

Dirac Electronic Optics in Graphene Devices

이길호 (Gil-Ho Lee)

Department of Physics, POSTECH

Quantum Electronics

- Study of quantum mechanical behavior of electrons in solids

Fundamental Science

Classical Mechanics



Mechanical Engineering

Electromagnetics



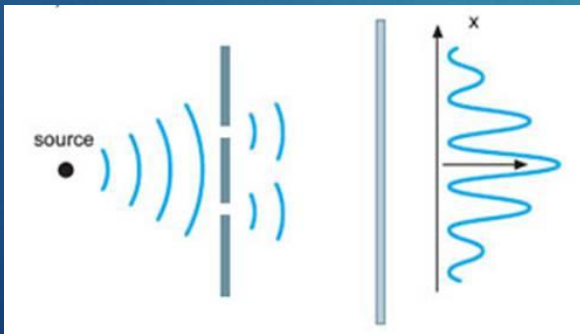
Electrical Engineering

Quantum Mechanics

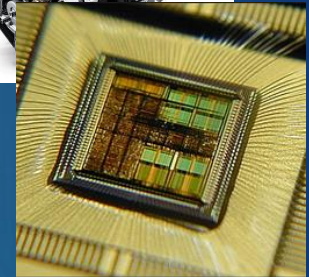


Quantum Engineering

e.g.) quantum coherence,
superconductivity



e.x.)
Engine



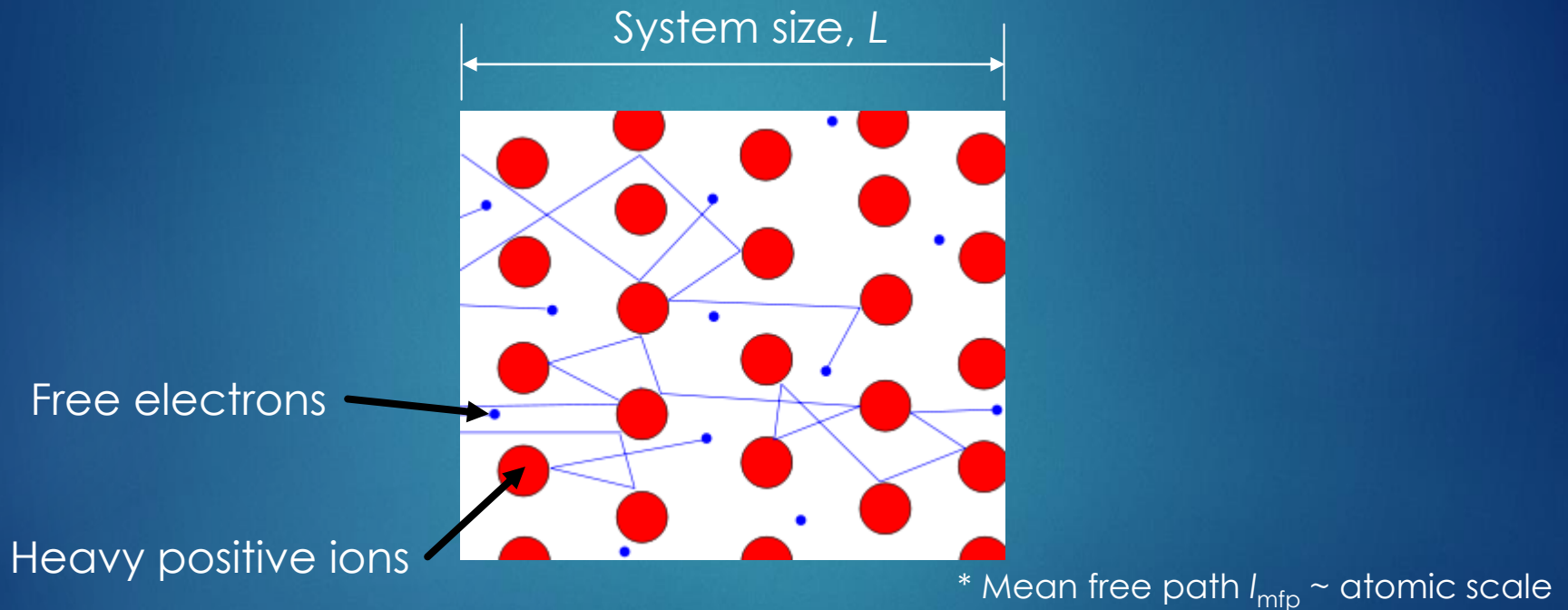
e.x.)
CPU



e.x.)
Quantum Computers

How Electrons Move in Solid

- Drude's view (1900) : Electrons keep colliding with ions.



Diffusive system ($l_{\text{mfp}} \ll L$): Resistance $\propto L$

Bloch Wave by F. Bloch in 1928

➤ Electrons in solid feels periodic potentials, $V(x + a) = V(x)$

$$\Psi_k(x) = u_k(x)e^{ikx}, \text{ where } u_k(x + a) = u_k(x)$$

, $E = E_k$: Multiple solutions assigned by k correspond to **Bands**.

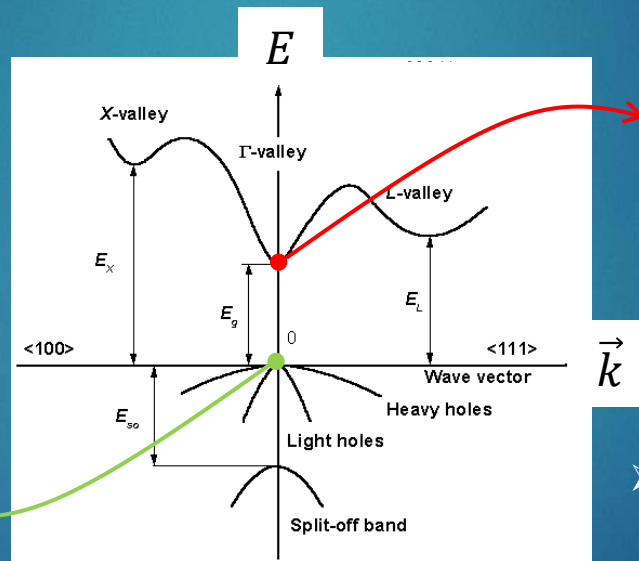
e.g.) **Band structure**
of Germanium

$$\frac{1}{m^*} = \frac{1}{\hbar^2} \frac{\partial^2 E}{\partial k^2}$$

“Hole-like quasiparticles”

Charge: e

$$m^* < 0$$



$$m^* > 0$$

Charge: e

“Electron-like quasiparticles”

➤ They bend opposite direction under same magnetic field.

Real electron in complicated interactions $\approx$ ‘Quasiparticle’ in free space

A photograph of a colorful playground with slides and climbing structures, set against a backdrop of green trees. A semi-transparent dark rectangle is overlaid on the right side of the image, containing the text. A solid green rectangle is in the top right corner.

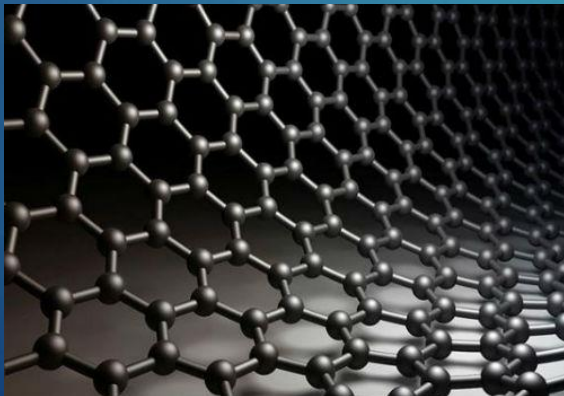
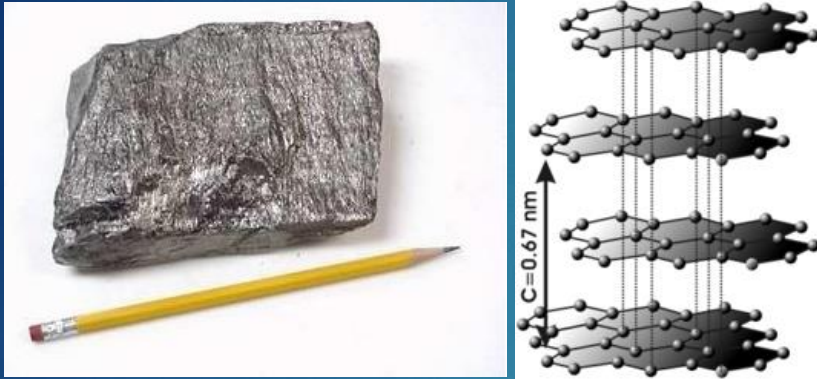
Graphene

-Playground for
condensed-
matter physicists

Graphite

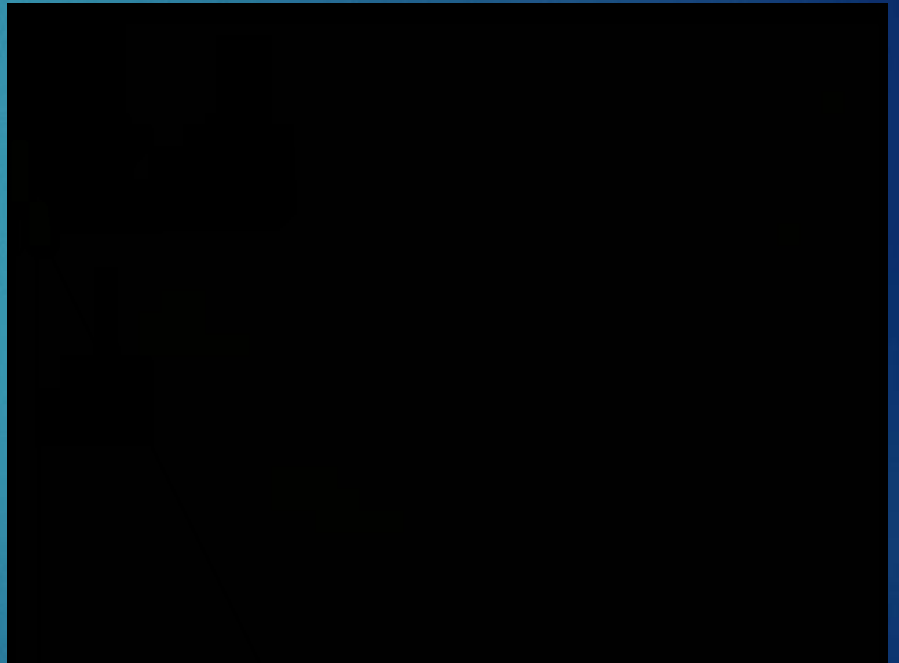
2-Dimensional Van der Waals materials

Graphite (흑연)



Graphene (그래핀)

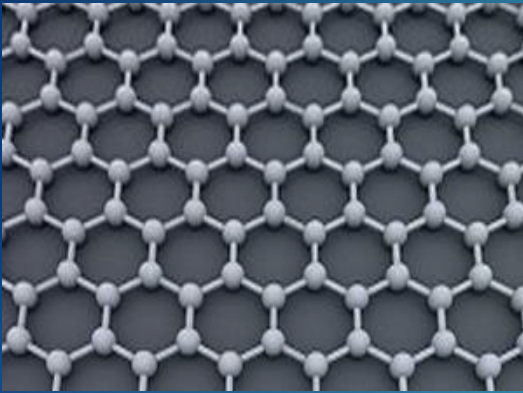
[Isolation of graphene with Scotch tape]
(2004)



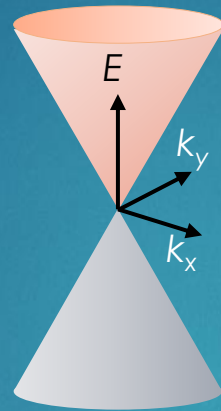
[Youtube, Prof. Kostya Novoselov and Dr. Peter Blake]

Graphene

Monoatomic layer
of carbon atoms



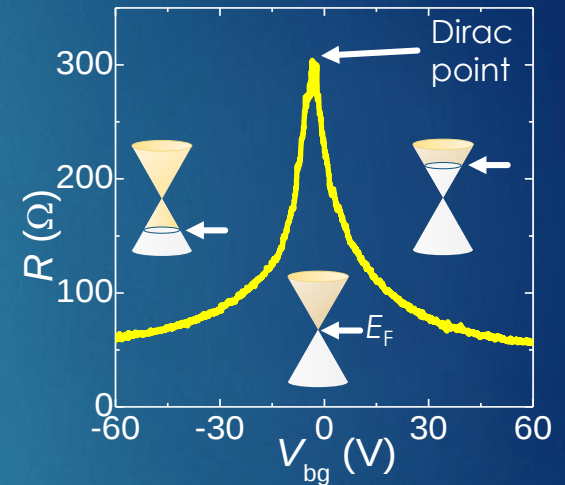
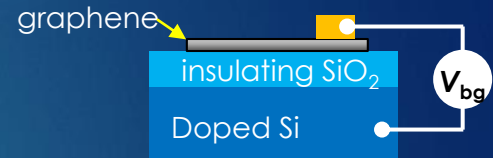
Electronic structure



