On Dualities in QFTs and String theory

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POSTECH

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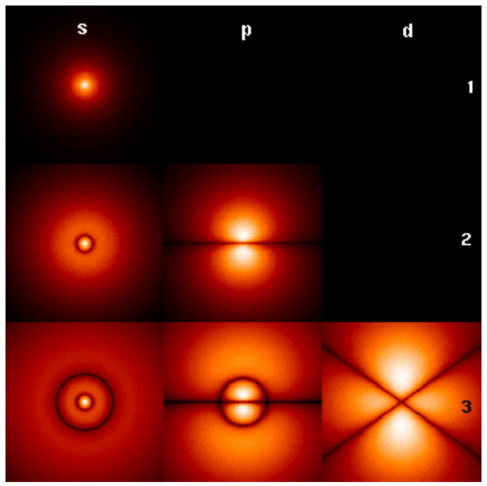
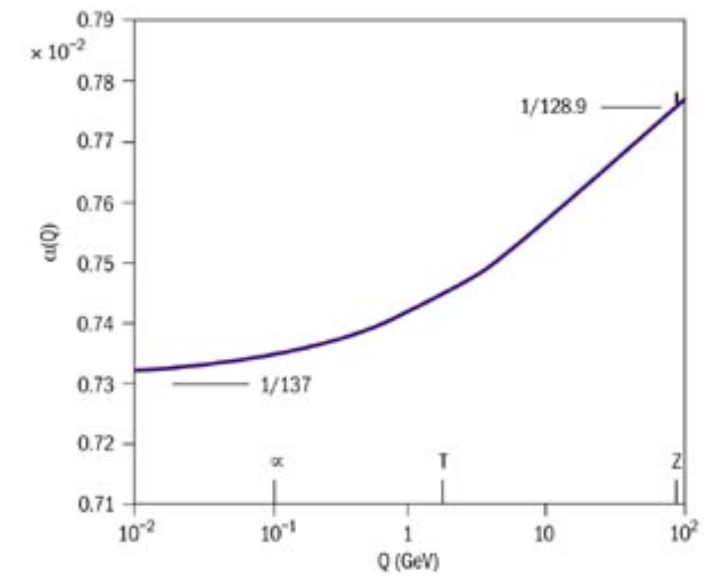
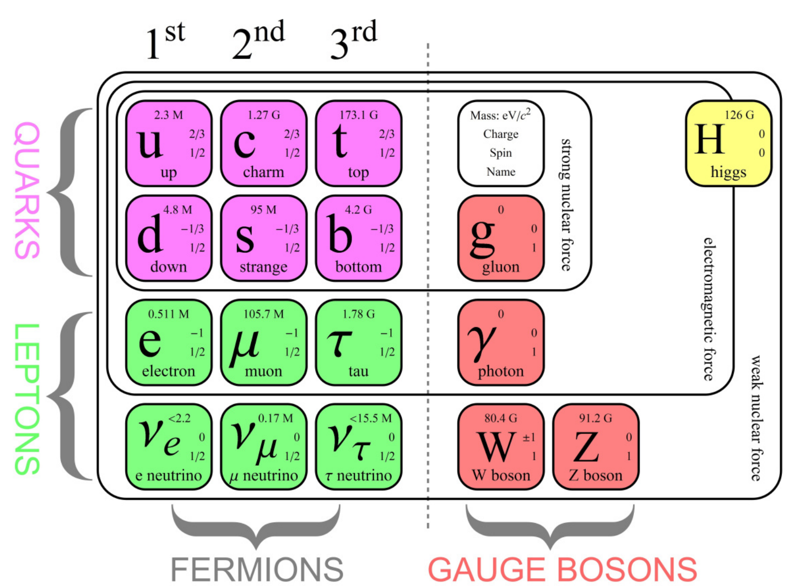
Colloquium at SNU



Electron density in Hydrogen Atom

Renormalization of fine structure constant in QED

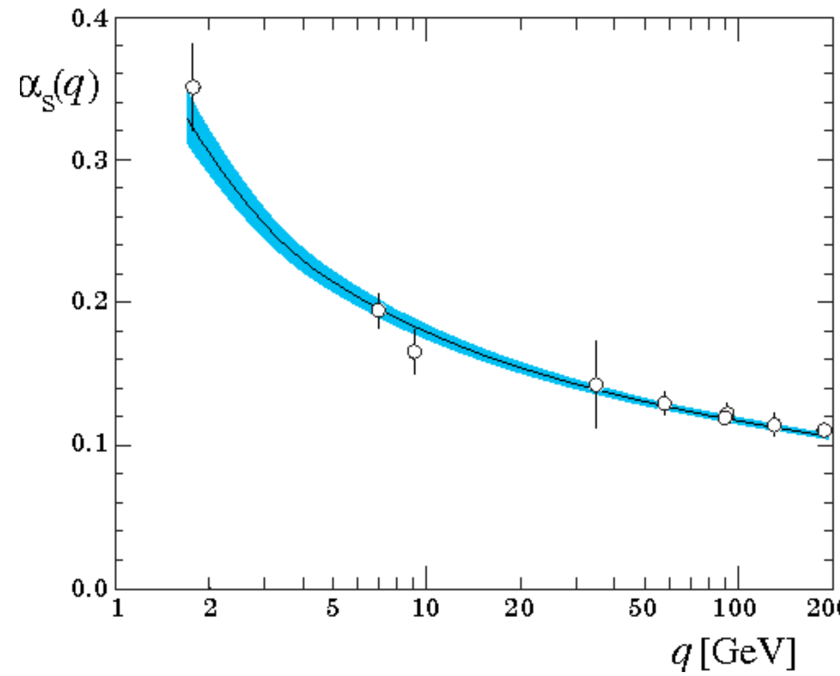




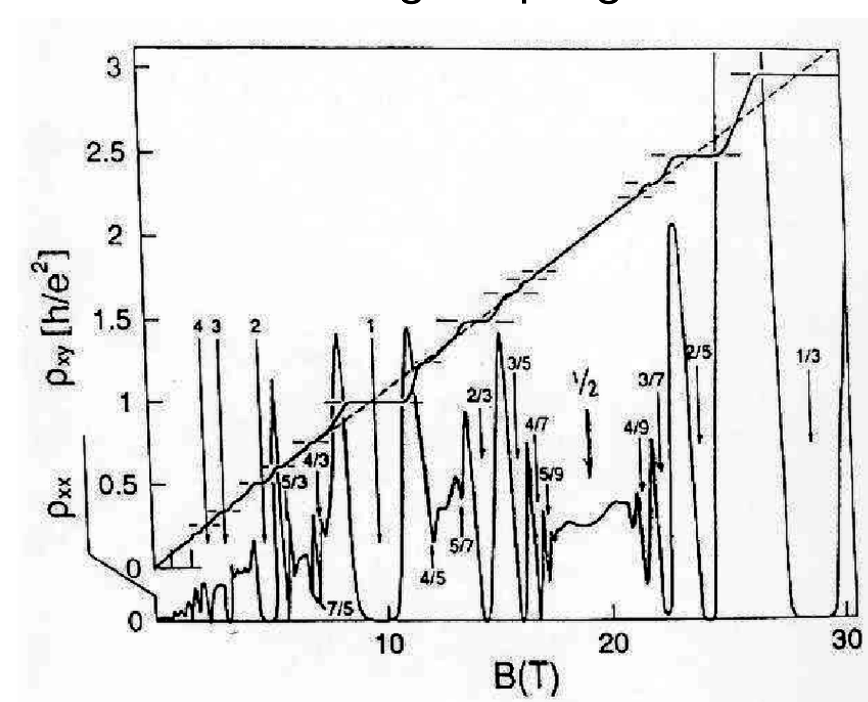
Standard Model

However, "strong dynamics" in Quantum systems (QM or QFT) cannot in general be understood by using perturbative approaches.

