

Particle Physics: Past, Present, and the Future

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(Pusan National Univ.)

May 9 2018 @ Seoul National Univ.

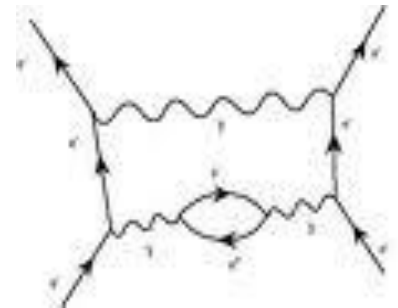
素粒子 물리학이란?

- 자연을 구성하는 기본입자는 무엇이며, 그들 사이의 기본 상호작용은 무엇인가를 연구.
- 복잡해 보이는 수 많은 자연현상들을 간단한 개념과 수식으로 이해하고자 함. (Beauty in Physics)
- 원자론적 세계관에 기반을 둔, 현시대의 가장 “Modern스러운” 학문.

Two Columns in Particle Physics

- QUANTUM FIELD THEORY

quantize Matter (cl.mech.), Field (E.M.),
E.M. + Rel.Q.M. = QED



- SYMMETRY (group theory)

Lorentz sym. $SO(3,1)$, $U(1)_{EM}$,

Isospin (p,n) $SU(2)$, $SU(3)$, ... , $SO(10)$, E_8 ,

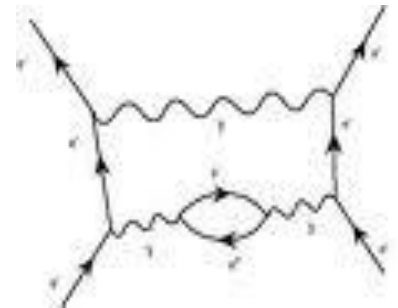
Gauge sym., Chiral sym., ...

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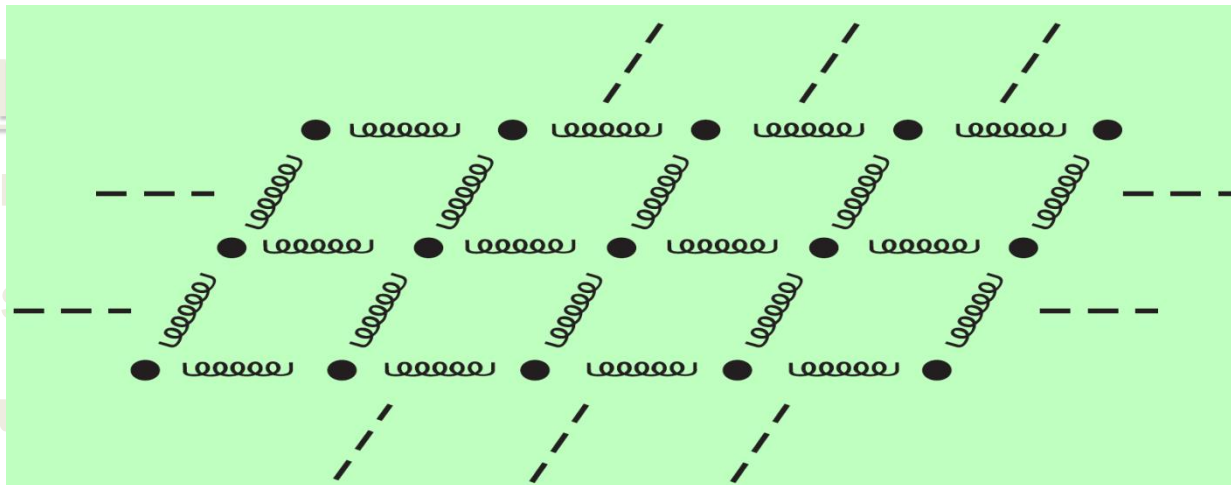


- SYMMETRY

Local

Isospin

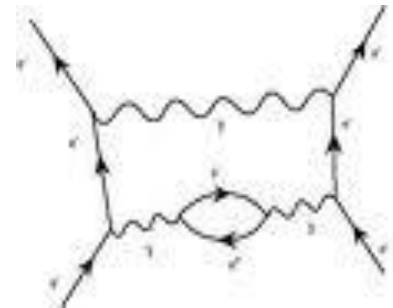
Gauge



Two Columns in Particle Physics

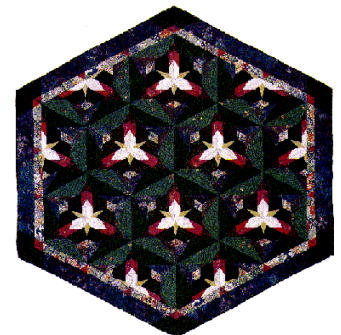
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Four Interactions in Nature

- Gravitational Interaction (by graviton)
[equivalence principle]
- Electro-Magnetic Interaction (by photon)
- Strong Interaction (by gluons) [confinement]
- Weak Interaction (by W, Z) [Higgs mechanism]

Special Rel. & E.-M. Intr.

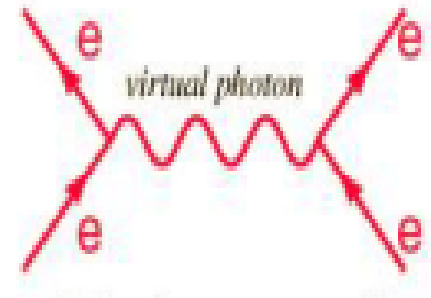
- Maxwell's theory $(E^i, B^i) = F_{\mu\nu}$

$$\partial_\mu F^{\mu\nu} = J^\nu$$

1. Special Relativity: $SO(3,1)$

$$X^\mu = (t, \mathbf{x}), \quad [t^2 - \mathbf{x}^2 = \text{const.}]$$

$$P^\mu = (E, \mathbf{P}), \quad [E^2 - \mathbf{P}^2 = m^2]$$



2. $U(1)$ gauge theory $A_\mu' = A_\mu + \partial_\mu \Lambda$

→ Yang-Mills Theory in 50s' $\partial_\mu \rightarrow \partial_\mu + iqA_\mu$

$SU(2)$ gauge theory for $(p, n), \pi$

(wrong but,,)

Special Rel. & E.-M. Intr.

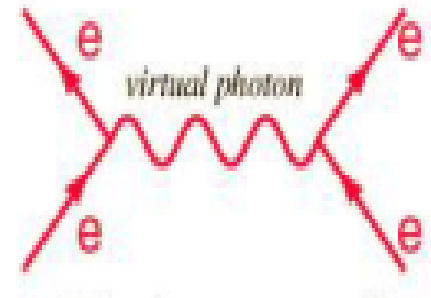
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Relativistic Q.M.

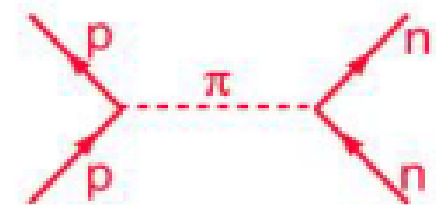
1. Klein-Gordon eq. ?
2. **spinor** (and gamma matrix) 의 발견
 $\{\gamma_\mu, \gamma_\nu\} = 2g_{\mu\nu}$ in $SO(2n)$
3. **anti-particle** 의 발견 $e^- \longleftrightarrow e^+$
4. Dirac's Sea , Prob. Ampl.? \rightarrow Q.F.T.

Strong Interaction

- Heisenberg's **SU(2) Isospin** (**p, n**)
+ hyper charge
 - Yukawa's Pi meson : \leftarrow Q.E.D.
force mediator among nucleons
 - discovery of thousands of mesons and baryons in 1950's
 \rightarrow quark model : $SU(3)_f, (u,d,s) = 3$

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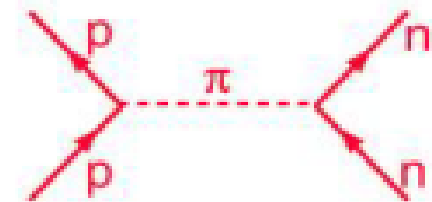
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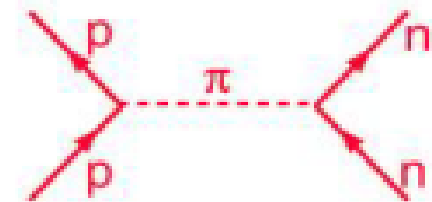
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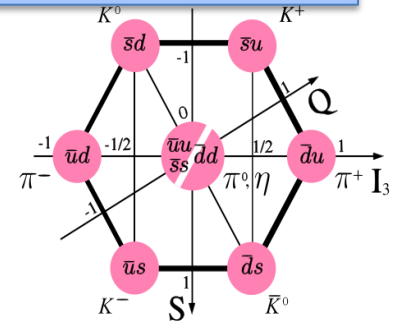
between nucleons

- $Q_{em} = T_3 + Y/2$ mesons and baryons in

→ quark model : **SU(3)_f**, (**u,d,s**) = 3

Strong Interaction

- $3 \times 3^* = 8 + 1$: explain well the **meson** octet



• $3 \times 3 \times 3 = 10 + 8 + 8 + 1$: baryons

only 10 ($s=3/2$) + 8 ($s=1/2$) observed.