Dirac cone

Linear band structure
 $E \propto k$ (similar to Light)

Electrically tunable



[Other properties]

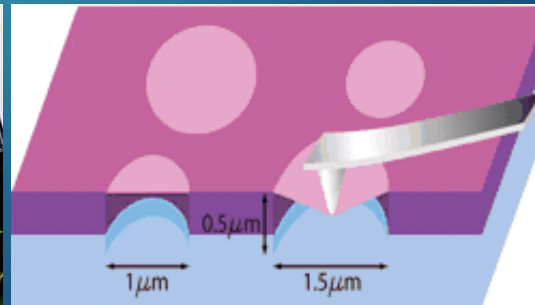
Atomically thin



Chemically inert



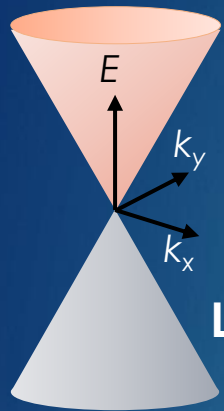
Mechanically strong



Transparent,
Flexible &
Conductive



Relativistic Dirac Fermion



Dirac cone

$$E^2 = (pc)^2 + (\cancel{mc^2})^2$$

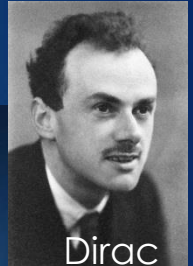
Linear band structure

$$E \propto \pm k = \pm p/\hbar$$



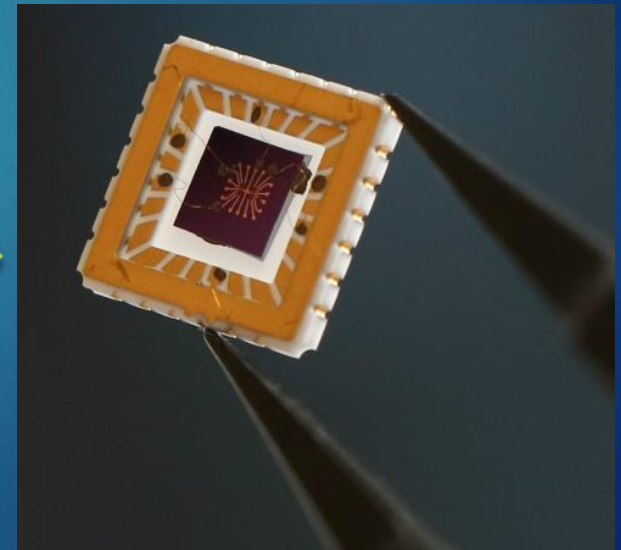
$$(i\hbar\partial - m)\psi = 0$$

Quantum Mechanics + Special Relativity



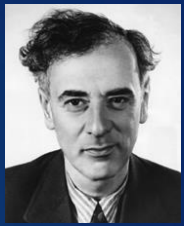
Dirac

Studying on relativistic quantum mechanics using nano-electronic devices



[LHC]

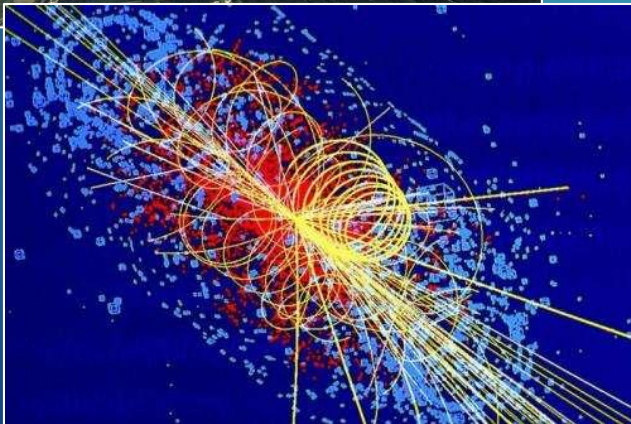
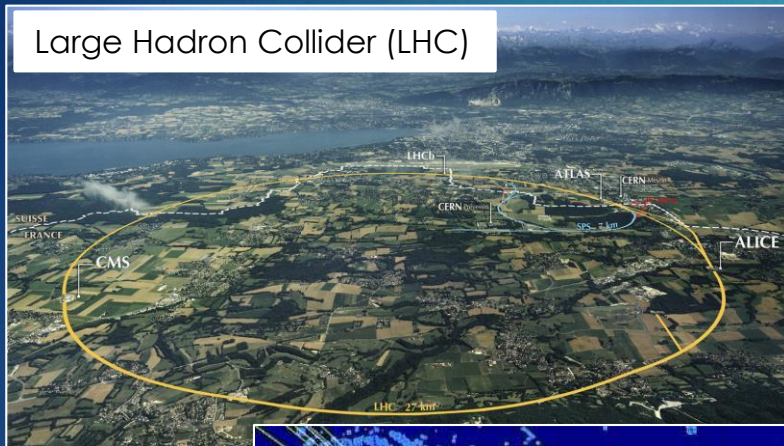
Quasi-Particles in Solid



[Lev Landau]

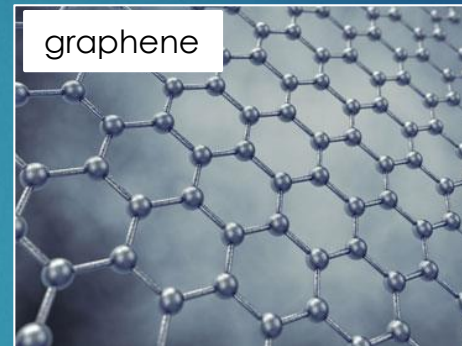
● “Real”-particles in particle physics

High-energy creates particles



● “Quasi”-particles in solid physics

“Engineering” a new kind of particles



Examples)

- Electron/hole-like
- Phonon
- Bogoliubon
- Exciton
- Trion
- Holon
- Spinon
- Magnon
- Phason
- Polaron
- Roton
- Soliton
- Dirac Fermion
- Majorana fermion
- Parafermion
- ...

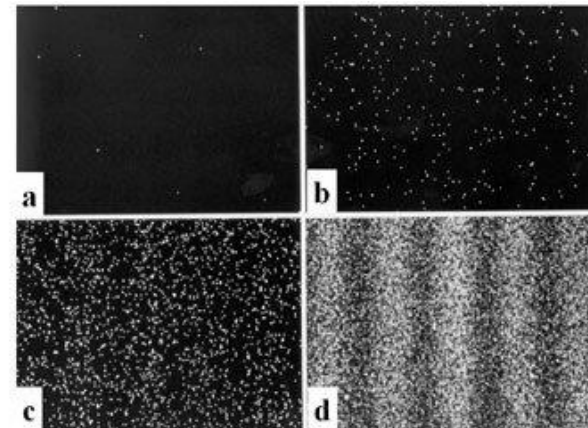
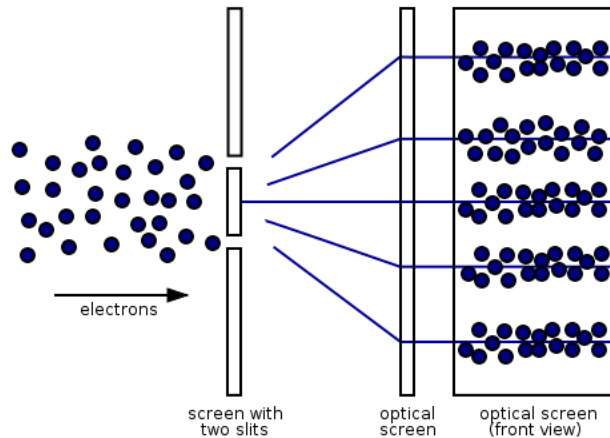
- ✓ More control
- ✓ Confined in solid

Searching for **New Fundamental Particle**

Dirac Electronics Optics

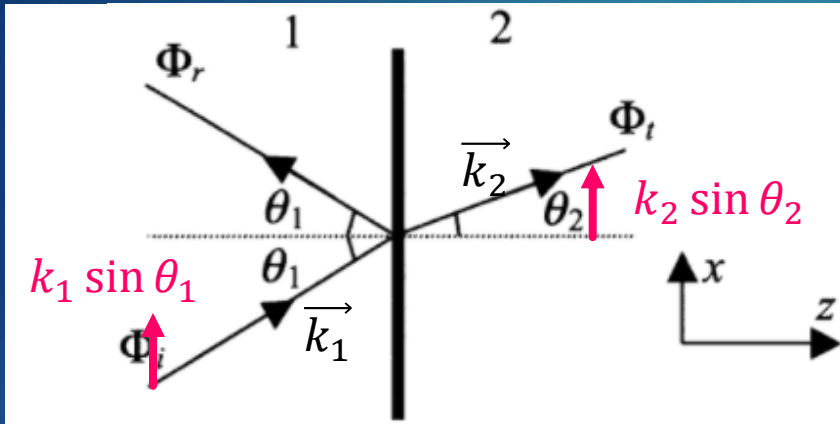
Wave-particle Duality

ELECTRON IS A
Wave!



Single-electron Build-up of Interference Pattern

Electronic Optics

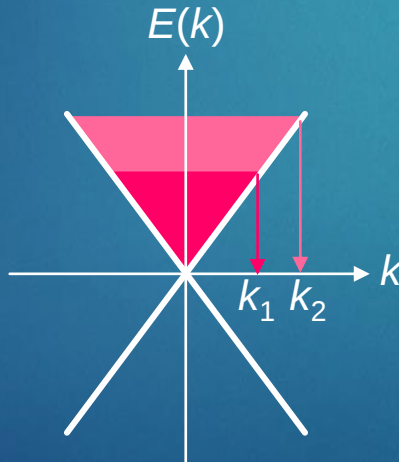


Momentum conservation
along x-direction:

$$k_1 \sin \theta_1 = k_2 \sin \theta_2$$

or, $n_1 \sin \theta_1 = n_2 \sin \theta_2$

$$n \equiv k/k_0, \quad k_0 = k \text{ in vacuum}$$

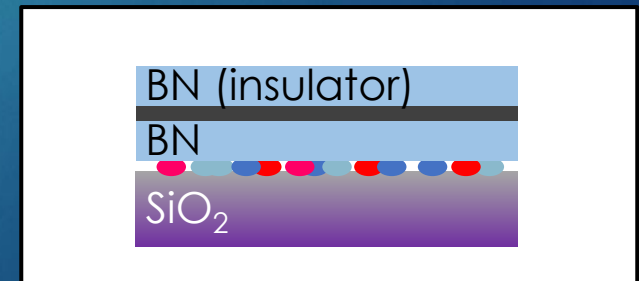
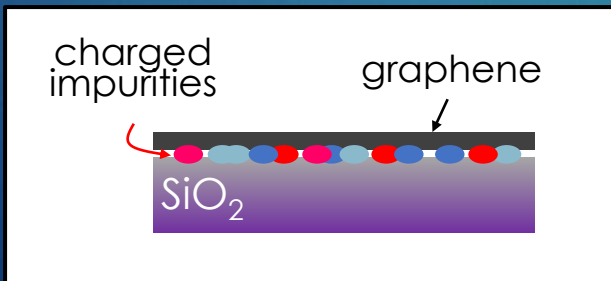


Refractive index n is gate-tunable.