- Quark confinement in QCD
- High Tc superconductor
- Fractional quantum Hall effect
- Black hole thermodynamics



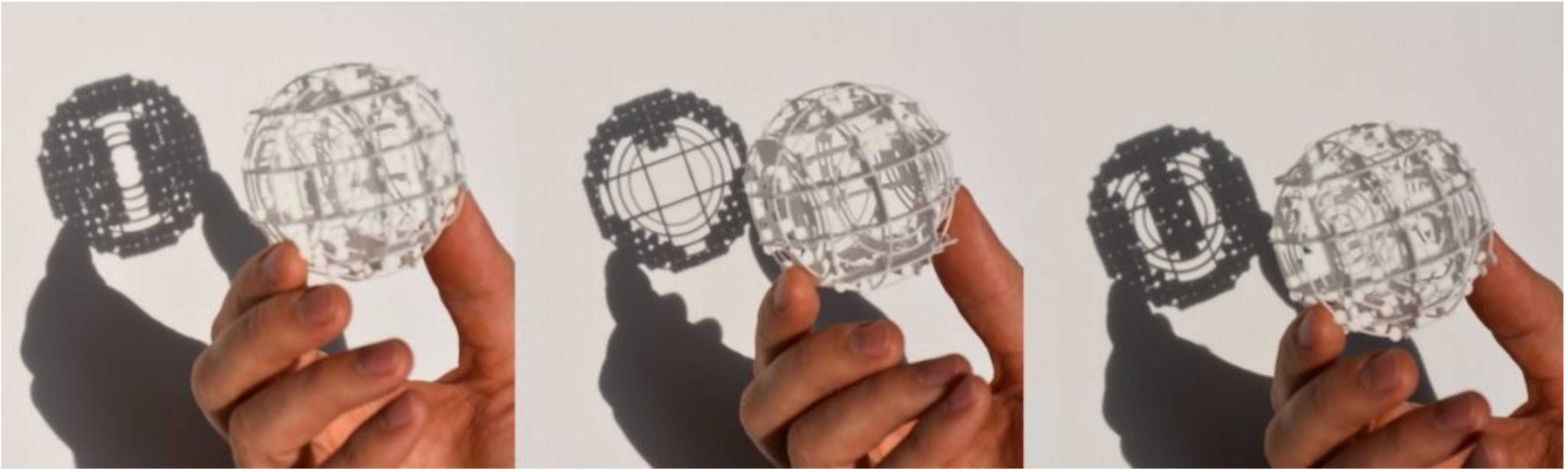
QCD running coupling constant



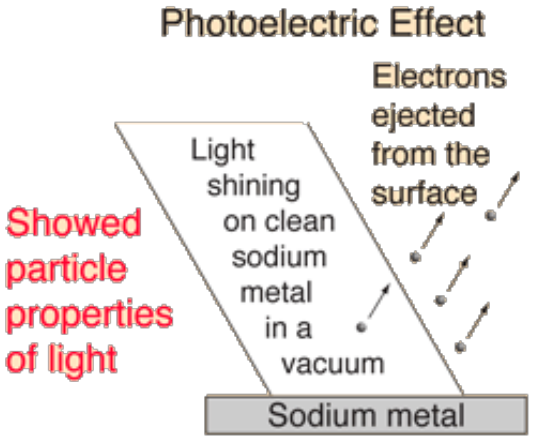
Fractional QHE

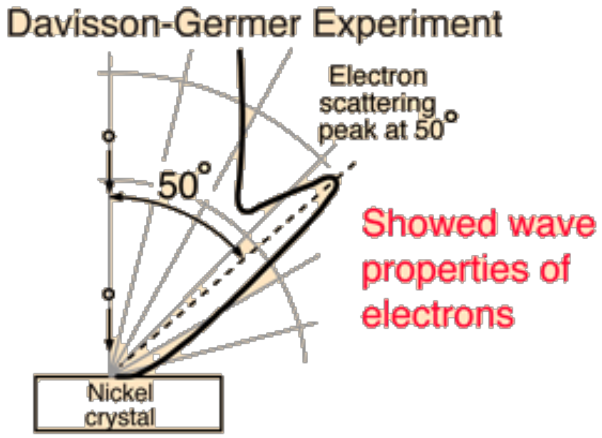
Duality

- Strong dynamics in quantum systems can be studied by using "duality"!
- Two (or more) different descriptions of same physics $\longrightarrow$ **Duality**



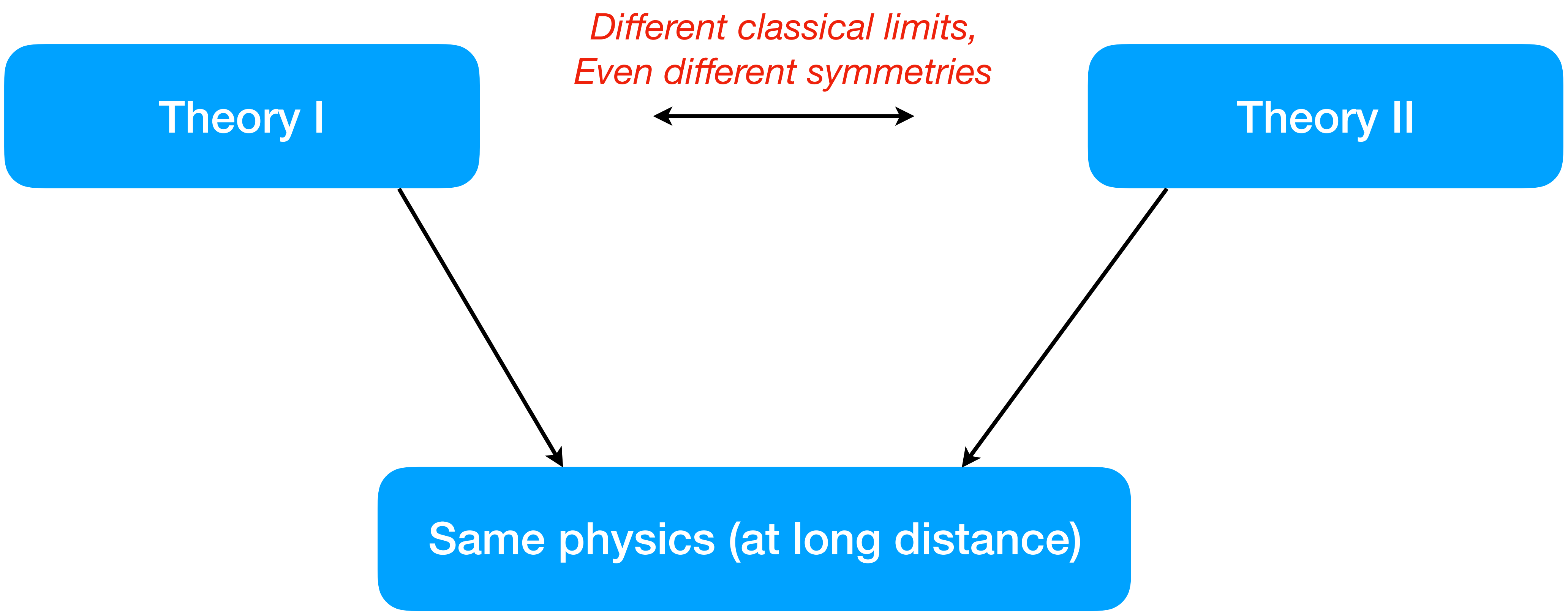
- Particle - Wave duality in QM





Duality

- Two (or more) different descriptions of same physics



- Different descriptions provide different insights in physics.

