[Colloquium, October 31st 2018]

Topology and Geometry in Quantum Matter

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POSTECH

Disclaimer:

It is mainly for undergraduates and young graduate students.

If you are an expert, you can sit back and relax.

Contents:

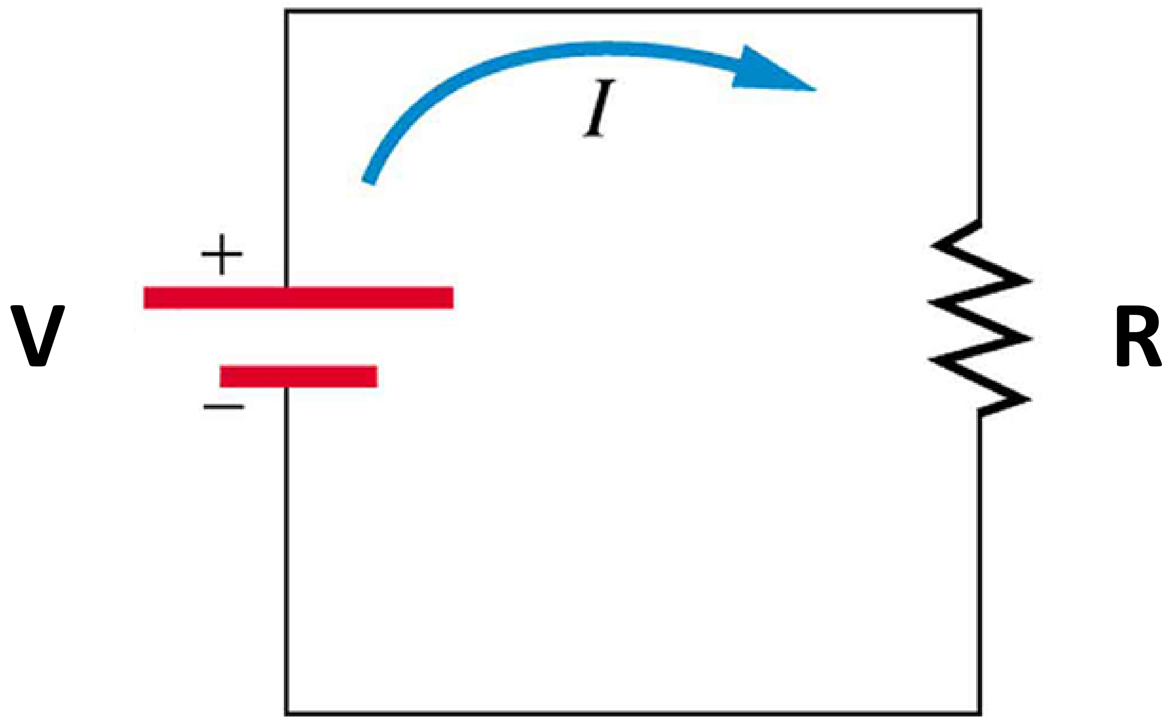
1. **Introduction:** Quantum Matter & Topological States

2. **Geometry in Topological Quantum Matter**

3. **Outlooks and Conclusions**

1. Introduction

What is Condensed Matter Physics?

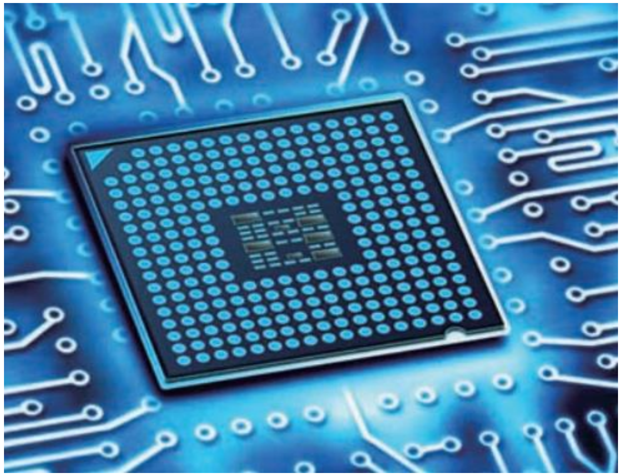


V = IR

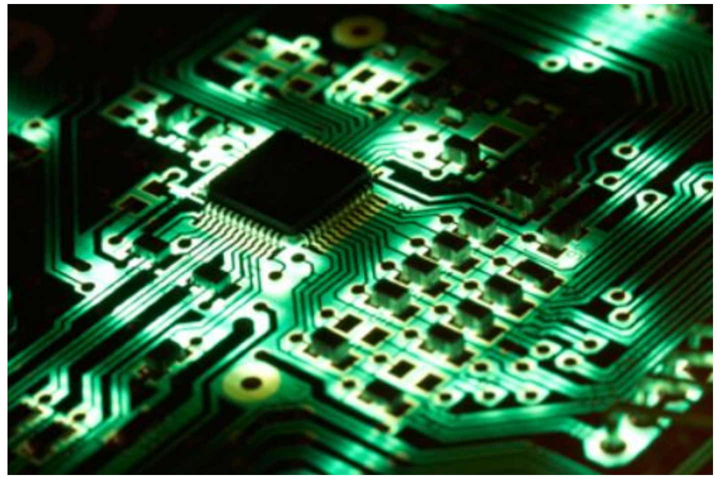
Simplest Version: $R = R(\rho_e, T, P, \cdots) = f$**(materials, temperature** $\cdots$ **)**

Seemingly Straightforward & Uninteresting (!)

Some of you think: Condensed Matter Physics is about…



Semiconductors



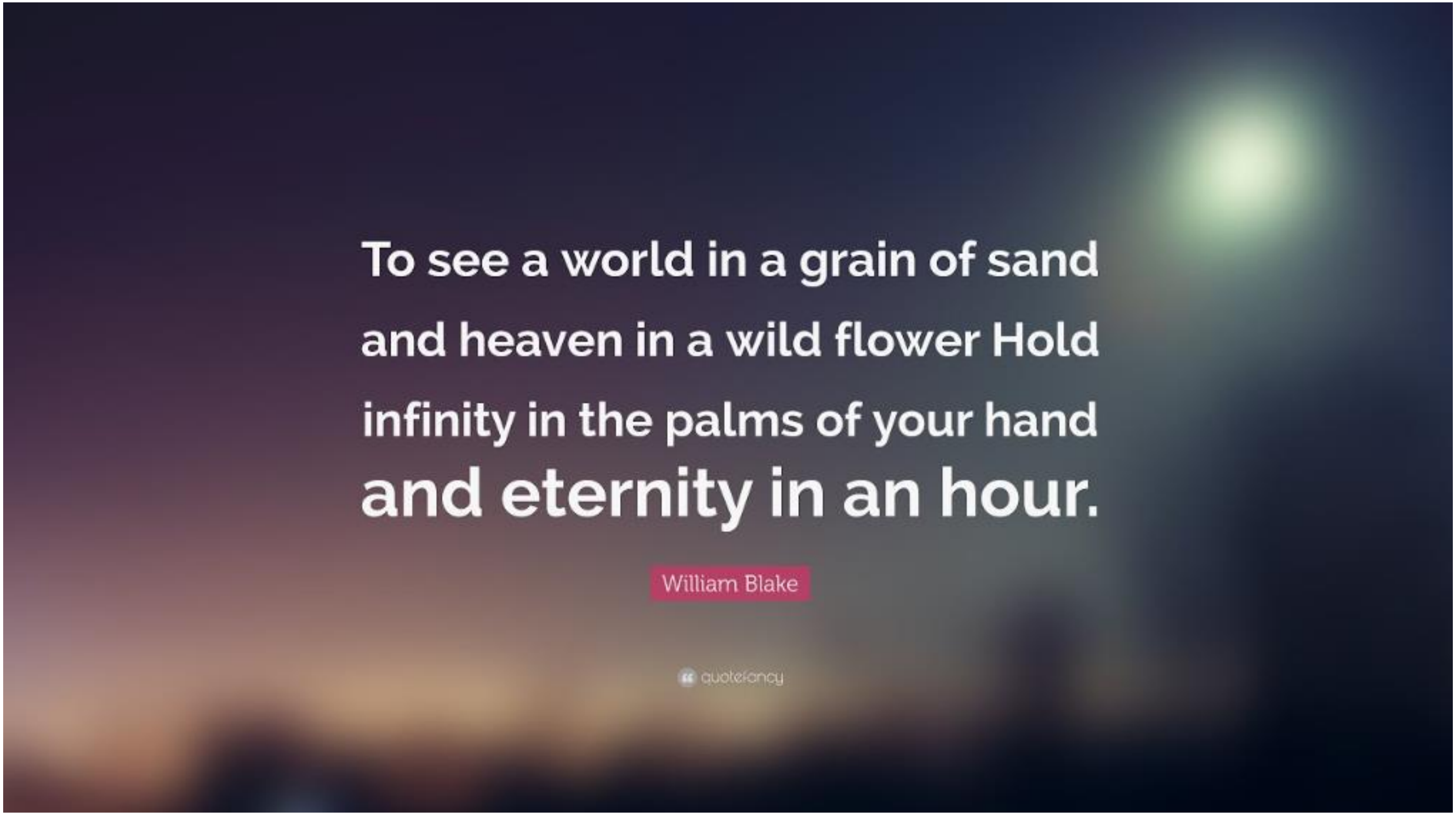
Electronics

Money

…and it is boring and too "down to the earth"

But, is it really?

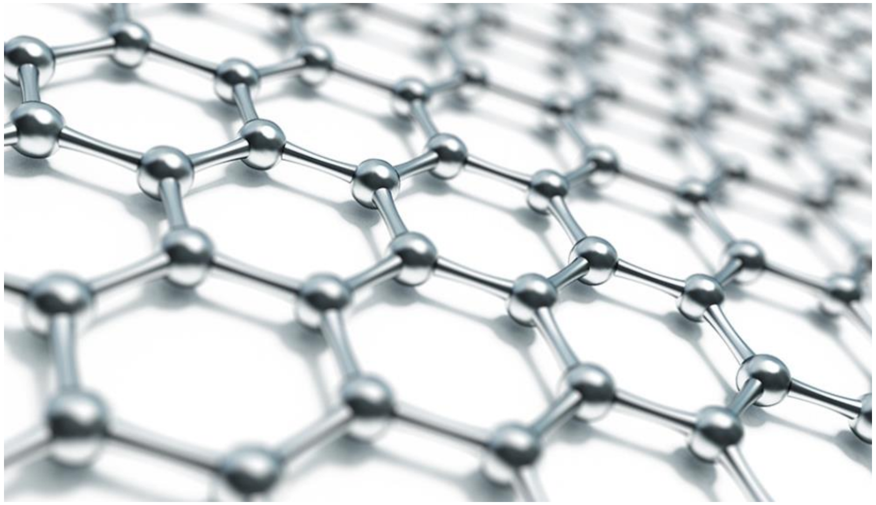
To me, it is more like:



"Seeing the Universe in a Grain of Sand"

Let’s look inside “a grain of sand”

Seeing the Universe from a grain of sand:



Graphene (Carbon Atoms)

Paul Dirac (1902-1984), 1933 Nobel Prize

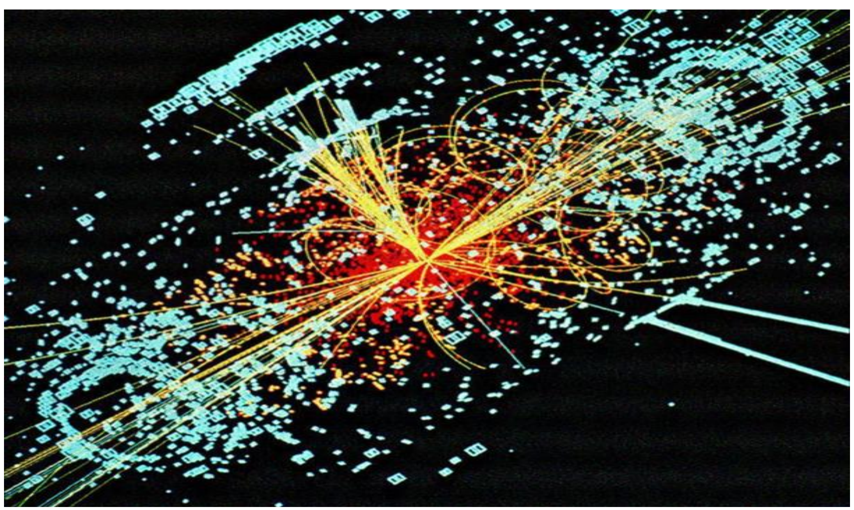
The Quantum Theory of the Electron.

By P. A. M. Dirac, St. John's College, Cambridge.