Optics v.s. (Dirac) Electronic Optics

	Optics	(Dirac) Electronic Optics
Particles	Photon	(Dirac) Electrons
Type	Bosons	Fermions
Charge	No charge	Negative or positive charge
Refractive index	Given by materials	Gate-tunable
Wave length	1000 nm	10 nm
System size	1 m	1 μ m

Cleaning Graphene for Electronic Optics

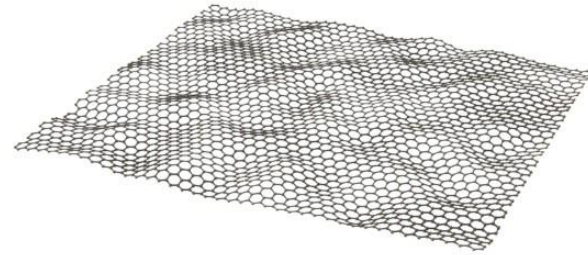


Layer-by-Layer Stacking

Early days,

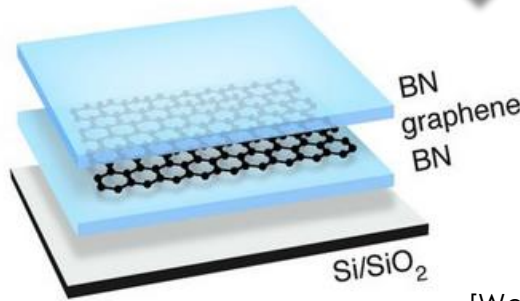
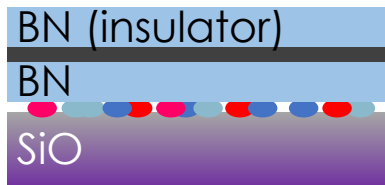
1. Impurities on substrate

2. Rough surface of substrate

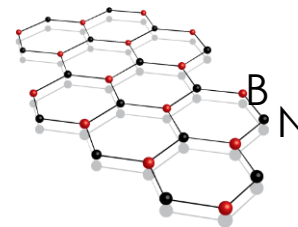


* Mean free path $l_{\text{mfp}} \sim 100 \text{ nm}$

Recently,



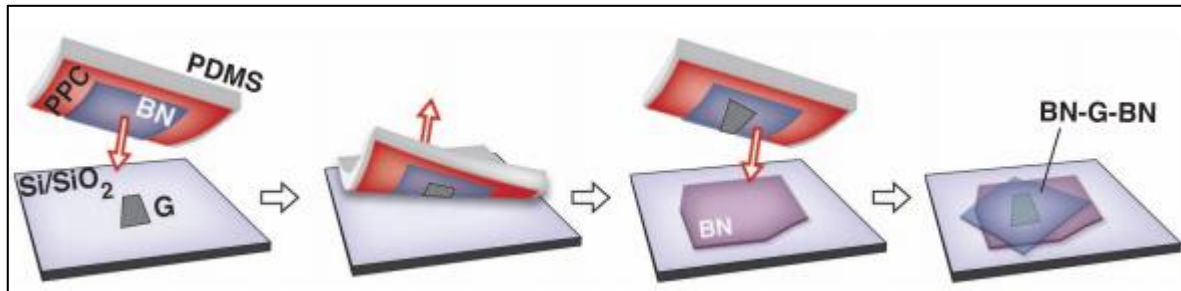
hexa-BN



[Wang et al., Science(2013)]

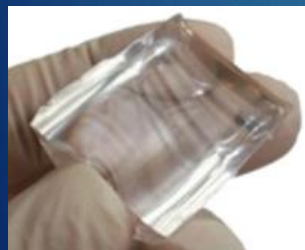
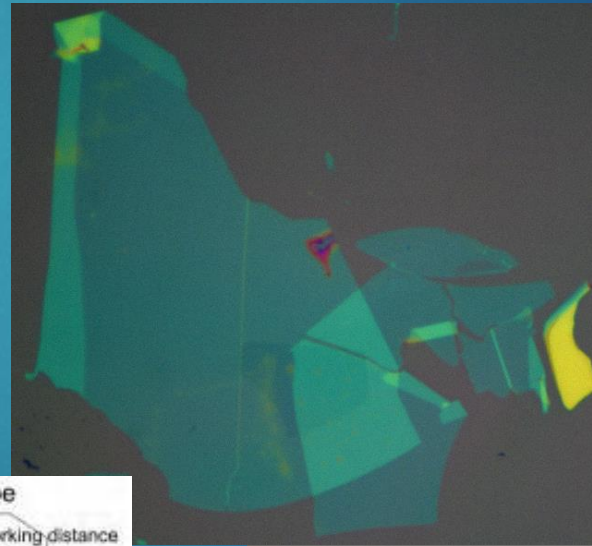
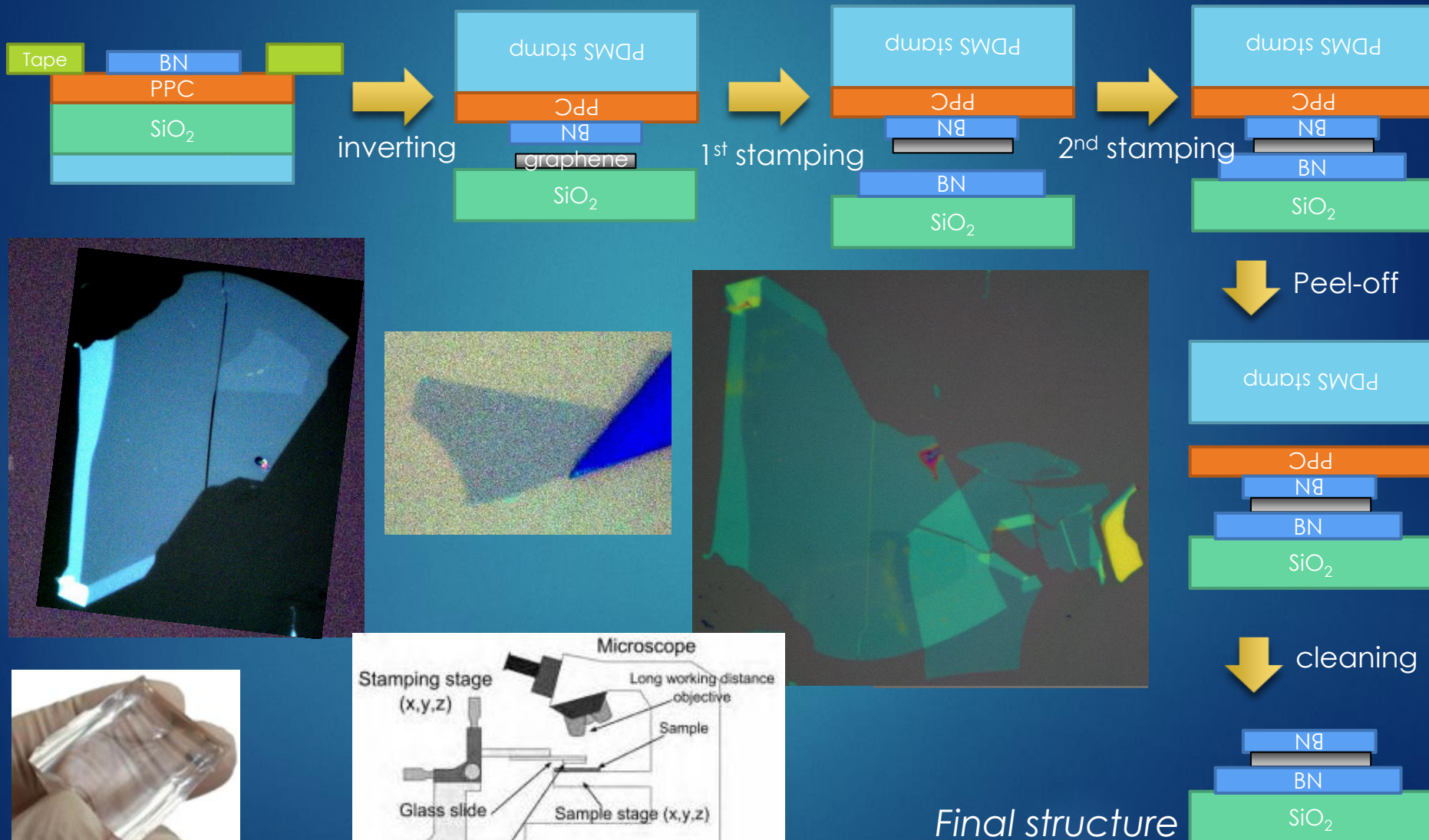


Transfer stage in POSTECH

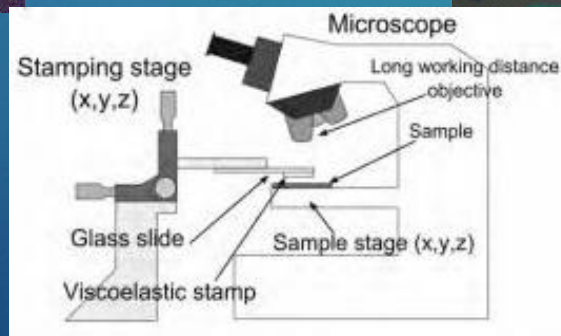


* Mean free path $l_{\text{mfp}} > 10 \mu\text{m}$

Dry-Transfer Technique



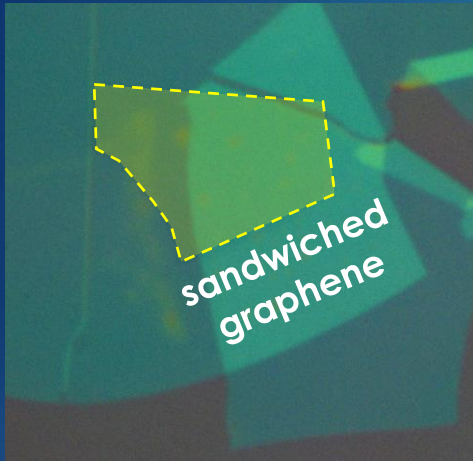
PDMS: transparent polymer



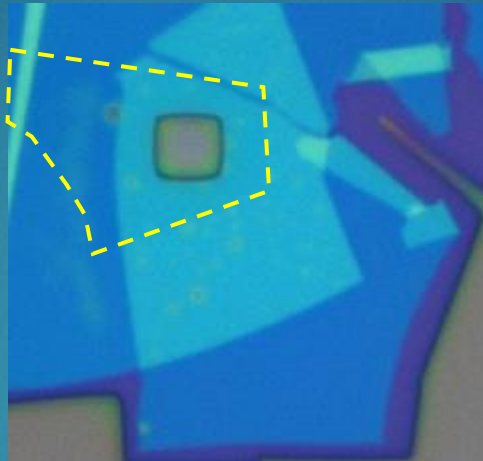
Final structure

Edge Contacts

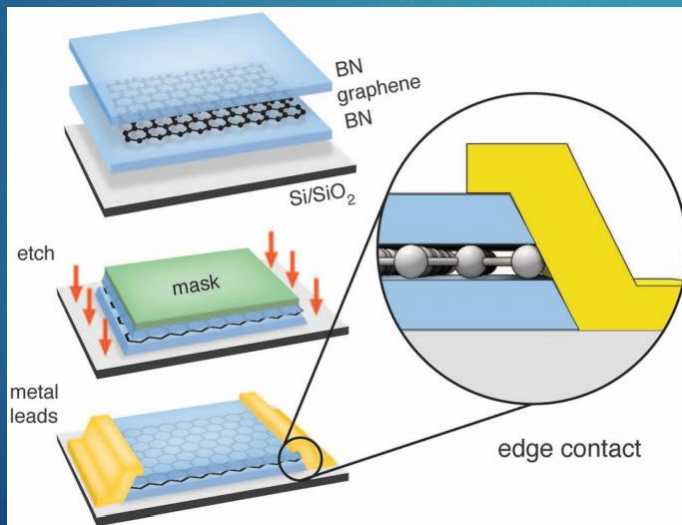
BN/Graphene/BN structure



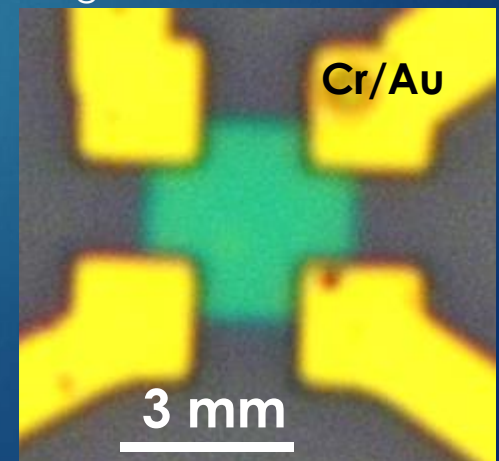
Masking



Reactive Ion Etching



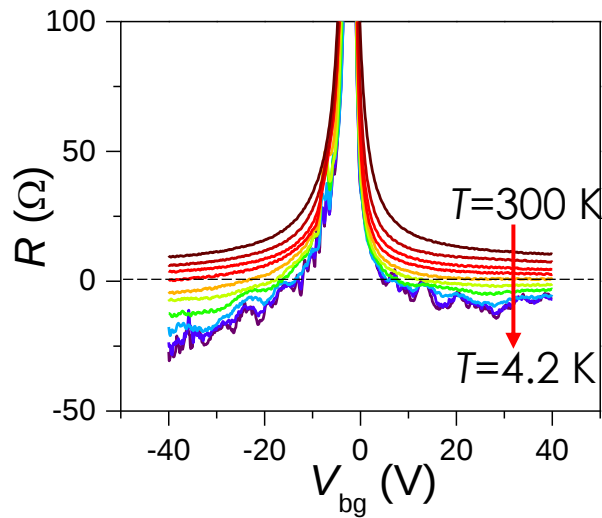
Edge contact



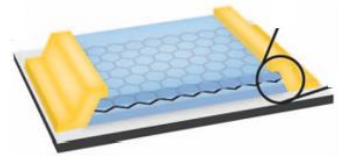
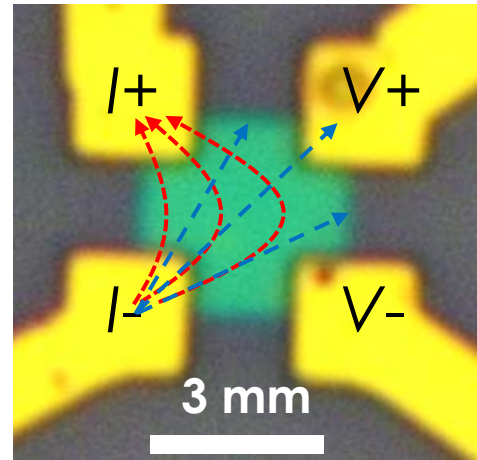
[L. Wang *et al.*, *Science* **342**, 614 (2013)]