- Strong-Weak duality : **String coupling** ⟷ **Weak coupling**

Contents

- Introduction
- Dualities in $d+1$ $(d \leq 2)$ dimensions : Simple duality examples
- Dualities in $d+1$ $(d \geq 3)$ dimensions : More dualities
- Gauge/Gravity duality
- Conclusion

Duality

that I found in "Sanya Nanshan Temple" in January

"Door of non-duality"

"Three sided statue of Guanyin Buddha" -> **Triality** ?

Dualities in lower dimensions

Duality of free Harmonic Oscillator

Free Harmonic Oscillator in quantum mechanics

$$\hat{H} = \frac{\hat{P}^2}{2m} + \frac{1}{2}m\omega^2\hat{X}^2 \qquad \text{with} \quad \hat{P} = -\hbar\frac{\partial}{\partial X}$$

- Duality exchanges $\hat{X} \leftrightarrow \hat{P}$ given by Fourier transformation

$$(\hat{X}, \hat{P}) \;\rightarrow\; \left(\frac{\hat{P}'}{m\omega}, -m\omega\hat{X}'\right)$$

- Duality maps $\hat{H}$ to itself !

Duality in 1+1d free bosons

Action for a compact free boson $\phi(x) \sim \phi(x) + 2\pi$

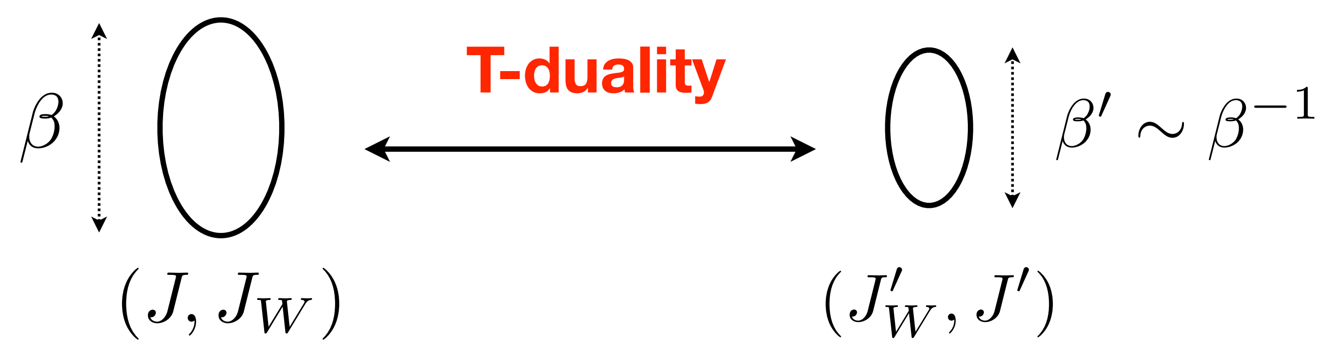
$$S = \int d^2x \, \frac{\beta^2}{2} (\partial_\mu \phi)^2$$

Equation of motion : $\partial^\mu \partial_\mu \phi(x) = 0$

Conserved currents ($\partial_\mu j^\mu = 0$) : $J^\mu = \beta^2 \partial^\mu \phi$ for $\phi(x) \to \phi(x) + c$

$$J_W^\mu = \frac{1}{2\pi} \epsilon^{\mu\nu} \partial_\nu \phi$$ from Bianchi id. $\epsilon^{\mu\nu} \partial_\mu \partial_\nu \phi = 0$

- Duality by $\partial^\mu \phi \leftrightarrow \epsilon^{\mu\nu} \partial_\nu \phi'$ leads to same action with $\beta \to \frac{1}{\beta}$.



- Equation of motion becomes trivial in dual theory of ϕ' .

Bosonization of 1+1d QFT

- Chiral fermions ψ_+, ψ_- can be written as functions of chiral bosons ϕ_+, ϕ_-

$$\psi_+ \sim e^{-i\phi_+} \ , \quad \psi_- \sim e^{i\phi_-}$$

- Ex) Massless Thirring model is dual to a free boson *[Thirring 1958], [Luscher, Peschel 1974], [Coleman 1975]*

$$S = \int d^2x \ i\bar{\psi}\gamma^\mu\partial_\mu\psi - g(\bar{\psi}\gamma^\mu\psi)^2 \quad \longleftrightarrow \quad S = \int d^2x \ \frac{\beta^2}{2}(\partial_\mu\phi)^2$$

with $\beta^2 = \frac{1}{4\pi} + \frac{g}{2\pi^2}$

- "Boson" and "Fermion" are not fundamental concepts in duality.

Free boson $\longleftrightarrow$ Interacting fermion

Simplest duality in 2+1d Maxwell theory

$$S = -\frac{1}{4e^2}\int d^3x\ F_{\mu\nu}F^{\mu\nu}$$ with

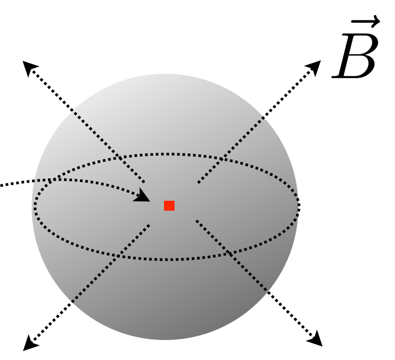
$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$

A_μ: U(1) gauge field

$A_\mu(x) \ \rightarrow \ A_\mu(x) + \partial_\mu \Lambda(x)$

Topological $U(1)_{\text{top}}$ current $J^\mu_{\text{top}} = \frac{1}{4\pi}\epsilon^{\mu\nu\rho}F_{\nu\rho}$ from Bianchi id. $\epsilon^{\mu\nu\rho}\partial_\mu F_{\nu\rho} = 0$

Charged object is magnetic monopole OP $\mathcal{M}(x)$



- Dual to a free theory of a compact scalar field σ by $F^{\mu\nu} \sim \epsilon^{\mu\nu\rho}\partial_\rho\sigma$.
- Monopole is implemented explicitly in dual theory as $\mathcal{M}(x) \sim e^{i\sigma(x)}$.
- Gauge symmetry can disappear or emerge, so it's not fundamental.

