Road to find needles

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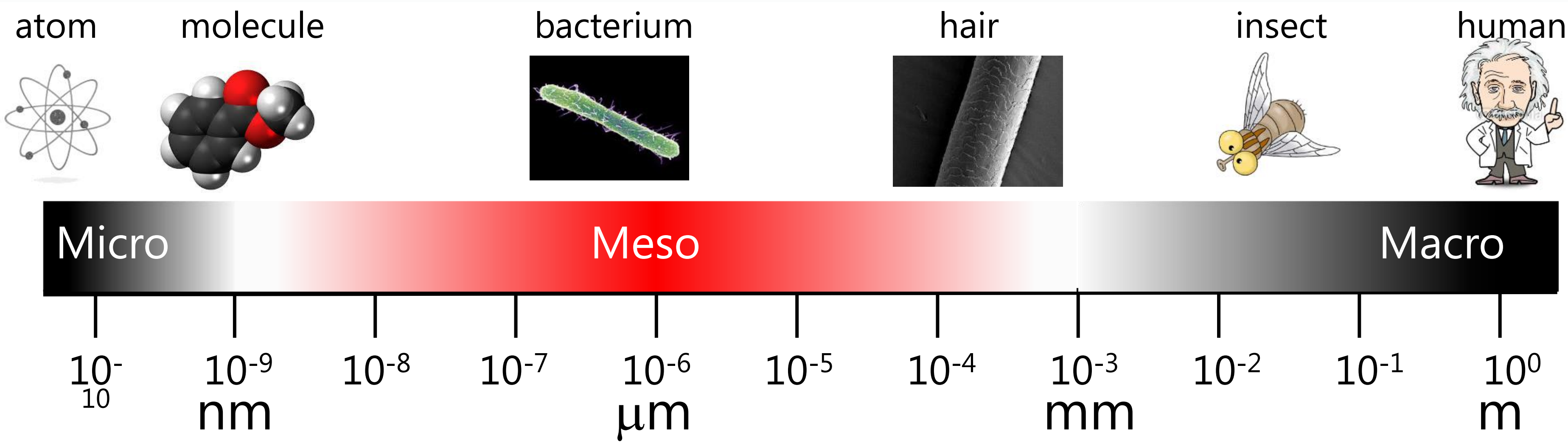
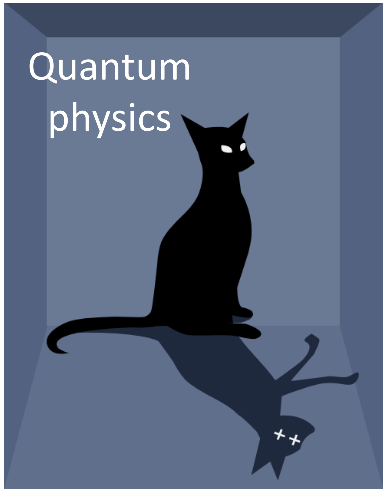
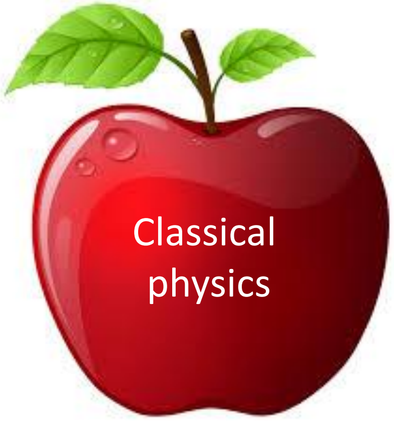
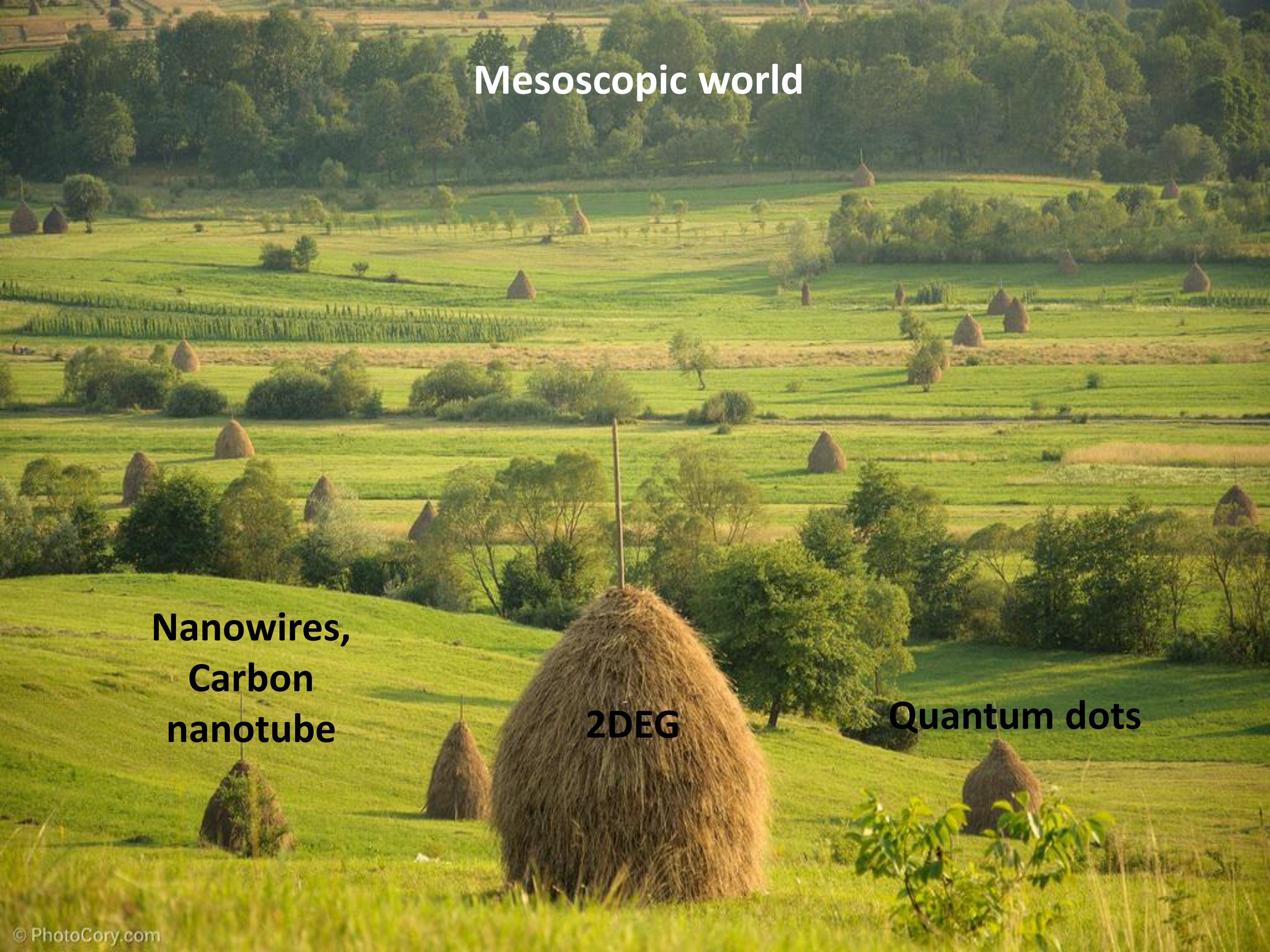
"Looking for a needle in a haystack"
• What needle am I looking for?
• Which haystack need I search for?
• How to find it?

Mesoscopic physics



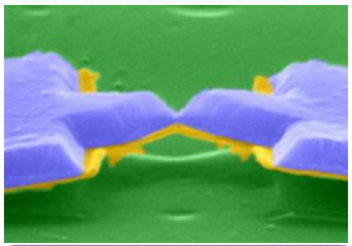




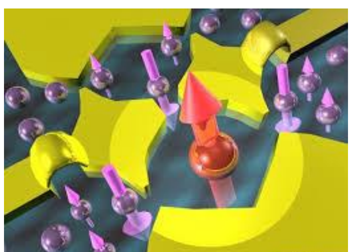

Mesoscopic world
Nanowires, Carbon nanotube
2DEG
Quantum dots
© PhotoCory.com

Topics in Mesoscopic physics

0D



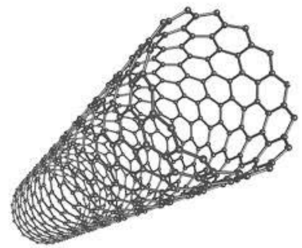
Single molecules



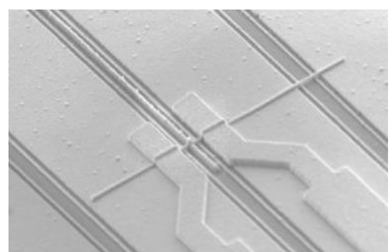
Quantum dots

Coulomb blockade
Kondo effect

1D



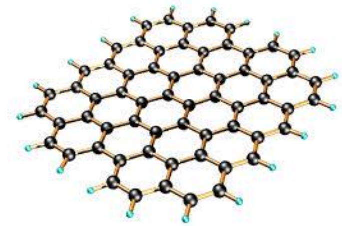
Carbon Nanotube



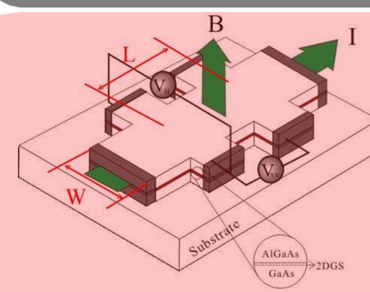
Nanowires

Luttinger liquid
Majorana fermion

2D



Graphene



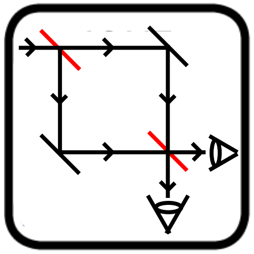
2DEG

Ballistic transport
Quantum Hall effect

Today's keywords

Why low dimension (2DEG)?

Why quantum Hall?

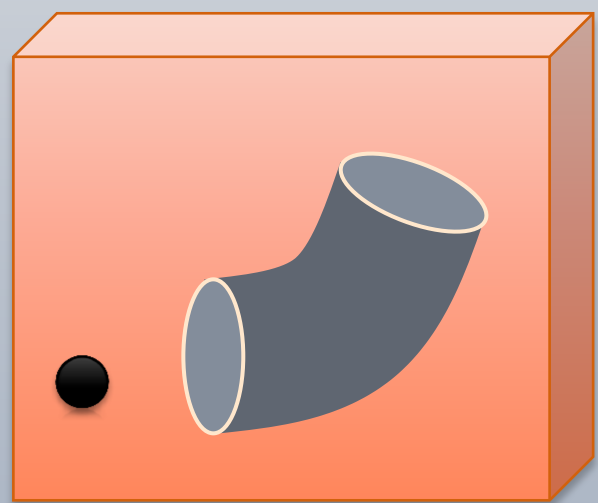


Why interferometry?