Ballistic Transport in Graphene

Temperature dependence of R



Negative resistance



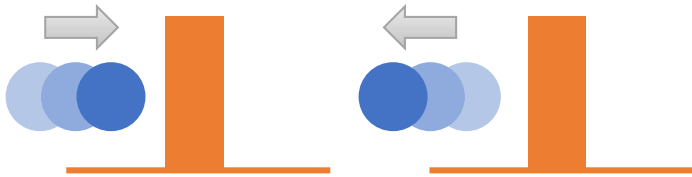
quasiparticle trajectory

- - - - - $\rightarrow$: diffusive case
- - - - - $\rightarrow$: ballistic case

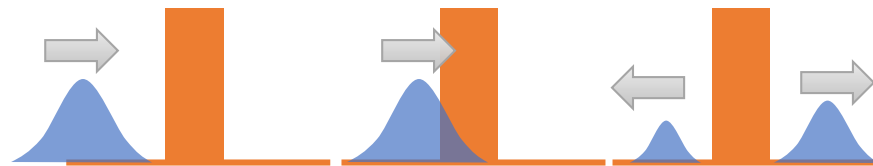
Klein Paradox by Oskar Klein in 1929

Klein paradox

Classical picture



Quantum picture



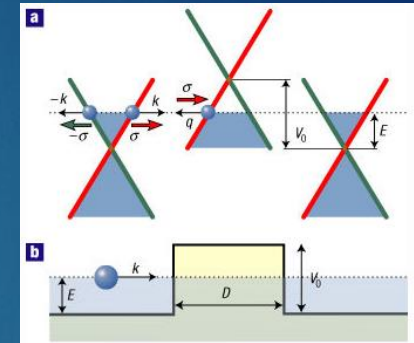
Relativistic Quantum picture (Klein tunneling)



100 %

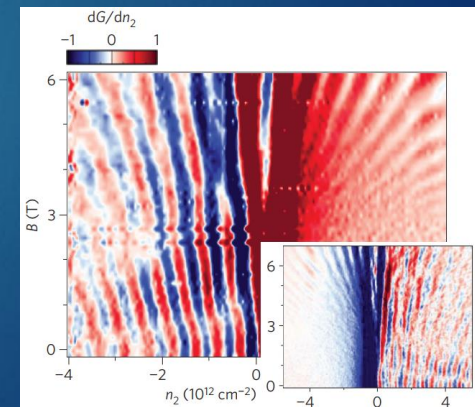
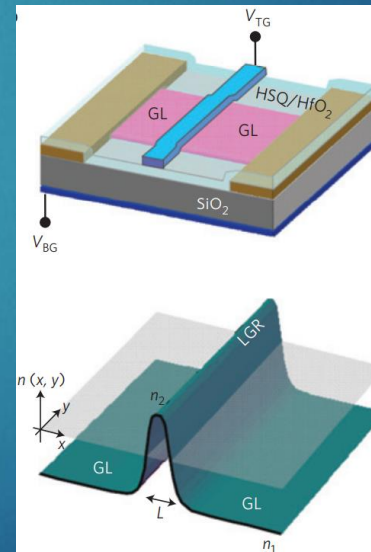
Klein tunneling in graphene

Theory



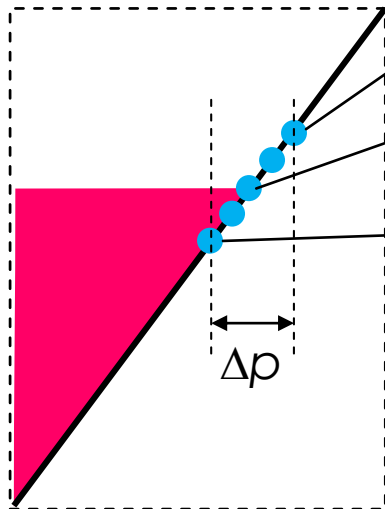
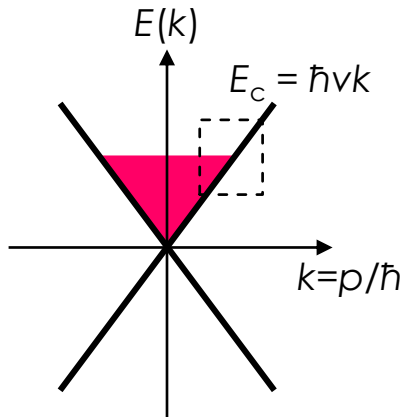
[Katsnelson et al., *Nature Phys.* (2006)]

Experiment



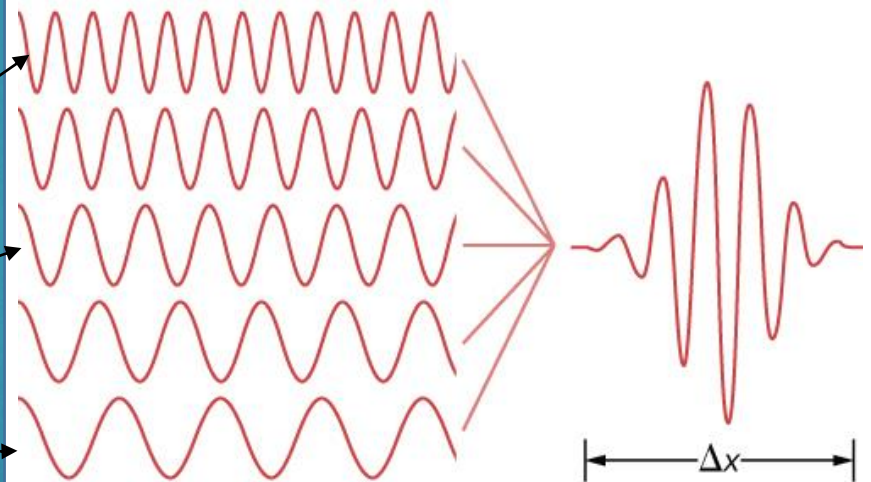
[A. Young and P. Kim, *Nature Phys.* (2009)]

Wave Packet



Several plane waves

Wave packet



Heisenberg Uncertainty Principle

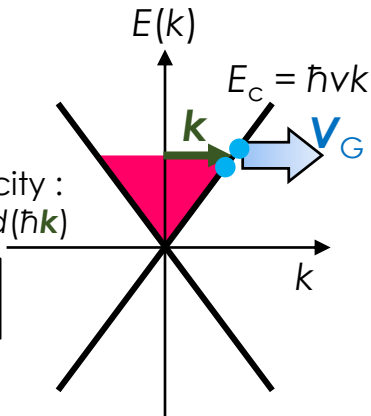
$$\Delta x \Delta p = \hbar/2$$

Group Velocity

n-doped

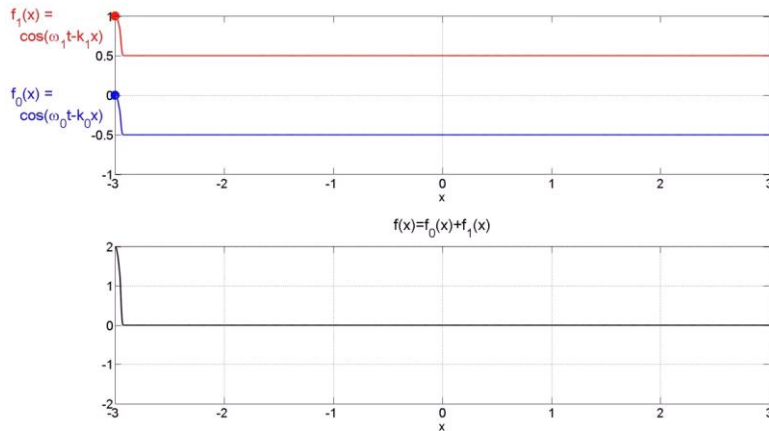
- Group velocity :
 $V_G = dE(k)/d(\hbar k)$

$$V_G = v\hat{k}$$



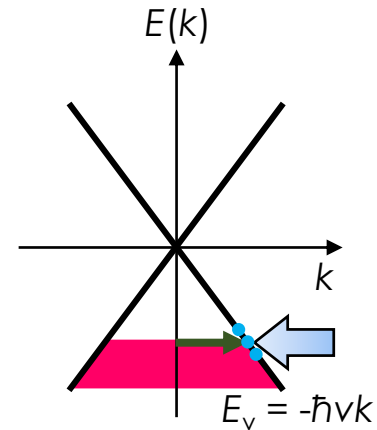
$$\Psi \sim e^{ikx - iEt/\hbar}$$

Bigger k has higher energy
(Shorter λ) (Faster)



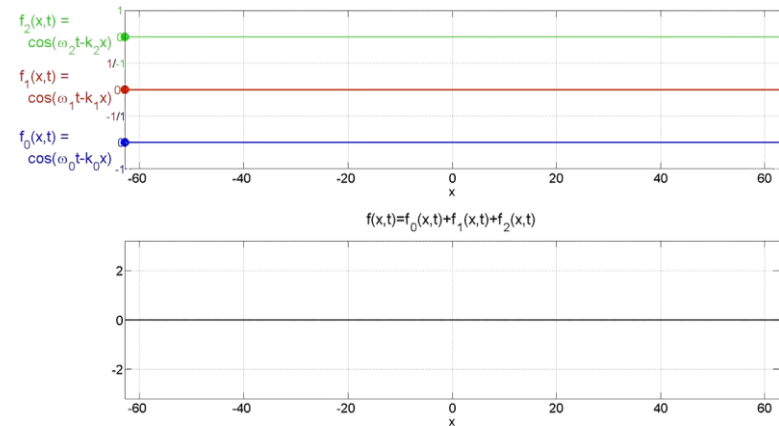
p-doped

* $v \sim c/300$



$$V_G = -v\hat{k}$$

Bigger k has lower energy
(Shorter λ) (Slower)

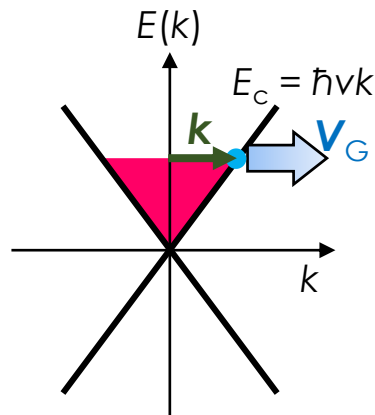


Negative Refraction

By V. G. Veselago (1967)