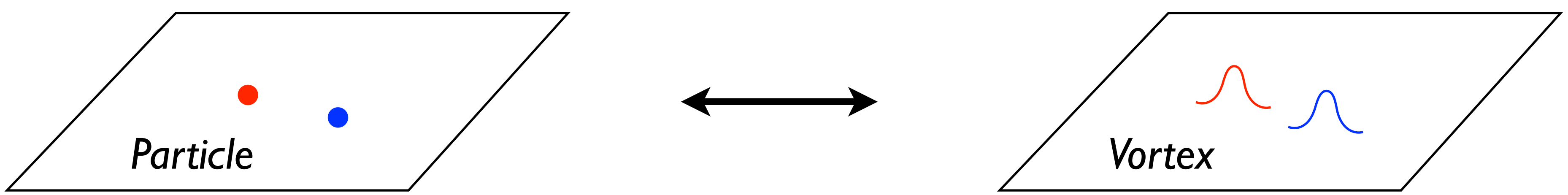
Particle-Vortex duality in 2+1d

	Particle theory	Vortex theory
Lagrangian	$\|D_A\phi\|^2 - \|\phi\|^4$	$\|D_a\Phi\|^2 - \|\Phi\|^4 + \frac{1}{2\pi}adA$
$U(1)$ current	j_ϕ	$\frac{1}{2\pi}\epsilon_{\mu\nu\lambda}\partial^\nu a^\lambda$
Operator	ϕ	$\mathcal{M}_a$
Mass deformation	$-m\|\phi\|^2$	$m\|\Phi\|^2$

A : background gauge field

a : dynamical gauge field

[Peskin 1978], [Dasgupta, Halperin 1981]



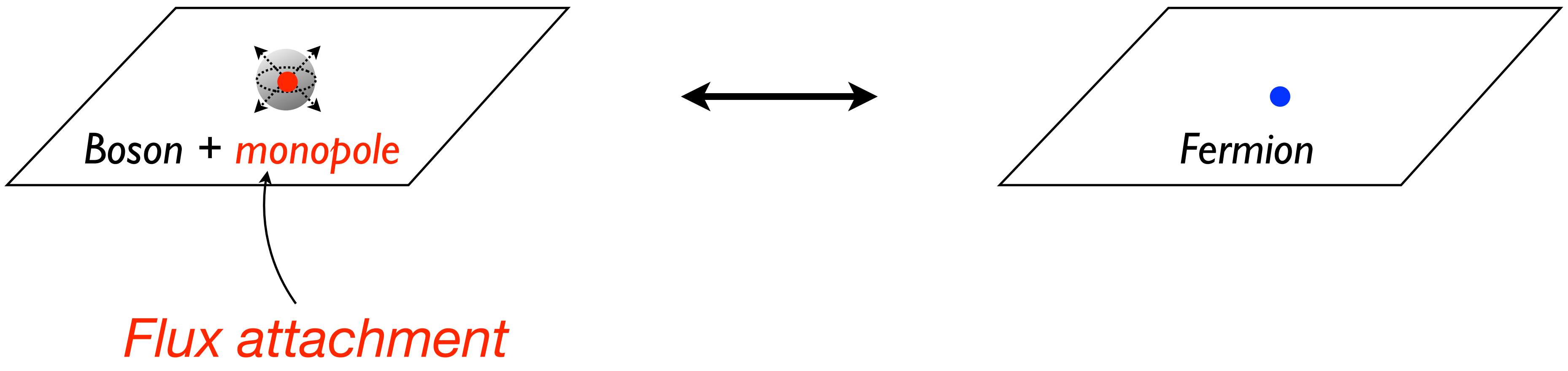
- Non-perturbative "soliton" is mapped to elementary particle.
- Two distinguished phases $\begin{cases} m > 0 & : \text{Mass gap} \\ m < 0 & : \text{Superfluid} \end{cases}$
- Emergent gauge symmetry of "a" in "vortex" theory.

Scalar field ϕ coupled to U(1) gauge field with Chern-Simons coupling is dual to a free Fermion ψ!

$Lagrangian$	$\lvert D_a\phi\rvert^2 - \lvert\phi\rvert^4 + \frac{1}{4\pi}ada$	$i\bar{\psi}\gamma^\mu\partial_\mu\psi$
$\mathrm{U}(1)$ $current$	$\frac{1}{2\pi}\epsilon_{\mu\nu\rho}\partial^\mu a^\nu$	j_ψ
$Operator$	$\mathcal{M}_a\phi$	ψ

"Bosonization"

[Wilczek, Zee 1983], [Polyakov 1988]



- "Spin" is not fundamental concept under duality!

Dualities in higher dimensions

Duality in 3+1d Maxwell theory

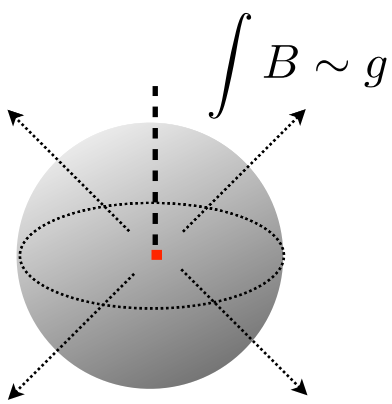
Maxwell equation without charge source

$$\nabla \cdot E = 0 \qquad \nabla \cdot B = 0$$
$$\nabla \times E = -\frac{\partial B}{\partial t} \qquad \nabla \times B = \frac{\partial E}{\partial t}$$

- Electric-magnetic duality : $(E, B) \ \rightarrow \ (B', -E')$ or $F^{\mu\nu} \ \rightarrow \ \epsilon^{\mu\nu\lambda\rho} F'_{\lambda\rho}$