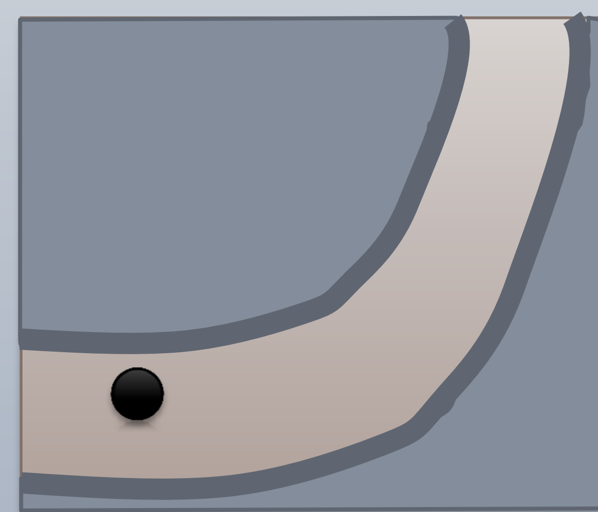
Physics in 2-D

Manipulating electrons' path is easier in 2D

$$A_{\text{å}} = e^{i(k_x x + k_y y + k_z z)}$$

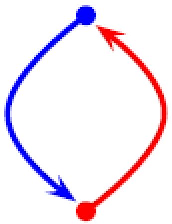


$$A_{\text{å}} = e^{i(k_x x + k_y y)}$$

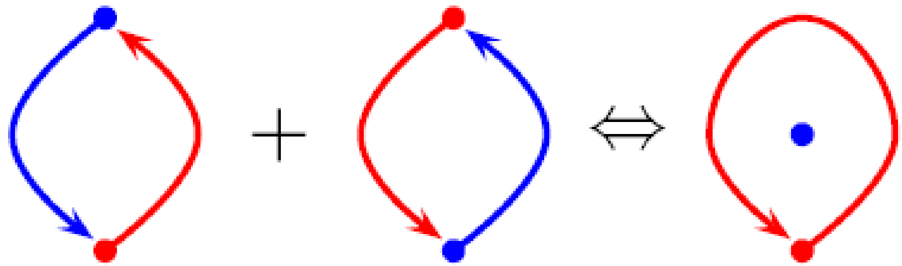


Anyon

Adiabatic exchange



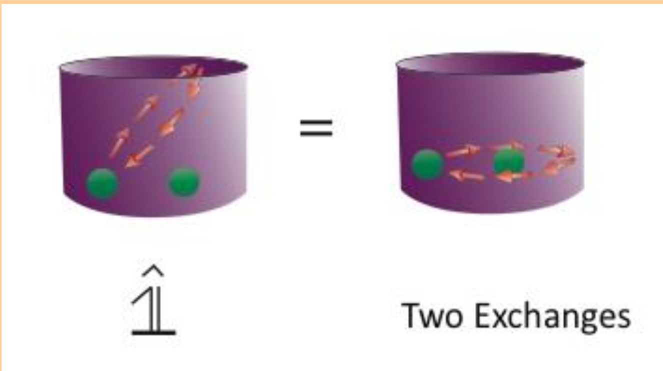
$$\left|Y(1,2)\right\rangle = e^{iq}\left|Y(2,1)\right\rangle$$



$$\left|Y(1,2)\right\rangle = e^{i2q}\left|Y(1,2)\right\rangle$$

Anyon

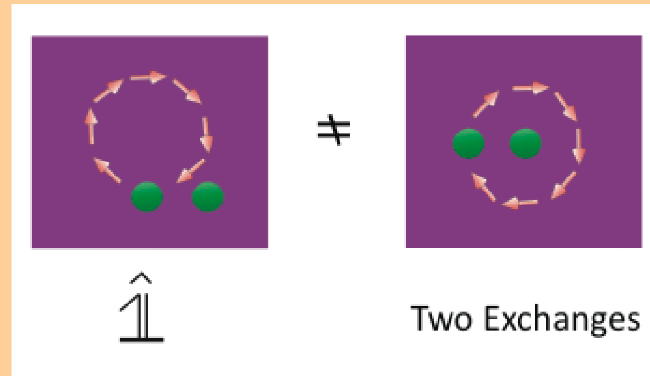
3-D



$q = 0$ for Boson

$q = p$ for Fermion

2-D

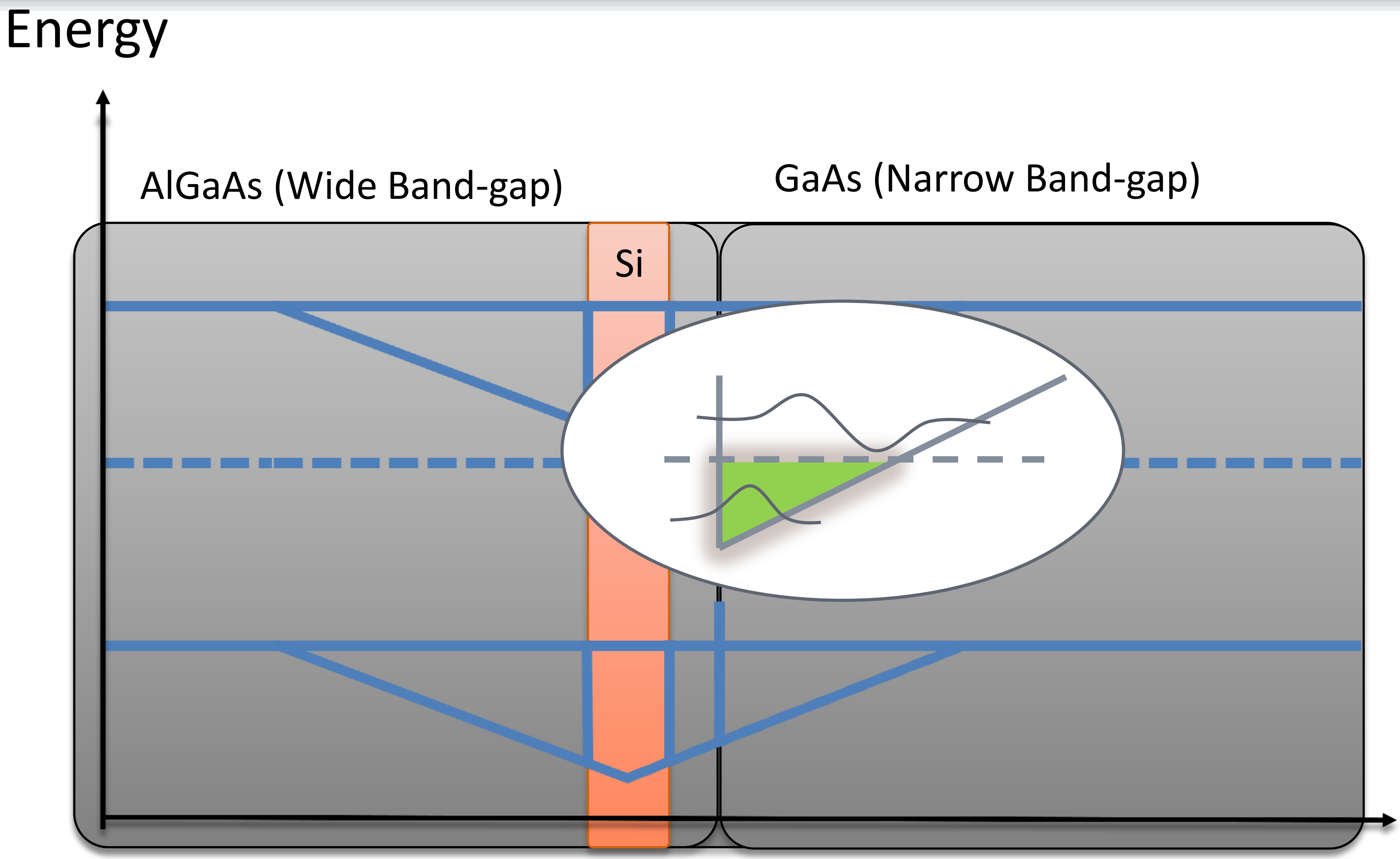


$0 £ q £ p$ for **Anyon**

Excitations in FQHE are strong candidates to be anyon

BUT, Never proved yet experimentally !!

2-dimensional electron gas (2DEG)



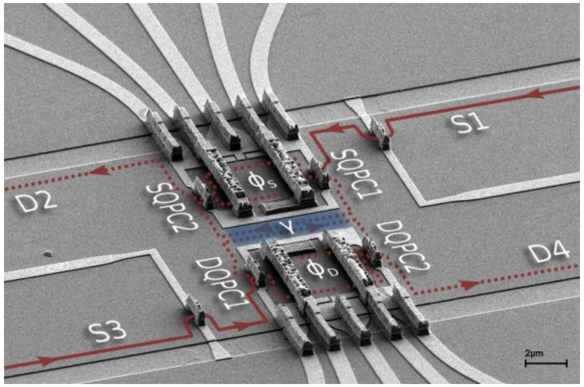
Why is 2D world exciting ?

High mobility, long mean free path, long coherence length

➡ Ballistic transport, electron interference

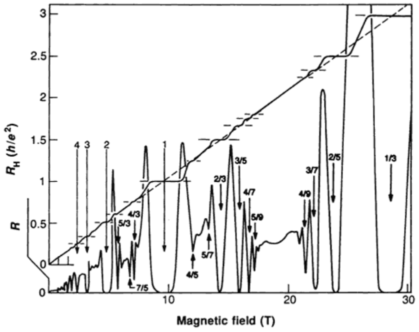
Easy electrostatic control by gate

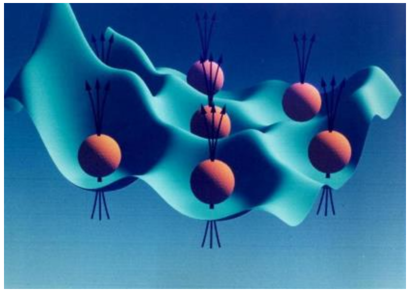
➡ Building complicated structure



Unique phenomena

➡ FQHE, anyon

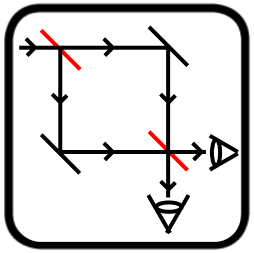




Today's keywords

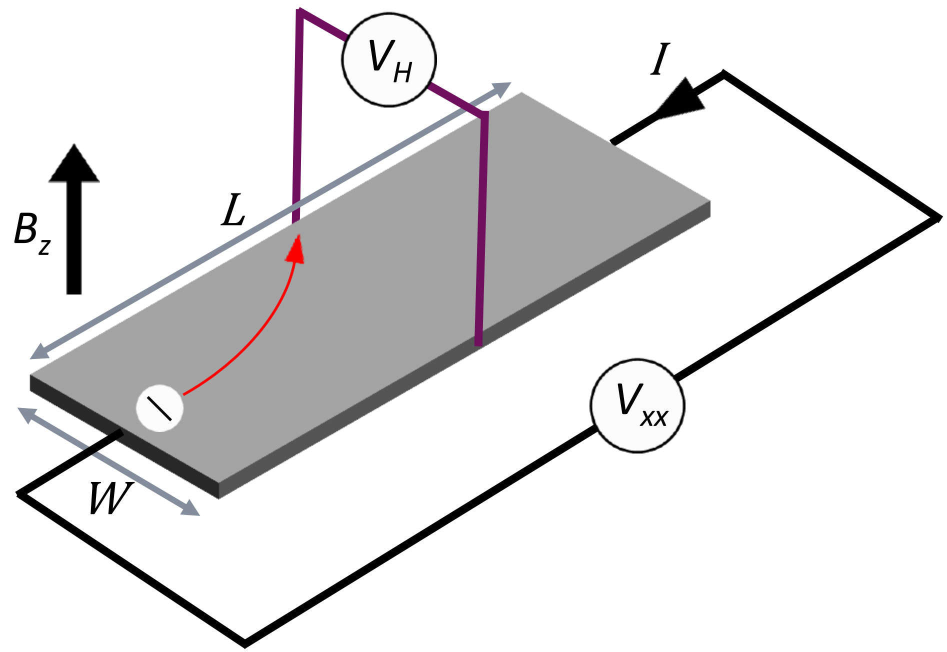
Why low dimension (2DEG)?

