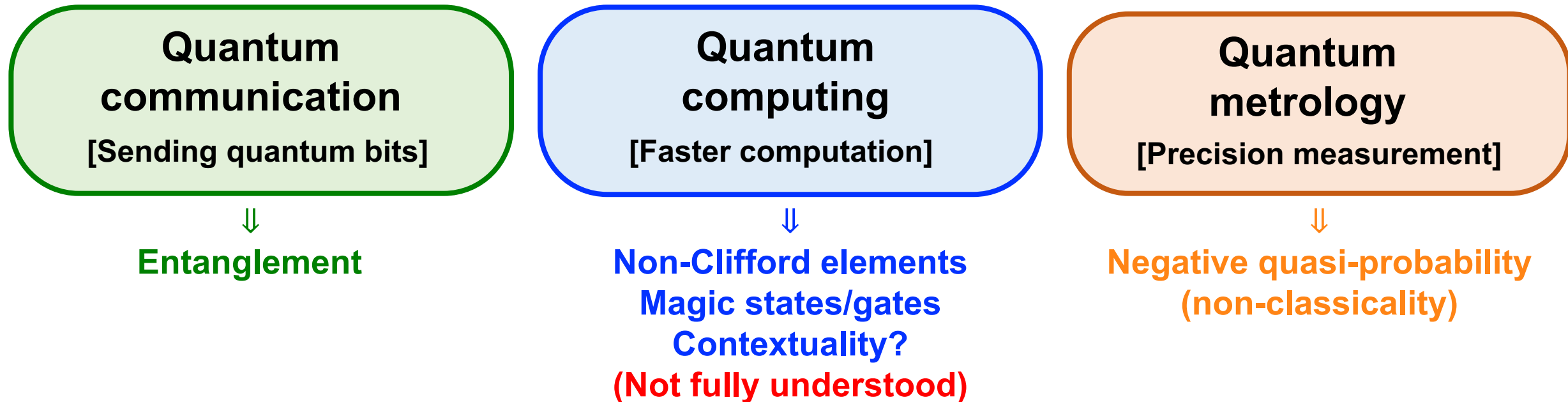
Task	Free operation	Free states	Quantum resources
Quantum communication	LOCC	Separable states ($\rho_{AB} = \sum_i p_i \rho_A^{(i)} \otimes \rho_B^{(i)}$)	Entanglement
Quantum metrology	Linear interferometer	Classical states (positive distribution)	Non-classicality
Quantum computing	Clifford gates	Stabilizer state (positive distribution)	Contextuality

Coherence
Non-Gaussianity
⋮

For an overview: [Rev. Mod. Phys. 91, 025001 (2019)]

Summary

- Quantum resource: Something that we can use to do a **“non-classical” task**
- For different quantum technologies, we need different types of quantum resources



- Open questions
 - ✓ How to efficiently distill/manipulate useful quantum resources (quantum thermodynamics)
 - ✓ How to protect quantum information efficiently against noise (quantum error-correction)

Thank you