Ambipolar band structure w/o gap

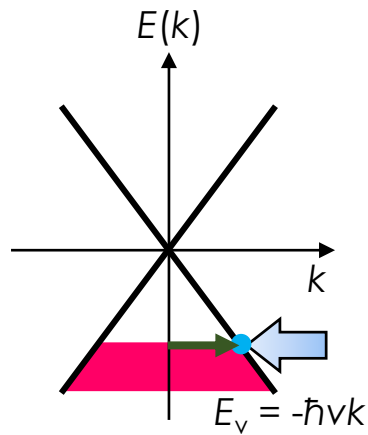
n-doped



- Group velocity :
 $\mathbf{v}_G = dE(k)/d(\hbar\mathbf{k})$

$$\hat{V}_G = v\hat{k}$$

p-doped

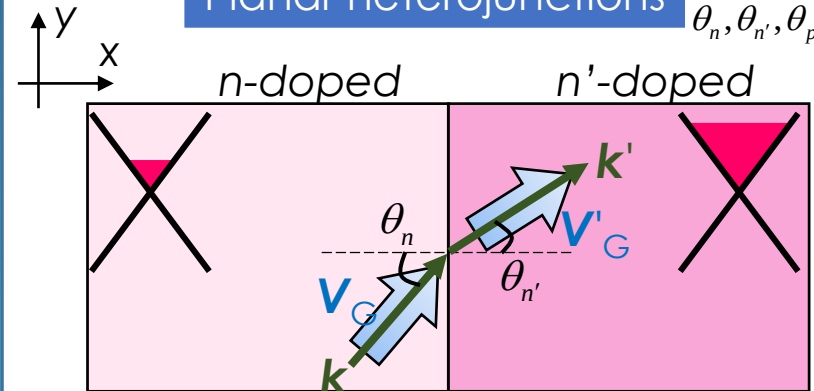


$$\hat{V}_G = -v\hat{k}$$

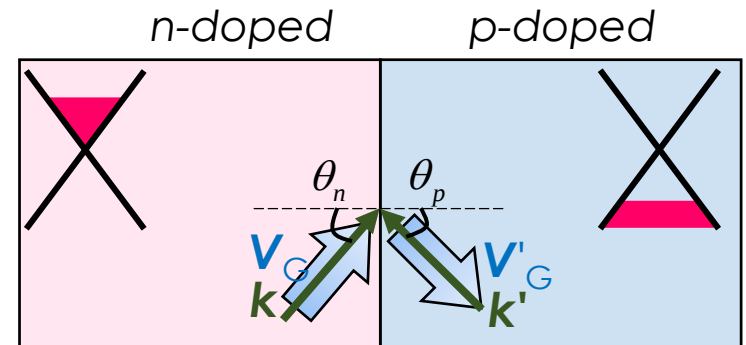
* $v \sim c/300$

Planar heterojunctions

$\theta_n, \theta_{n'}, \theta_p > 0$



$$k_n \sin \theta_n = k_{n'} \sin \theta_{n'}$$



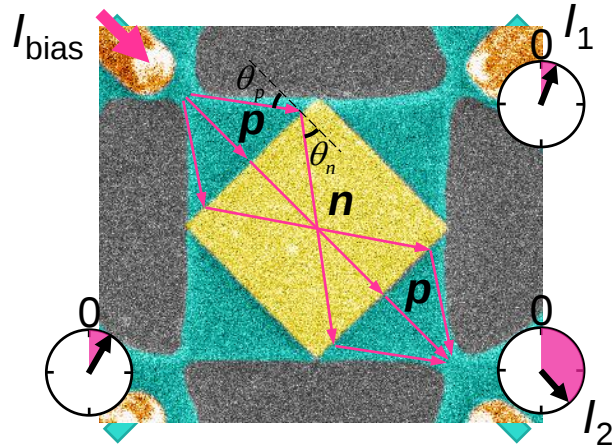
$$k_n \sin \theta_n = -k_p \sin \theta_p$$

$$\frac{\sin \theta_n}{\sin \theta_p} = -\frac{k_p}{k_n} = -\sqrt{\frac{\rho_p}{\rho_n}} \equiv n$$

[Cheianov, Fal'ko, & Altshuler, Science (2007)]

Experiments

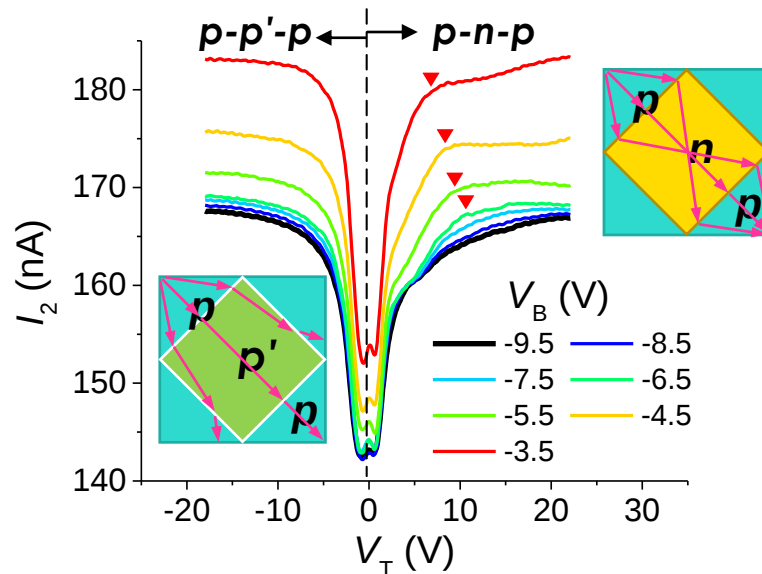
Veselago's lens



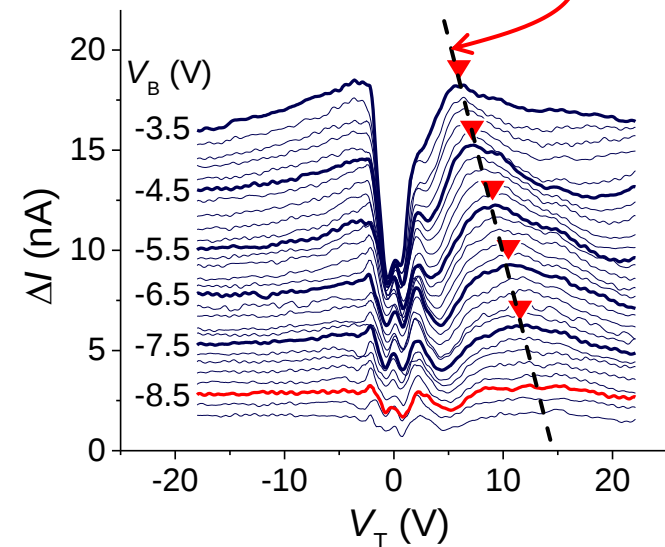
[**G.-H. Lee**, G.-H. Park, H.-J. Lee, *Nature Physics* (2015)]

$$\frac{\sin \theta_n}{\sin \theta_p} = -\frac{k_p}{k_n} = -\sqrt{\frac{\rho_p}{\rho_n}} \equiv n$$

Draining current



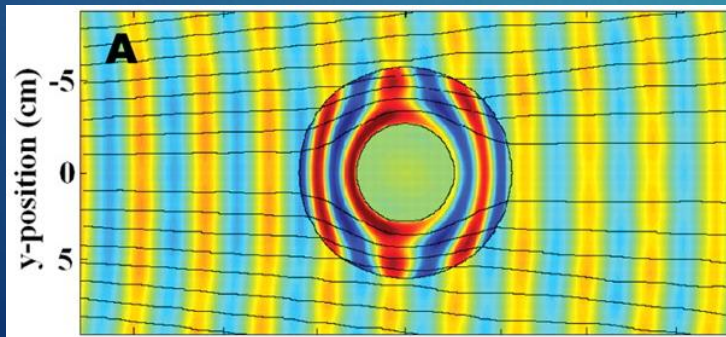
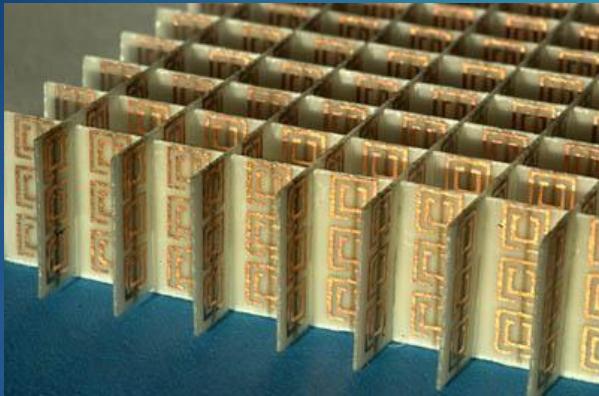
Background subtracted



Comparing to Metamaterials

Meta-materials: Reparative structures smaller than wavelength

e.g.) Microwave $\lambda \sim 10$ cm,
Electrons, $\lambda_F \sim 10$ nm,

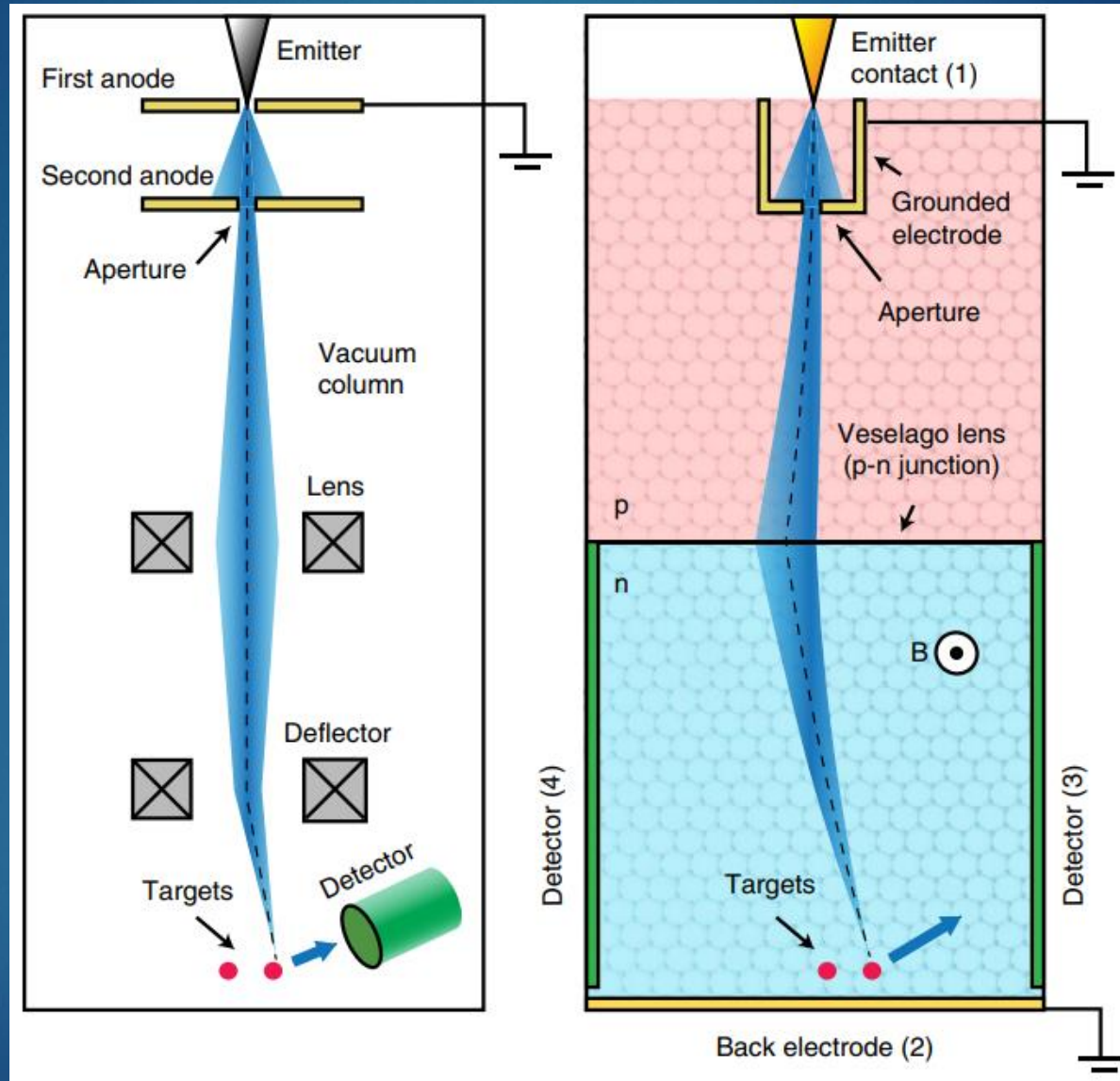


[D. Schurig *et al.*, Science 2006]

Invisible blanket

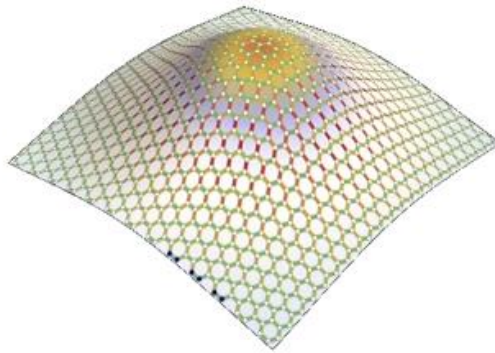


Dirac Fermion Microscope



Stretched Graphene

- Relativistic particle in a curved space

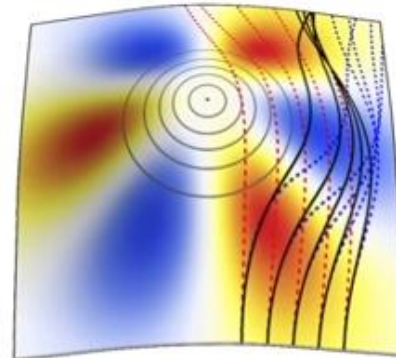
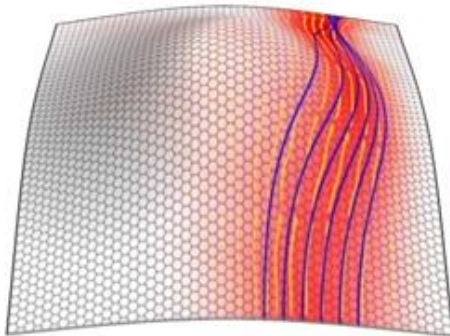
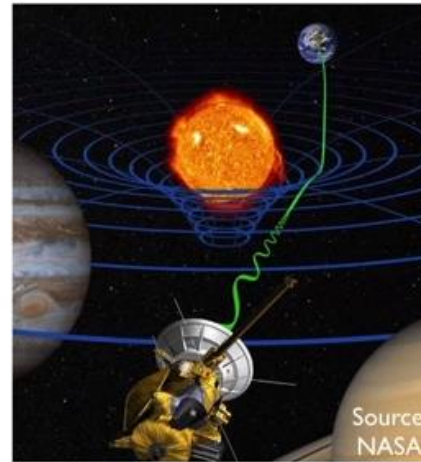