$\psi = \phi(\text{orbit}) \times \chi(\text{spin}) \times l(\text{internal}) \times C(\text{color})$

→ SU(3) color is necessary!! [Han, Nambu]

They first proposed the $SU(3)_c$ gauge theory,
introducing the 8 gluons → Q.C.D.

Strong Interaction

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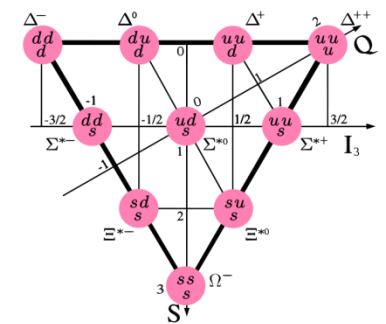
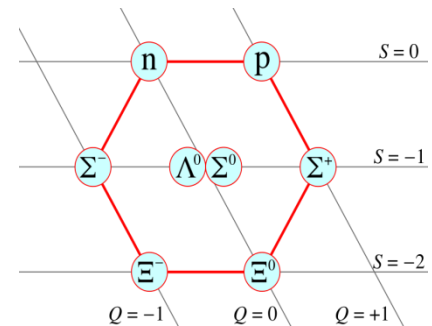
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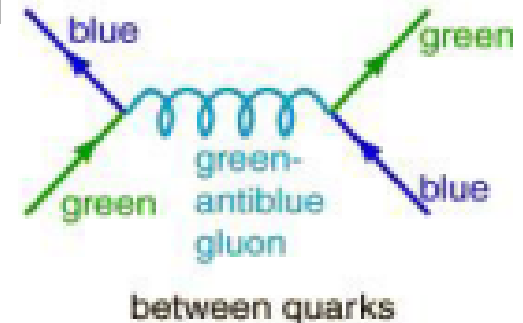
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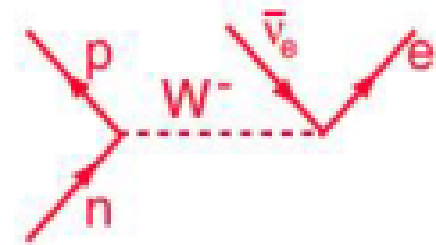


Weak Interaction

- Fermi Interaction for β -decay $n \rightarrow p^+ + e^- + \bar{\nu}^c$

$$H = G_F J^\mu J_\mu^+ \leftarrow \text{Q.E.D.}$$

$$J_{c.c.}^\mu = \bar{u}\gamma^\mu(1 - c\gamma_5)d + \bar{\nu}\gamma^\mu(1 - c\gamma_5)e$$



- V-A (i.e. $c=1$) \rightarrow Chiral theory
- Non-Renorm. theory \rightarrow Gauge theory

Weak Interaction

- Heisenberg's SU(2) Isospin (p,n)

→ (u,d) in terms of quarks

SU(2) doublet for leptons (ν, e⁻) ?

- SU(2)_L × U(1)_Y gauge theory, (ν_L, e_L⁻), e_R⁻
- Spontaneous sym. Breaking (Higgs mech.)
for masses of W, Z, matter → U(1)_{em}
- Renorm. of gauge theory proved

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A MODEL OF LEPTONS*

Steven Weinberg†

Laboratory for Nuclear Science and Physics Department,
Massachusetts Institute of Technology, Cambridge, Massachusetts

(Received 17 October 1967)

Leptons interact only with photons, and with the intermediate bosons that presumably mediate weak interactions. What could be more natural than to unite these spin-one bosons into a multiplet of gauge fields? Standing in the way of this synthesis are the obvious differences in the masses of the photon and intermediate meson, and in their couplings. We might hope to understand these differences by imagining that the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum. However, this raises the specter of unwanted massless Goldstone bosons.² This note will describe a model in which the symmetry between the electromagnetic and weak interactions is spontaneously broken, but in which the Goldstone bosons are avoided by introducing the photon and the intermediate-boson fields as gauge fields.³ The model may be renormalizable.

We will restrict our attention to symmetry groups that connect the observed electron-type leptons only with each other, i.e., not with muon-type leptons or other unobserved leptons or hadrons. The symmetries then act on a left-handed doublet

$$L = \begin{bmatrix} \frac{1}{2}(1 + \gamma_5) \begin{pmatrix} \nu_e \\ e \end{pmatrix} \end{bmatrix} \quad (1)$$

and on a right-handed singlet

$$R = \begin{bmatrix} \frac{1}{2}(1 - \gamma_5) \end{bmatrix} e. \quad (2)$$

The largest group that leaves invariant the kinematic terms $-\bar{L}\gamma^\mu \partial_\mu L - \bar{R}\gamma^\mu \partial_\mu R$ of the Lagrangian consists of the electronic isospin \tilde{T} acting on L , plus the numbers N_L , N_R of left- and right-handed electron-type leptons. As far as we know, two of these symmetries are entirely unbroken: the charge $Q = T_3 - N_R - \frac{1}{2}N_L$, and the electron number $N = N_R + N_L$. But the gauge field corresponding to an unbroken symmetry will have zero mass,⁴ and there is no massless particle coupled to N ,⁵ so we must form our gauge group out of the electronic isospin \tilde{T} and the electronic hypercharge $Y = N_R + \frac{1}{2}N_L$.

Therefore, we shall construct our Lagrangian out of L and R , plus gauge fields \tilde{A}_μ and B_μ coupled to \tilde{T} and Y , plus a spin-zero doublet

$$\varphi = \begin{pmatrix} \varphi^0 \\ \varphi^- \end{pmatrix} \quad (3)$$

whose vacuum expectation value will break \tilde{T} and Y and give the electron its mass. The only renormalizable Lagrangian which is invariant under \tilde{T} and Y gauge transformations is

$$\begin{aligned} \mathcal{L} = & -\frac{1}{4}(\partial_\mu \tilde{A}_\nu - \partial_\nu \tilde{A}_\mu + g\tilde{A}_\mu \times \tilde{A}_\nu)^2 - \frac{1}{4}(\partial_\mu B_\nu - \partial_\nu B_\mu)^2 - \bar{R}\gamma^\mu (\partial_\mu - ig'B_\mu)R - L\gamma^\mu (\partial_\mu - ig\tilde{A}_\mu - i\frac{1}{2}g'B_\mu)L \\ & - \frac{1}{2}[\partial_\mu \varphi - ig\tilde{A}_\mu \cdot \tilde{T}\varphi + i\frac{1}{2}g'B_\mu \varphi]^2 - G_e(\bar{L}\varphi R + \bar{R}\varphi^\dagger L) - M_1^2 \varphi^\dagger \varphi + h(\varphi^\dagger \varphi)^2. \end{aligned} \quad (4)$$

We have chosen the phase of the R field to make G_e real, and can also adjust the phase of the L and Q fields to make the vacuum expectation value $\lambda = \langle \varphi^0 \rangle$ real. The "physical" φ fields are then φ^-