- Charge sources under duality

  Electron with charge e $\longleftrightarrow$ Monopole with charge g

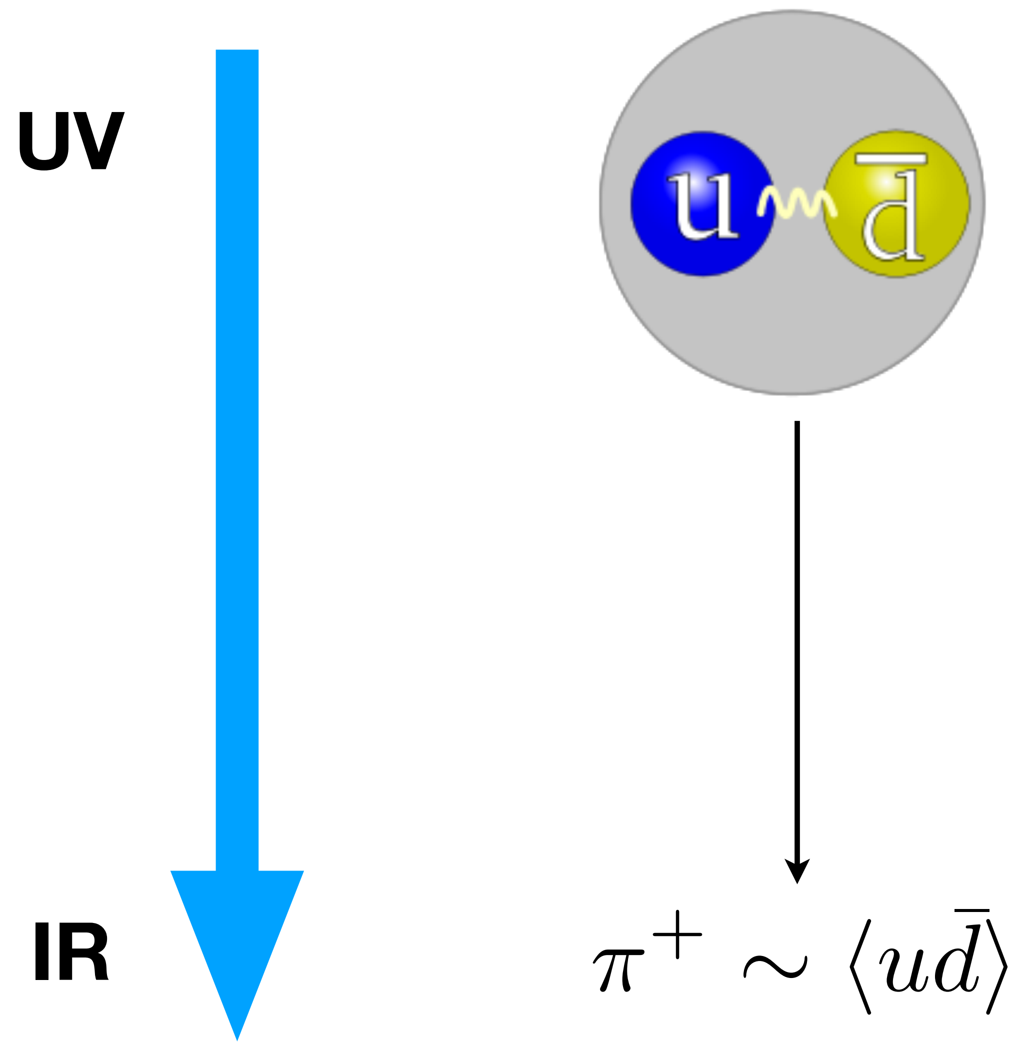


Dirac monopole

- Strong/Weak duality

  A_μ with e $\longleftrightarrow$ A'_μ with $g \sim e^{-1}$ due to charge quantization $e \cdot g = 2\pi\hbar$

- QCD describes strong forces between quarks and gluons at short distance.
- On the other hand, physics at long distance is described by Chiral perturbation theory of mesons such as pions $\pi^{\pm}, \pi^0$.

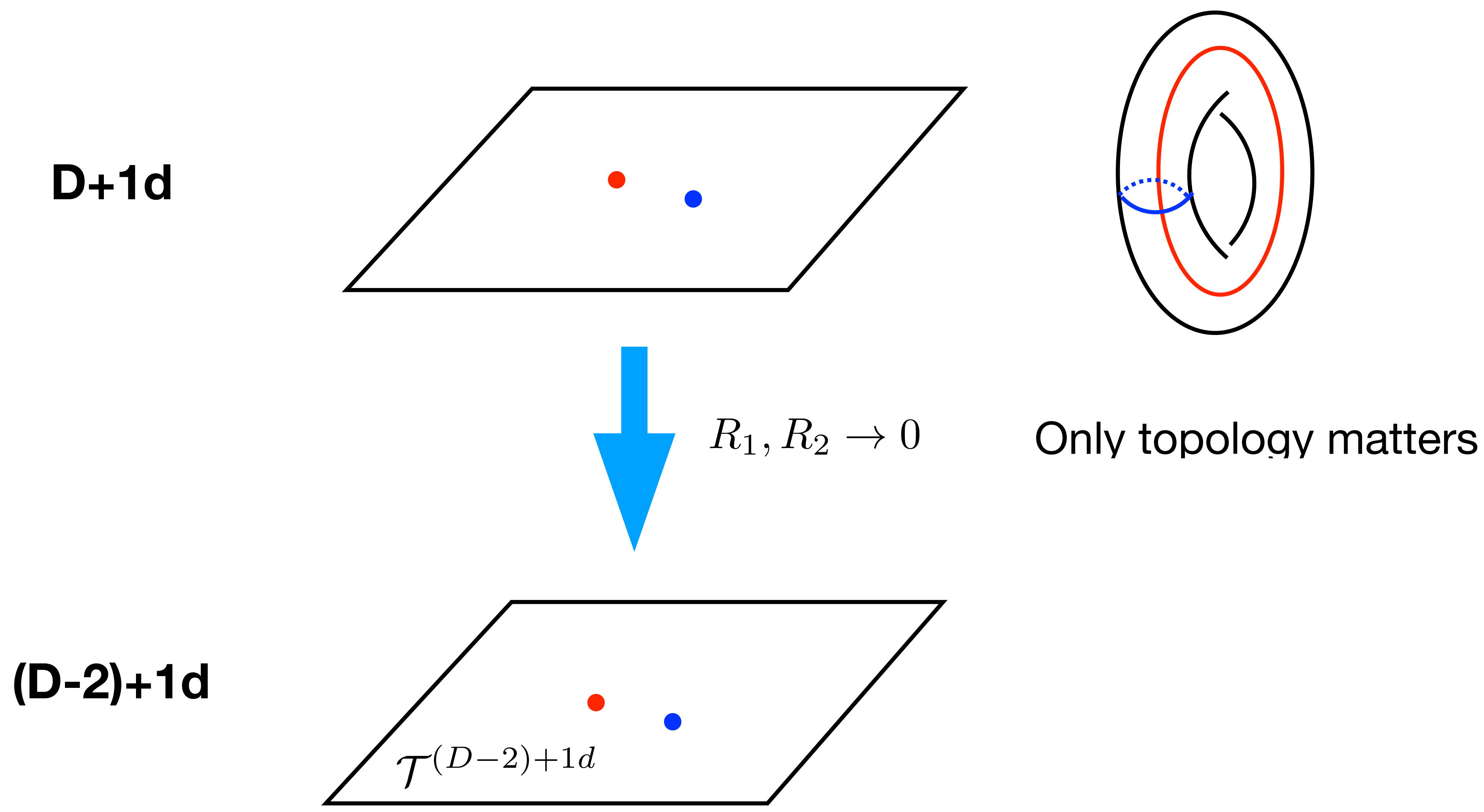


$$\pi^+ \sim \langle u\bar{d} \rangle$$

- Degrees of freedom at short distance $\neq$ those at long distance.