- Condensed matter approach:

- ▶ tight-binding Hamiltonian
- ▶ NEGF method

- General relativistic approach:

- ▶ relativistic point particles
- ▶ curved space, pseudo-magnetic field



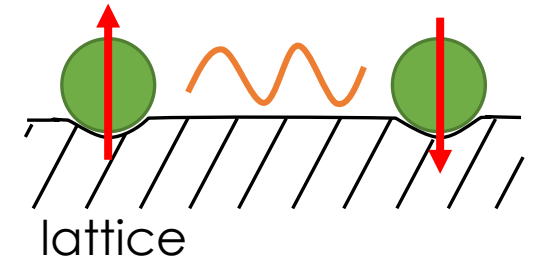
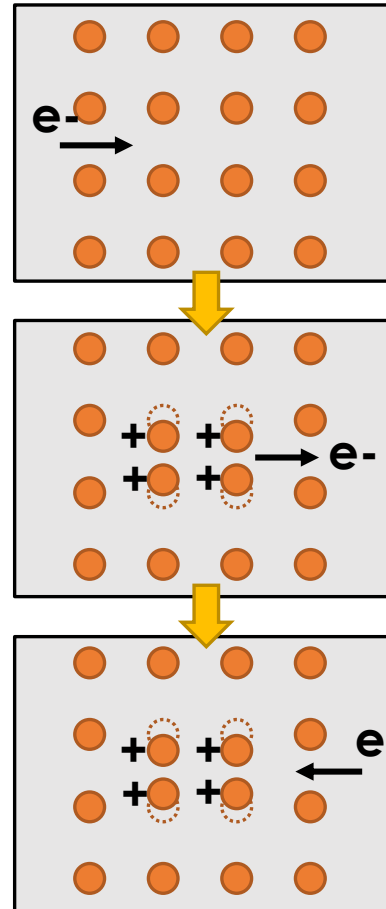
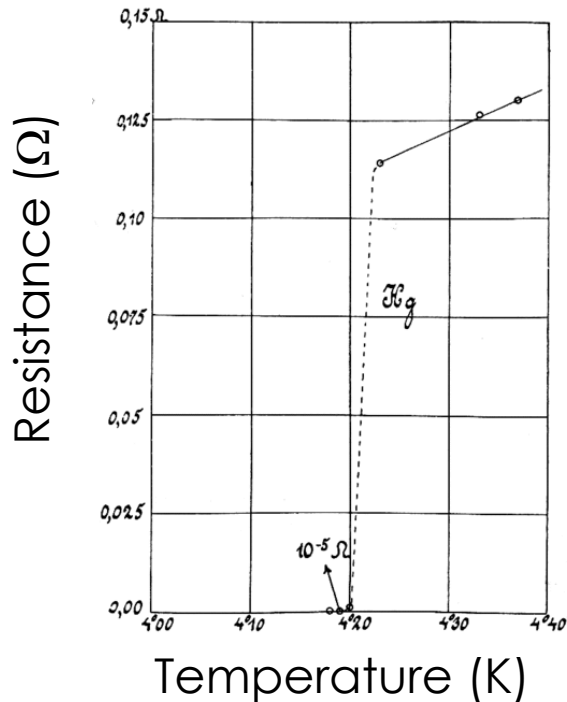
Brief Introduction to Superconductivity

Superconductivity

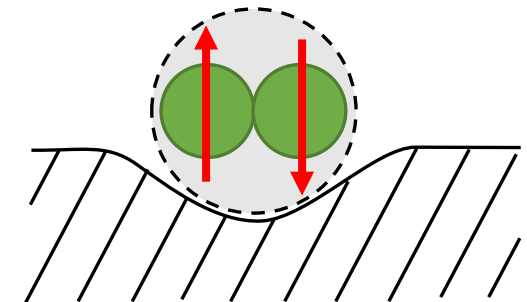
- **Superfluidity** of boson-like **Cooper pairs** made up of two fermions



[H. K. Onnes, 1911]



Cooper pair



in BCS superconductor

Spin-singlet (anti-sym.)

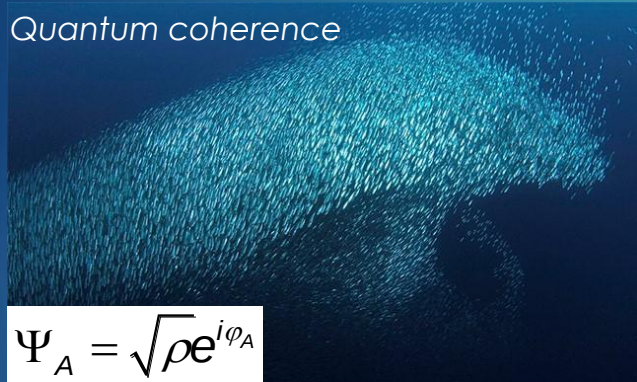
$$|S\rangle = \frac{1}{\sqrt{2}} (|\uparrow\downarrow\rangle - |\downarrow\uparrow\rangle)$$

Orbital : s-wave (sym.)

Macroscopic Quantum Phenomena

All 10^{23} electrons in superconductor behaves as a single quantum object.

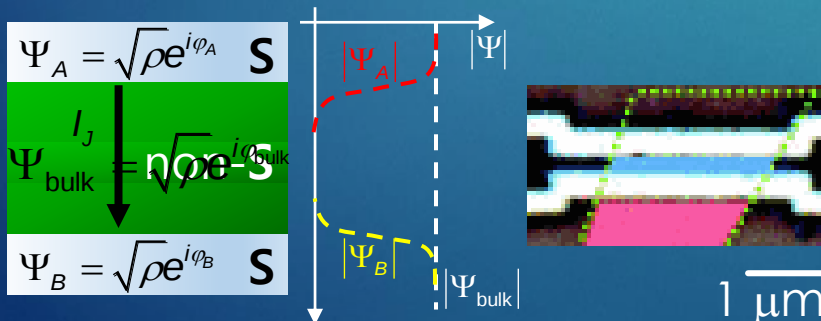
Quantum coherence



$$\Psi_A = \sqrt{\rho} e^{i\varphi_A}$$

e.g.) Josephson junction

- Macroscopic quantum device



Application for Quantum Computer

Using **superposition** of quantum mechanics

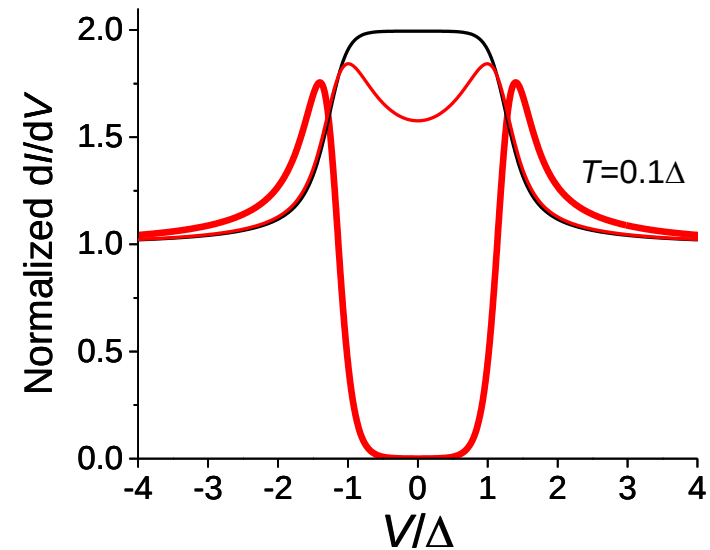
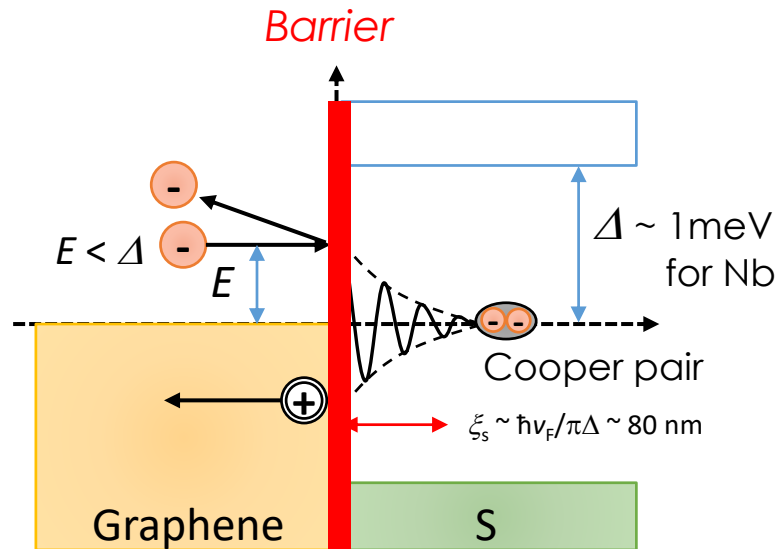
$$\frac{1}{\sqrt{2}} |\text{cat}\rangle + \frac{1}{\sqrt{2}} |\text{cat}\rangle$$

- Classical computer: $f : 00 \rightarrow f(00)$
 $f : 01 \rightarrow f(01)$
 $f : 10 \rightarrow f(10)$
 $f : 11 \rightarrow f(11)$
- Quantum computer:
 $U_f : \frac{1}{\sqrt{4}} (|00\rangle + |01\rangle + |10\rangle + |11\rangle)$
 $\rightarrow \frac{1}{\sqrt{4}} (U_f |00\rangle + U_f |01\rangle + U_f |10\rangle + U_f |11\rangle)$

Quantum parallelism gives exponential computation power.

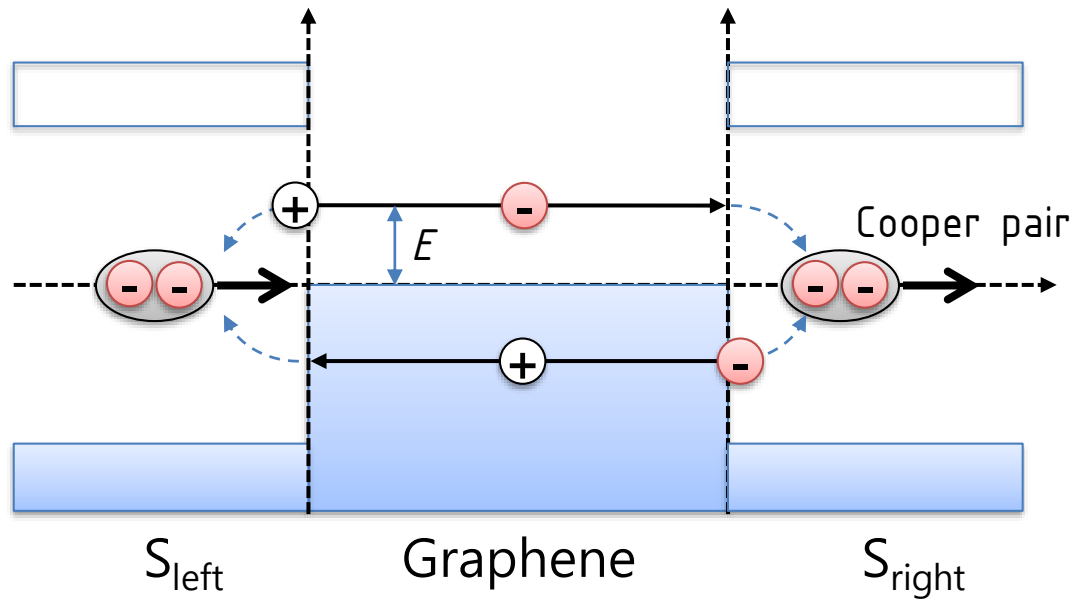
Google, IBM, Intel, Microsoft, etc. are actively working on it.

Andreev Reflection



Andreev-reflected hole is highly coherent with incident electron.