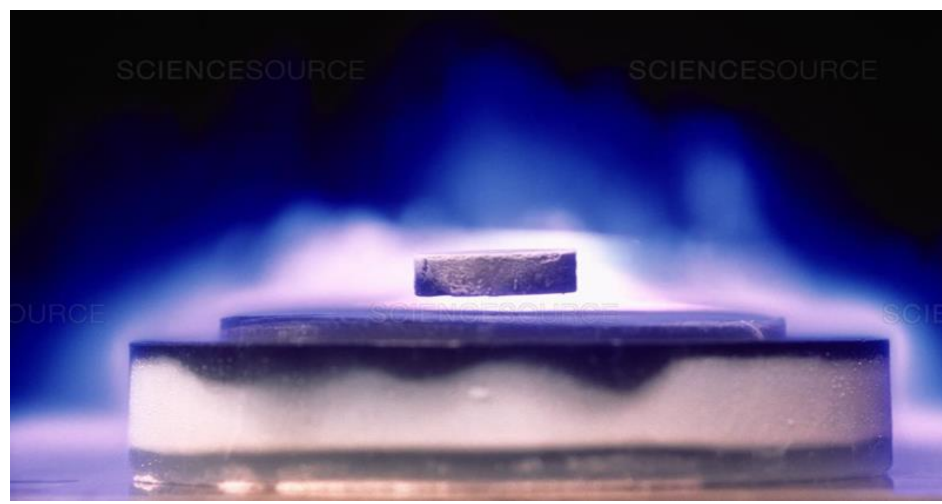
(Communicated by R. H. Fowler, F.R.S.—Received January 2, 1928.)

The new quantum mechanics, when applied to the problem of the structure of the atom with point-charge electrons, does not give results in agreement with experiment. The discrepancies consist of "duplexity" phenomena, the observed number of stationary states for an electron in an atom being twice the number given by the theory. To meet the difficulty, Goudsmit and Uhlenbeck have introduced the idea of an electron with a spin angular momentum of half a quantum and a magnetic moment of one Bohr magneton. This model for the electron has been fitted into the new mechanics by Pauli,* and Darwin,† working with an equivalent theory, has shown that it gives results in agreement with experiment for hydrogen-like spectra to the first order of accuracy.

Dirac's Electrons



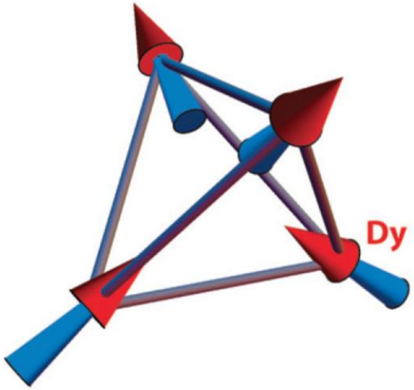
Higgs Particles from LHC



Superconductors

Seeing the Universe and Beyond:

Magnetic Monopole:



Magnetic monopoles in spin ice

C. Castelnovo[1], R. Moessner[1,2] & S. L. Sondhi[3]

[Nature (2008)]

New Fermions (due to the lack of Lorentz invariance)

Beyond Dirac and Weyl fermions: Unconventional quasiparticles in conventional crystals

Barry Bradlyn[1,*], Jennifer Cano[1,*], Zhijun Wang[2,*], M. G. Vergniory[3], C. Felser[4], R. J. Cava[5], B. Andrei Bernevig[2,†]

[Science (2016)]

Useful inputs from High-Energy Physics:

Three Lectures On Topological Phases Of Matter

Edward Witten [arxiv:1510.07698 (2015)]

In recent years, a number of fascinating new applications of quantum field theory in condensed matter physics have been discovered. For an entrée to the literature, see the review articles

...and there are a lot more [Names in Red color = HEP theorists].

A Duality Web in 2+1 Dimensions and Condensed Matter Physics

[**Seiberg**, Senthil, Wang, **Witten** (2016)]

Fermionic symmetry protected **topological** phases and **cobordisms**

[**Kapustin,** Thorngren, **Turzillo,** Wang (2016)]

An SYK-Like Model Without Disorder

[**Witten** (2016)]

Chen, GYC, **Faulkner**, Fradkin (2015)

GYC, **Parrikar**, You, **Leigh**, and Hughes (2015)

...and much more.

Fundamental Importance of Condensed Matter Physics

"Classification/Understandings of Phases of Matter in Universe"

$\boldsymbol{T > 0}$: Liquid, Solid, Gas, Magnetism, Superconductors

$\boldsymbol{T = 0}$: **How many phases of matter are there? One or more? Why?**

Note: 2nd Law of Thermodynamics says "Entropy $\rightarrow$ 0 as $T \rightarrow 0$"

Modern Condensed Matter Physics

Condensed Matter Systems as “Lens” toward “Nature”

“Grain of a Sand”: What you practically study

“Nature”: What you really learn

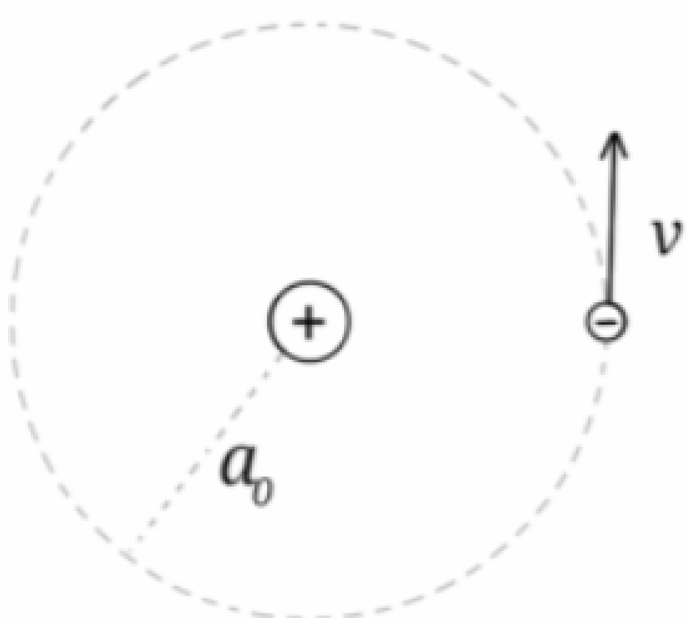
Today, we will use a very particular Lens:

Topology and Geometry in Quantum Matter

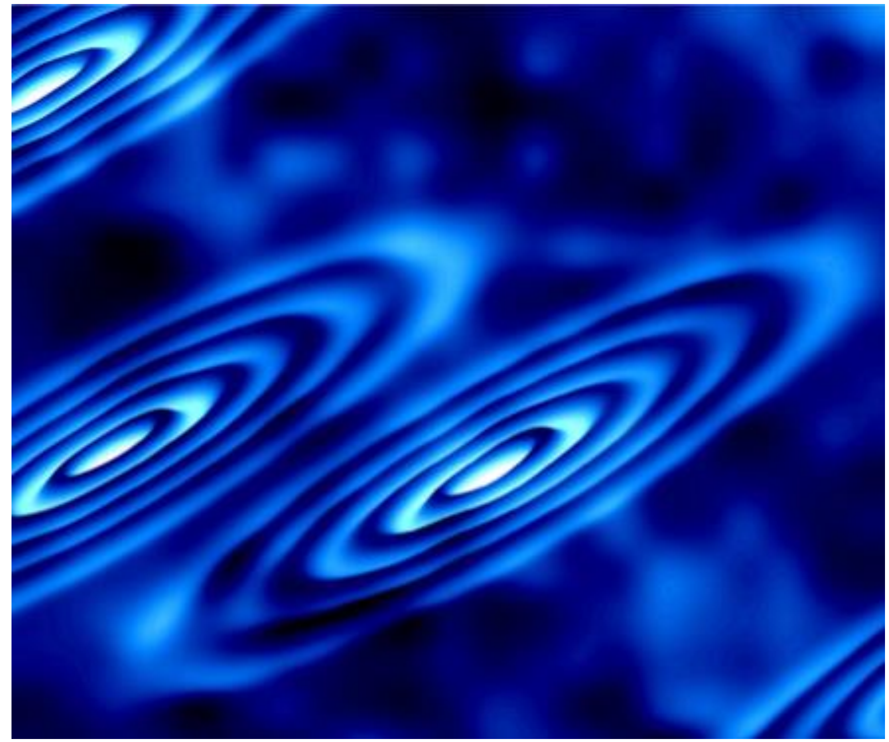
Why topology, geometry, quantum important?

"Quantum" Electrons in Solids 1:

Electrons in Solids are "Quantum Mechanical"



Classical Electron



Electrons' Orbits on Bismuth

[STM from Yazdani's group, Science (2016)]

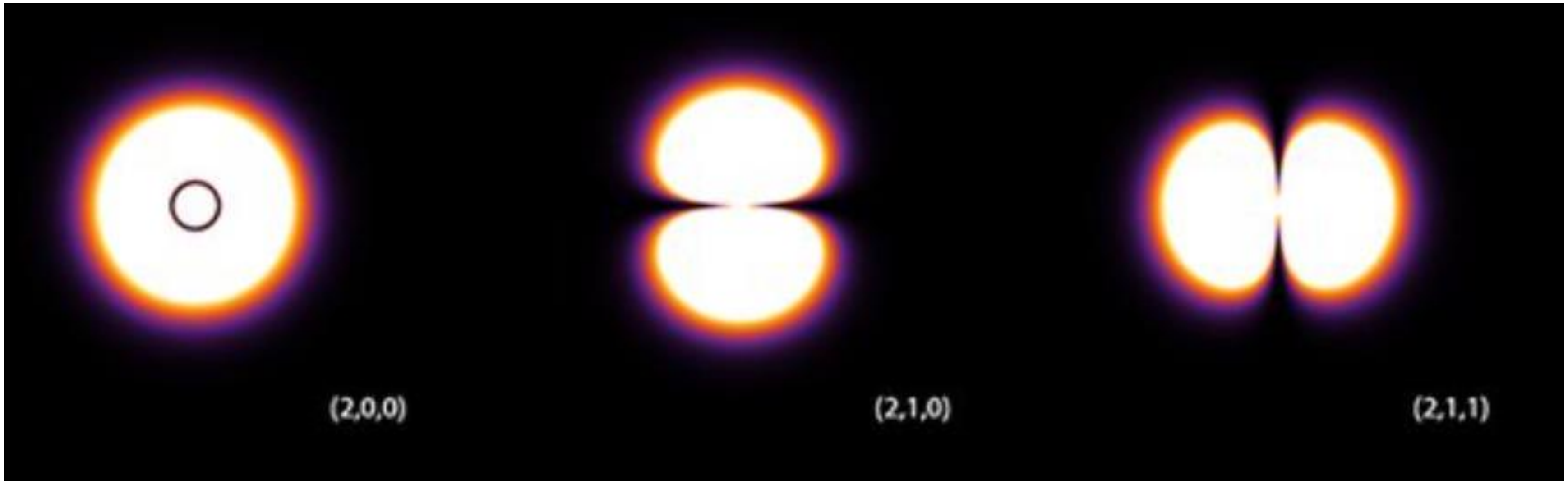
What's different in Quantum Mechanics?