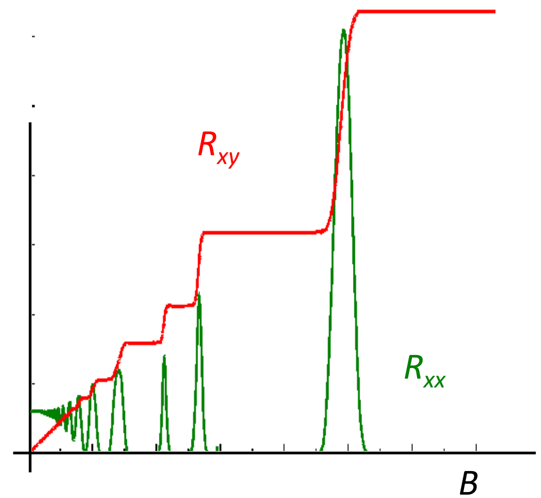
Why quantum Hall?



Why interferometry?

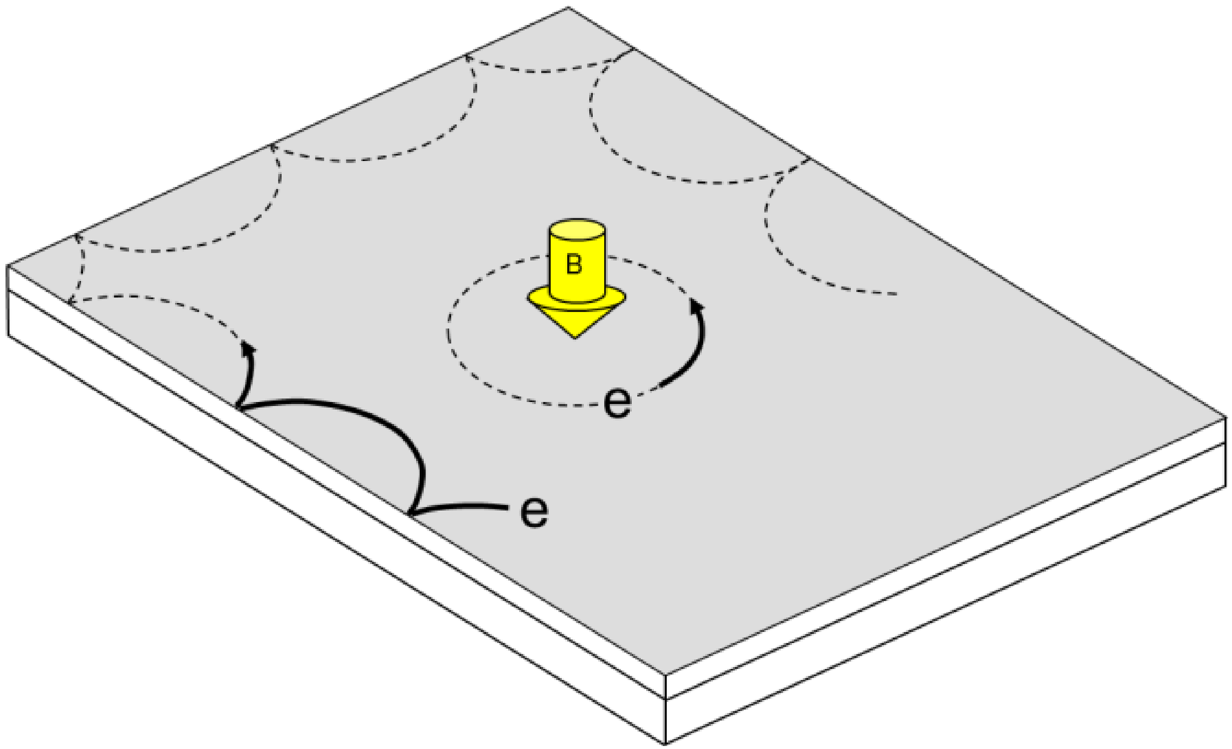
Classical Hall effect





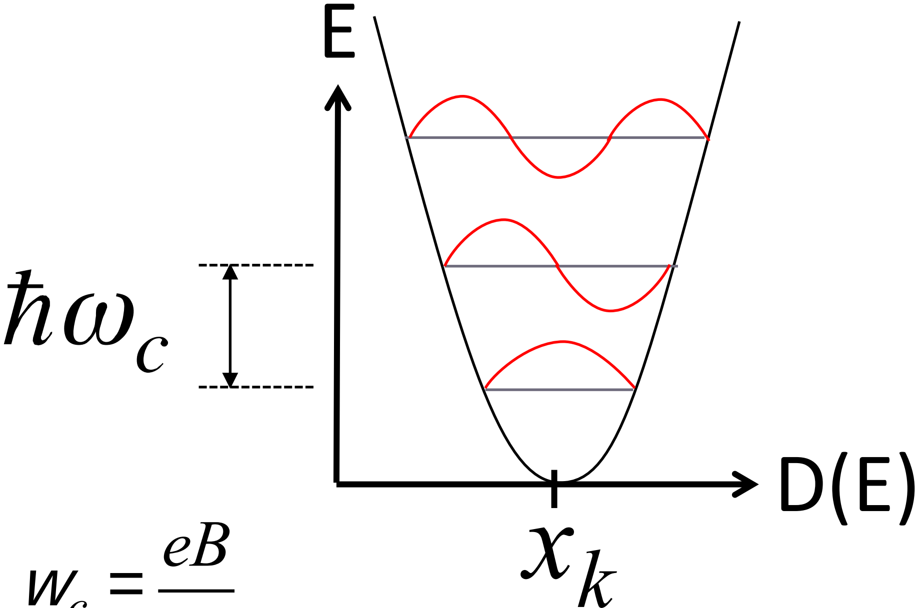
$$R_{xy} = \frac{B}{en} \qquad R_{xx} = \frac{L}{W}\frac{1}{nem}$$