Idea of Compactification

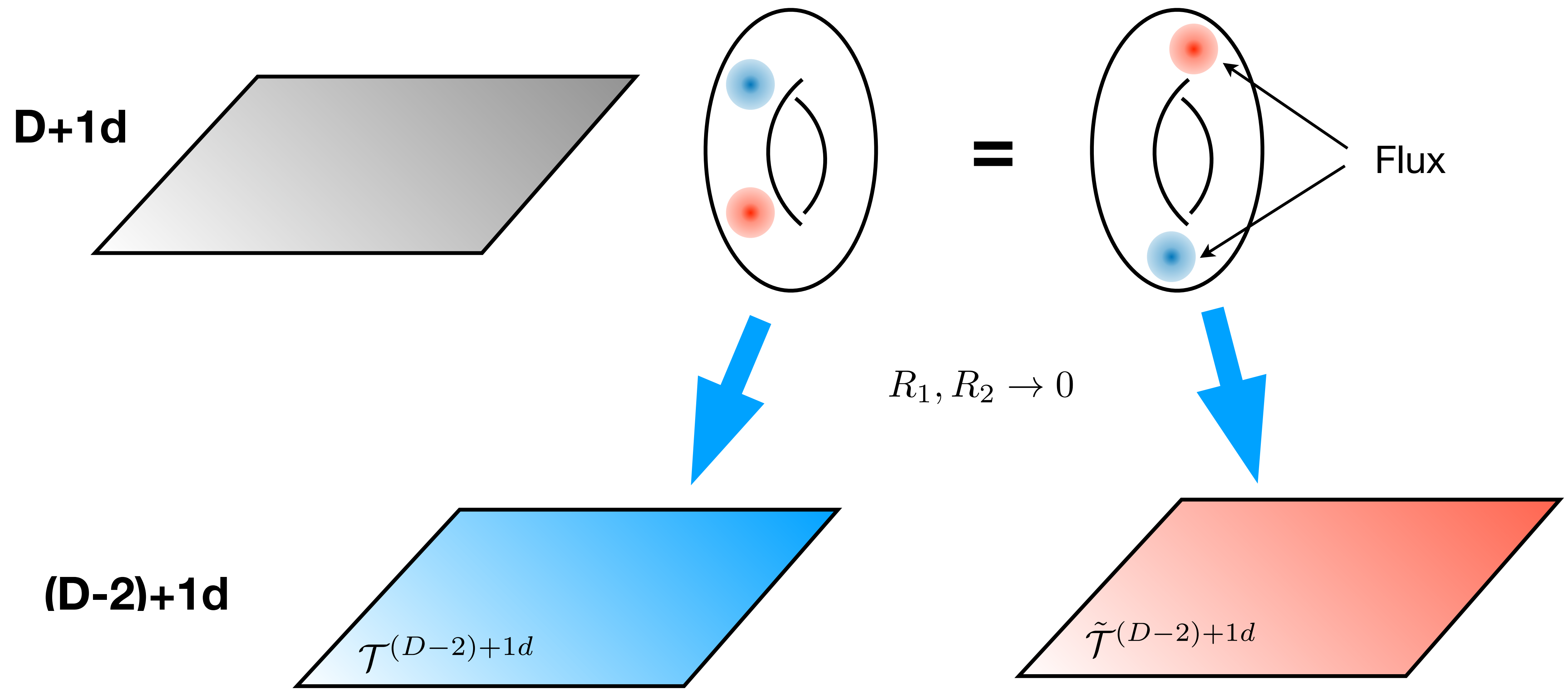
- D+1d QFT compactified on a torus leads to (D-2)+1d QFT.



- Duality exchanging $\bullet \longleftrightarrow \bullet$ comes from $R_1 \longleftrightarrow R_2$ on torus.

Idea of Compactification

- D+1d QFT on a torus with fluxes leads to a family of (D-2)+1d QFTs.



- Duality exchanging $\mathcal{T} \longleftrightarrow \tilde{\mathcal{T}}$ comes from ● $\longleftrightarrow$ ● on torus.

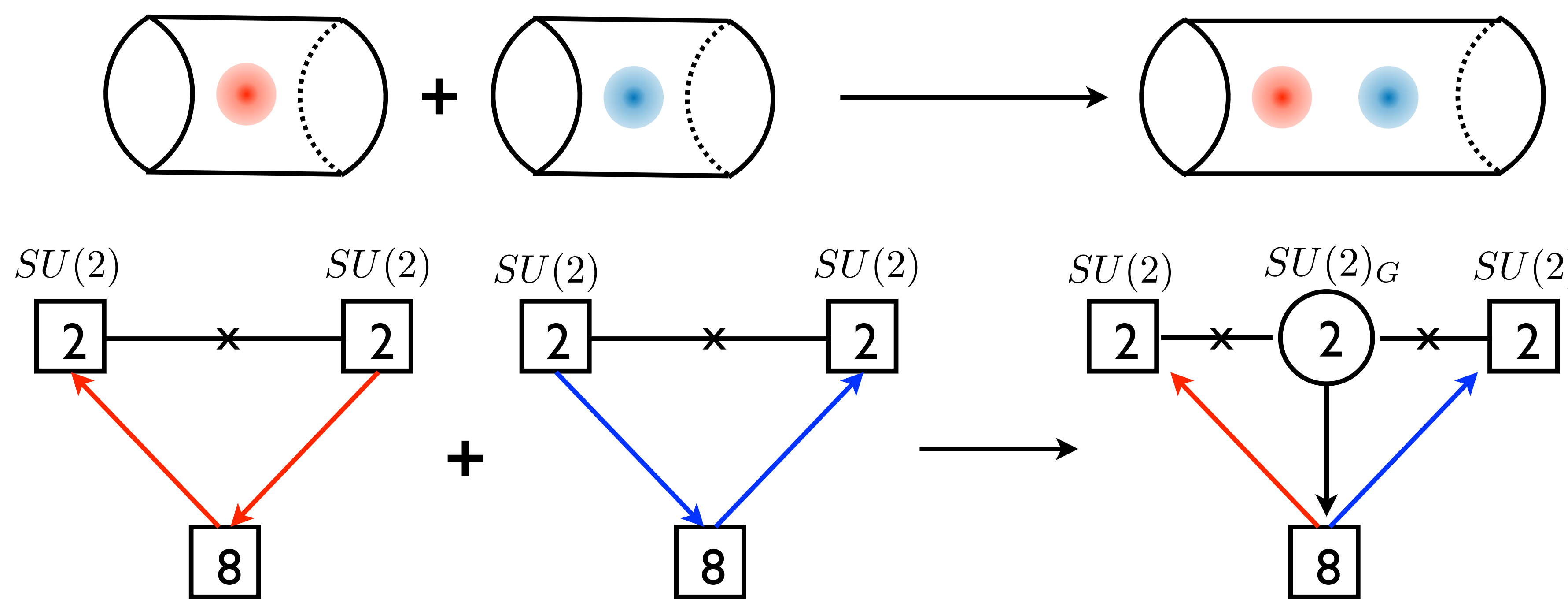
5+1d E-string theory and compactification

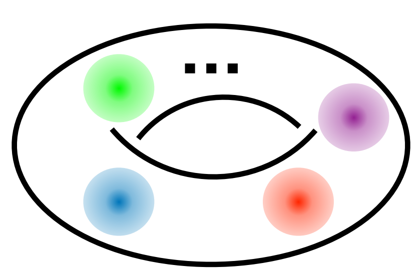
- 5+1d E-string theory *[Witten 1995], [Ganor, Hanany 1996],…*
  - Minimal supersymmetric conformal theory with E_8 global symmetry

- LEGO-like construction of Lagrangians for such 3+1d QFT from 5+1d E-string theory compactified on a torus with E_8 fluxes. *[H-C Kim, Razamat, Vafa, Zafrir 2017]*

LEGO blocks for fluxes

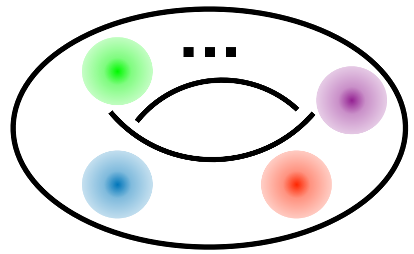
3+1d QFT operators



- Generic flux on torus 

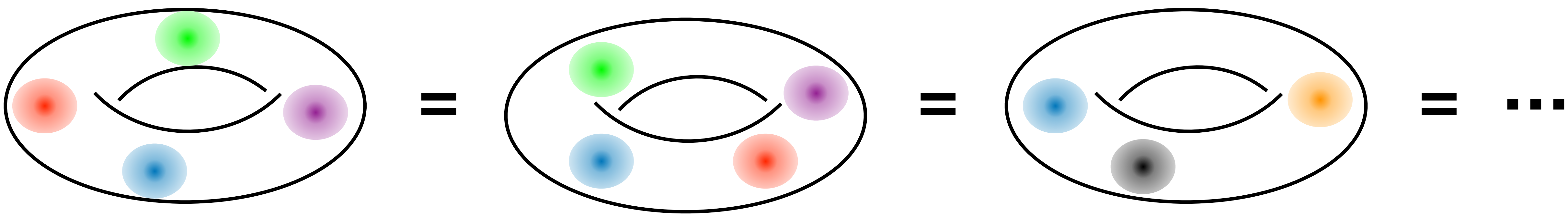
 gives rise to infinite number of 3+1d QFTs