"Quantum" Electrons in Solids 2:

Quantum = Not a point in Space

We can talk about "Shape/Structure" of Electrons' Orbits in Space

E.g. in Hydrogen:



Striking Consequence of

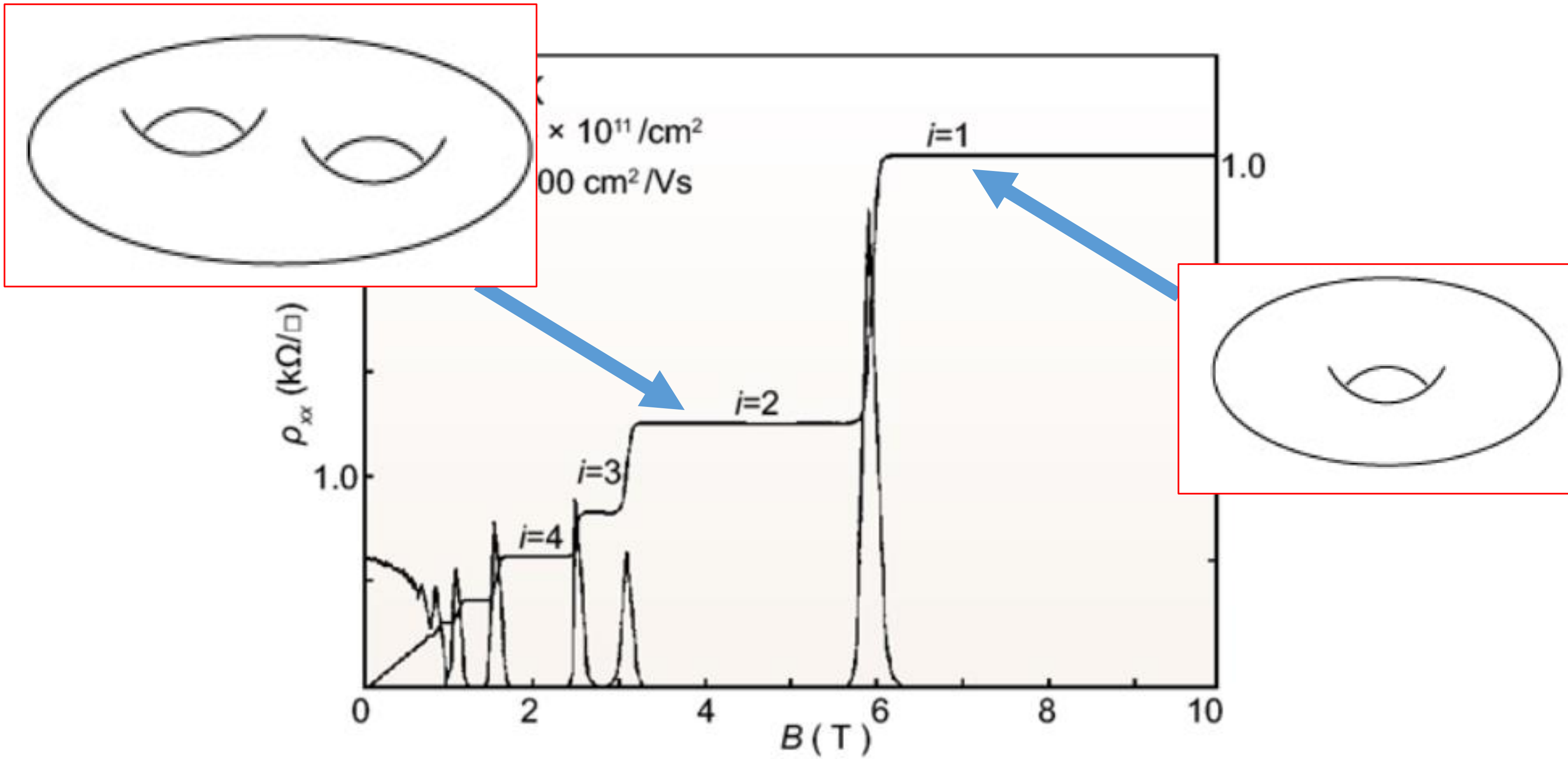
Non-trivial Shapes (= Topology) of Electrons' Orbits

Topological States

: Universal physics depends solely on "topology"

Ex: quantum Hall states (2-dim. electron gas under magnetic field)

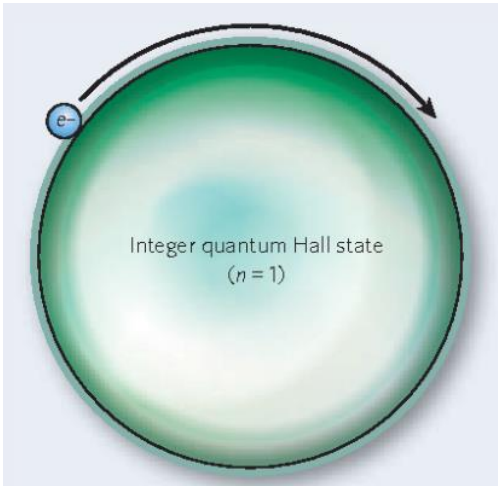
$$J_x = \rho_{xy} E_y \quad \text{and} \quad \rho_{xy} = \frac{h}{e^2} \times \frac{1}{N}$$

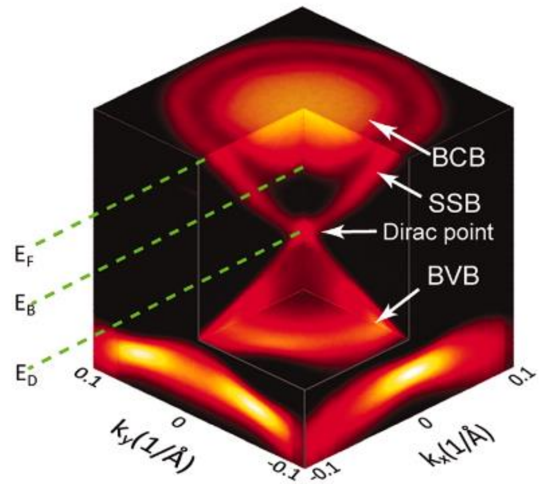


Different Universal Properties = Different Topology

Topology appears not only in the resistivity:

For example: Metallic Boundary States





	Quantum Hall states	Topological Band Insulator
Bulk:	2d system	3d system
Boundary:	1d chiral fermion	2d Dirac fermion

Topology manifests as robust metallic channels at Boundary

Robust Topology:

Topological phases cannot distinguish the following geometries:

=

=

"Only the global topology (of space, wavefunction, Hamiltonian) matters."

[disorders, chemical details, surface conditions etc, shouldn't matter]

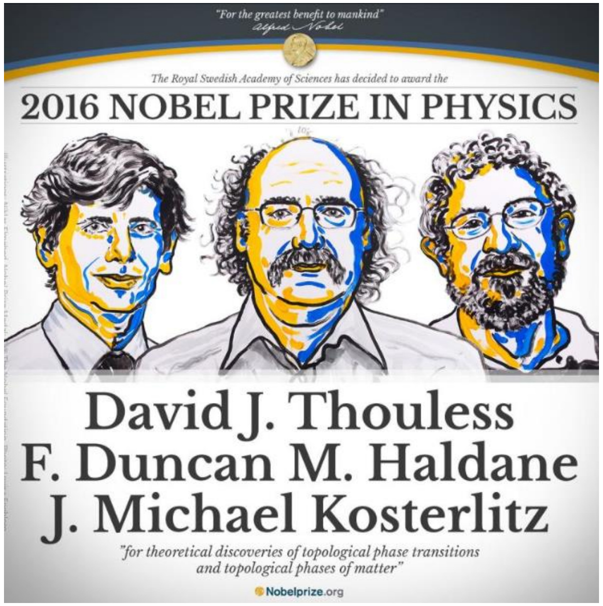
Metallic boundary states, and Resistivity/Conductivity

are Universal as far as "topology" doesn't change

= Overall shape of electrons' orbits

Topological State *is* the Central Theme in Modern Physics

Nobel Prize (2016)



…and other prizes

APS Buckley Prize (2017)

Dirac Medal (2015)

New Horizon Prize (2016, 2012)

2. Topology and Geometry

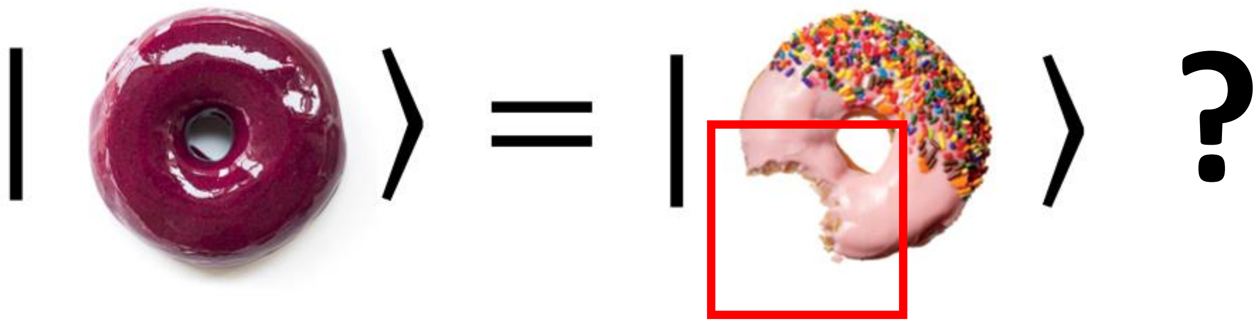
in Quantum Matter

Extra information about Space:

Crystal symmetry & its deformations

Are all the topological states truly "*topological*"?

[= common belief]



Geometric features

Not Entirely True.

Topological states see "Geometry"

Not only seeing, but there are intricate interplays !

Topological Order

Spatial Symmetry

Geometry

Examples:

Anisotropic Quantum Hall effects and Higher-Order Topological Insulators

and many others (we will come back later)

Not only seeing, but there are intricate interplays !

Topological Order

Spatial Symmetry

Geometry

Examples:

Anisotropic Quantum Hall effects and Higher-Order Topological Insulators

and many others (we will come back later)

Anisotropic Quantum Hall States

& Geometry in Quantum Hall States

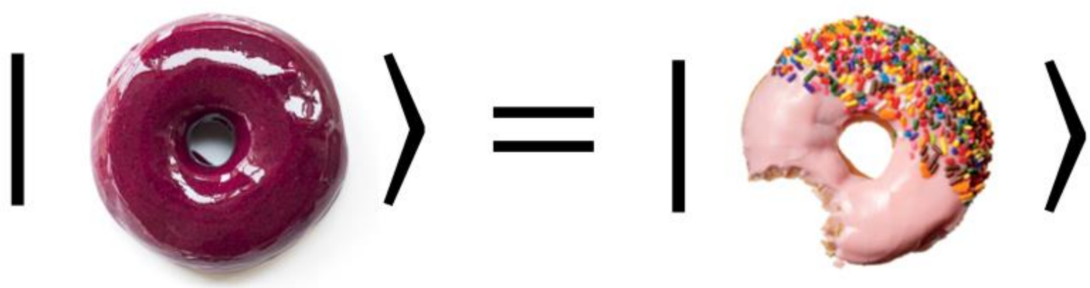
Quantum Hall states:

1. Electrons under uniform magnetic field

2. Exotic: Emergent fractional excitations with fractional statistics

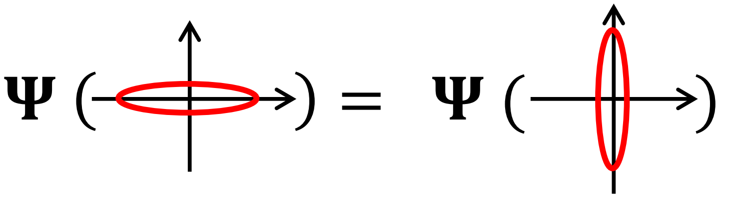
3. "Birth place" for "Topological Phase"

4. Topological: successful theoretical descriptions imply

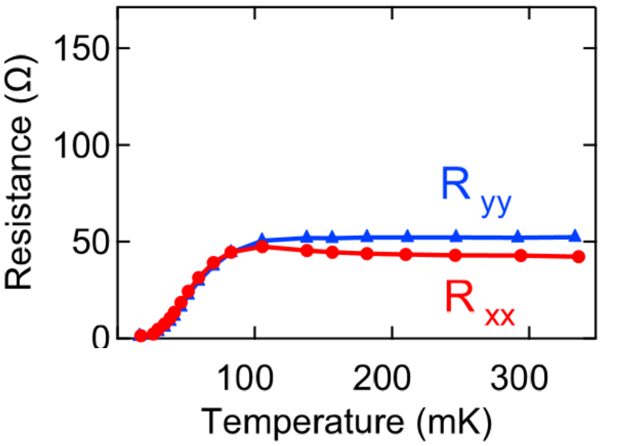


The wave-functions are independent of geometry !

A consequence of being "topological" is:



[rotational symmetry]



Theory supports: "composite particle theory"

Composite Bosons:

Zhang, Hansson, Kivelson (1989)
[**cited: 1000 times**]

Wen (1992, 1995, 2004)
[**cited: 1200 times**]

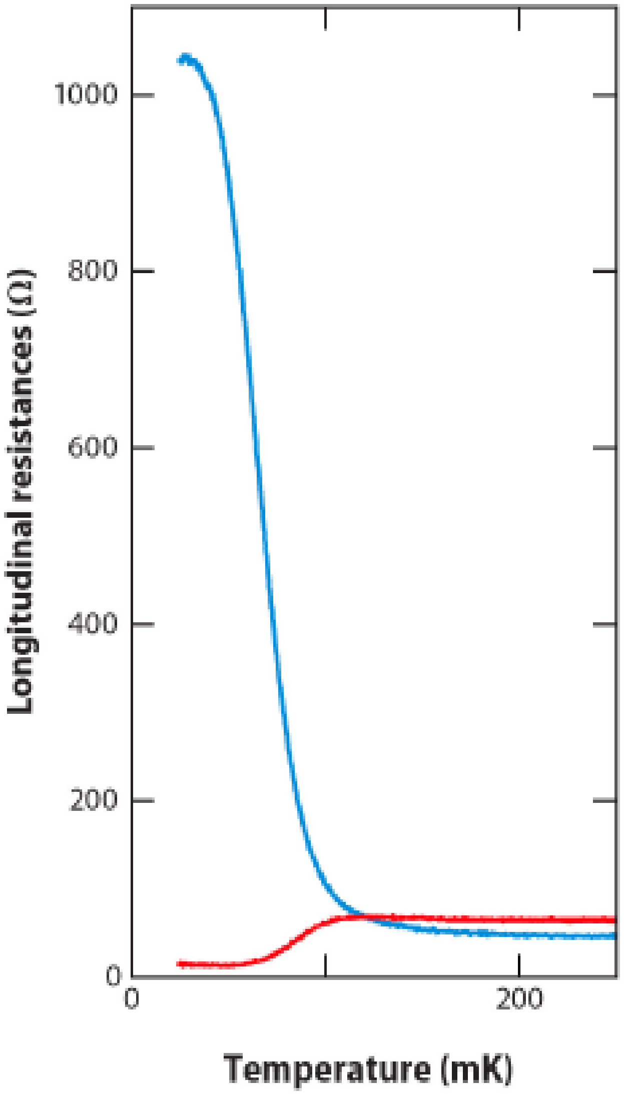
Composite Fermions

Jain (1989)
[**cited: 2200 times**]

Lopez, Fradkin (1991, 2013)
[**cited: 1700 times**]

..and many other papers or textbooks.