Quantum Hall effect



Quantum Hall effect

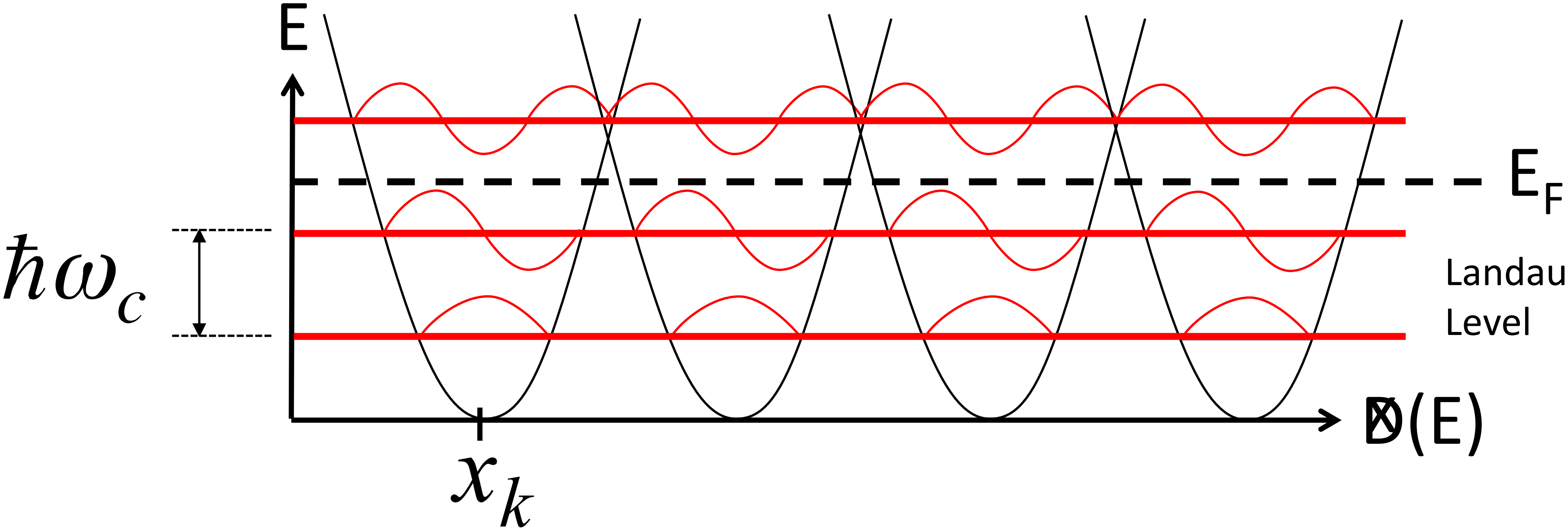
Quantum Hall effect



$$w_c = \frac{eB}{m}$$

Quantum Hall effect

Quantum Hall effect



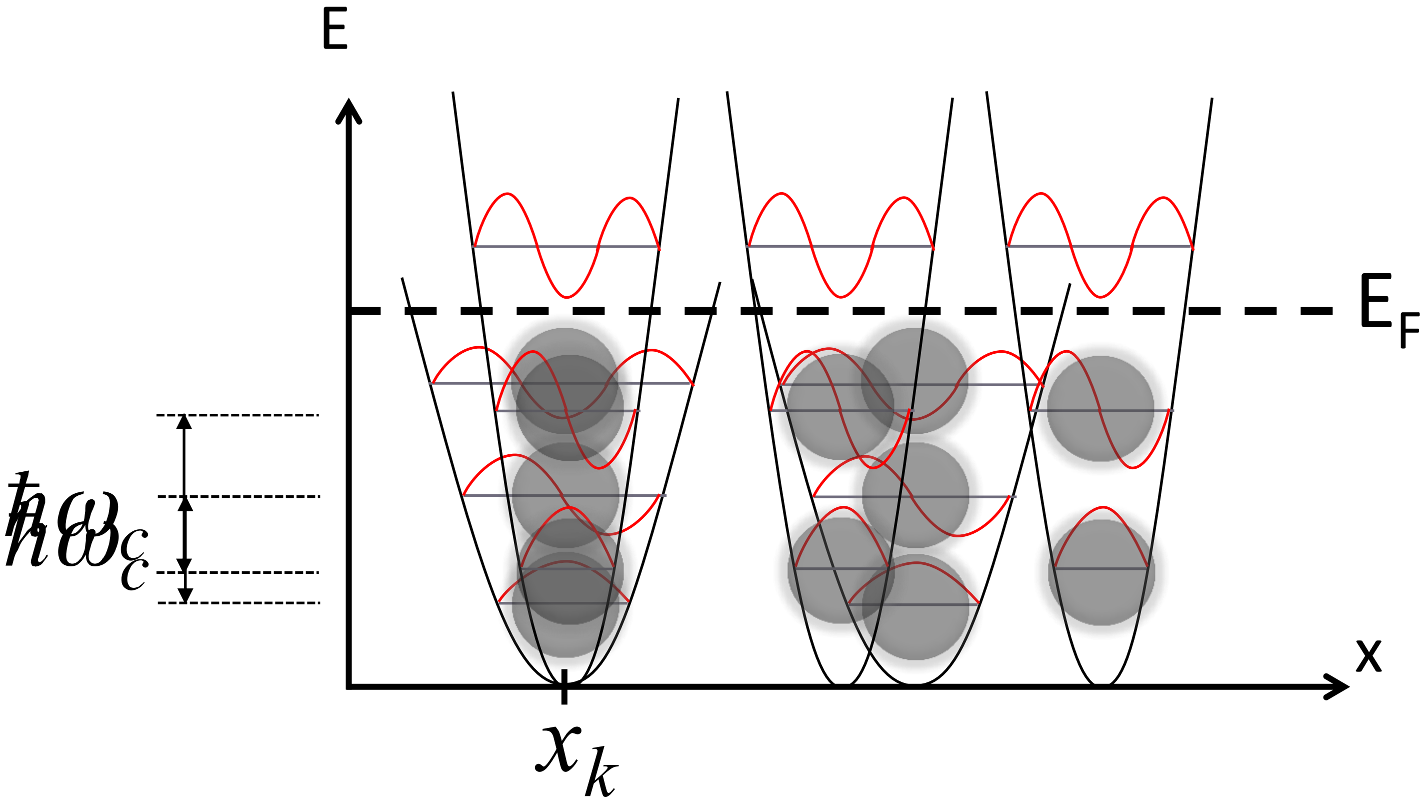
filling factor ν = number of filled LL

= number of electron / flux quantum

Fermion vs. Boson

vs.

Quantum Hall effect



B increase→ degeneracy increase

Edge states in QHE

Recipe:

2 Spatial Dimensions

\+

Confinement

\+

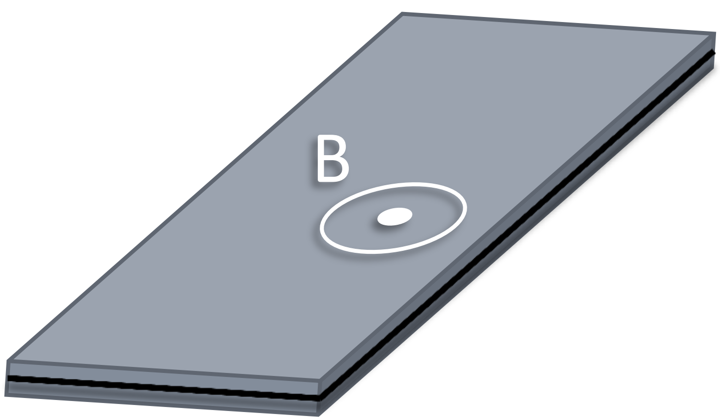
Magnetic Field

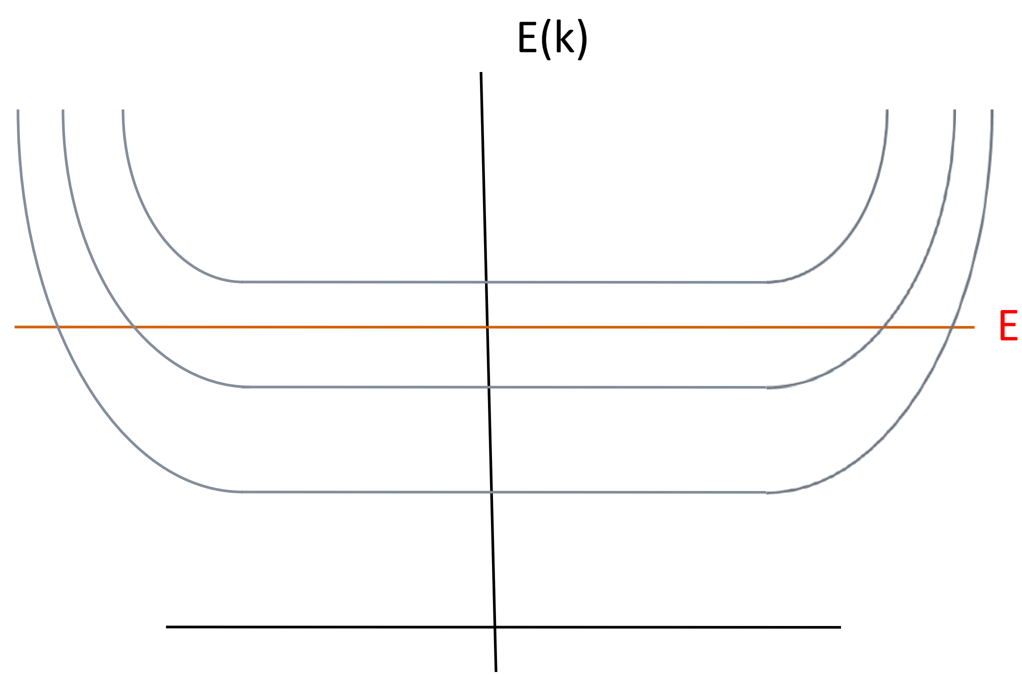
=

QHE

$$E(n,x) = \hbar\omega_c(n + 1/2) + V(x)$$

Landau level $$w_c = \frac{eB}{m}$$

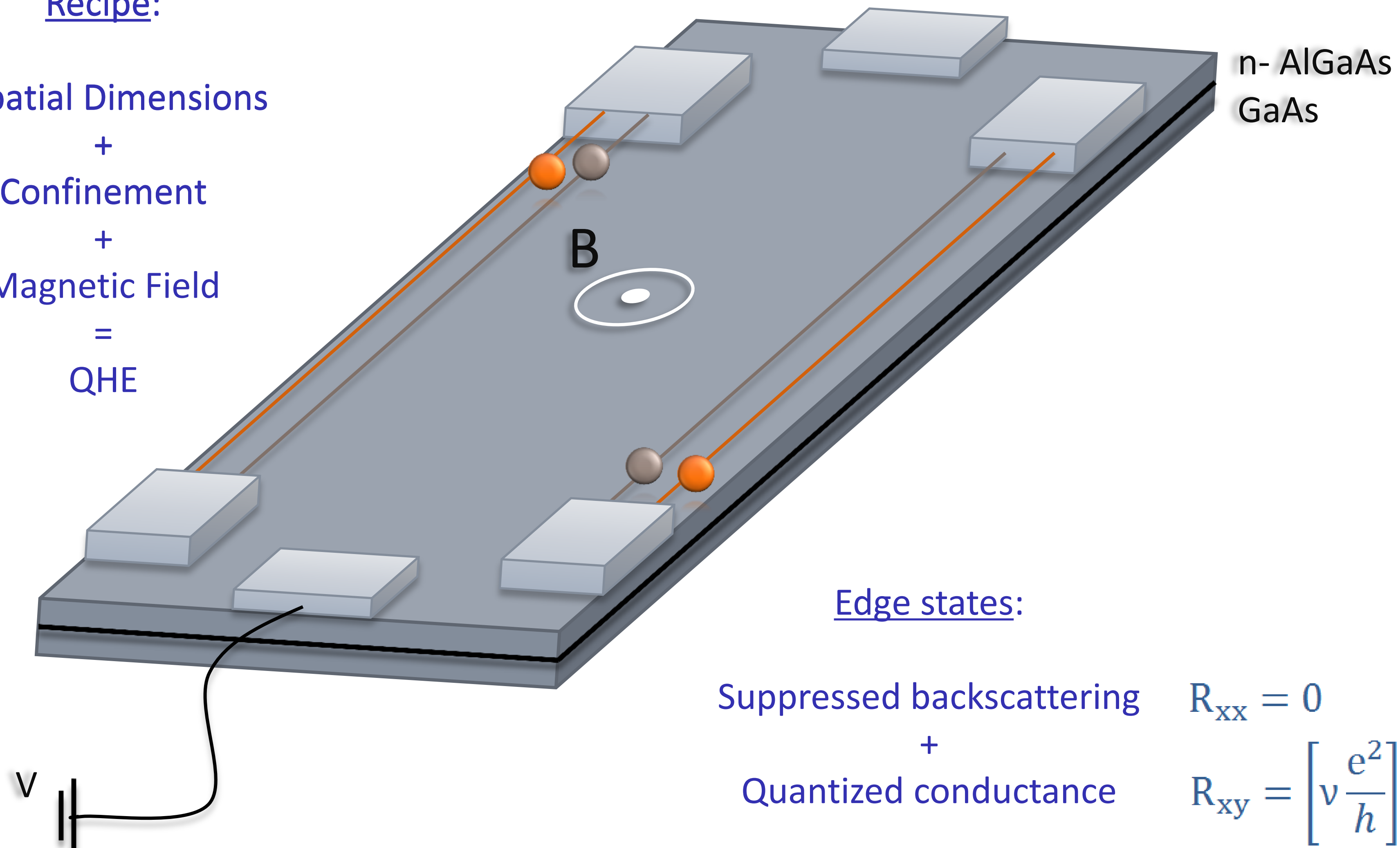




Edge states in QHE

Recipe:

2 Spatial Dimensions
+
Confinement
+
Magnetic Field
=
QHE



Edge states:

Suppressed backscattering $\quad R_{xx} = 0$
+
Quantized conductance $\quad R_{xy} = \left[\nu \frac{e^2}{h} \right]^{-1}$

Edge states in QHE

ordinary 2D metal

quantum Hall edge states
chiral 1D metal

- Easy to engineer electron path
- No back scattering
- The quantum hall effect helps us mimic lasers

Quantum Hall effect

VOLUME 45, NUMBER 6 PHYSICAL REVIEW LETTERS 11 AUGUST 1980

New Method for High-Accuracy Determination of the Fine-Structure Constant Based on Quantized Hall Resistance

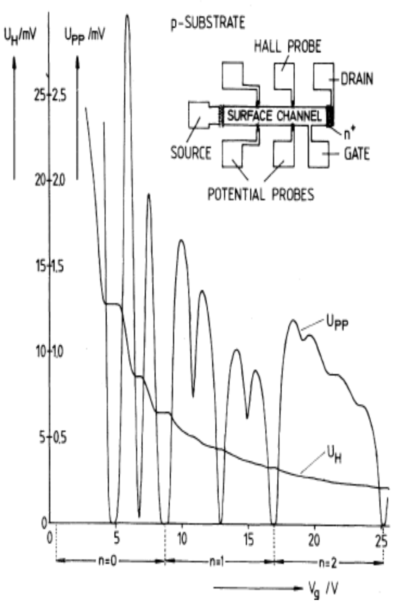
K. v. Klitzing
Physikalisches Institut der Universität Würzburg, D-8700 Würzburg, Federal Republic of Germany, and Hochfeld-Magnetlabor des Max-Planck-Instituts für Festkörperforschung, F-38042 Grenoble, France