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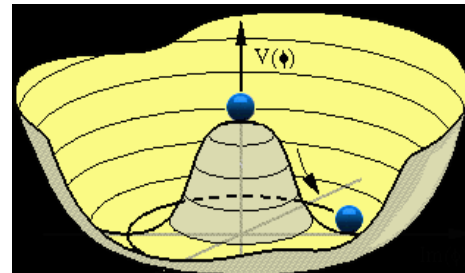
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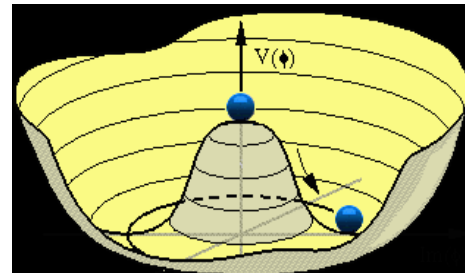
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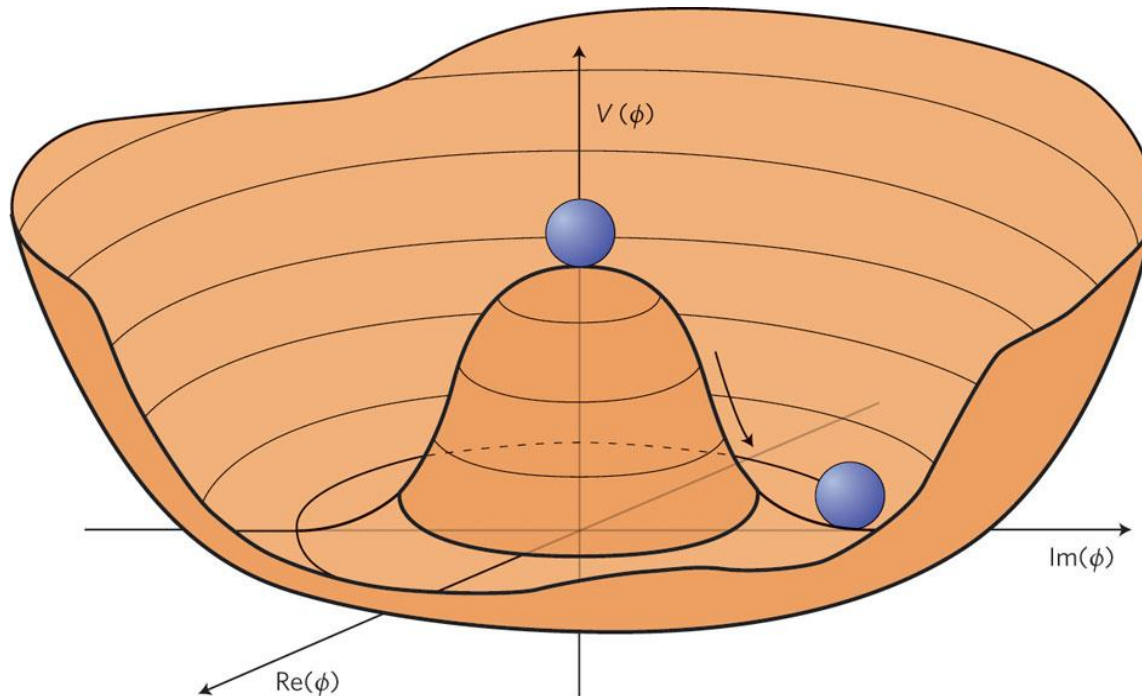
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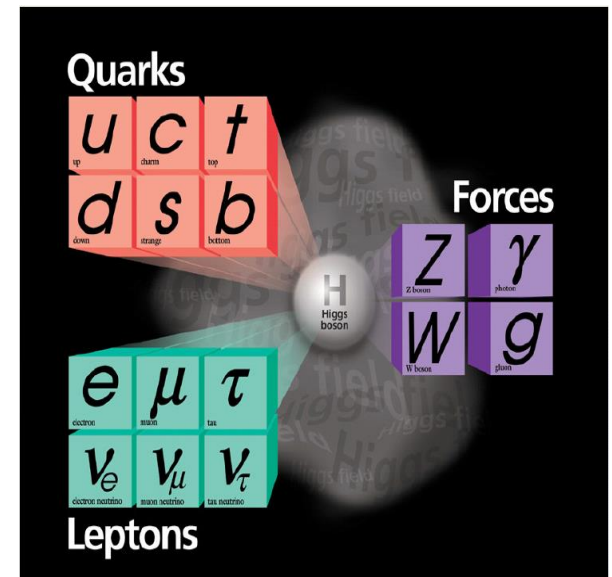
$$V(H) = -\mu^2 |H|^2 + \lambda |H|^4$$



Standard Model

Quantum Field Theory with

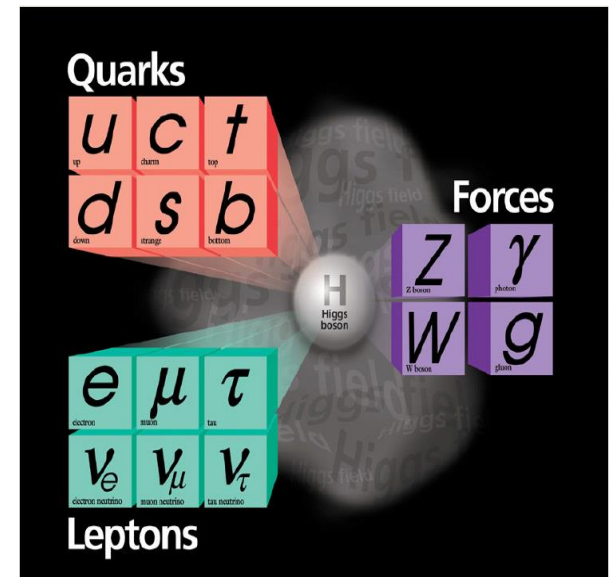
- $SU(3)_C \times SU(2)_L \times U(1)_Y$
Gauge Sym. (gauge bosons)
- 3 families of chiral fermions
("quarks, leptons")
- 1 elementary scalar field
("Higgs")



Standard Model

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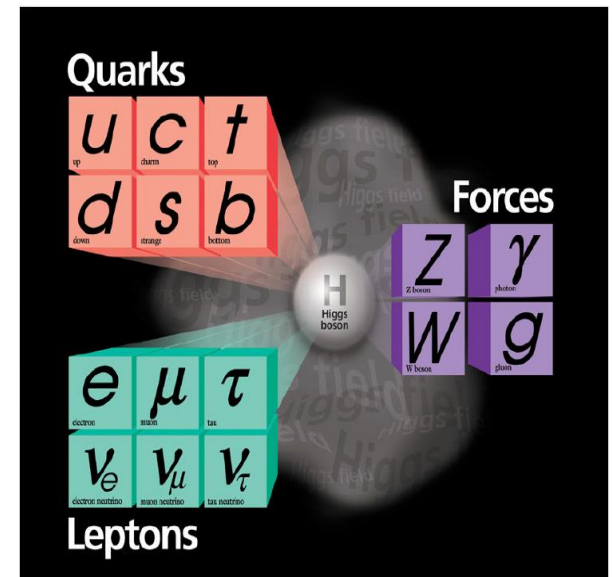
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Standard Model

Quantum Field Theory with

- $SU(3)_c \times SU(2)_L \times U(1)_Y$
Gauge Sym. (gauge bosons) → Well-tested
- 3 families of **chiral fermions**
("quarks, leptons") → Well-tested
- 1 **elementary scalar field**
("Higgs") $SM \rightarrow QCD \times U(1)_{em}$ → Found in 2013

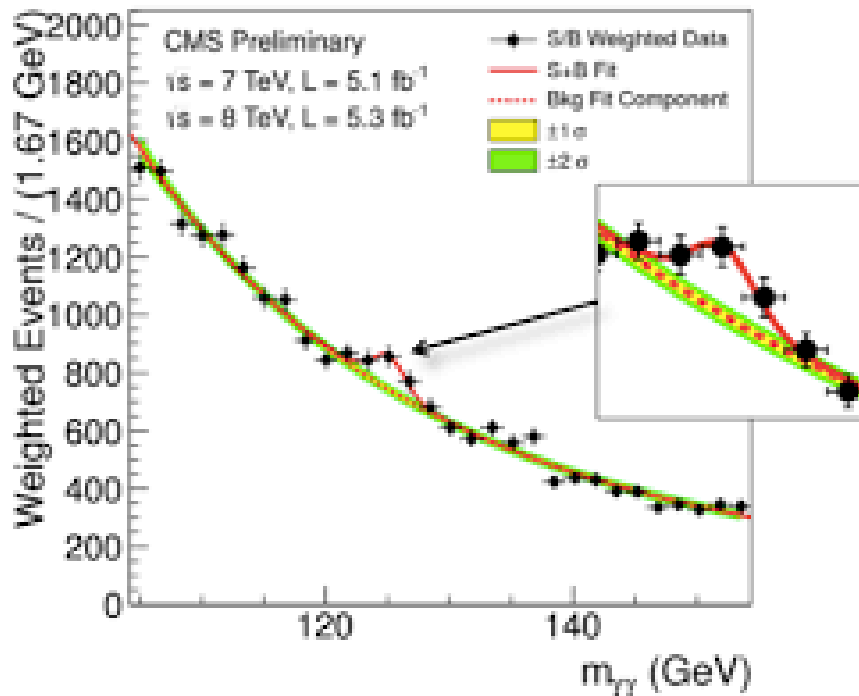
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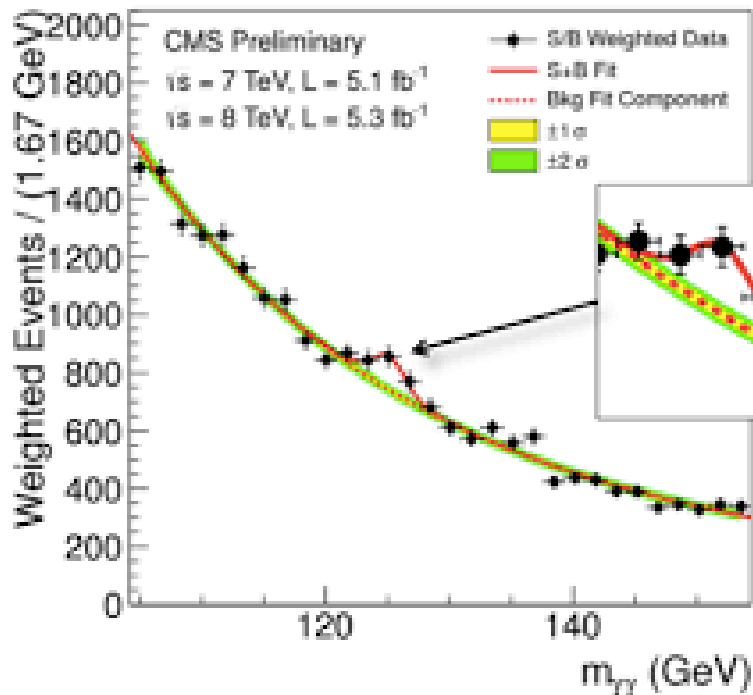
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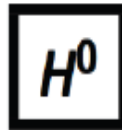
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→ **Found in 2013**

Standard Model



- 1 elementary scalar field
 (“Higgs”) $\text{SM} \rightarrow \text{QC}$



$$J = 0$$

Mass $m = 125.09 \pm 0.24 \text{ GeV}$

Full width $\Gamma < 1.7 \text{ GeV}, \text{CL} = 95\%$

H^0 Signal Strengths in Different Channels

See Listings for the latest unpublished results.