Anisotropic electronic states



[Eisenstein (2011)]

Found in a few different quantum Hall systems

[**Review:** Fradkin, Kivelson, Eisenstein, Mackenzie(2010)]

No stable R_{xy} plateau

Topological (stable R_{xy} plateau) & rotational symmetry

and

Anisotropic & no stable R_{xy} plateau

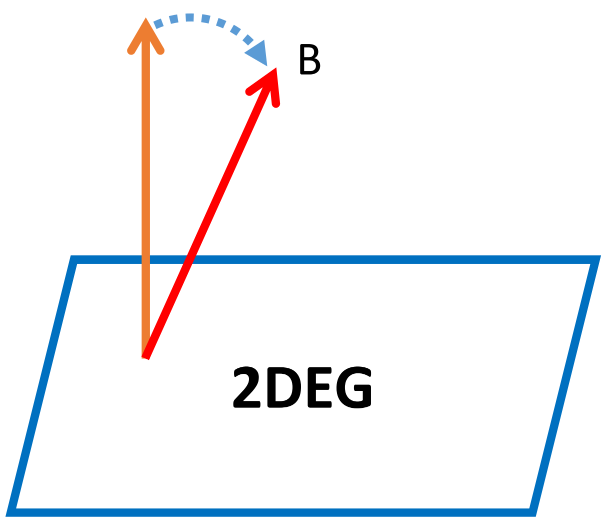
"Topology" and "anisotropy" never live together !

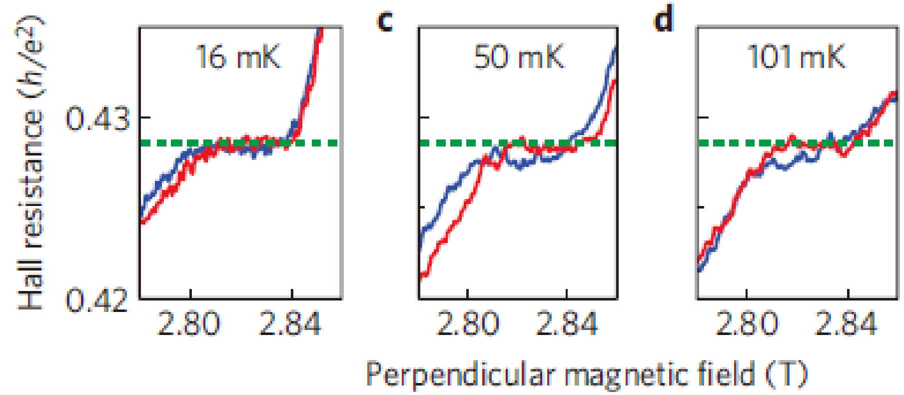
There has been an exception…

Evidence for a fractionally quantized Hall state with anisotropic longitudinal transport

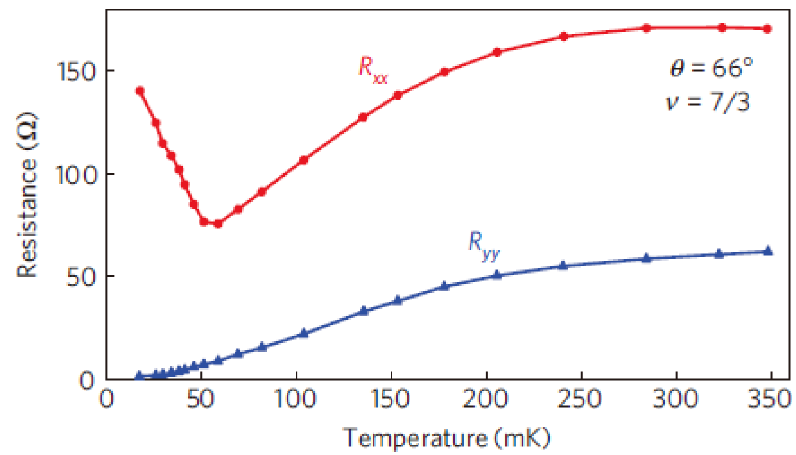
Jing Xia[1*†], J. P. Eisenstein[1], L. N. Pfeiffer[2] and K. W. West[2] [Nature physics, 2011]

Tilting the field by θ

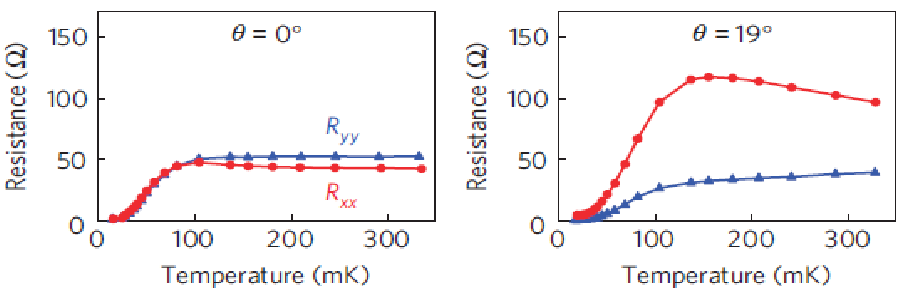




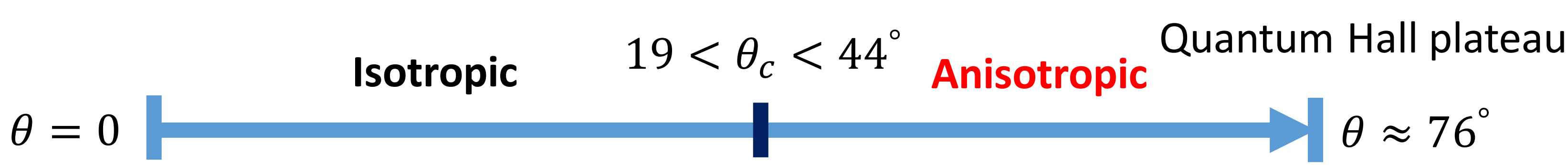
[Quantum Hall plateau ($\theta = 66^\circ \le 76^\circ$)]



[typical behavior of anisotropy ($\theta = 66^\circ$)]



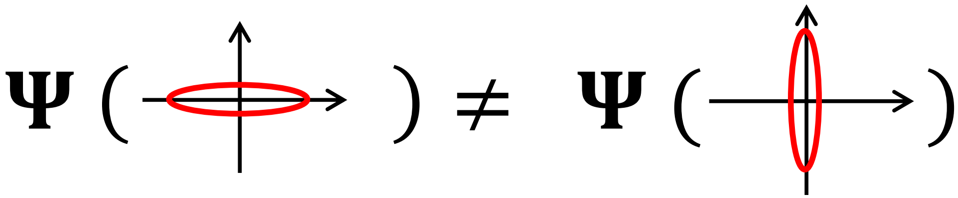
[rotational symmetric]



Evidence for a fractionally quantized Hall state with anisotropic longitudinal transport

Jing Xia[1*†], J. P. Eisenstein[1], L. N. Pfeiffer[2] and K. W. West[2] [Nature physics, 2011]

"Topological" state (stable R_{xy}) with "Anisotropy"



A new form of quantum matter which has not been observed

1. How to describe such state?

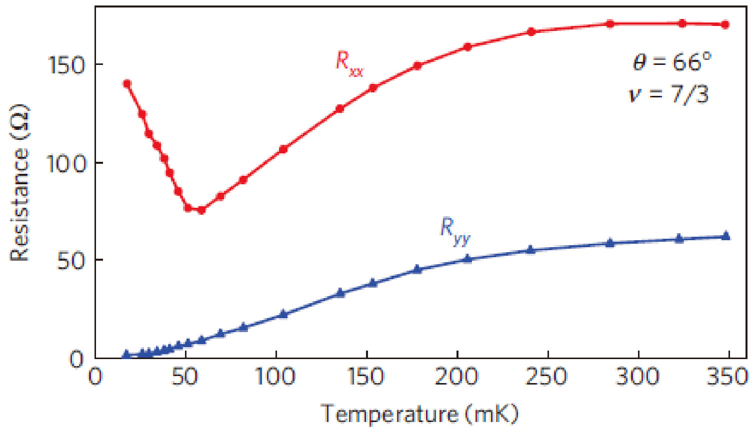
2. Any interplay of "topology" and "anisotropy"?

3. Characteristics of the transition?

…

We develop the new theoretical frameworks.

From experiments to theory:



[Xia et.al., Nat. Phys. (2011)]

Interpretation: "anisotropic effective mass δg_{ab}" of electrons

$$H = \frac{1}{2m}\,(D_i\,\Psi)^*\;\; g^{ij}\,\left(D_j\Psi\right) \text{ with } D_\mu = \partial_\mu + iA_\mu$$

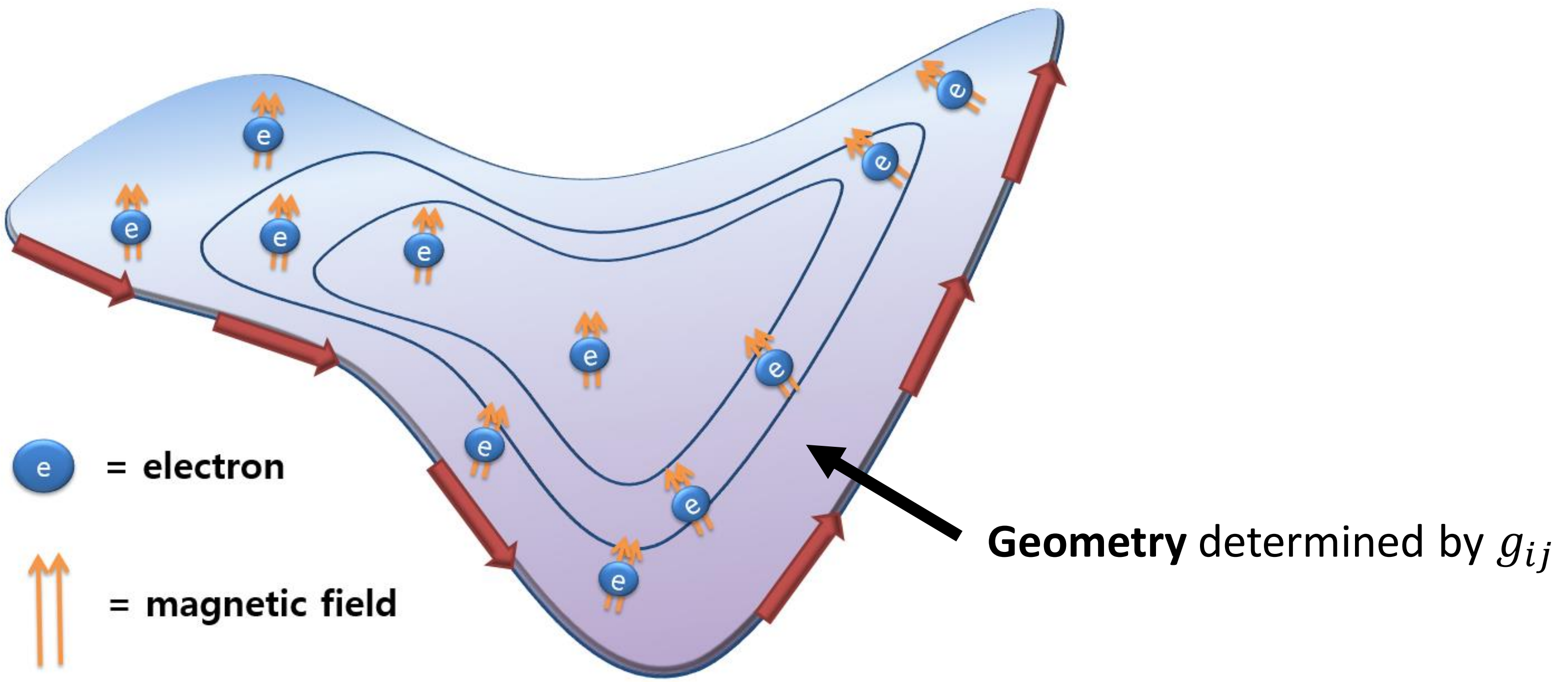
$$g^{ij} = \begin{bmatrix} 1+\delta g_{xx} & \delta g_{xy} \\ \delta g_{xy} & 1-\delta g_{xx} \end{bmatrix}$$ **with δg=order parameter of anisotropy**