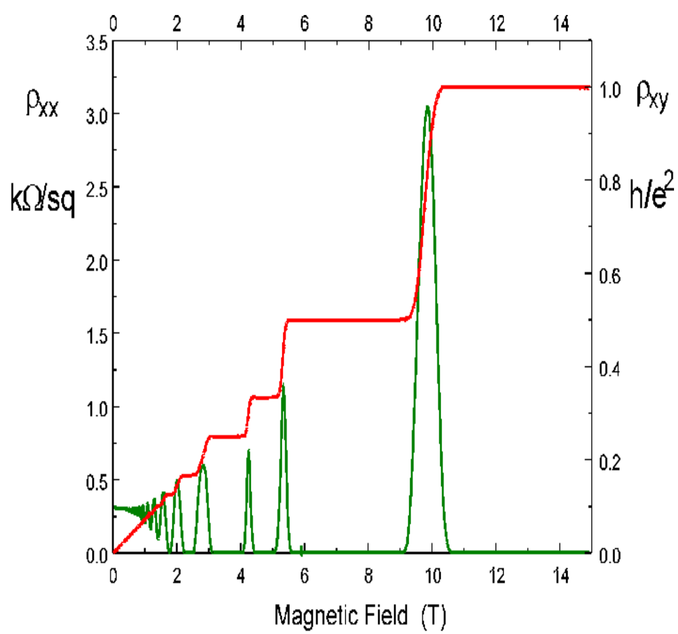
and

G. Dorda
Forschungslaboratorien der Siemens AG, D-8000 München, Federal Republic of Germany

and

M. Pepper
Cavendish Laboratory, Cambridge CB3 0HE, United Kingdom
(Received 30 May 1980)



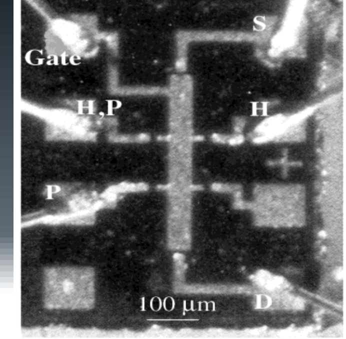


integer quantum Hall effect

$$R_{xy} = \left(\nu \frac{e^2}{h}\right)^{-1}$$

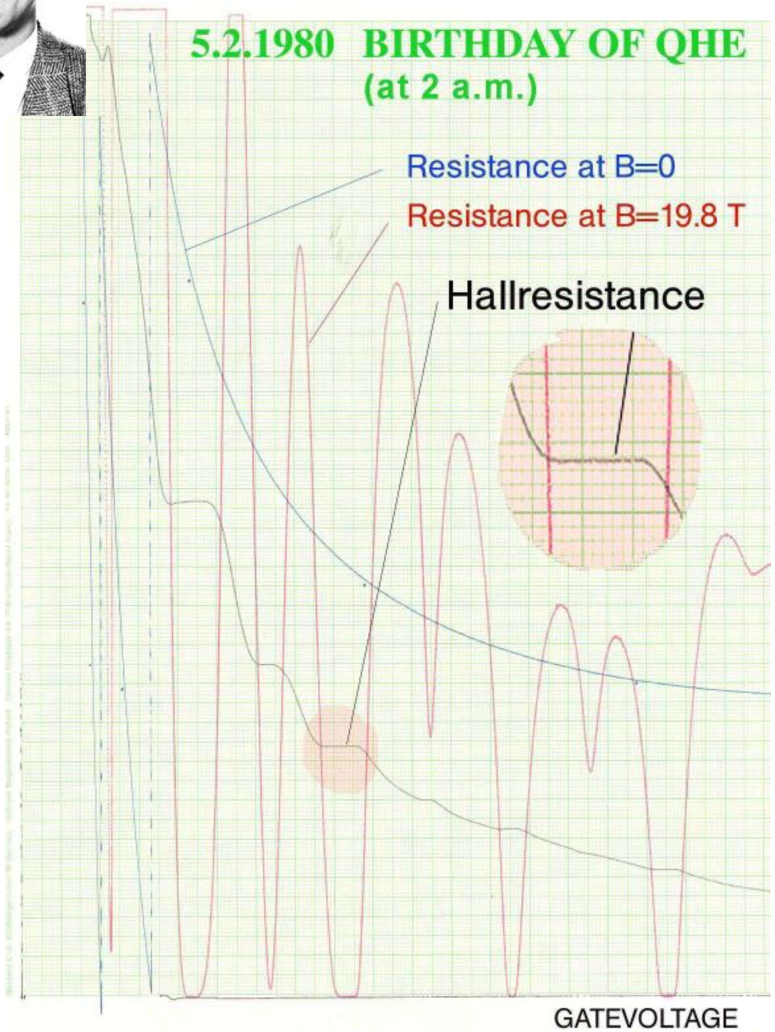
$$R_{xx} = 0 \text{ Ohm}$$

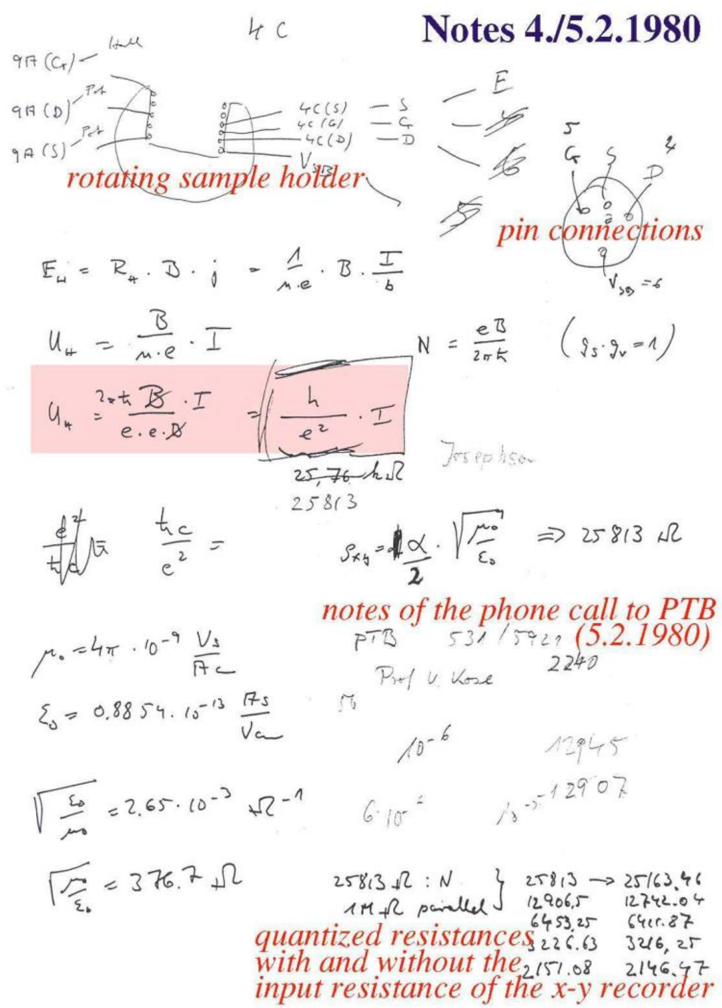
Quantum Hall effect



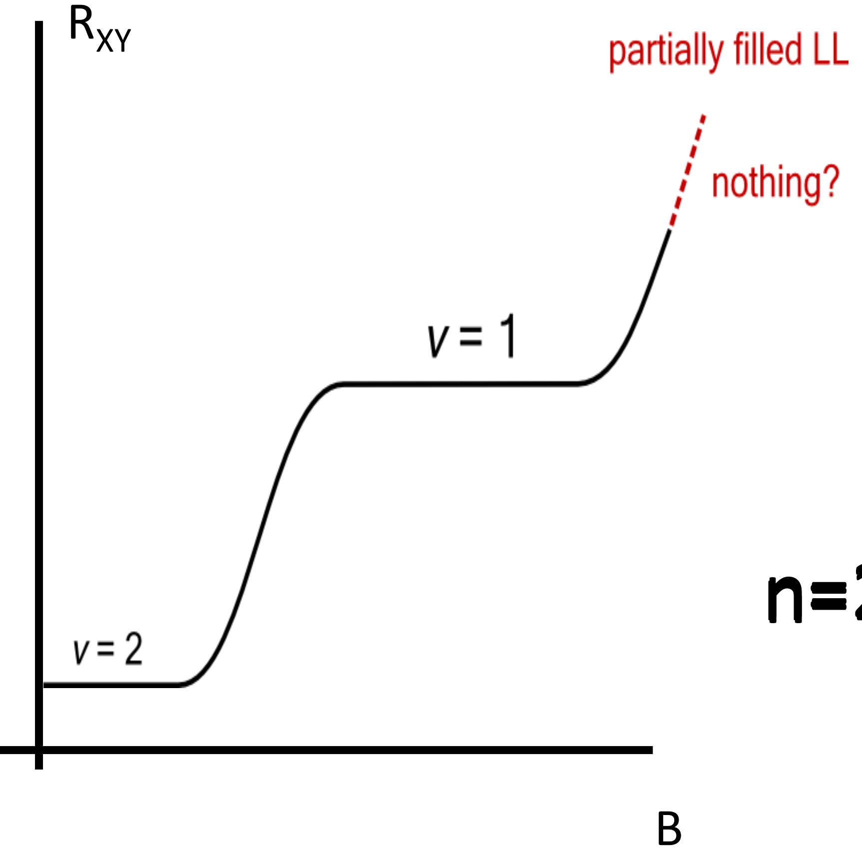
Nobel Prize in physics 1985

" for the discovery of quantized Hall effect"