Combined Final States = 1.10 ± 0.11

$$W W^* = 1.08^{+0.18}_{-0.16}$$

$$Z Z^* = 1.29^{+0.26}_{-0.23}$$

$$\gamma\gamma = 1.16 \pm 0.18$$

$$b\bar{b} = 0.82 \pm 0.30 \quad (S = 1.1)$$

$$\mu^+\mu^- < 7.0, \text{CL} = 95\%$$

$$\tau^+\tau^- = 1.12 \pm 0.23$$

$$Z\gamma < 9.5, \text{CL} = 95\%$$

$$t\bar{t}H^0 \text{ Production} = 2.3^{+0.7}_{-0.6}$$

Standard Model

- Extremely successful for $L > 10^{-16}$ cm
- Starting point of (theoretical) physics
“beyond SM”

Beyond Standard Model

- but not fully satisfactory as a fundamental theory

1. not incorporate Gravity

2. Gauge hierarchy Probl.

3. gauge forces not unified

4. No dark matter

Beyond Standard Model

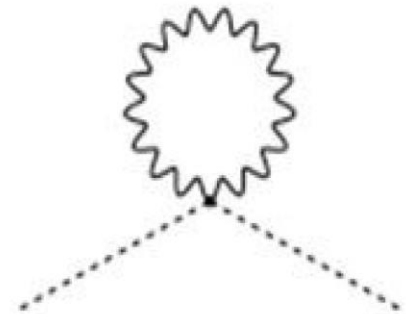
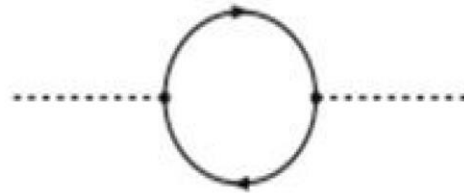
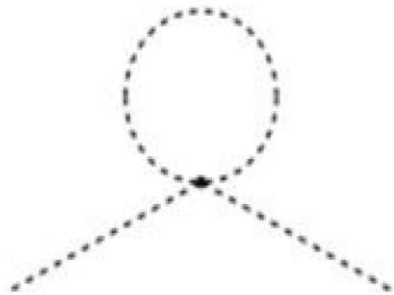
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1. **not incorporate Gravity** → **SUGRA, String th.**
2. **Gauge hierarchy Probl.** → **SUSY, technicolor,,**
3. **gauge forces not unified** → **Grand Unif.**
4. **No dark matter** → **SUSY**

Gauge Hierarchy Problem

The Higgs potential is fully renormalizable, but...

Loop corrections to the Higgs boson mass...



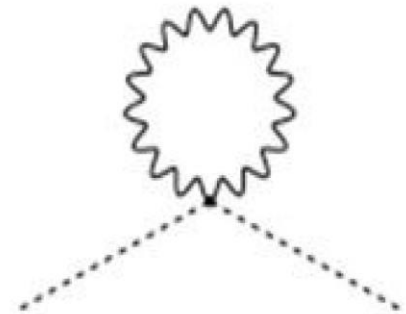
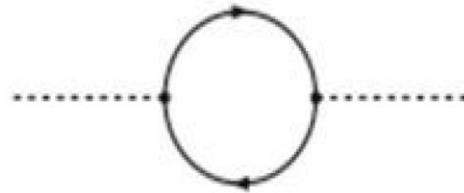
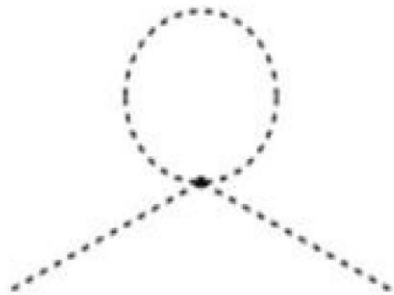
...are quadratically divergent:

$$\Delta m^2 \propto \int^{\Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \sim \frac{\Lambda^2}{16\pi^2}$$

Gauge Hierarchy Problem

The Higgs potential is fully renormalizable, but...

Loop corrections to the Higgs boson mass...



...are quadratically divergent:

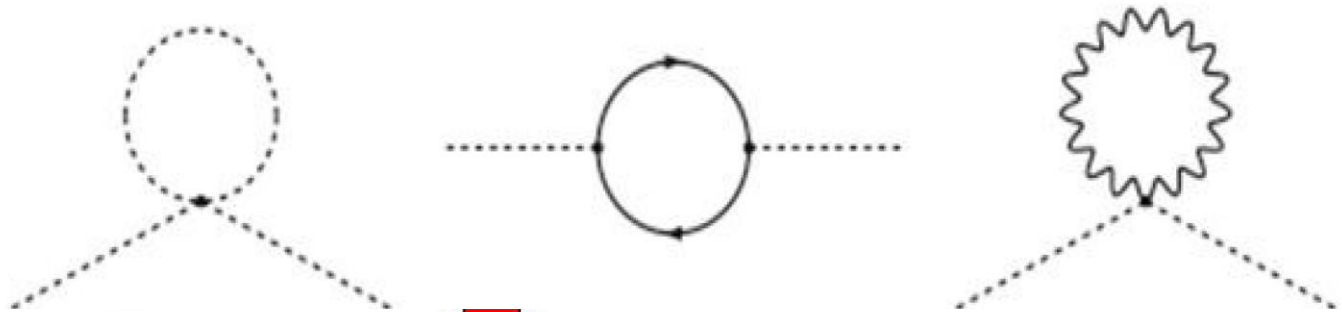
$$\Delta m^2 \propto \int^{\Lambda} \frac{d^4 k}{(2\pi)^4} \frac{1}{k^2} \sim \frac{\Lambda^2}{16\pi^2}$$

Fine Tuning < 10% for $\Lambda > 1.5$ TeV.

(= $\Delta m^2 / m^2$)

Gauge Hierarchy Problem

The Higgs potential is fully renormalizable, but...
 Loop corrections to the Higgs boson mass...



$$\approx - (200 \text{ GeV})^2$$

$$\approx - (200 \text{ GeV})^2 \cdot 10^{32}$$

$$\text{for } \Lambda = 10^3 \text{ GeV}$$

$$\text{for } \Lambda = 10^{19} \text{ GeV}$$

$$m_H^2 = m_{0H}^2 + \delta m_H^2$$

$$m_H \approx 100 - 200 \text{ GeV}$$

$$\approx + (200 \text{ GeV})^2 \cdot 10^{32}$$

$$\approx - (200 \text{ GeV})^2 \cdot 10^{32}$$

UV Theory at TeV scale ?

- From experiment, Higgs mass is 126 GeV.
- In SM, Higgs receive mass correction from *fermion* loops

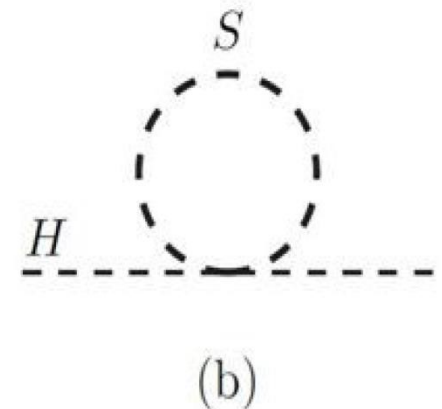
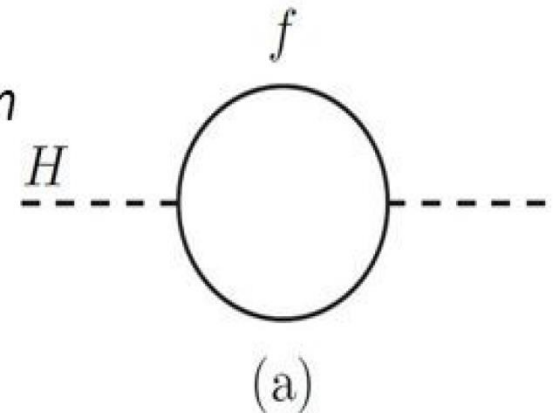
$$\Delta m_h^2 = -\frac{|\lambda_f|^2}{8\pi^2} \Lambda_{UV}^2 + \dots$$

“Hierarchy problem”:

Higgs bare mass = $10^{19} - 126$ GeV

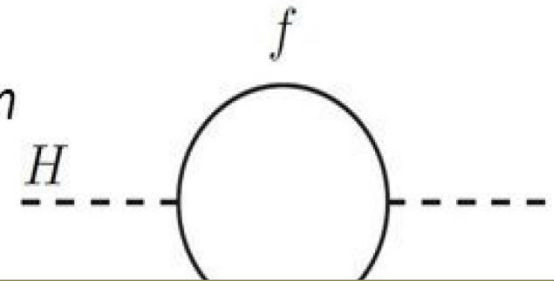
- If we introduce a new scalar field, then the correction to mass term is:

$$\Delta m_h^2 = \frac{\lambda_S}{16\pi^2} \Lambda_{UV}^2 + \dots$$



UV Theory at TeV scale ?