You, GYC, and Fradkin, PRX (2014)

From experiments to theory:

$$H = \frac{1}{2m}\,(D_i\,\Psi)^*\ \ g^{ij}\,\left(D_j\Psi\right) \text{ with } D_\mu = \partial_\mu + iA_\mu$$

Equivalent to a regular quantum Hall state on curved space

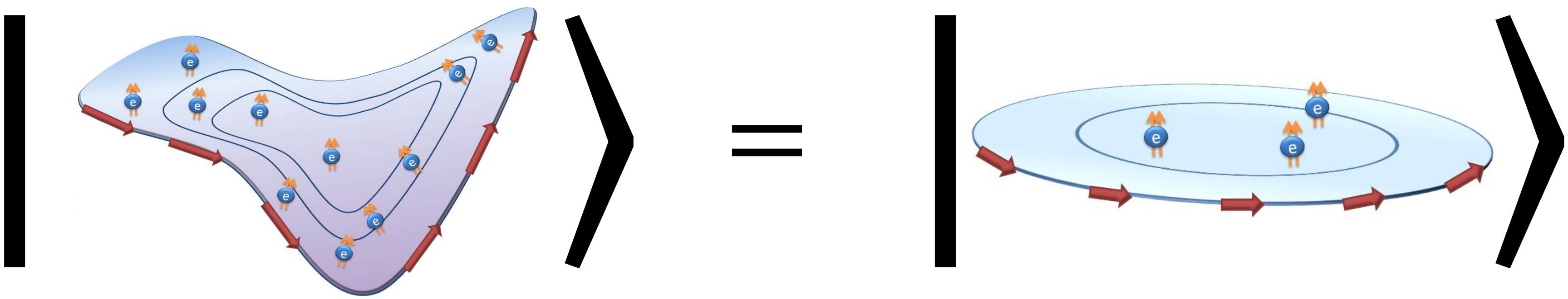


How quantum Hall states "respond" to geometry g_{ij}?

You, GYC, and Fradkin, PRX (2014)

The standard composite particle theory fails:

“Topological”



$$g_{ij} \neq \delta_{ij} \qquad\qquad g_{ij} = \delta_{ij}$$

[isotropic and flat]

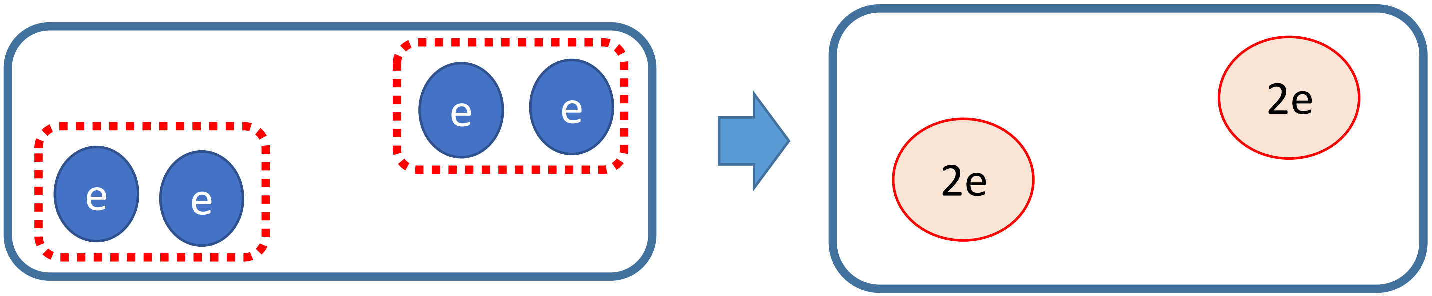
The standard theory cannot explain physics “sensitive to g_{ij}”

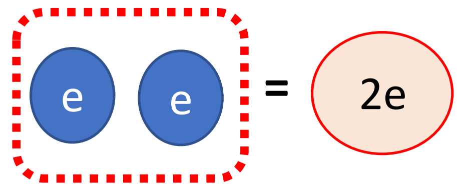
We need to develop a new theory!

GYC, You, and Fradkin (2014) [See also unpublished results in my invited talks at APS March Meeting (2015)]

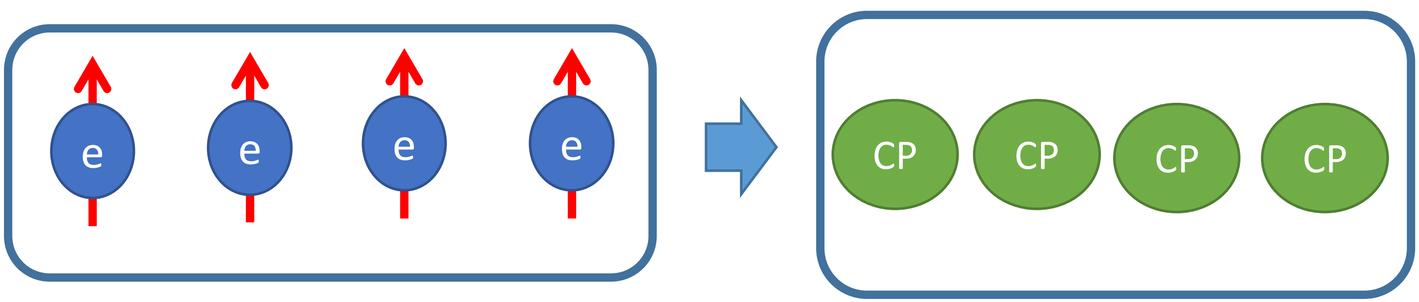
Standard Composite Particle Theory

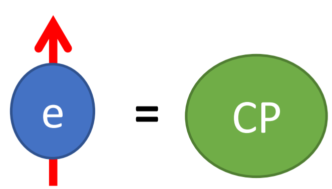
Landau-Ginzburg theory of BCS superconductors:



Cooper pair: 

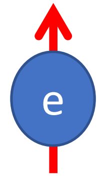
Composite particle theory of quantum Hall effects:



Flux attachment : 

, “composite particle”

Composite Particle Theory:

This "composite particle"  **leads to…**

A simple theory:

Electromagnetic gauge

$$L = \bar{\rho} A_0 - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + A_\mu \boxed{\frac{\epsilon^{\mu\nu\lambda}}{2\pi} \partial_\nu b_\lambda} + \cdots$$

J^μ: electron current

known as the Chern-Simons effective theory

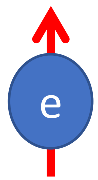
Topological: no data about geometry g_{ij}

[Zhang, Hansson, Kivelson (1989), Wen (1992), (1995)]

New Composite Particle Theory:

New theory:

Standard theory

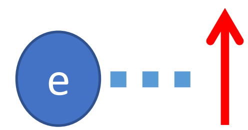


Point particle

Not working correctly!

New theory

[GYC, You, and Fradkin (2014, 2016)]



"Stick-like" object

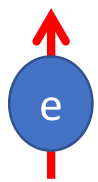
[cf. "Framing"]

GYC, You, and Fradkin (2014) [See also unpublished results in my invited talks at APS March Meeting (2015)]

New Composite Particle Theory:

Difference between the two?

Standard theory



Point particle

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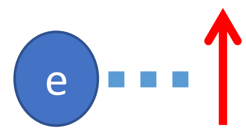
New theory

[GYC, You, and Fradkin (2014, 2016)]



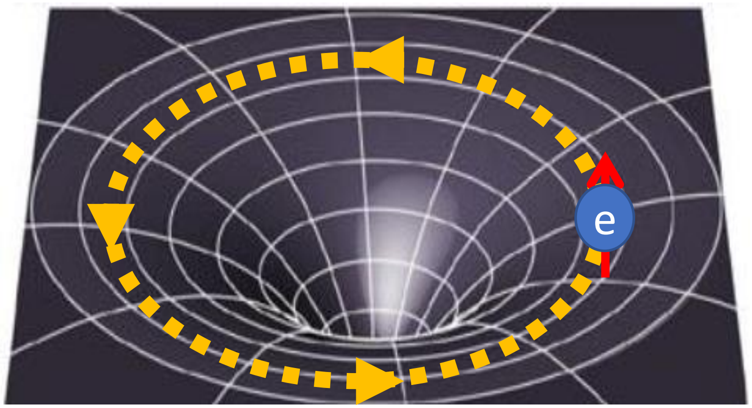
“Stick-like” object

[cf. “Framing”]

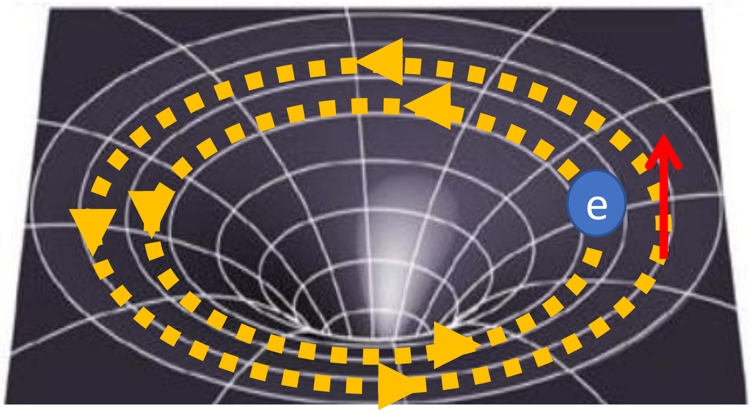
GYC, You, and Fradkin (2014) [See also unpublished results in my invited talks at APS March Meeting (2015)]

Curved space with curvature

Standard composite particle theory
[point particle]

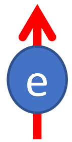


New composite particle theory
[Stick-like structure]

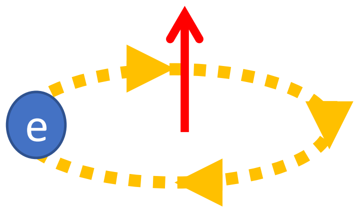


[GYC, You, and Fradkin (2014)]

From the rest frame of the composite particle:



Nothing interesting happens.

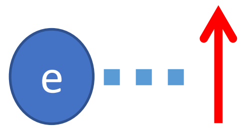


Electron goes around the flux !
["internal rotation"]

There is an additional Berry phase from curvature of the geometry.

The new composite particle sees the geometry g_{ij}

New Composite Particle Theory:

New "composite particle"  **leads to…**

A geometrical theory:

Electromagnetic gauge

$$L = \bar{\rho}\left(A_0 + \frac{3}{2}\boldsymbol{\omega_0}\right) - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + (A_\mu + \frac{3}{2}\boldsymbol{\omega_\mu})\boxed{\frac{\epsilon^{\mu\nu\lambda}}{2\pi}\partial_\nu b_\lambda} + \cdots$$

Spin connection: $\boldsymbol{\omega_\mu}$

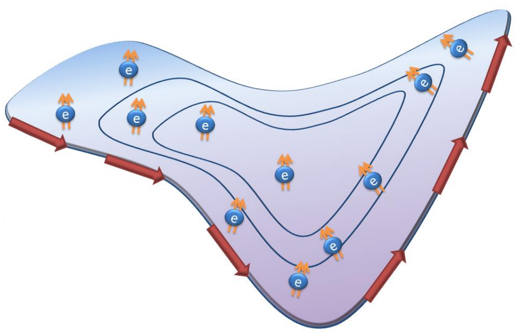
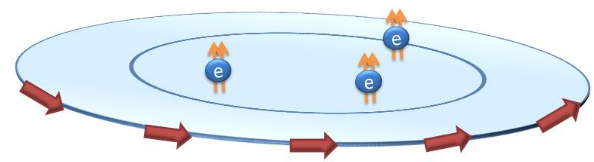
J^μ: electron current

[a function of g_{ij}]

New Composite Particle Theory:

$$L = \bar{\rho}\left(A_0 + \frac{3}{2}\boldsymbol{\omega_0}\right) - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \left(A_\mu + \frac{3}{2}\boldsymbol{\omega_\mu}\right) J^\mu + \cdots$$

Dependence on geometry ω_μ explicitly

$\Big|$  $\Big\rangle \neq \Big|$  $\Big\rangle$

$g_{ij} \neq \delta_{ij}$ $\qquad$ $g_{ij} = \delta_{ij}$

This theory can explain physics associated with anisotropy g_{ij}

Back to Experiments from Theory

0. Applied to the anisotropic state:

Evidence for a fractionally quantized Hall state with anisotropic longitudinal transport

Jing Xia[1*†], J. P. Eisenstein[1], L. N. Pfeiffer[2] and K. W. West[2] [Nature physics, 2011]

$$L = \bar{\rho}\left(A_0 + \frac{3}{2}\omega_0\right) - \frac{3}{4\pi} b_\mu \partial_\nu b_\lambda \epsilon^{\mu\nu\lambda} + \left(A_\mu + \frac{3}{2}\omega_\mu\right)\frac{\epsilon^{\mu\nu\lambda}}{2\pi}\partial_\nu b_\lambda + \cdots$$