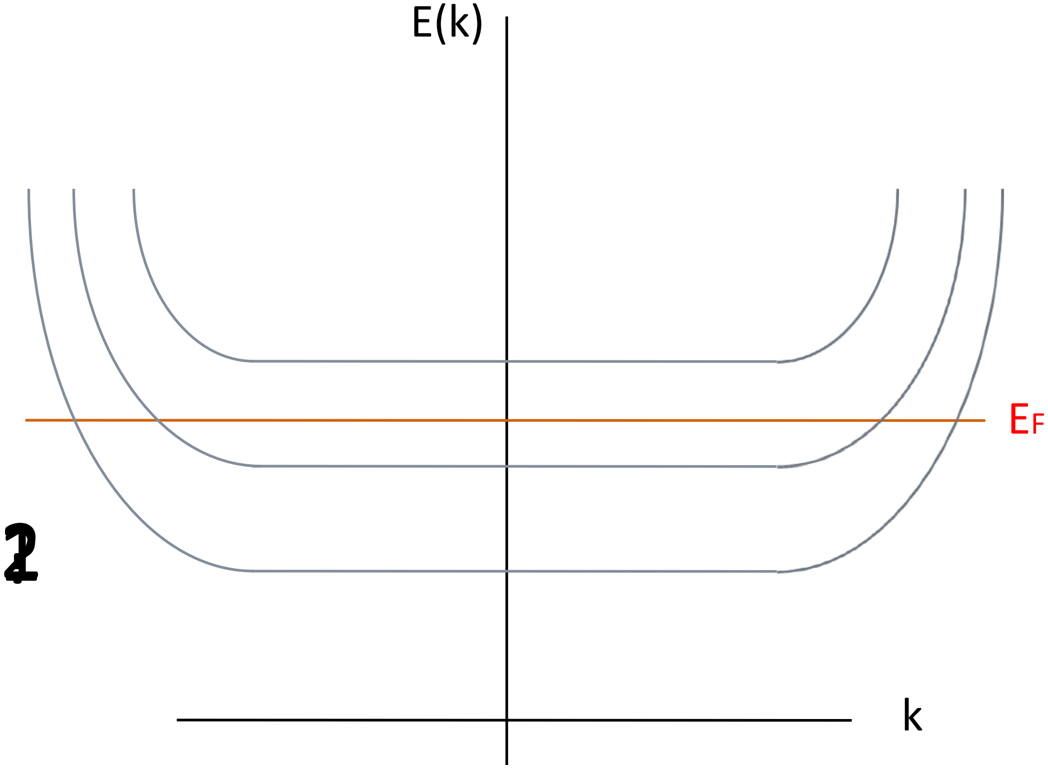




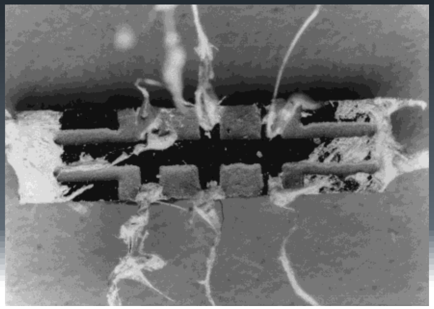
cleaner 2D electrons + Low T + Strong B



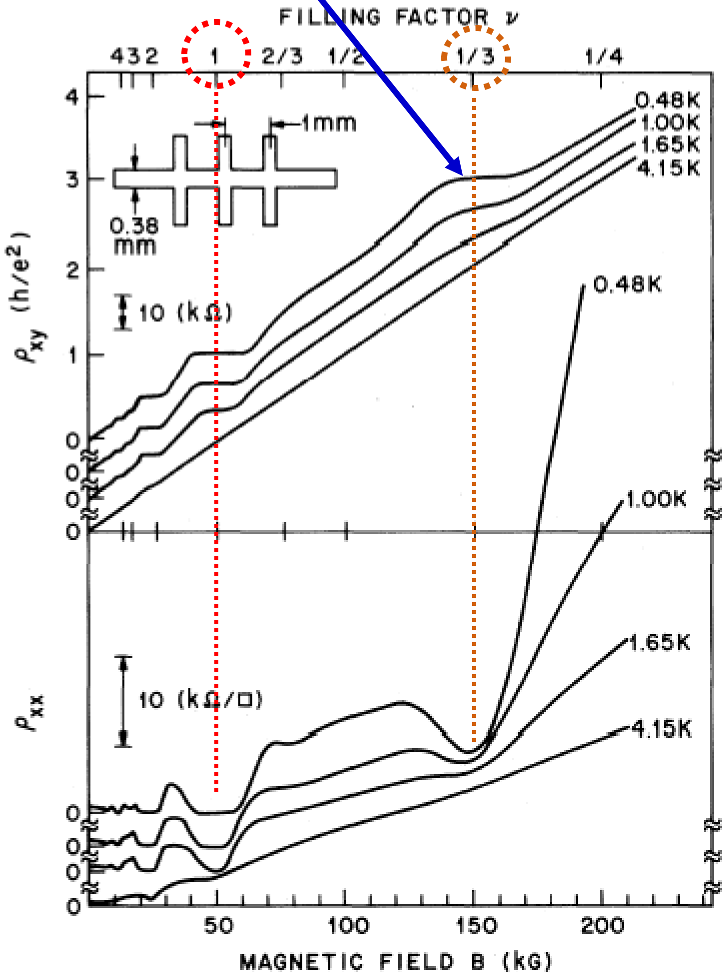
n=2



Fractional QHE



$R_H = (e^2/3h)^{-1}$



VOLUME 48, NUMBER 22 PHYSICAL REVIEW LETTERS 31 MAY 1982

Two-Dimensional Magnetotransport in the Extreme Quantum Limit

D. C. Tsui,[a],[b] H. L. Stormer,[a] and A. C. Gossard
Bell Laboratories, Murray Hill, New Jersey 07974
(Received 5 March 1982)

tion. Our observation of a quantized Hall resistance of $3h/e^2$ at $\nu = \frac{1}{3}$ is a case where Laughlin's argument breaks down. If we attribute it to the presence of a gap at E_F when $\frac{1}{3}$ of the lowest Landau level is occupied, his argument will lead to quasiparticles with fractional electronic charge of $\frac{1}{3}$, as has been suggested for $\frac{1}{3}$-filled quasi one-dimensional systems.[21]

Fractional QHE

VOLUME 50, NUMBER 18 PHYSICAL REVIEW LETTERS 2 MAY 1983

Anomalous Quantum Hall Effect: An Incompressible Quantum Fluid with Fractionally Charged Excitations

R. B. Laughlin
Lawrence Livermore National Laboratory, University of California, Livermore, California 94550

(Received 22 February 1983)

This Letter presents variational ground-state and excited-state wave functions which describe the condensation of a two-dimensional electron gas into a new state of matter.

PACS numbers: 71.45.Nt, 72.20.My, 73.40.Lq

$$\Psi_m(Z_1, Z_2, .., Z_N) = \prod_{i<j}^{N} (Z_i - Z_j)^m \exp\left[-\frac{1}{4l^2} \sum_{k}^{N} |Z_k|^2 \right]$$

Nobel Prize in physics 1998
" for the discovery of a new form of quantum fluid with fractionally charged excitations"

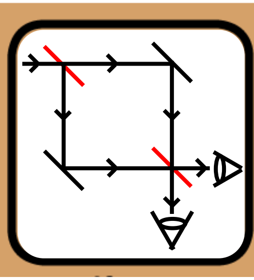
Why Quantum Hall ?

- The quantum hall effect helps us mimic lasers in quantum optics
  - Electrons directed along definite paths flexible design, definite acquired phase
  - No back-scattering insensitive to impurities
- The quasiparticles in the fractional QHE regime is expected to be anyon

Today's keywords

Why low dimension (2DEG)?

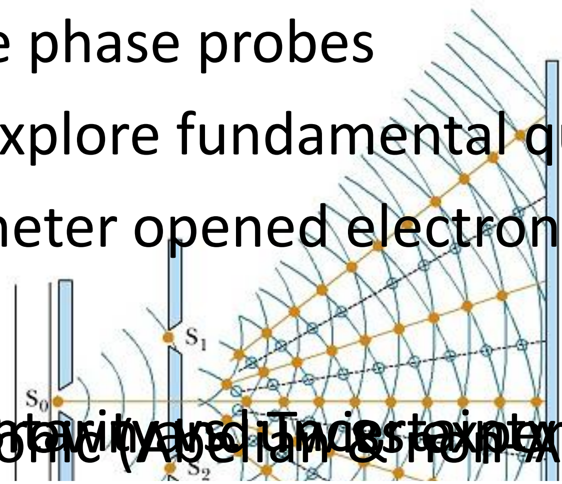
Why quantum Hall?



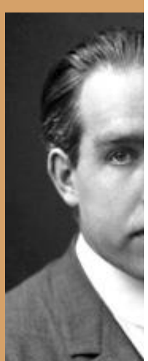
Why interferometry?

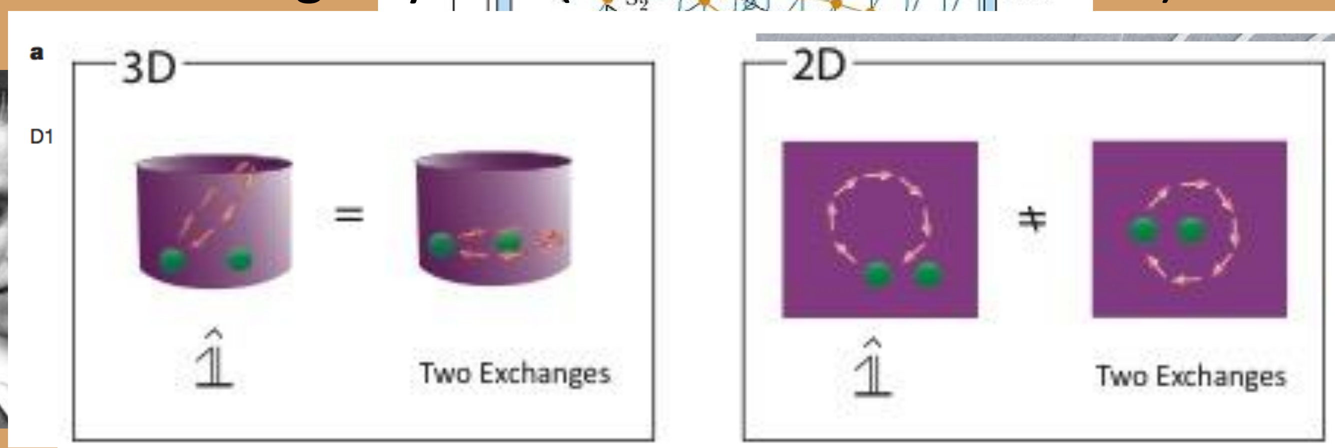
Why interferometry?

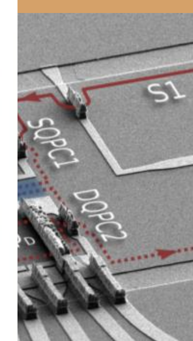
- Interferometers are phase probes
- Good platform to explore fundamental quantum physics
- Electron interferometer opened electron quantum optics



Probing anyonic (Abelian & non-Abelian) statistics



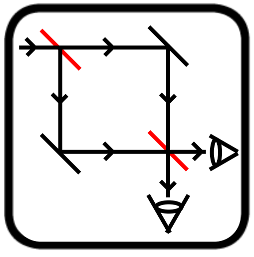




I. Neder *et al.* Nature (2007)

Dong-In Chang *et al.*, Nat. Phys. (2008)   E. Weisz , H. K. Choi *et al.*, Science (2014)

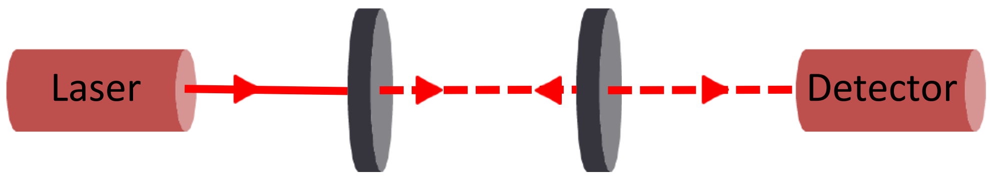
Electronic interferometers



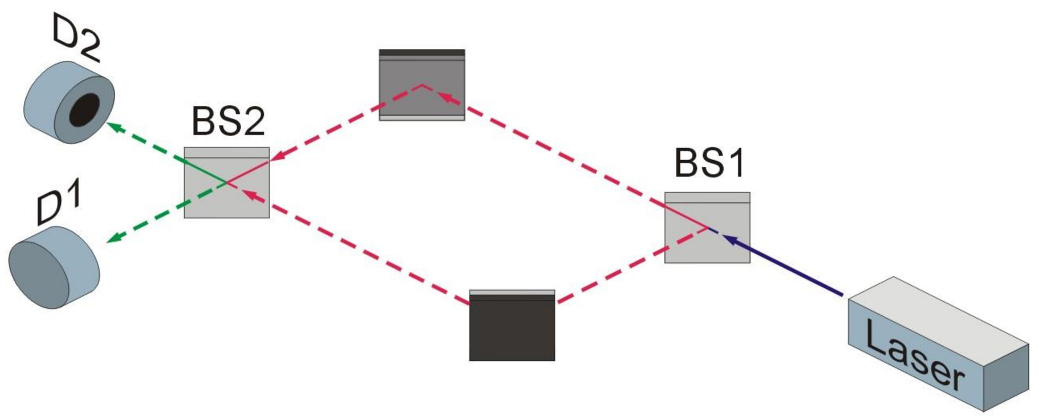
How do we construct electronic interferometers?