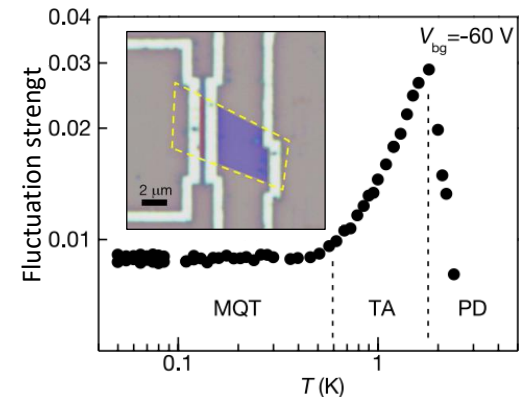
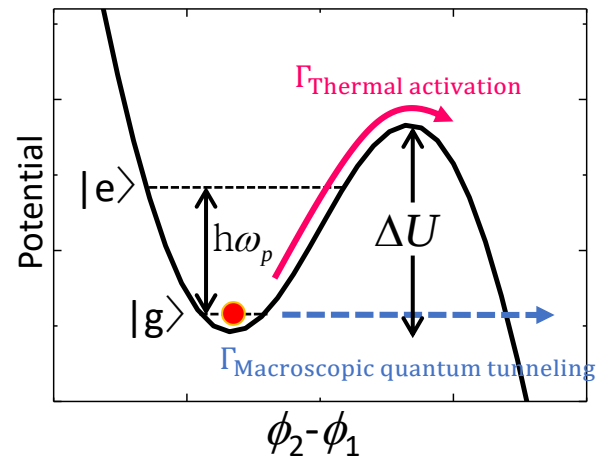
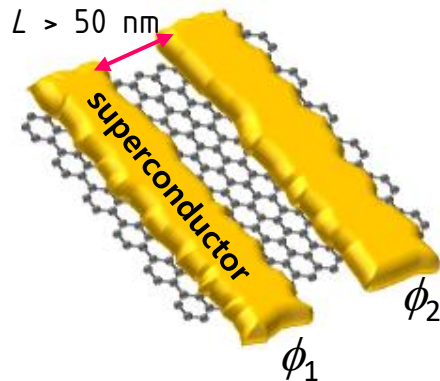
Josephson Junction



Cooper pairs are transferred *coherently*.

Examples of Hybrid Devices (1)

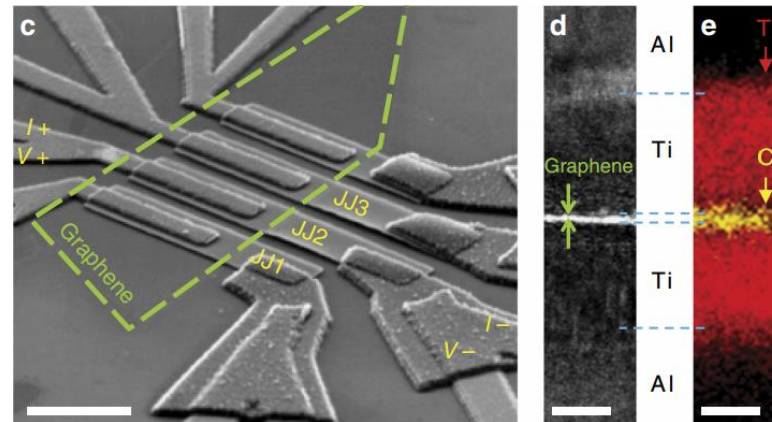
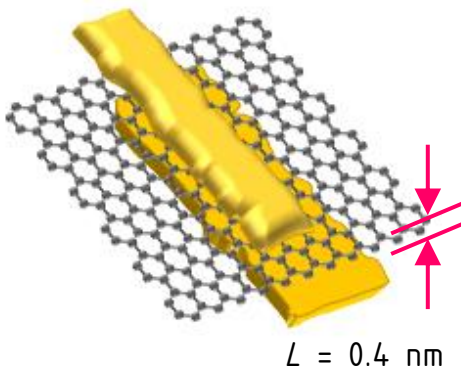
Gate-tunable Macroscopic Quantum Tunneling



[G.-H. Lee et al., PRL(2011)]

[D. Jeong, G.-H. Lee et al., PRB(2011)]

First Short-Ballistic Josephson Junction

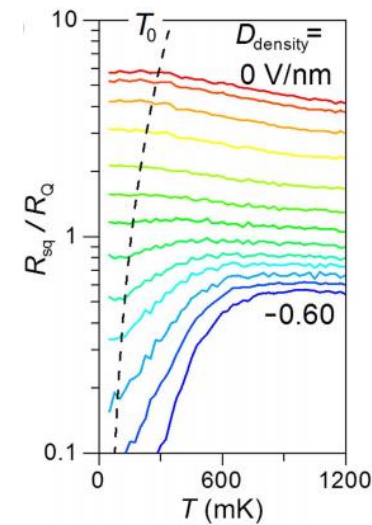
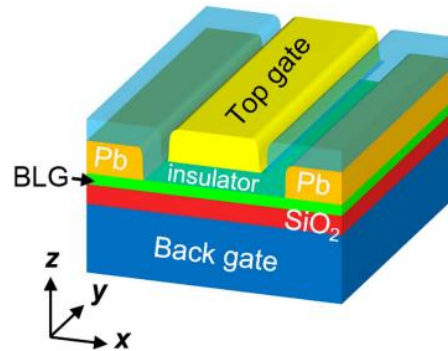
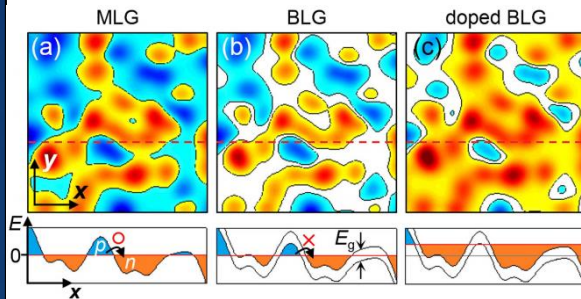


[G.-H. Lee et al., Nature Comm. (2015)]

[G.-H. Lee et al., APEX (2013)]

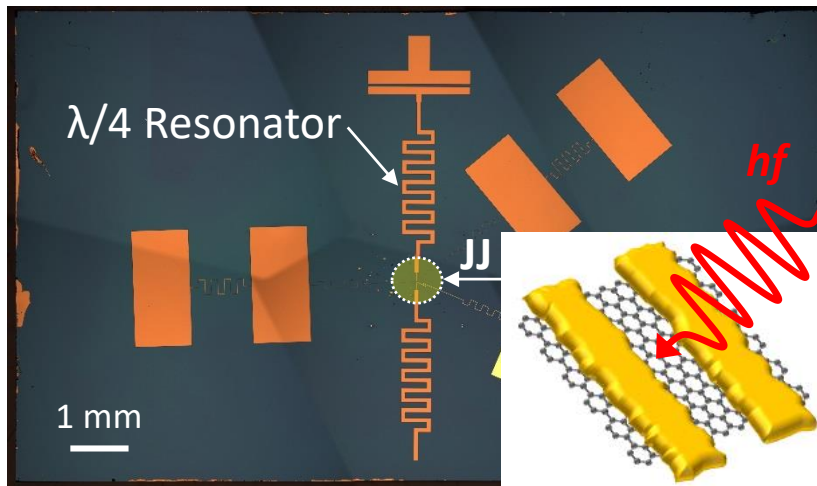
Examples of Hybrid Devices (2)

Superconductor-Insulator Transition

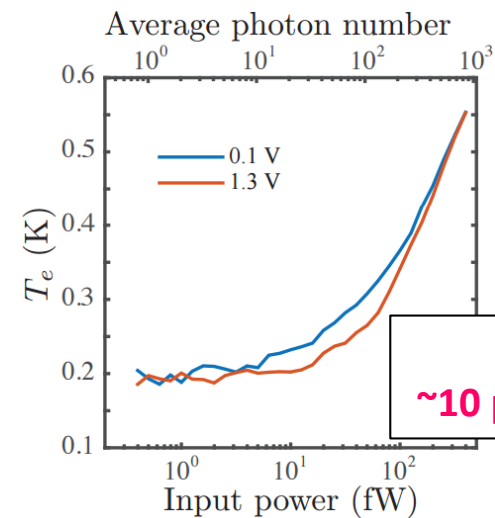


[G.-H. Lee et al., Scientific Report (2015)]

Microwave Single Photon Detector



Crucial step towards quantum information



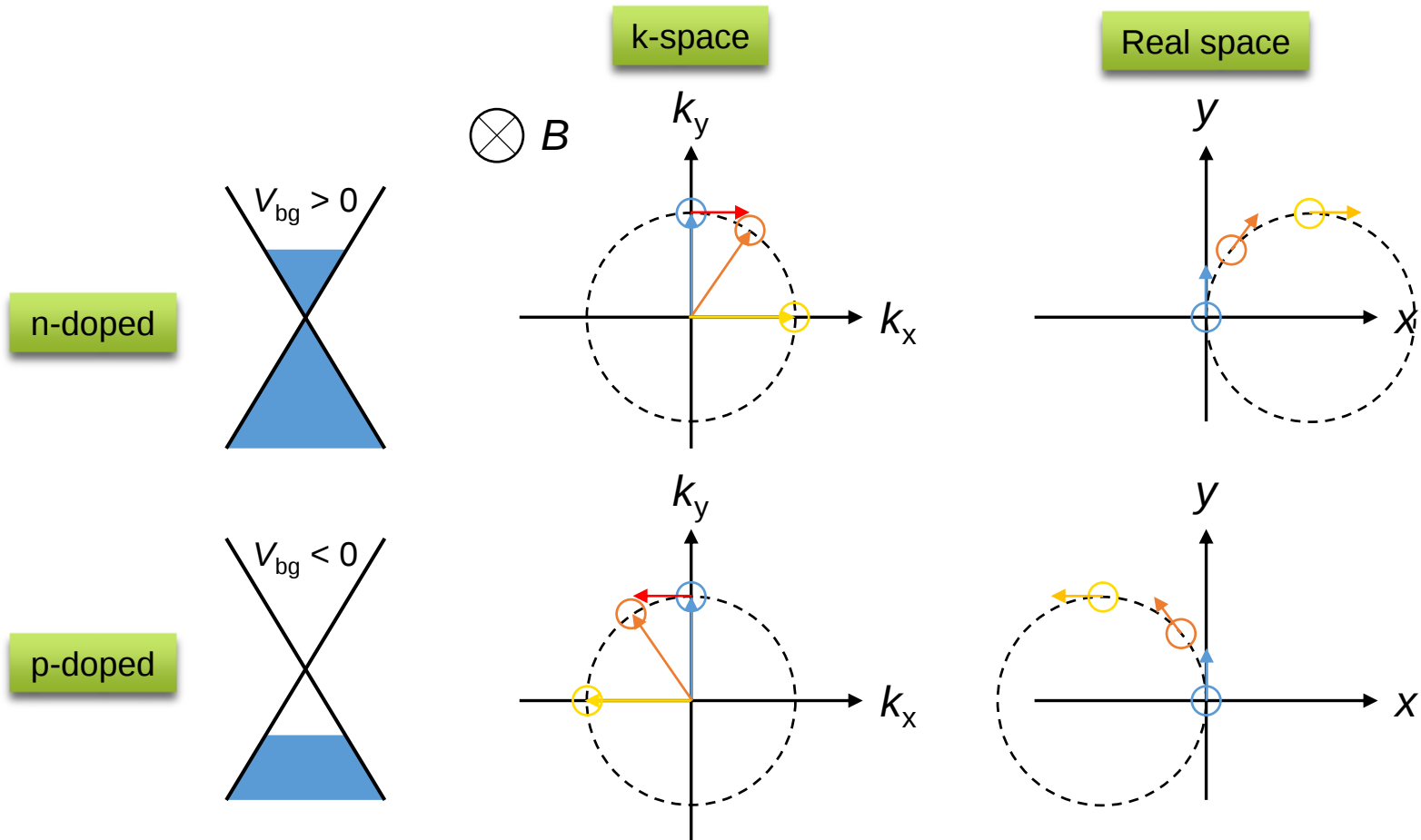
Sensitivity :
~10 photons (@8GHz)

[G.-H. Lee, et al., (In preparation)]

Visualizing Andreev-pairs (Magnetic Focusing)