The anisotropic state has:

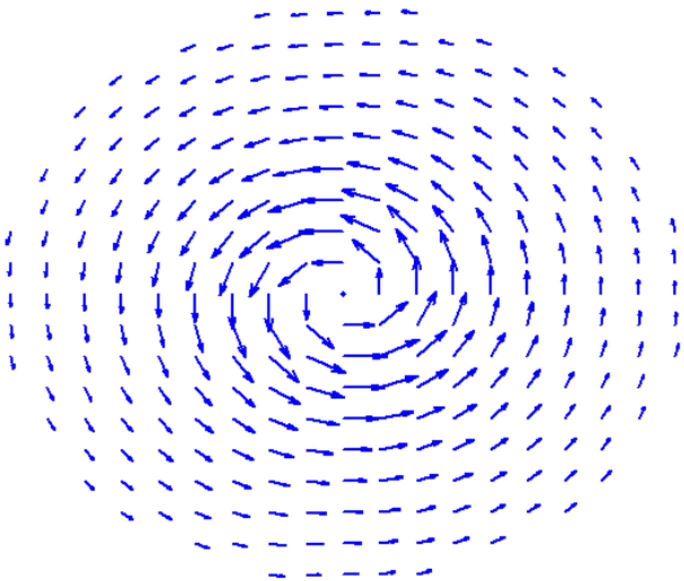
1. Well-quantized R_{xy} plateau

2. Electronic charge-$\frac{1}{3}$ fractional excitation

GYC, You, and Fradkin (2014); You, GYC, and Fradkin (2014)

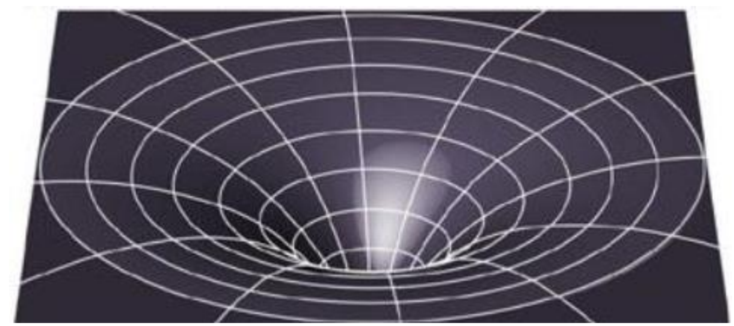
1. New fractional excitation in anisotropic states

Disclinations: vortex-like defects in $\overrightarrow{\Phi}\sim(\delta g_{xx},\delta g_{xy})$



Configuration of $\overrightarrow{\Phi}$

=



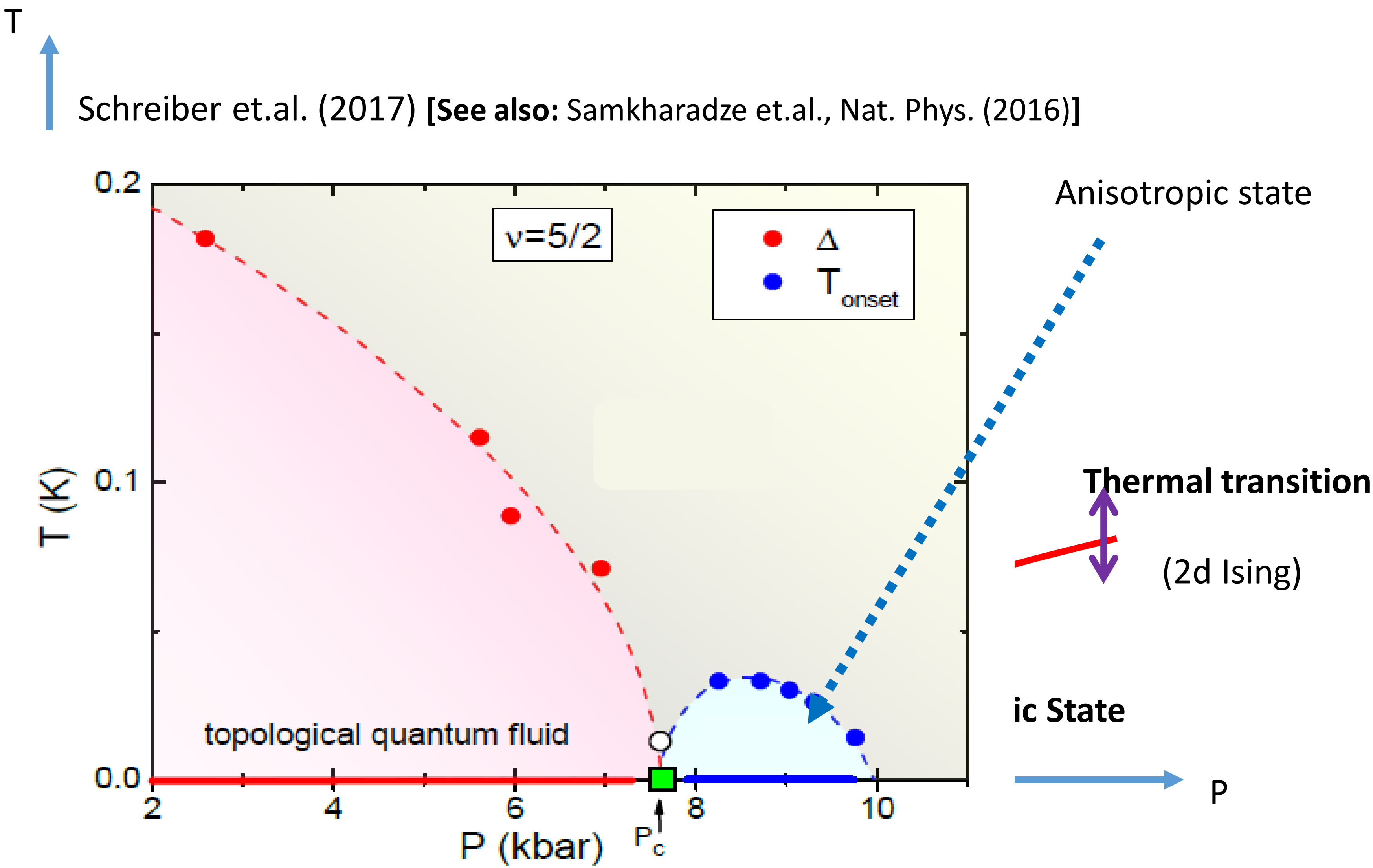
Electrons see the defect as the curvature source.

Disclinations carry the fractional charge and statistics !

This is a new fractional excitation which exists only inside anisotropic states.

GYC, Parrikar, You, Leigh, and Hughes, PRB (2014)

2. Phase diagram & Quantum Critical Phenomena:



[Unusual z=2 quantum criticality]

You, GYC, and Fradkin, PRB (2016); GYC, and Fradkin, *in preparation*

3. Other signatures of symmetry-broken quantum Hall states:

From the theory, we can also derive:

3. Softening of collective modes at the transition

4. Transport signatures

- Anisotropic DC conductivities
- Peak in the AC conductivity measurement

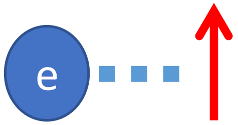
GYC and Fradkin, *in preparation*

So far, we have shown that:

1. New forms of quantum Hall states with unexpected anisotropy are found.

[**Ref.** Xia et. al., (2011), Samkharadze et.al.,(2016), Schreiber et.al. (2017)]

2. We have introduced the New composite particle theory

..which change the nature of composite particles into a "stick" 

GYC, You, and Fradkin, PRB (2014) Gromov, GYC, You, Abanov, and Fradkin, PRL (2015) You, GYC, and Fradkin, PRB(2016)

3. We have predicted several exotic experimental signatures of anisotropic states

You, GYC, and Fradkin, PRX (2014) GYC, Parrikar, You, Leigh, and Hughes, PRB (2014) GYC, and Fradkin, *in preparation*

How general is this interplay?

Topological Order

Spatial Symmetry

Geometry

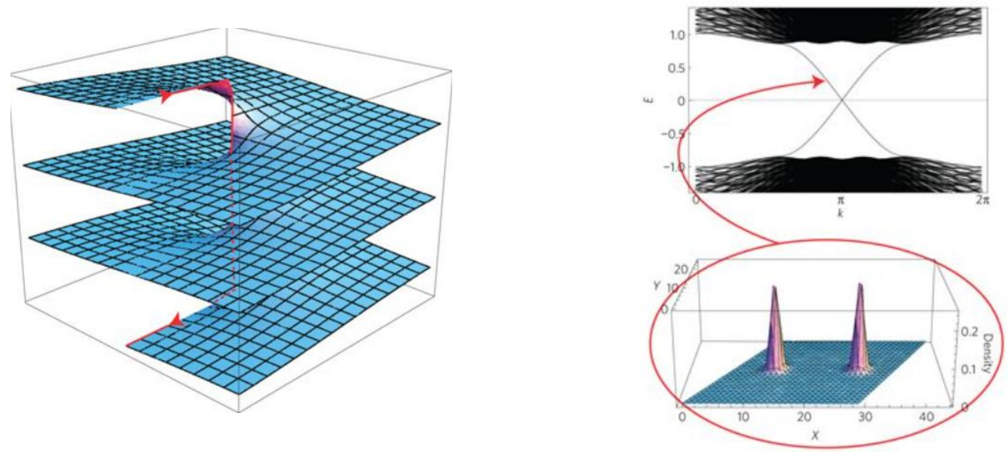
Examples:

Anisotropic Quantum Hall effects and Higher-Order Topological Insulators

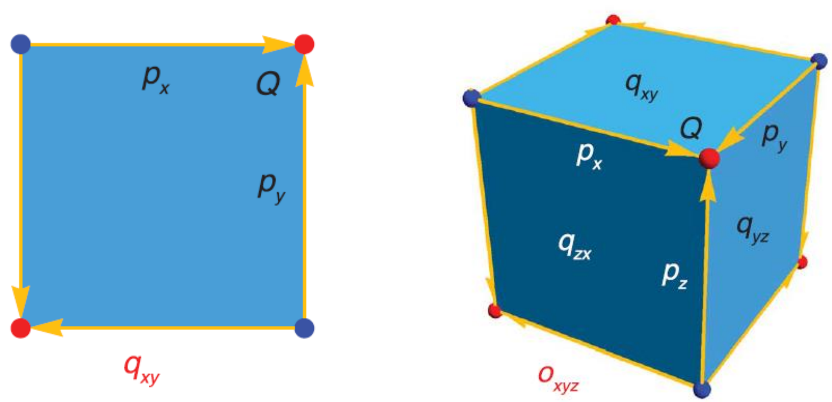
and many others (we will come back later)

Examples:

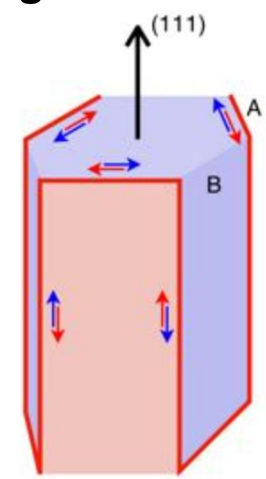
(1) Screw Dislocations in Topological Band Insulators [Ref. Ran et.al., Nat. Phys. (2009)]



(2) “Corner” of Boundary **(i.e., Curvature)** in **Higher-Order Topological Insulators**



[Ref. Benalcazar et.al., Science (2017)]

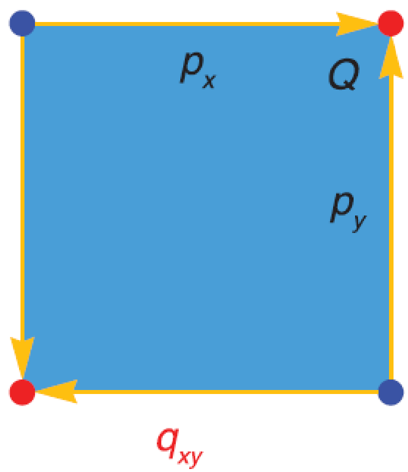


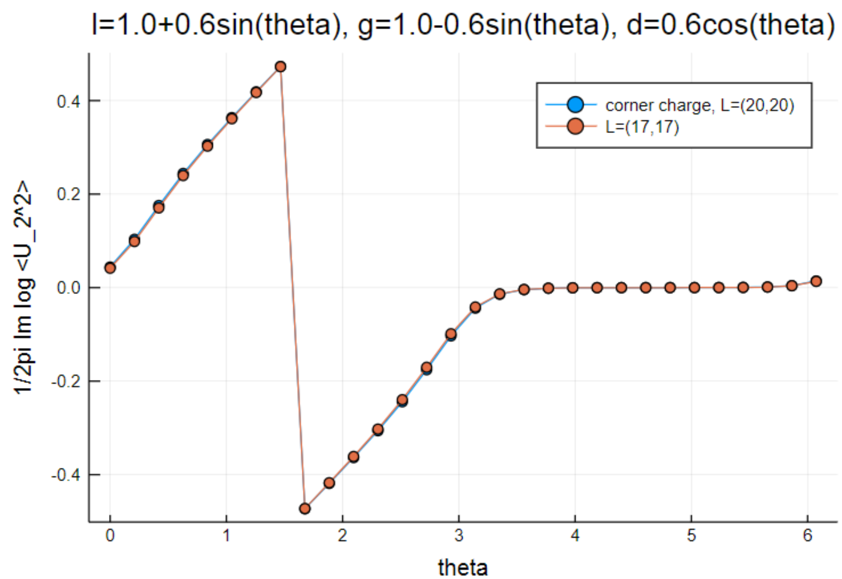
Bismuth [*Nat. Phys. 2018*]

General Theoretical Frameworks are *lacking* !