Optical interferometers

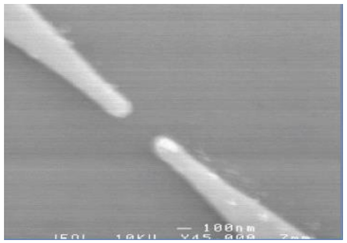
Fabry-Perot Interferometer (FPI)

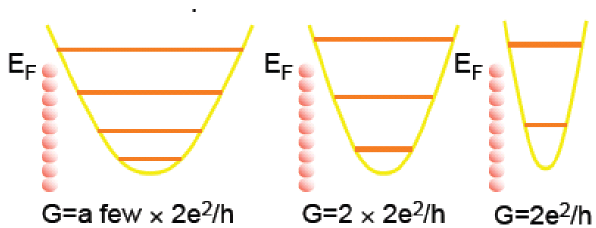


Mach-Zehnder Interferometer (MZI)



Quantum Point Contact



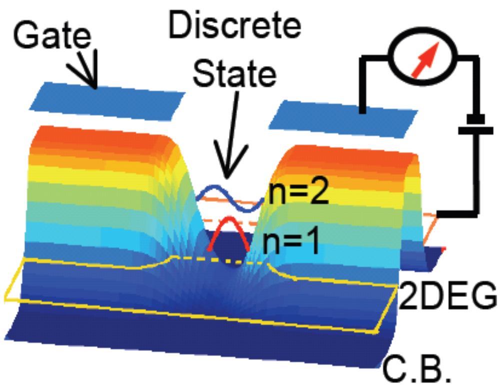


On the one hand:

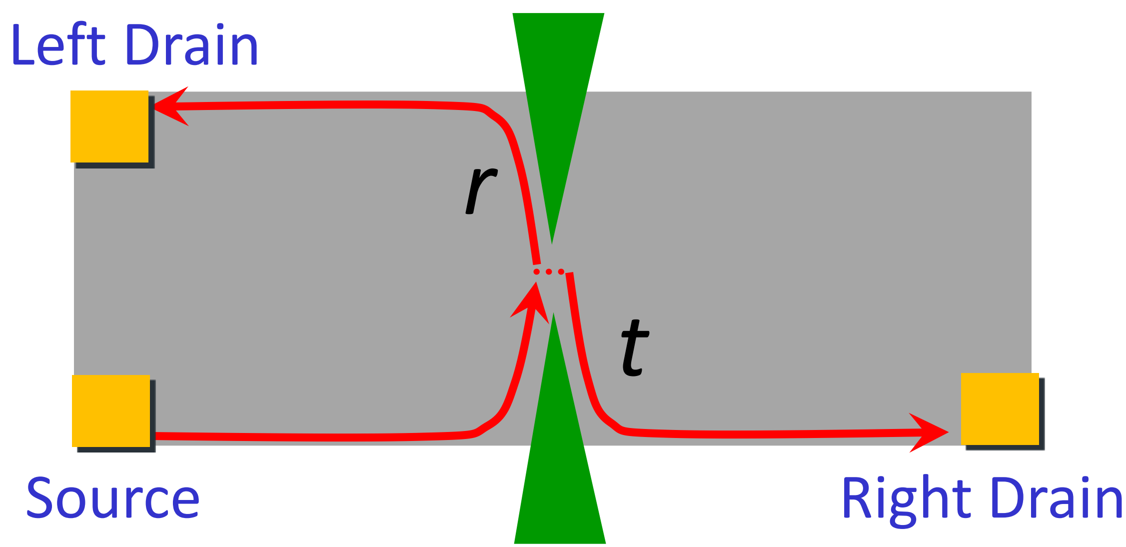
$$I_L = |r|^2 \times I,\quad I_R = |t|^2 \times I$$

On the other hand:

$$y = r \times y_L + t \times y_R$$

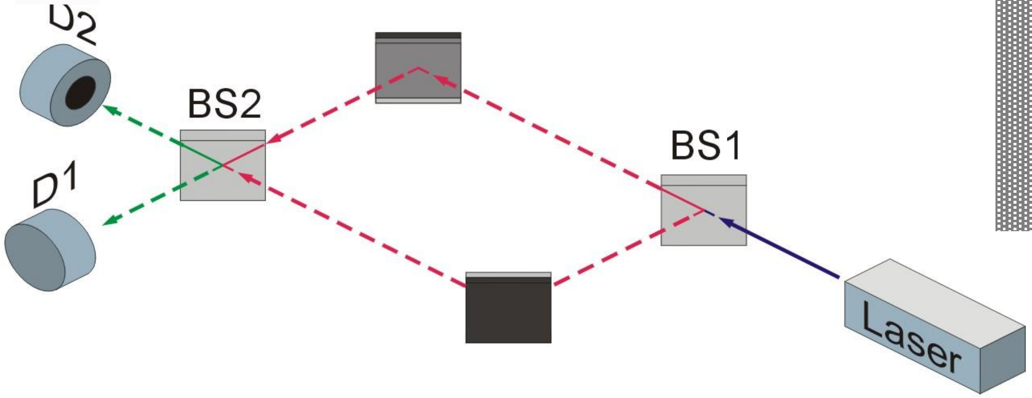


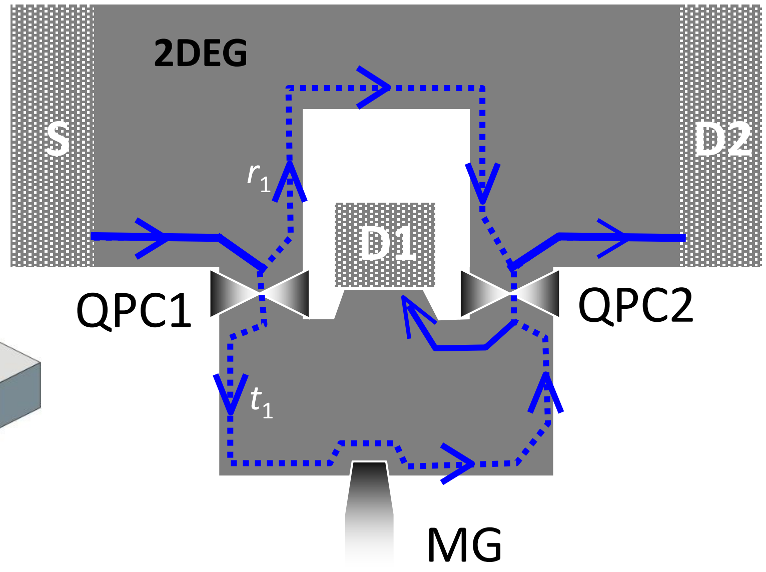
QPC with QHE edge state



- Conductance quantization
- QPC acts like as beam splitter

Realization of electronic MZI

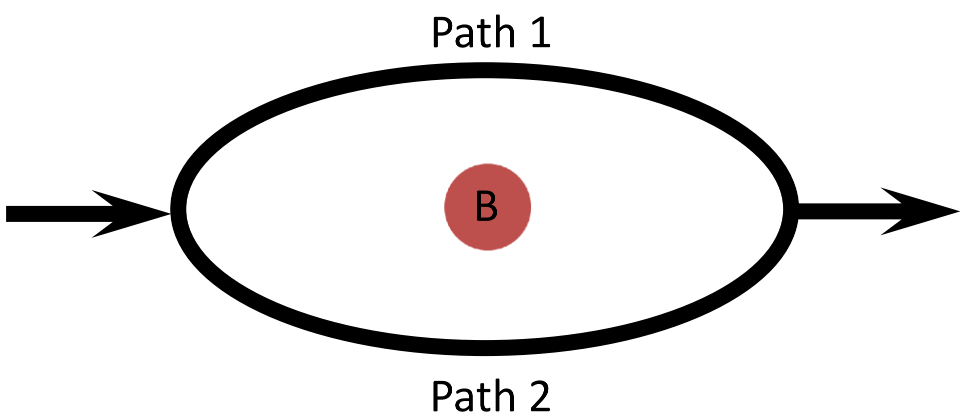




Photonic beam → quantum Hall edge state
Beam splitter → quantum point contact (QPC)

Yang Ji *et al.*, Nature **422**,415 (2003)