- From experiment, Higgs mass is 126 GeV.
- In SM, Higgs receive mass correction from *fermion* loops



$$\Delta m_h^2 = (\lambda_s/16\pi^2) \Lambda_{UV}^2 - (|\lambda_f|^2/8\pi^2) \Lambda_{UV}^2 + \text{Log corrections}$$

Need $(s, s^c) \leftrightarrow (f, f^c)$ and $\lambda_s = |\lambda_f|^2$.

➔ **Supersymmetry !!**

Naturalness Problem?

Is a fine-tuning really a problem?

**“ Explanation is to provide a framework,
in which the object looks trivial.”**

Supersymmetry

extend Poincare Sym.: $boson \leftrightarrow fermion$

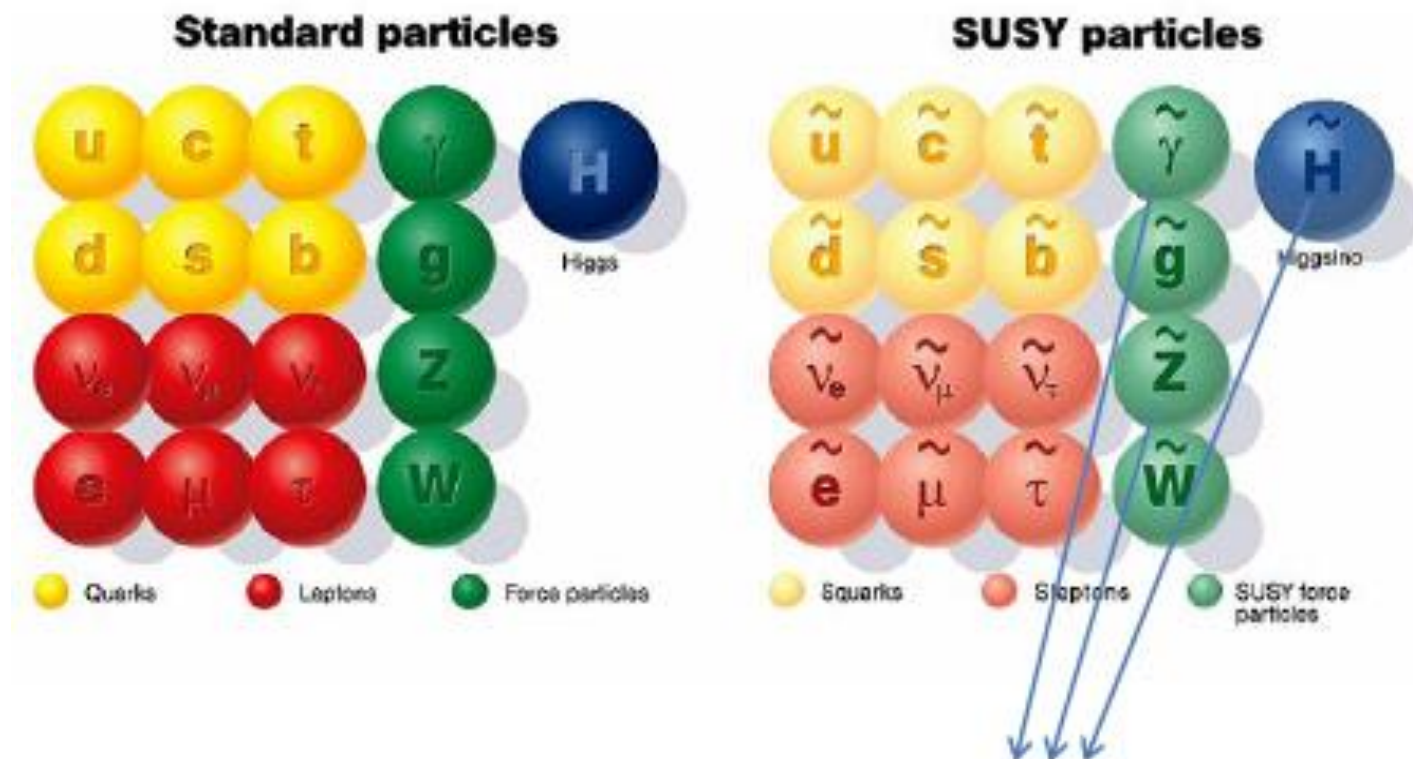
$$Q|fermion\rangle = |boson\rangle, \quad Q^\dagger|boson\rangle = |fermion\rangle$$

$$\{Q, Q^\dagger\} = 2P_\mu \sigma^\mu$$

$$N = 1 \text{ SUSY: } (g_{\mu\nu}, \psi_\mu)_g, \quad (A_\mu^a, \lambda^a)_V, \quad (\psi, \phi)_\chi$$

Minimal Supersymmetric SM (**MSSM**):
introduce super-partner for each SM field

Particle Contents in MSSM



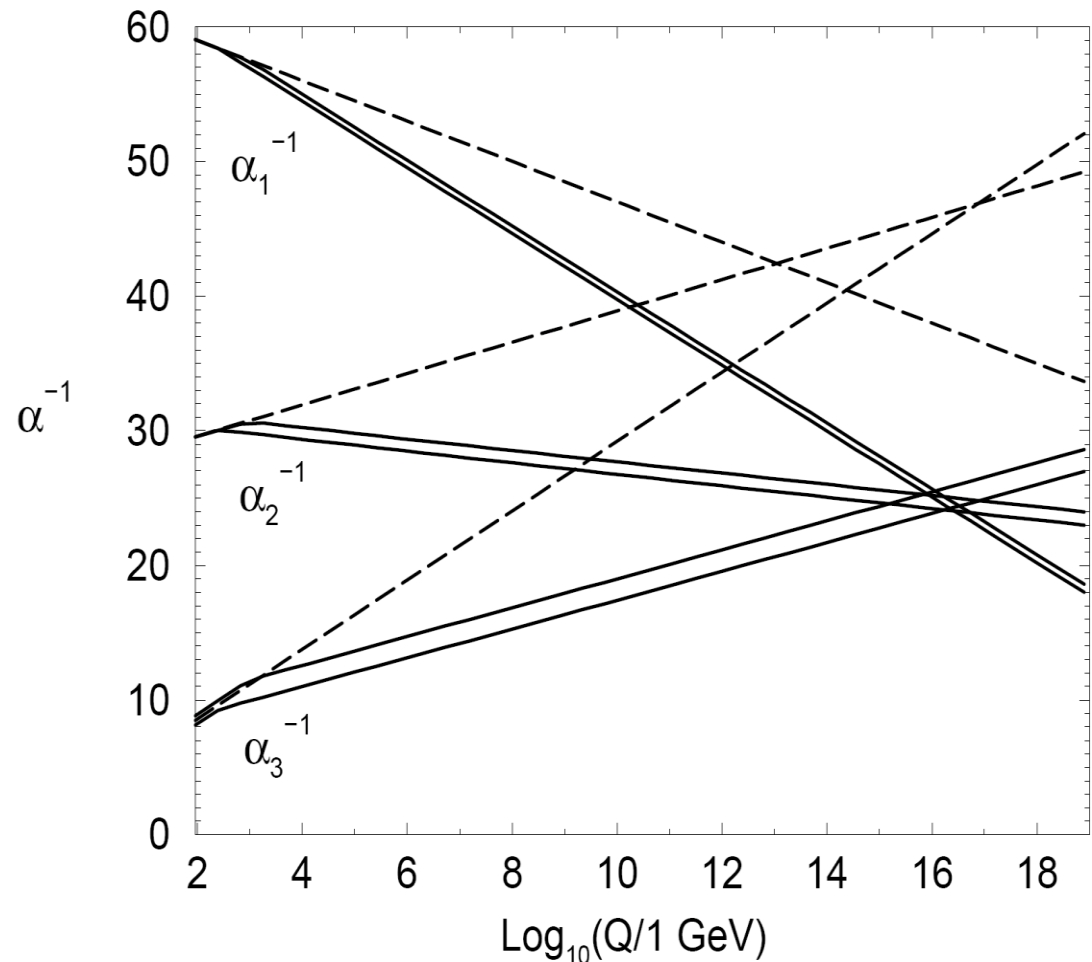
Neutralinos: natural DM candidates
with TeV mass and electroweak coupling