Consequently,

Characterization of Higher-Order Topological Insulator is still unclear





Ref. Byungmin Kang, Hyun Woong Kwon, Kwon Park, and **GYC**, in progress

We devised a new order parameter for many-body systems:

$$Q_{xy} = \frac{1}{2\pi} \mathrm{Im} \log \langle GS | U_2 | GS \rangle \quad \text{with} \quad U_2 = \exp\left(\frac{2\pi i}{L_x L_y} \sum xy\, \rho(x) \right)$$

Maybe, I will present this next time

Take-Home Message:

Geometry in Topological States = Non-Trivial

…and there are more to understand.

3. Outlooks and Conclusions

So far…

We have focused on a very particular theme:

"Geometry" of Topological Phases

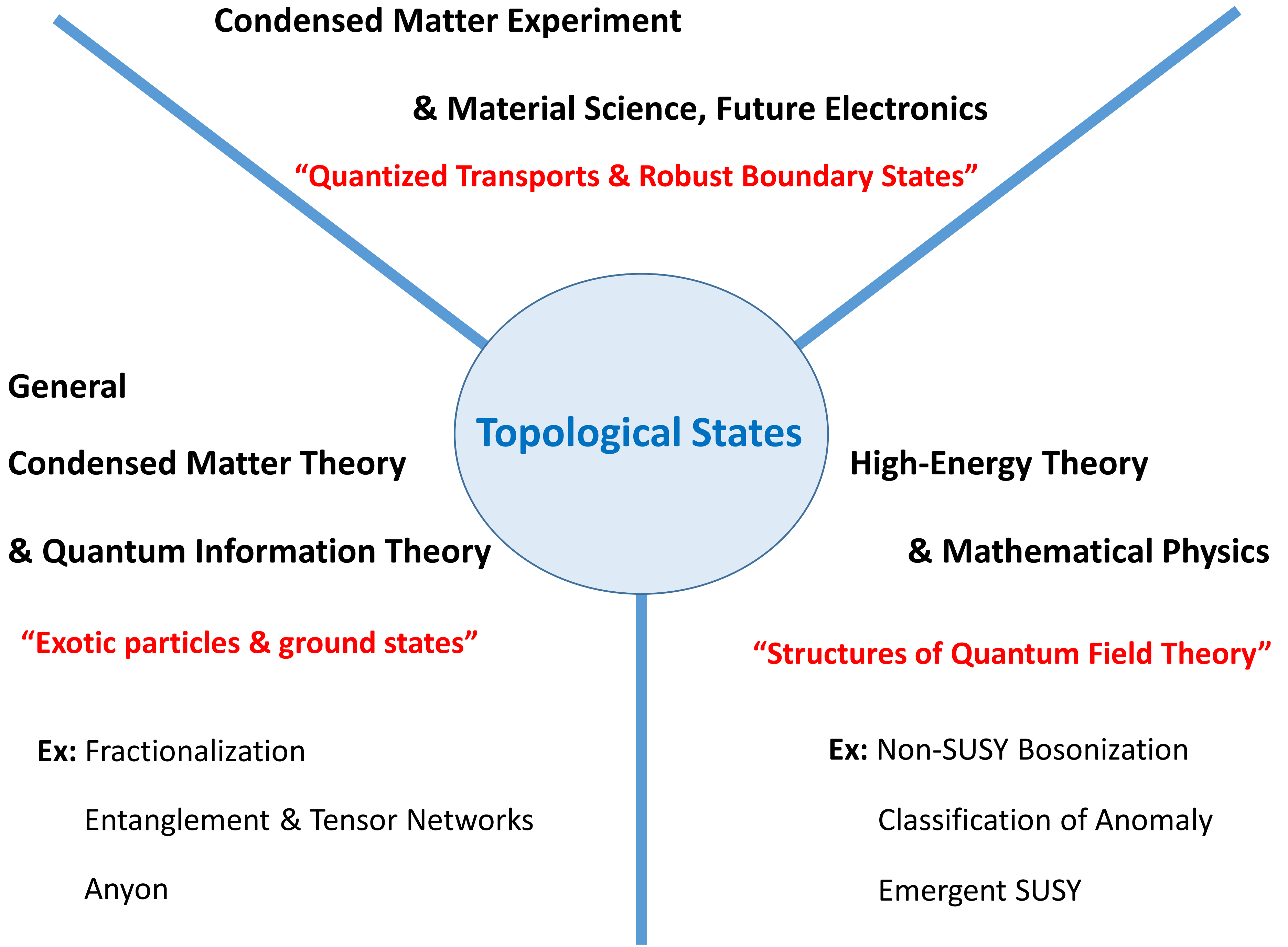
I hope you find this physics interesting.

Let's zoom out

…and estimate if they are really worthy to study

By "how it influences other fields"

[not always the best estimate]


Condensed Matter Experiment
& Material Science, Future Electronics
"Quantized Transports & Robust Boundary States"
Topological States
General
Condensed Matter Theory
& Quantum Information Theory
"Exotic particles & ground states"
Ex: Fractionalization
Entanglement & Tensor Networks
Anyon
High-Energy Theory
& Mathematical Physics
"Structures of Quantum Field Theory"
Ex: Non-SUSY Bosonization
Classification of Anomaly
Emergent SUSY

There are lots of important problems/themes to be explored!

1. Classifications of Topological States with generic symmetry

= Classifications of Anomaly with generic symmetry and matter contents

[in connection with HEP theory & Mathematical Physics]

2. Feasible Models & Experimental Realizations for Many Topological States

[in connection with Condensed Matter Experiment]

3. Stable realizations/symmetry constraints for Non-abelian Anyons

[General Condensed Matter Theory/Quantum Information]

4. Non-Equilibrium Topological States & Classifications

[General Condensed Matter Theory]

5. Generic Frameworks for Topological Phase Transitions

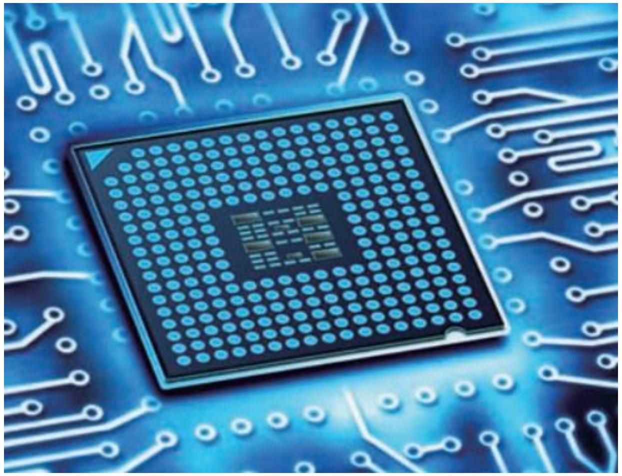
[General Condensed Matter Theory/HEP theory]

…and so on.

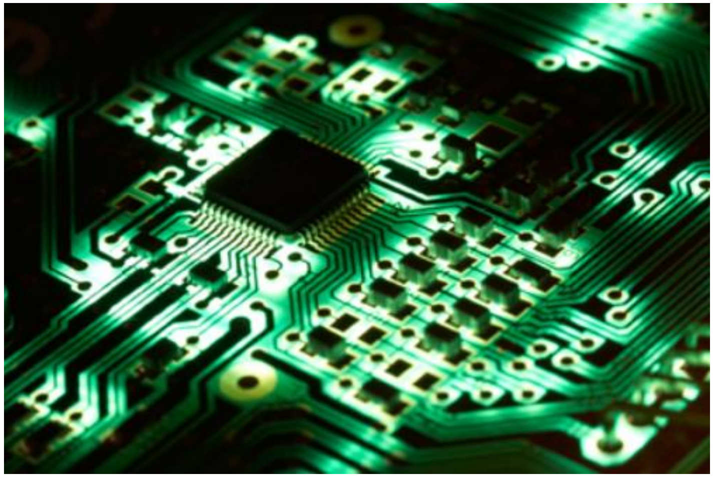
So, for young students:

There are huge chances to contribute academically!

You may also find:



Better Semiconductors



Better Electronics

More Money

...as a bonus !

Conclusions:

1. (Loosely-defined) Topological Phase:

= Universal physics is determined by topology of wavefunctions

2. Topological Phase is also “Geometric”

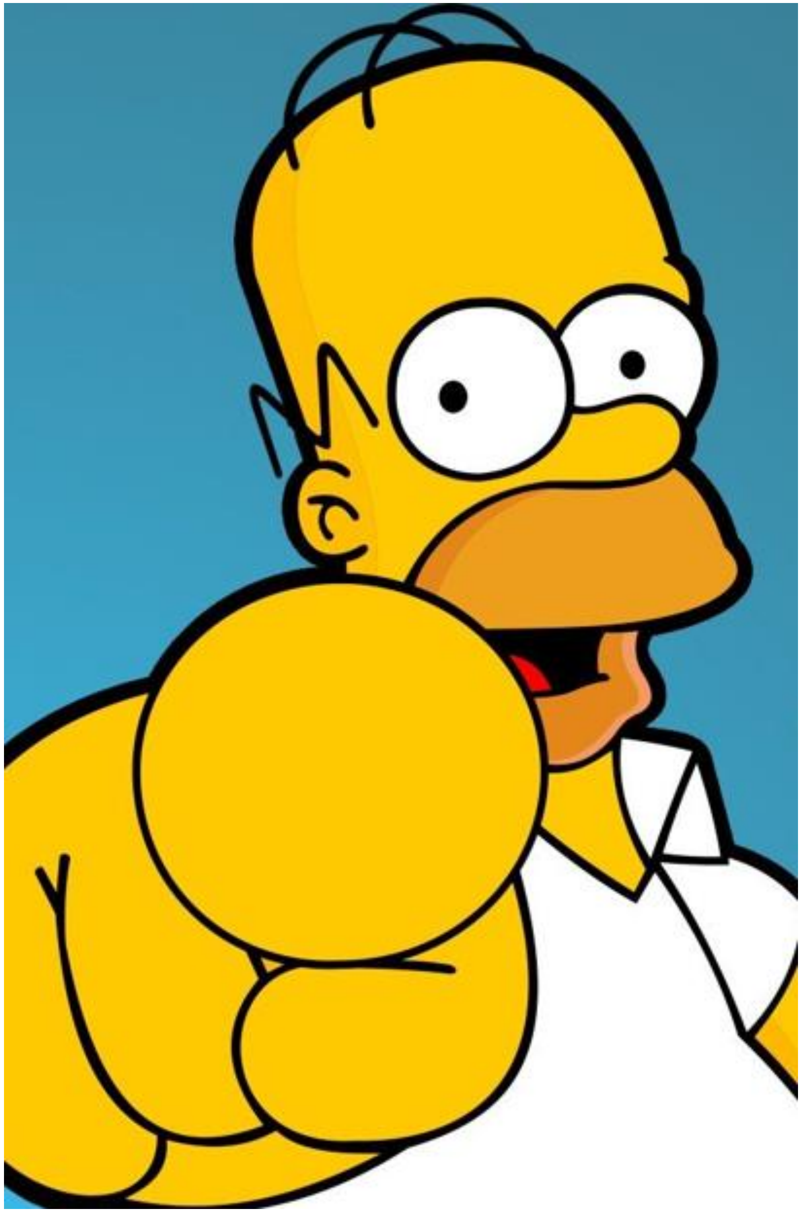
and this introduces new physics

Ex: new fractional excitations, unconventional quantum phase transitions and so on

3. Study of Topological Phase is “interdisciplinary”

Condensed Matter Theory/Experiment, High-energy Theory, Quantum Information Theory

Finally, for the students:



I want you to join me !

Thanks !