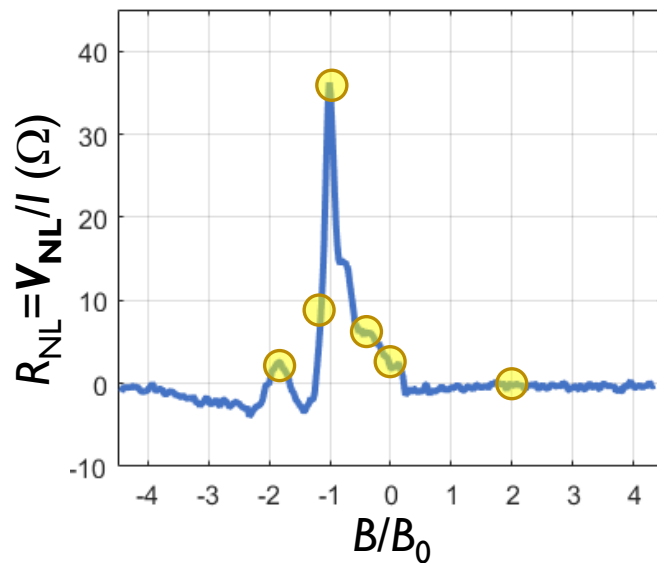
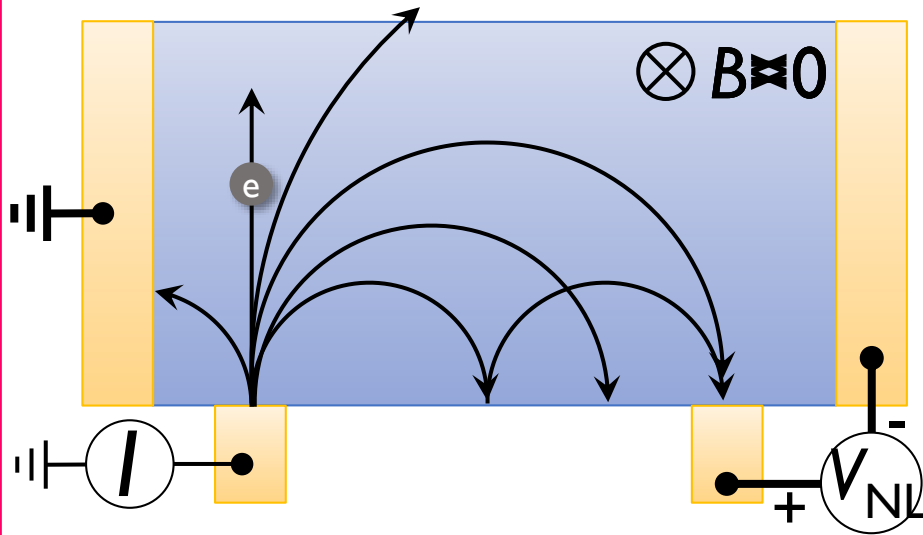
Electrons under Magnetic Field

$$\vec{F} = q\vec{v}_g \times \vec{B} = d(\hbar\vec{k})/dt$$

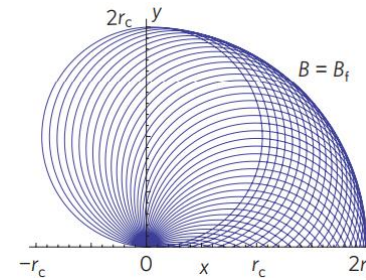


Cyclotron effective mass : $m^* > 0$ for CB, $m^* < 0$ for VB

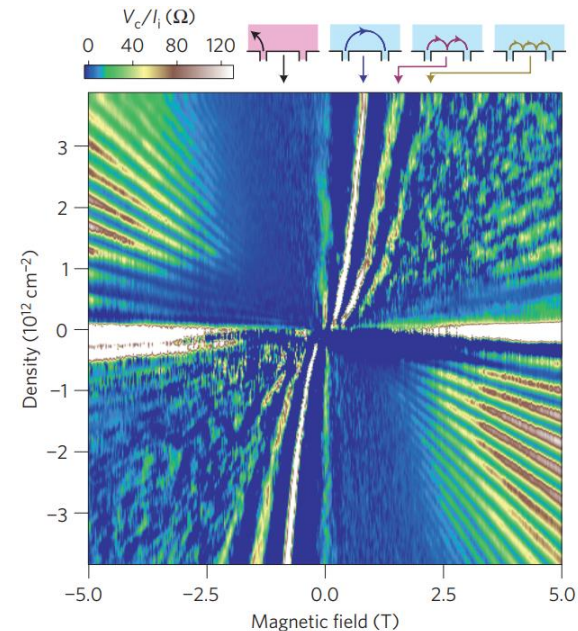
Steering Electron Beam



Electronically tunable magnetic focusing

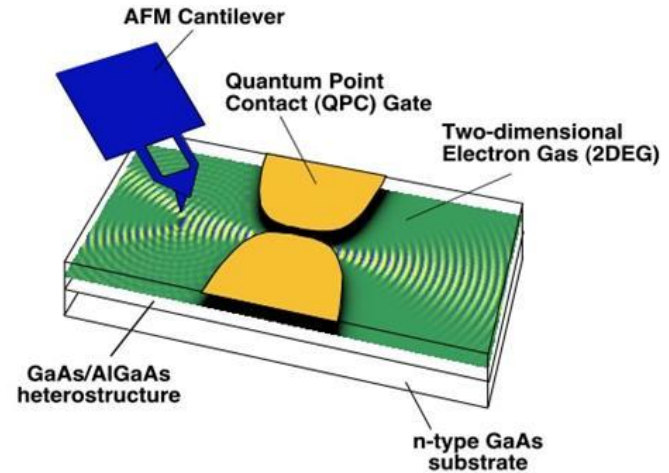
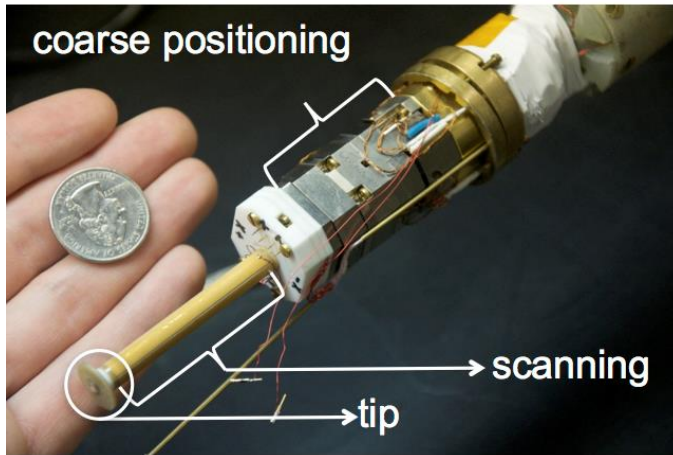


$$B_f^{(p)} = \left(\frac{2\hbar k_F}{eL} \right) p = \left(\frac{2\hbar \sqrt{\pi n}}{eL} \right) p$$



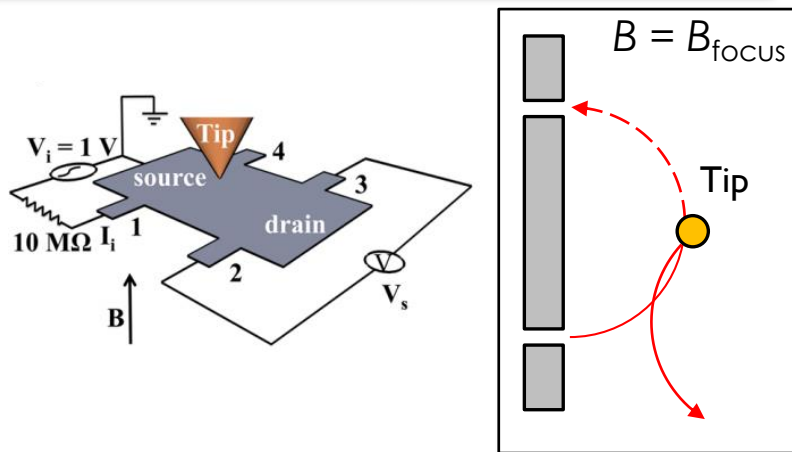
[Taychatanapat et al., Nature Phys. (2013)]

Imaging Electron Path

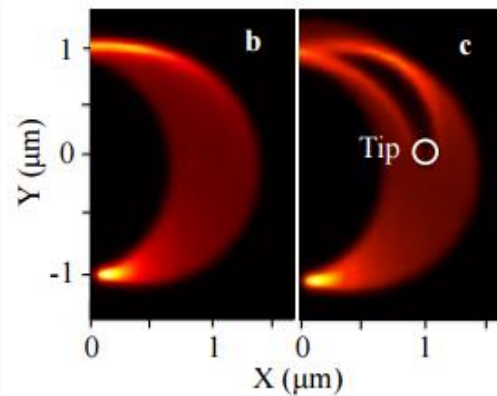


[Goldhaber-Gordon group, Westervelt group]

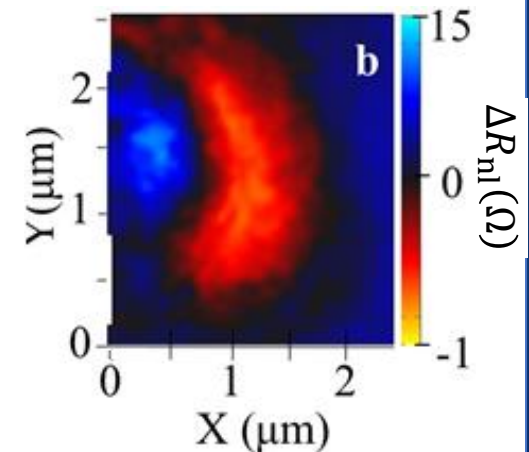
Imaging bent electrons in graphene



Simulation

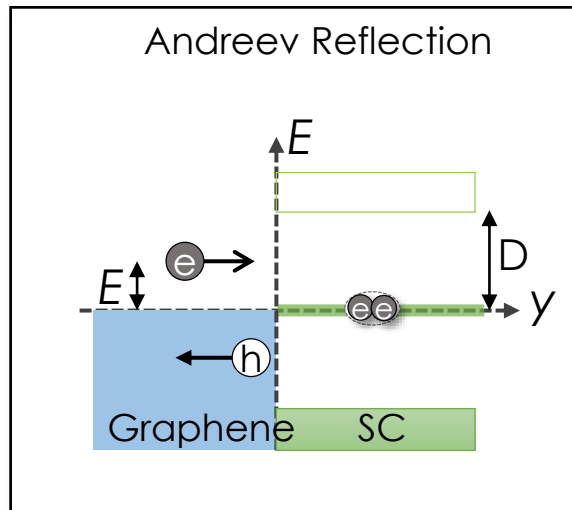
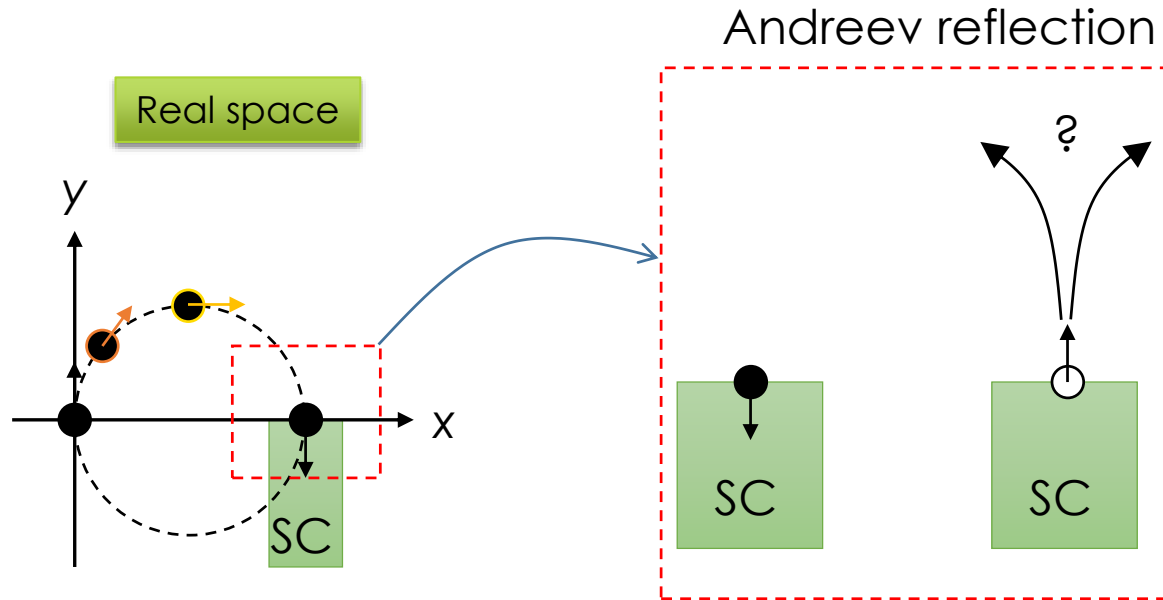


Measurement



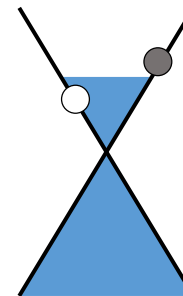
[S. Bhandari, G.-H. Lee, et al., Nano Lett., (2016)]

Which Way to Go?



Hint,

n-doped



Andreev reflected hole
 = Hole in CB ($m^* < 0$)
 = Absence of electron in CB ($m^* > 0$)

Summary

- ▶ Dirac Electronic optics in high-quality graphene
 - Study on relativistic quantum mechanics in solid system.
- ▶ Magnetic focusing in graphene
 - Playground to investigate behavior of Andreev pairs under magnetic field
 - Scanning gate microscopy images path of quasi-particles
- ▶ Quantum Hall with superconductor
 - Promising candidate to realize elusive non-Abelian anyon in 2D platform.