RG evolution of $\{g_3, g_2, g_1\}$

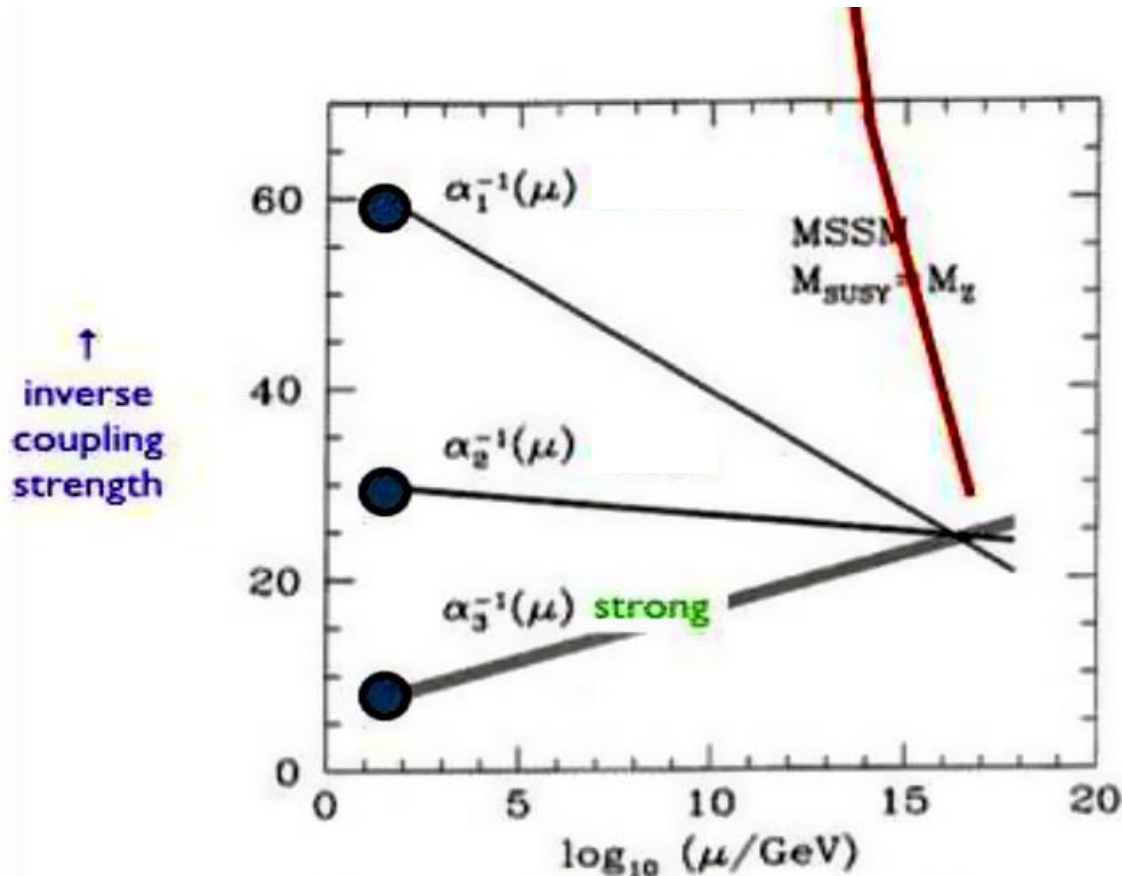
By including
the contributions
by the **super-partners**,

$\{g_3, g_2, (5/3)^{1/2}g_Y\}$
are unified
at 10^{16} GeV.

[See the solid lines.]



RG evolution of $\{g_3, g_2, g_1\}$



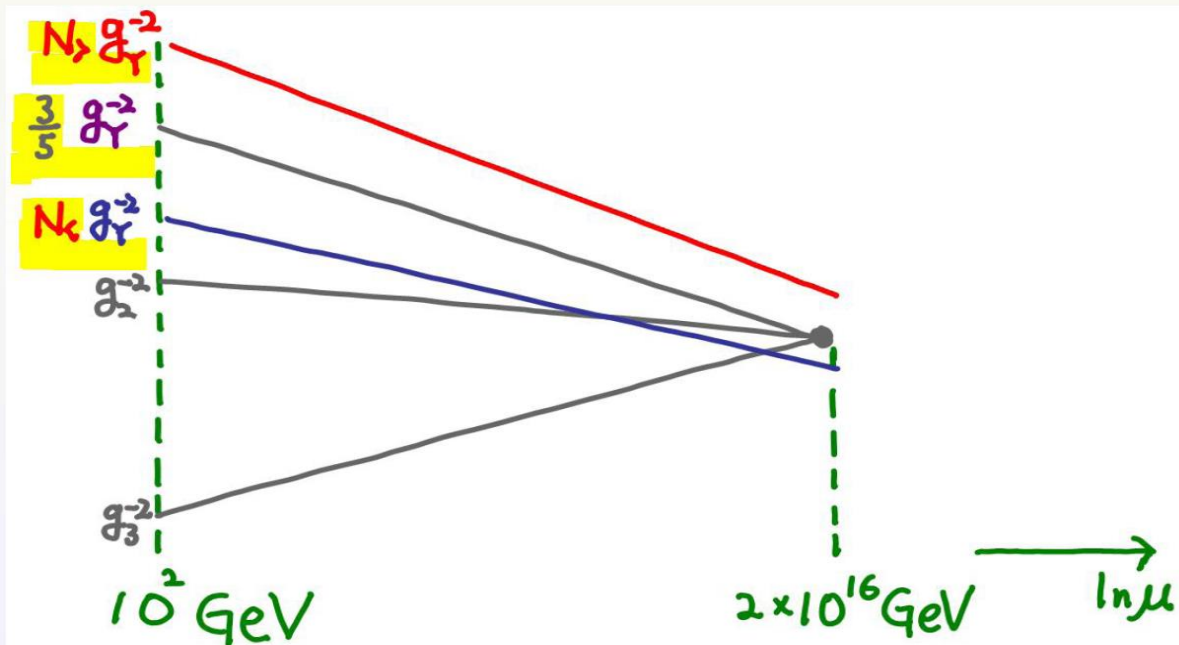
Gravity fits too!
(roughly)

large energy, short distance →

Why $g_1^2 = (5/3) g_Y^2$?

$$iD_\mu = i\partial_\mu + \left(\frac{g_Y}{\sqrt{N}} \right) \left(\sqrt{N} \ Y \right)$$

\sqrt{N} : arbitrary , $Y[Q] = \frac{1}{6}$, $Y[u^c] = \frac{-2}{3}$, $Y[d^c] = \frac{1}{3}$, \dots



Why $g_1^2 = (5/3) g_Y^2$?

If $Y \in T_{SU(5)} , T_{SO(10)} , \dots ,$

$$\begin{aligned} \text{Tr } T_{SU(5)} &= 0 , \\ \text{Tr } T_{SU(5)} T_{SU(5)} &= \frac{1}{2} , \end{aligned}$$

$$T_{SU(5)}^D = \sqrt{\frac{3}{5}} \begin{pmatrix} 1/3 & & & & \\ & 1/3 & & & \\ & & 1/3 & & \\ & & & -1/2 & \\ & & & & -1/2 \end{pmatrix} \equiv \sqrt{\frac{3}{5}} Y ,$$

when $SU(5)$ is broken to $SU(3)_c \times SU(2)_L \times U(1)_Y$.

MSSM seems to be in GUT.

- $SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5), SO(10)$
- $\{g_3, g_2, \sqrt{\frac{5}{3}}g_Y\}$ unified at 10^{16} GeV
 $\rightarrow \sin^2 \theta_W^0 = \frac{g_Y^2}{g_2^2 + g_Y^2} = \frac{3}{8}$
- $\{u^c, Q, e^c\} \subset \mathbf{10}, \quad \{d^c, L\} \subset \bar{\mathbf{5}}, \quad \nu^c = \mathbf{1} \quad \text{in } SU(5)$
 $\{\mathbf{10}, \bar{\mathbf{5}}, \mathbf{1}\} \quad \text{in } SU(5) \quad \subset \quad \mathbf{16} \text{ [spinor]} \quad \text{in } SO(10)$
- Additional $SU(5)$ multi-plets do NOT spoil the gauge coupling unification.
 $\rightarrow \sin^2 \theta_W \approx 0.23$ at 10^2 GeV

MSSM seems to be in GUT.

$$\bar{\mathbf{5}}_F = \begin{pmatrix} d_1^c \\ d_2^c \\ d_3^c \\ e^- \\ -\nu_e \end{pmatrix} \quad \mathbf{10}_F = \left(\begin{array}{ccc|cc} 0 & u_3^c & -u_2^c & u_1 & d_1 \\ -u_3^c & 0 & u_1^c & u_2 & d_2 \\ u_2^c & -u_1^c & 0 & u_3 & d_3 \\ \hline -u_1 & -u_2 & -u_3 & 0 & e^+ \\ -d_1 & -d_2 & -d_3 & -e^+ & 0 \end{array} \right)$$

- $\{u^c, Q, e^c\} \subset \mathbf{10}$, $\{d^c, L\} \subset \bar{\mathbf{5}}$, $\nu^c = \mathbf{1}$ in $SU(5)$
 $\{\mathbf{10}, \bar{\mathbf{5}}, \mathbf{1}\}$ in $SU(5) \subset \mathbf{16}$ [spinor] in $SO(10)$
- Additional $SU(5)$ multi-plets do NOT spoil the gauge coupling unification.
 $\rightarrow \sin^2 \theta_W \approx 0.23$ at 10^2 GeV

MSSM seems to be in GUT.