Feel free to email me at:

gilyoungcho@postech.ac.kr

More formally, I can compute:

The quantum amplitude $P[C]$ **for the composite particle to move along curve C**

$$L = \frac{1}{4\pi \cdot \boldsymbol{k}} \epsilon^{\mu\nu\lambda} a_\mu \partial_\nu a_\lambda - j^\mu a_\mu$$

[Witten (1989), Polaykov (1988), Dunne, Jackiw, and Trugenberger (1989)]

$$\mathrm{P[C]} \propto [\mathrm{Linking}] \cdot \mathbf{[Torsion]}$$

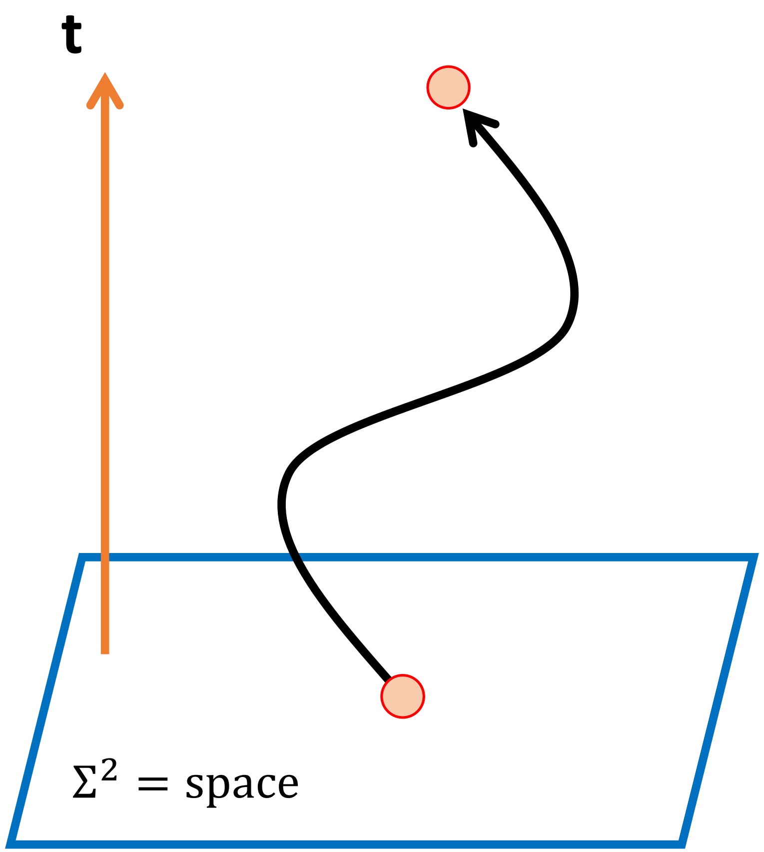
$$\propto \exp\left(-i \int_C \boldsymbol{dx^\mu} \left[\boldsymbol{A_\mu} + \frac{\boldsymbol{k}}{\mathbf{2}} \boldsymbol{\omega_\mu}\right]\right)$$

…couple directly to $\boldsymbol{\omega_\mu}$ as like $\boldsymbol{A_\mu}$!

[**consistent with spin-statistics relations**]

The standard approach gives:

$$P_{\mathrm{old}}[C] \propto \exp\left(-i \int_C \boldsymbol{dx^\mu}\, \boldsymbol{A}_\mu\right)$$

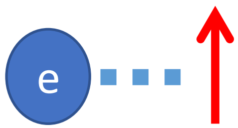


Composite particle moving along C

GYC, You, and Fradkin (2014) [See also unpublished results in my invited talks at APS March Meeting (2015)]

(Conjecture) Inside topological states:

Electrons "see" geometry through emergent "**frame**"

Instead of point particle, 

Stick *is realized.*

Consequence:

(1) Geometric Defects = Magnetic Fluxes

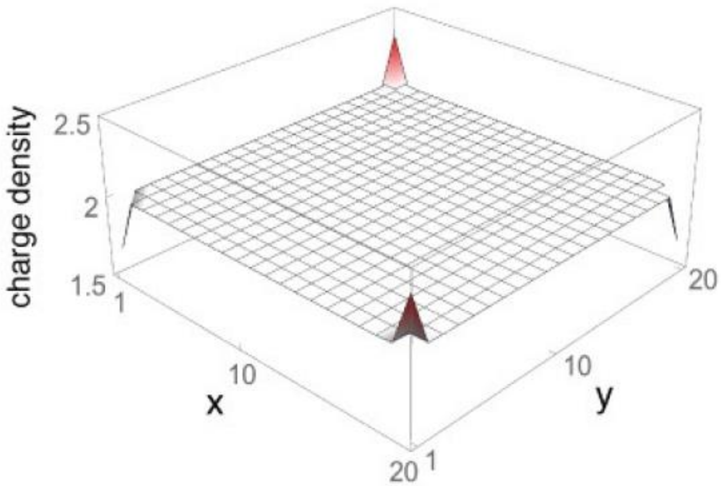
…which localize "fractional excitations" in topological states

(2) Geometry affects Electronic Structures

e.g., transport, collective modes and so on

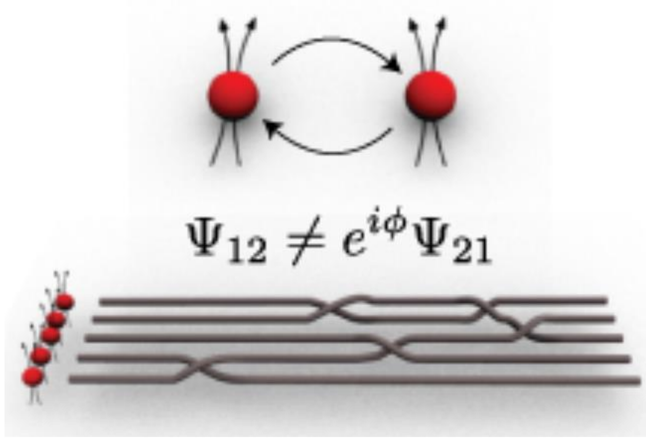
Topology appears not only in the resistivity:

2. Emergent "Fractional" Modes:



[Ref: Quantized Multipoles, Science, 2017]

Charge at "boundary" = $\frac{e}{2}$



Majorana fermion and **Anyons**

Fractional Statistics

[cf. quantum computations]

Note: "finite-N electrons" is charge-N and fermionic (if N odd)

bosonic (if N even)

⇨ **Fractionalization is *unique* in many-body "topological" states**

Activities around Topological States:

1. Efforts on Experimentally-relevant Systems

2. Developing New Theories

e.g. Structures of “theories”
in connection with HEPs
and mathematics

Topological

States

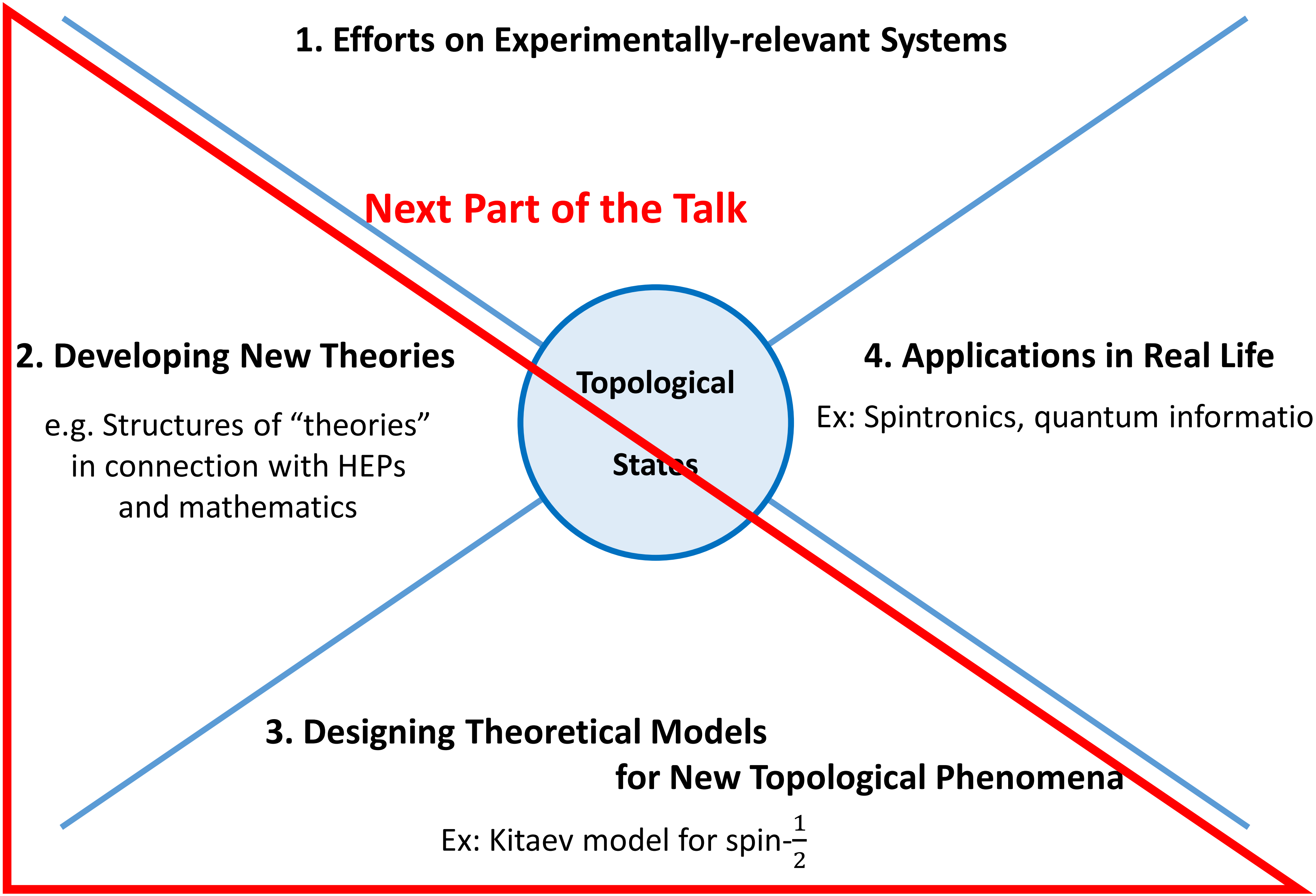
4. Applications in Real Life

Ex: Spintronics, quantum information

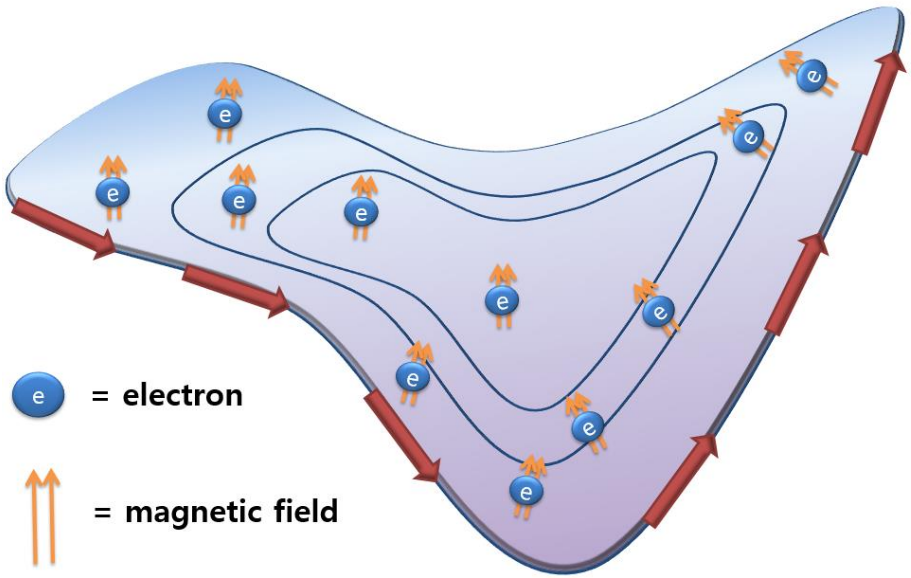
3. Designing Theoretical Models
for New Topological Phenomena

Ex: Kitaev model for spin-$\frac{1}{2}$

Activities around Topological States:

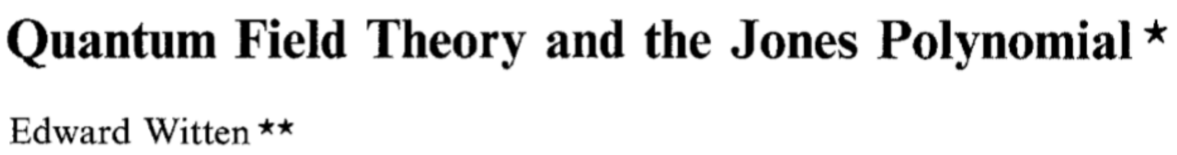


Remark:



This is the (condensed matter) realization of…

"Gravitational Anomaly (Framing Anomaly)"


Quantum Field Theory and the Jones Polynomial *

Edward Witten **

…and there are interpretations in condensed matter systems.