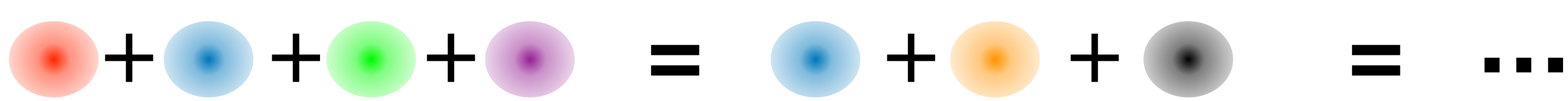
3+1d Dualities from 5+1d E-string theory

Generic flux on torus  gives rise to infinite number of 3+1d QFTs

3+1d QFTs only depend on topology and fluxes. *[H-C. Kim, Razamat, Vafa, Zafrir 2017]*

⟶ Infinitely many dualities !

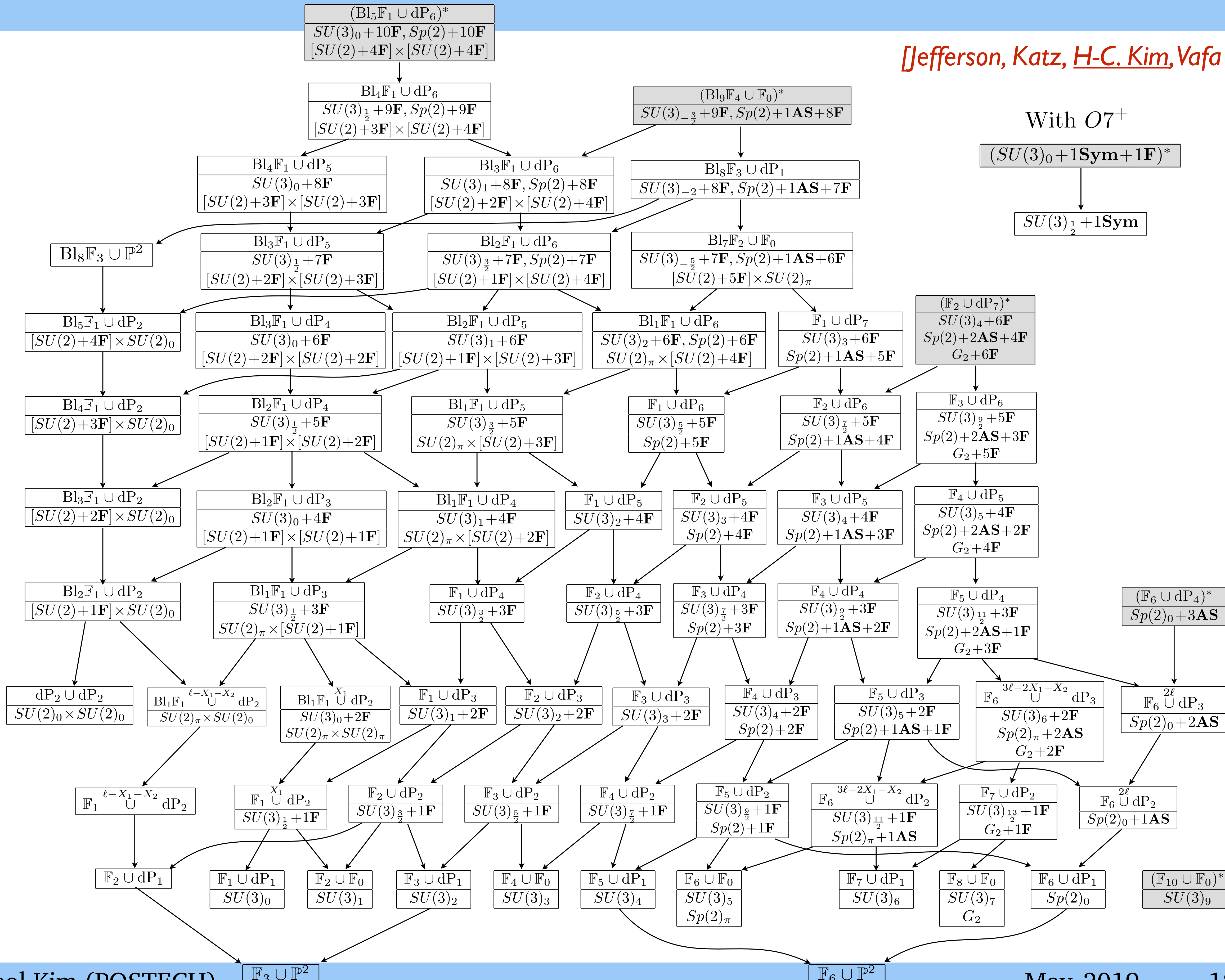


when 

- Higher dimensional QFTs can provide easy interpretations and also new predictions of lower dimensional physics such as dualities, enhanced symmetries.

Classification of 4+1d rank 2 QFTs and Dualities

[Jefferson, Katz, H-C. Kim, Vafa 2018]



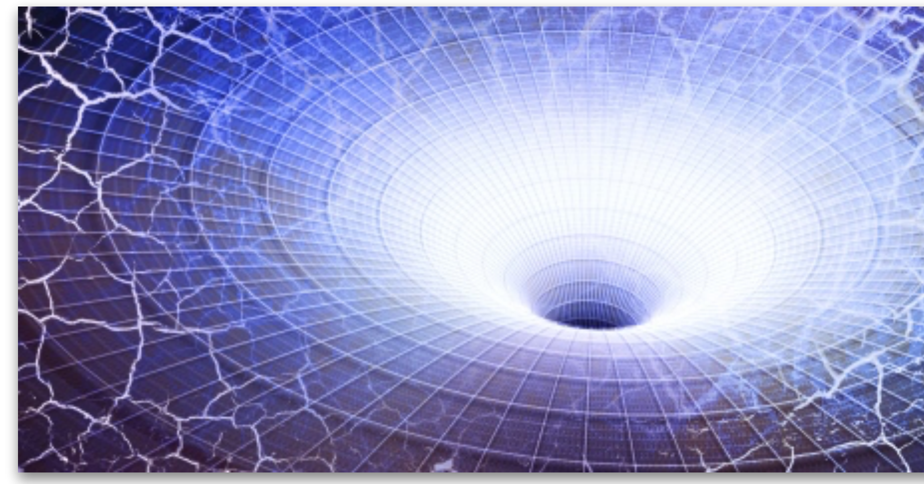
Gauge/Gravity Duality

Gravity

Spacetime is dynamical [*Einstein* 1915]

- In quantum world, spacetime geometry arises from quantization of graviton field (or metric field $g_{\mu\nu}$).