- $SU(3)_c \times SU(2)_L \times U(1)_Y \subset SU(5), SO(10)$
- $\{g_3, g_2, \sqrt{\frac{5}{3}}g_Y\}$ unified at 10^{16} GeV

So does a GUT naturally exist in our Nature?

$\{10, 5, 1\}$ in $SU(5) \subset 16$ [spinor] in $SO(10)$

- Additional $SU(5)$ multi-plets do NOT spoil the gauge coupling unification.

$$\rightarrow \sin^2 \theta_W \approx 0.23 \text{ at } 10^2 \text{ GeV}$$

MSSM seems NOT to be in GUT.

- Higgs Sector:

$$\{H_d, \bar{\mathbf{3}}\} \subset \bar{\mathbf{5}}_h$$

$$\{H_u, \mathbf{3}\} \subset \mathbf{5}_h \quad \text{in } SU(5)$$

$$\{\bar{\mathbf{5}}_h, \mathbf{5}_h\} \subset \mathbf{10}_h \quad \text{in } SO(10)$$

$\bar{\mathbf{3}}$ and $\mathbf{3}$ destroys the gauge coupling unification, and so should be removed from the low energy field spectrum.

→ Doublet/Triplet splitting problem

- Difficult to avoid the mass relation predicted in GUTs:

$$m_d = m_e$$

$$QH_d d^c + LH_d e^c \subset \mathbf{10} \bar{\mathbf{5}} \bar{\mathbf{5}}_h \text{ or } \mathbf{16} \mathbf{16} \mathbf{10}_h$$

GUT-like MSSM

- $G = SU(3)_c \times SU(2)_L \times U(1)_Y$: (SM)
- $\sin^2 \theta_W = \frac{3}{8}$: (GUT)
- $3 \times \{Q, d^c, u^c, L, e^c, \nu^c\} \subset 3 \times \mathbf{16}$ [SO(10) spinor]
: (GUT)
- $\{H_d, H_u\} \subset \mathbf{10}_h$ [SO(10) vector] : (GUT)
but D/T split : (SM)
- All other Matter fields are vector-like under G_{SM}
 \rightarrow superheavy
- $QH_d d^c + LH_d e^c$ not included in $\mathbf{10} \bar{\mathbf{5}} \bar{\mathbf{5}}_h$ or $\mathbf{16} \mathbf{16} \mathbf{10}_h$:
(SM)
- R-parity for stable proton and LSP

String model

- Higher dim. GUTs seem to be a good candidate, where GUT/SUSY breakings are associated with compactification mech.

But they are non-renormalizable.

They need to be embedded in string theory.

- The E8 x E8 heterotic string theory seems to be best for realistic model construction.

Heterotic String

Left mover (26 dim. bosonic string)

$$X_L^\mu(\sigma - \tau) : \mu = 0, 1, 2, \dots, 9,$$

$$X_L^I(\sigma - \tau) : I = 10, 11, \dots, 25 \text{ (gauge coordinate)}$$

$I = 10, \dots, 25$: compactified on an “even self-dual” lattice

→ Allowed Gauge Groups : $E_8 \times E'_8, SO(32)$

Right mover (10 dim. superstring)

$$X_R^\mu(\sigma + \tau) :$$

$$\psi_R^\mu(\sigma + \tau) : \text{NS sector (bosonic)}, \quad \text{R sector (fermionic)}$$

Heterotic String

Physical Massless States

Level-matching condition: $m_L^2 = m_R^2$ (\rightarrow tachyon free)

$$\tilde{\alpha}_{-1}^{\mu} |0\rangle_L \otimes \left\{ \begin{array}{l} |8_v\rangle_R \\ |8_s\rangle_R \end{array} \right\} : 10 \text{ dim. } \textit{SUGRA}$$

$$\left\{ \begin{array}{l} \tilde{\alpha}_{-1}^{\prime} |0\rangle_L \\ |p^2 = 2\rangle_L \end{array} \right\} \otimes \left\{ \begin{array}{l} |8_v\rangle_R \\ |8_s\rangle_R \end{array} \right\} : 10 \text{ dim. } \textit{SYM with } E_8 \times E_8'$$

Heterotic String

To reduce the space-time dim., and break SUSY and gauge sym.,

Orbifold Compactification

[Dixon, Harvey, Vafa, Witten '86]

- relatively simple, easy to analyze
- **CFT** is still useful for e.g. Yukawa coupling analysis.

$$\left\{ \begin{array}{l} \tilde{\alpha}_{-1}^{\prime} |0\rangle_L \\ |p^2 = 2\rangle_L \end{array} \right\} \otimes \left\{ \begin{array}{l} |8_v\rangle_R \\ |8_s\rangle_R \end{array} \right\} : \text{ 10 dim. SYM with } E_8 \times E_8'$$

Heterotic String

To reduce the space-time dim., and break SUSY and gauge sym.,

[Kim, Kim, Kye '07]

$$\vec{\phi} = \left(\frac{5}{12}, \frac{4}{12}, \frac{1}{12} \right) : Z_{12} \text{ orbifold}$$

$$V = \left(\frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4} \ \frac{1}{4}; \frac{5}{12} \ \frac{5}{12} \ \frac{1}{12} \right) \left(\frac{1}{4} \ \frac{3}{4} \ 0; 0^5 \right)'$$

$$W = \left(\frac{2}{3} \ \frac{2}{3} \ \frac{2}{3} \ \frac{-2}{3} \ \frac{-2}{3}; \frac{2}{3} \ 0 \ \frac{2}{3} \right) \left(0 \ \frac{2}{3} \ \frac{-2}{3}; 0^5 \right)'$$

$$G = [SU(3)_c \times SU(2)_L \times U(1)_Y \times U(1)^4] \times [SO(10) \times U(1)^3]'$$

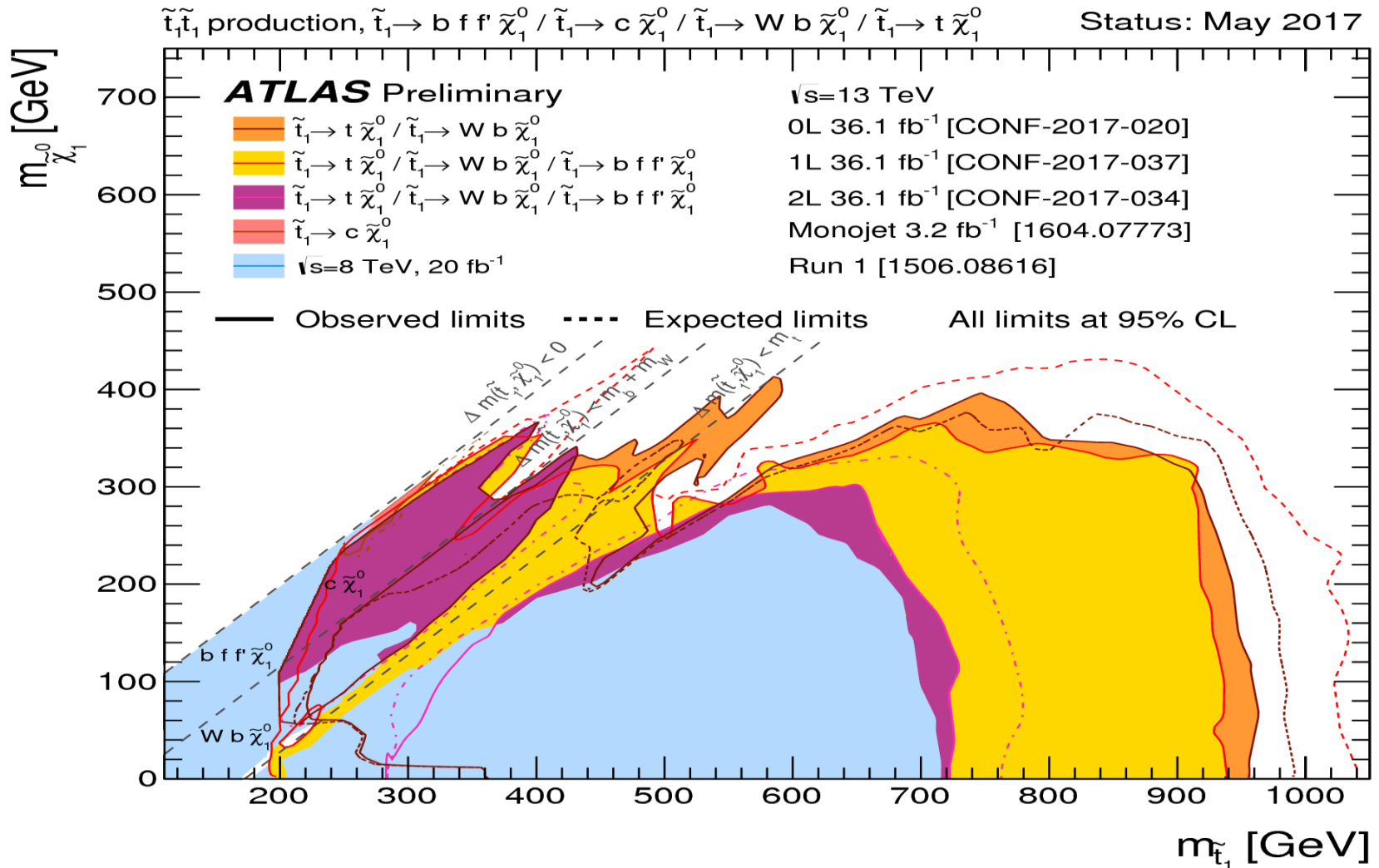
$$\implies [SU(8) \times U(1)] \times [SO(12) \times SU(2)]' \quad \text{in } D=6$$