However, Gravity is non-renormalizable.



https://www.nottingham.ac.uk/mathematics /research/mathematical-physics

SoMA-WordPress.com

- Thermodynamical entropy of black holes (BH)

$$S_{\text{BH}} = \frac{kA_{\text{BH}}}{4\hbar G_N}$$

A_{BH} : BH surface area

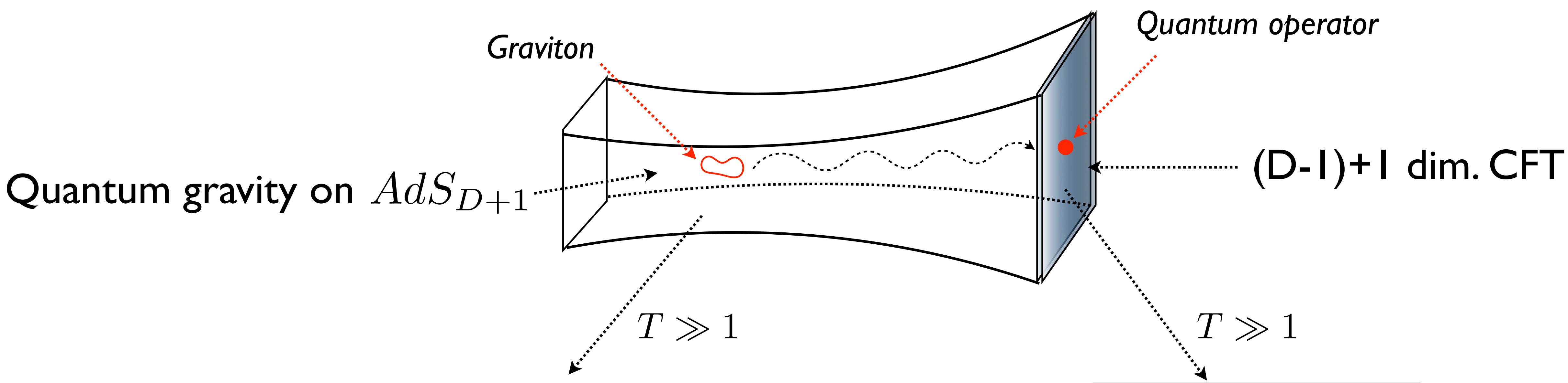
G_N : Newton constant

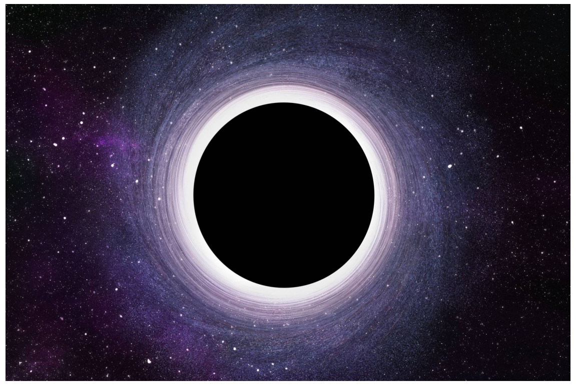
[*Bekenstein* 1973], [*Hawking* 1974]

Quantum gravity can be formulated in terms of degrees of freedom living on the boundary of the spacetime. $\longrightarrow$ **Holography**

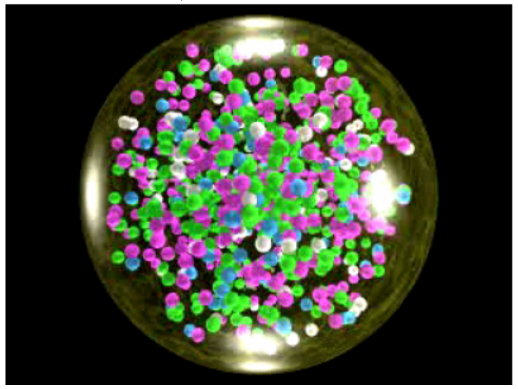
['t Hooft, Susskind]

- Quantum gravity in a D+1 dimensional curved "anti-de Sitter" spacetime (AdS space) is holographically described by (D-1)+1 dimensional conformal field theory (CFT) living on the boundary of AdS space. [Maldacena 1997]





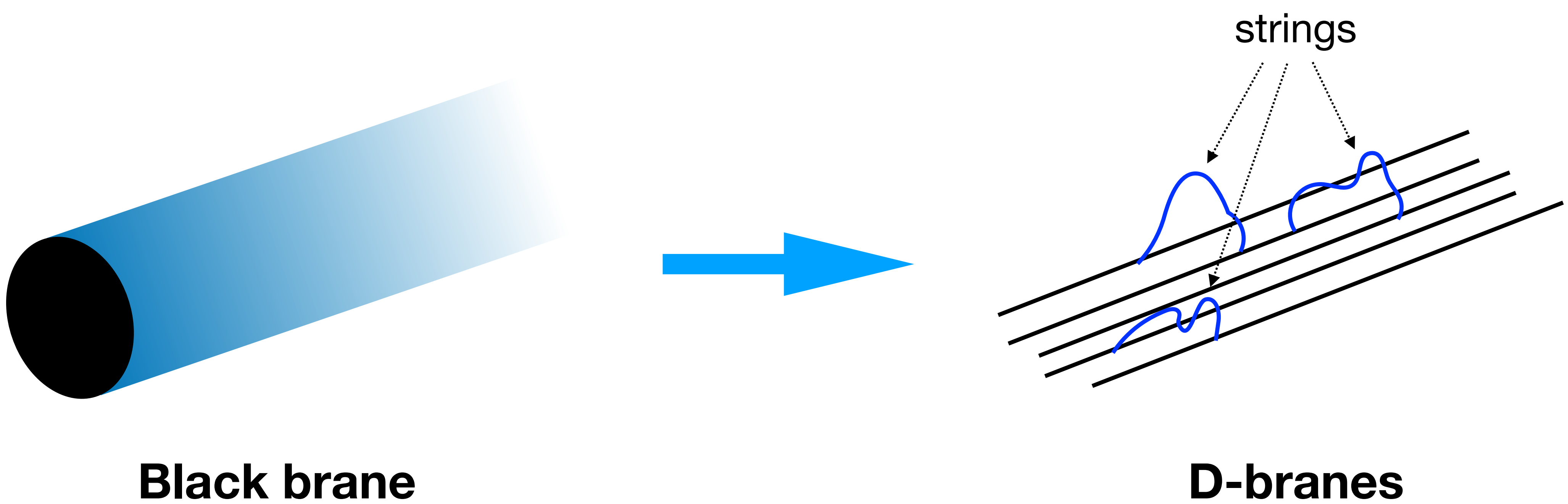
Black Hole



Quark-Gluon Plasma

Black Hole Entropy Counting

- Strominger and Vafa black hole counting (1996) :
  We can count black hole microstate and find black hole entropy S_{BH} by using dual theory living on D-branes.



Summary

- Duality is a fact that two or more seemingly different quantum systems describe the same physics.

- Strong-Weak duality can be used to understand strong dynamics in quantum systems.

- Many examples : Bosonization, Particle/Vortex duality, QCD vs Mesons, Gauge/Gravity duality, ...