by including KK modes above $\frac{1}{R}$: 6D SU(8) GUT

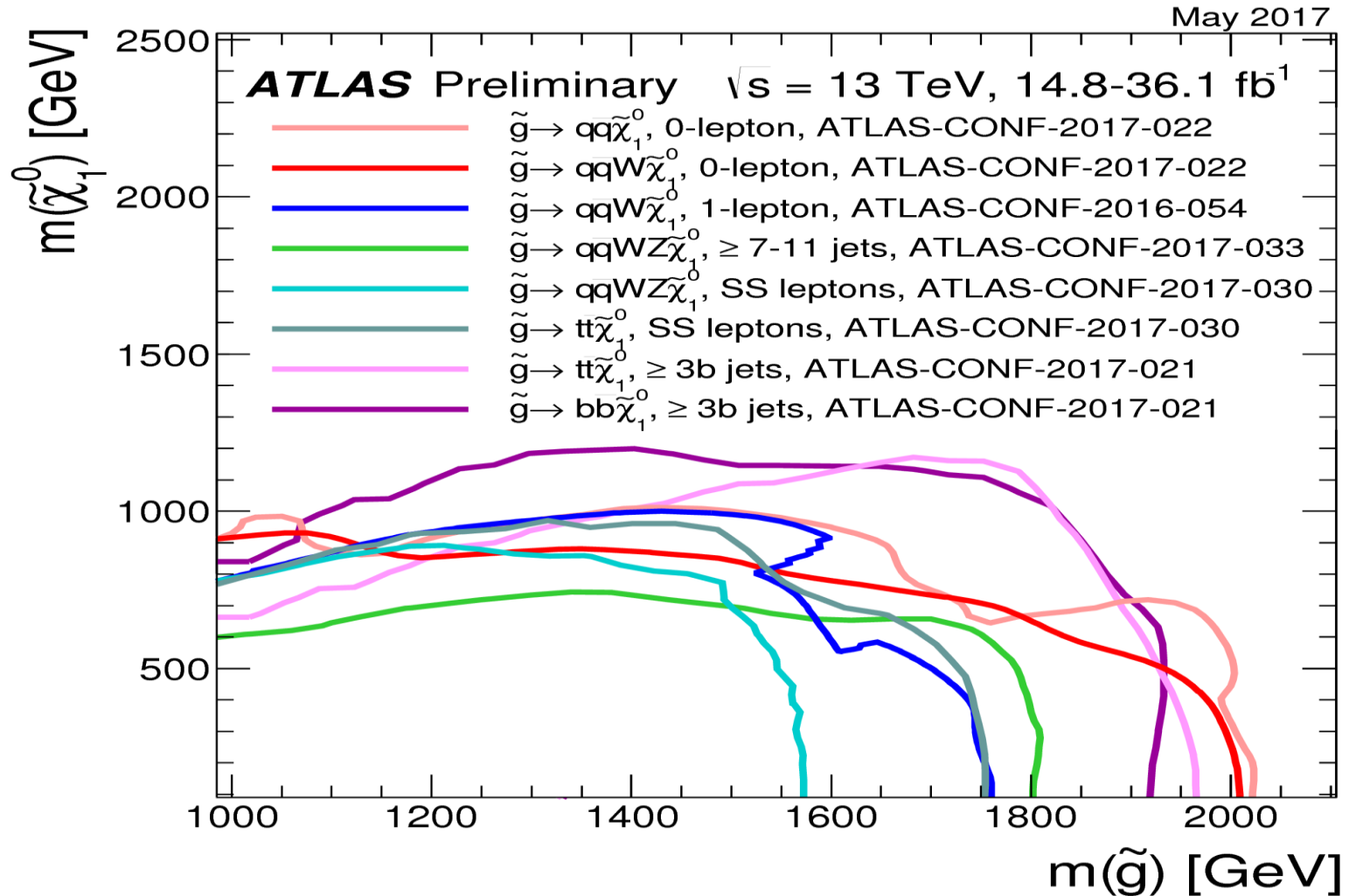
Supergravity model

- SUPERGRAVITY version of SM
 1. (classical) gravity interaction
 2. a soln. of the gauge hierarchy problem
 3. electroweak sym. breaking mech.
 4. Dark Matter candidate (LSP)
 5. gauge coupling unification (?)
→ implies a fundamental interaction?
 6. but Non-renormalizable → string theory

LHC constraints on SUSY (stop)



LHC constraints on SUSY (Gluino)



- The **naturalness** problem of **EW scale** and **Higgs boson mass** has been the most important issue for last four decades.
- The **MSSM** has been the most promising BSM candidate.
 - **No evidence** of **BSM** has been observed yet at LHC.
- **Theoretical puzzles** raised in the SM still remain **UNsolved**.
- **A barometer** of **the solution** to the naturalness problem is the **stop mass**.

The **stop mass** bound has been already **> 1 TeV**.

(The **gluino mass** bound has exceeded **> 2 TeV**.)

→ They start threatening the traditional status of SUSY as a solution to the naturalness problem of the EW phase transition.

- ATLAS and CMS have discovered the **SM(-like) Higgs with 125 GeV mass**, which is too heavy as a SUSY Higgs.
- According to the recent analyses, **10-20 TeV stop mass** is necessary **for the 125 GeV Higgs mass** (without a large stop mixing).

$$\Delta m_{h_u}^2|_{1\text{-loop}} \approx \frac{3|y_t|^2}{8\pi^2} \tilde{m}_t^2 \log\left(\frac{\tilde{m}_t^2}{\Lambda^2}\right) \left[1 + \frac{1}{2} \frac{A_t^2}{\tilde{m}_t^2}\right], \quad \frac{1}{2} m_Z^2 = \frac{m_{h_d}^2 - m_{h_u}^2 \tan^2 \beta}{\tan^2 \beta - 1} - |\mu|^2,$$

$$\Delta m_H^2|_{1\text{-loop}} \approx \frac{3m_t^4}{4\pi^2 v_h^2} \left[\log\left(\frac{\tilde{m}_t^2}{m_t^2}\right) + \frac{A_t^2}{\tilde{m}_t^2} \left(1 - \frac{1}{12} \frac{A_t^2}{\tilde{m}_t^2}\right) \right],$$

- ATLAS and CMS have discovered the **SM(-like) Higgs with 125 GeV mass**, which is too heavy as a SUSY Higgs.
- According to the recent analyses, **10-20 TeV stop mass** is necessary **for the 125 GeV Higgs mass** (without a large stop mixing).

**A fine-tuning of $10^{-3} - 10^{-4}$
seems to be unavoidable !! ??**

Science = Data ?



“Equalizer,”
nickname of Cot_45 (1872)

- Recently some new ideas (without SUSY) have been suggested to relax the gauge hierarchy problem.
- For UV completion, however, embedding them in SUSY also have been discussed.

- Recently some new ideas (without SUSY) have been suggested to relax the gauge hierarchy problem.
- For UV completion, however, embedding them in SUSY also have been discussed.

I attempt to address
the (little) hierarchy problem
in the **SUSY** framework.

Problems in SUSY models

Gravity Mediated SUSY Breaking mech.

μ and $B\mu$ terms are O.K.

But Flavor and CP problems would arise.

Gauge Mediated SUSY Breaking mech.

Flavor and CP problems are absent.

But μ and $B\mu$ problems would be serious.

On-going Project

- The **MSSM μ term** is **dynamically adjusted by singlets** such that the min. cond. of the Higgs is fulfilled .
- The **large VEV of singlets (μ term)** are efficiently controlled by a Higgs VEV of order 100 GeV.
- A relatively small soft mass of a singlet is responsible for the small $\langle H \rangle$ (or small M_Z).
- Mixings btw the Higgs and singlets constrain the allowed parameter space.

Summary

- Quantum physics and symmetries play essential roles in the construction of the SM.
- The SM is extremely successful, but has also some problems.
- Most new physics are stimulated to be developed in order to overcome them, and will be experimentally tested soon.

Summary

*Theoretical puzzles raised in the SM
still remain **UNsolved**.*

Thank you!