Aharonov-Bohm effect



$$\varphi_{path1} = \frac{e}{\hbar}\int_1 \vec{A}\cdot d\vec{x}$$

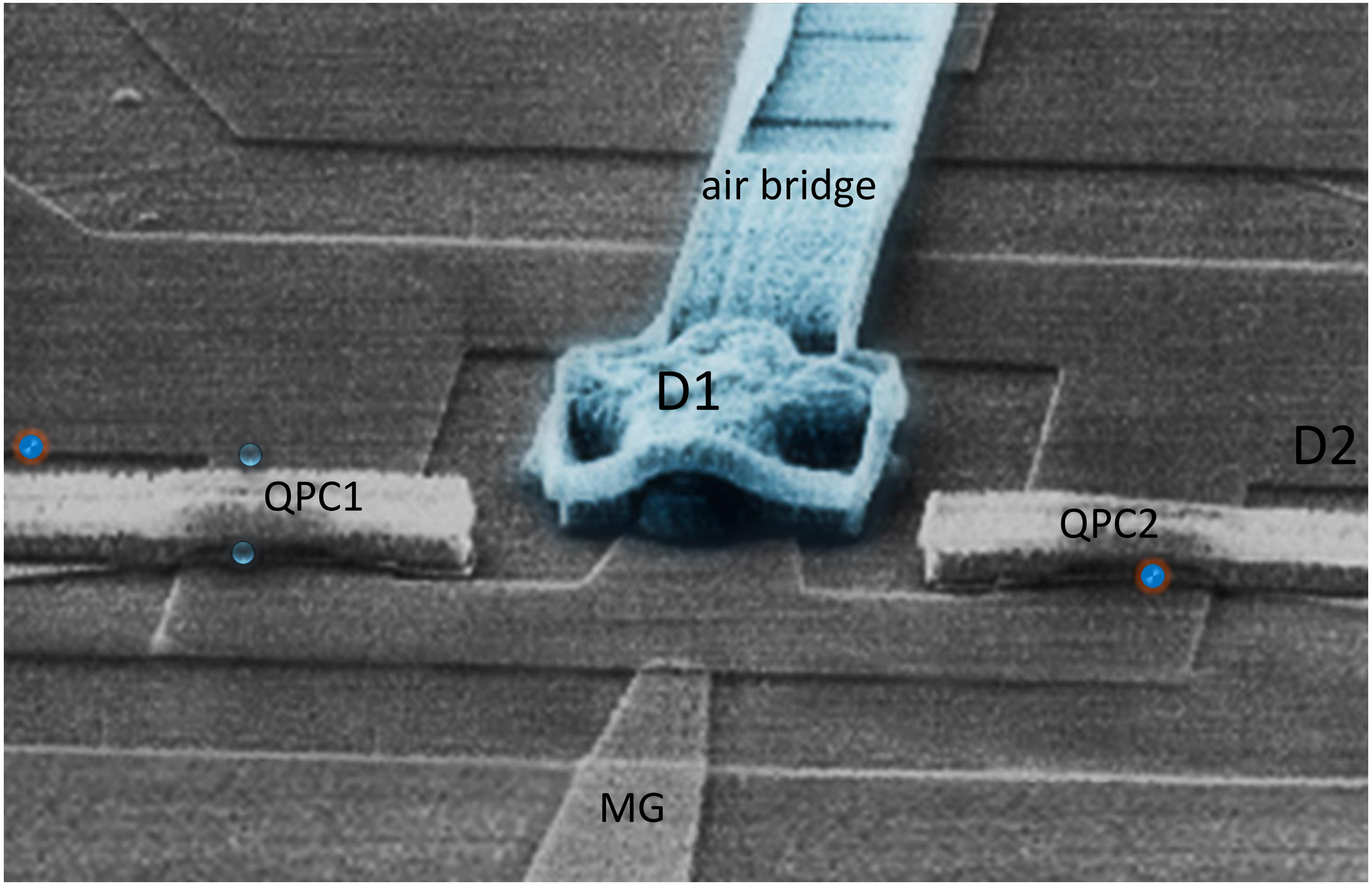
$$\varphi_{path2} = \frac{e}{\hbar}\int_2 \vec{A}\cdot d\vec{x}$$

$$\Psi = \psi_1 + \psi_2$$

$$\Psi = e^{i\varphi_{path1}}\psi_1 + e^{i\varphi_{path2}}\psi_2 = e^{i\varphi_{path1}}(\psi_1 + e^{i\varphi_{AB}}\psi_2)$$

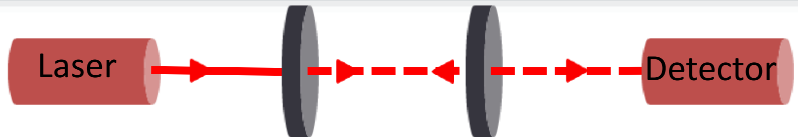
$$\varphi_{AB} = \varphi_{path2} - \varphi_{path1} = \frac{e}{\hbar}\oint \vec{A}\cdot d\vec{x} = \frac{e}{\hbar}\int \vec{B}\cdot d\vec{a} = \frac{e}{\hbar}\Phi = 2\pi\frac{\Phi}{\phi_0}$$

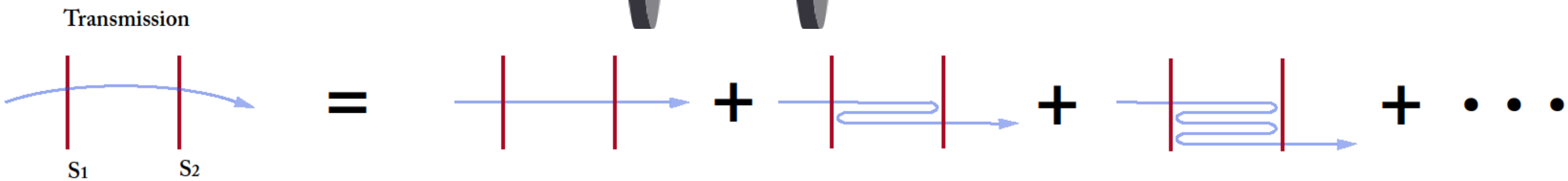
Realization of electronic MZI

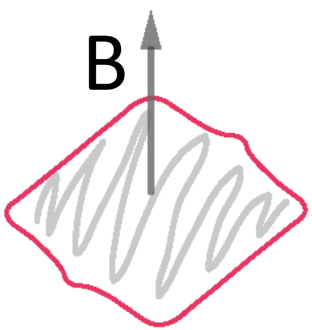


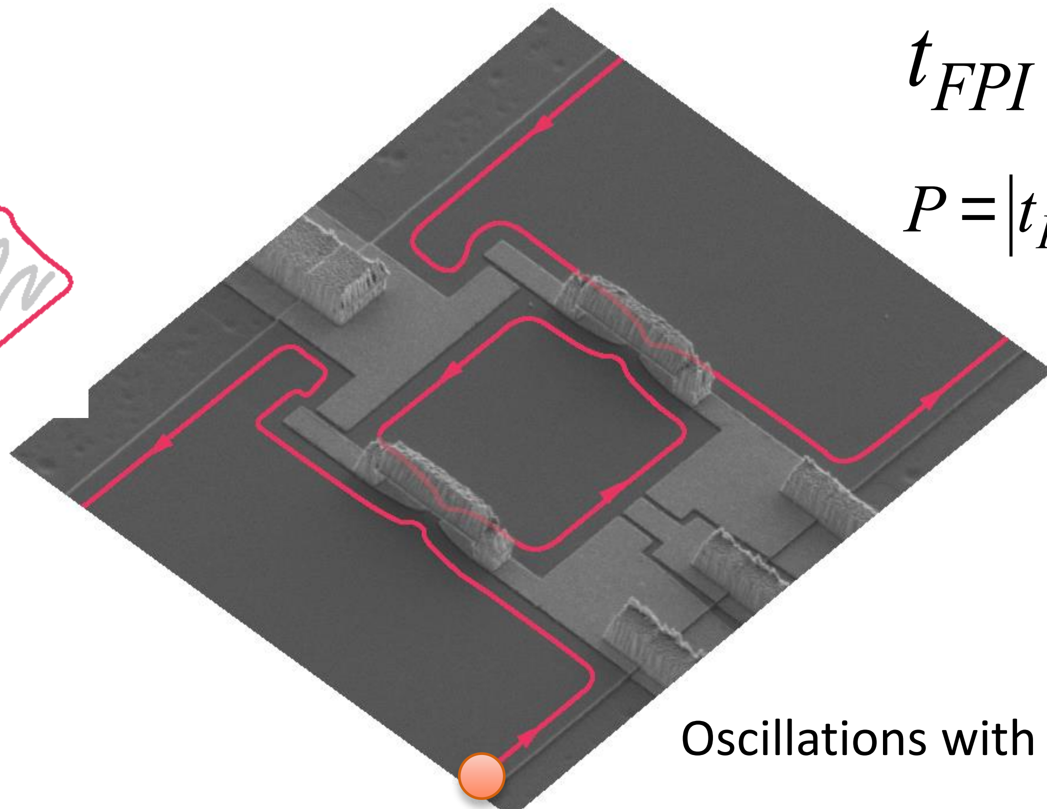
Yang Ji *et al.*, Nature **422**,415 (2003)

Electronic Fabry-Perot interferometer









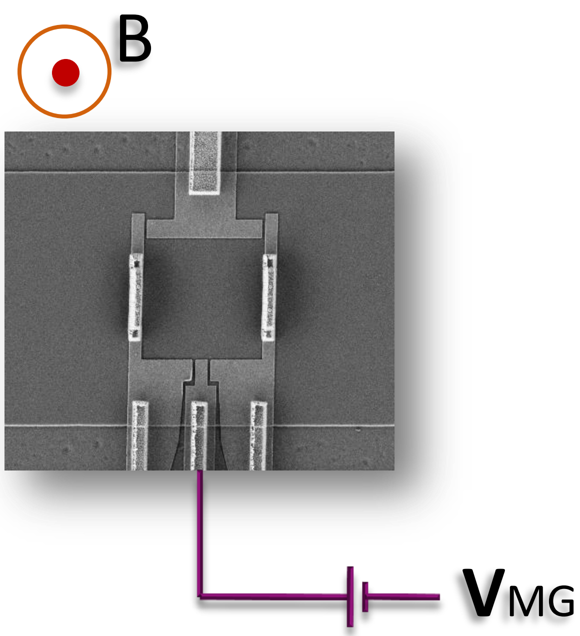
$$t_{FPI} = t_1 t_2 + t_1 r_2 r_1 t_2 e^{ij_{AB}}$$

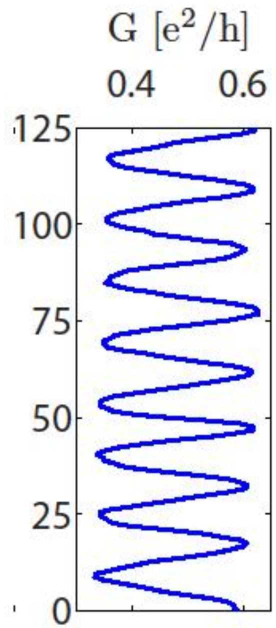
$$P = |t_{FPI}|^2 = T_0 + T\cos(j_{AB})$$

Oscillations with respect to AB phase

AB oscillation in FPI

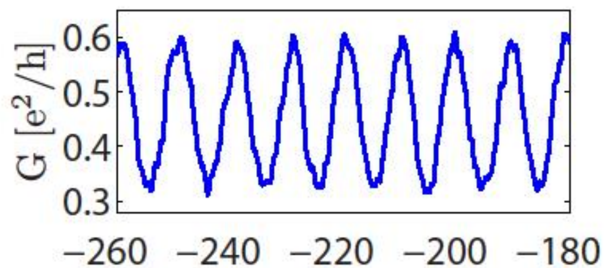
$$t_{FPI} = t_0 + t_1 e^{2piAB/f_0}$$



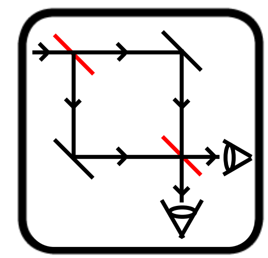


$$dB = f_0 / A$$

$$dA = f_0 / B$$



Zoom out



<u>Physics in 2D</u>

- Anyons
- Control e's path

<u>QHE</u>

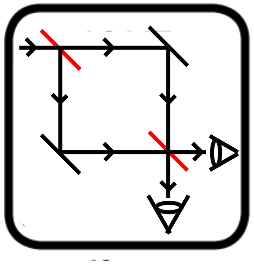
- FQHE: anyons & NA
- Definite path

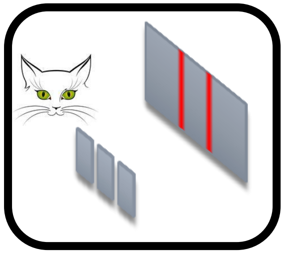
<u>Interferometry</u>

- Fundamental QM
- Probe statistics

What can we do with interferometers ?

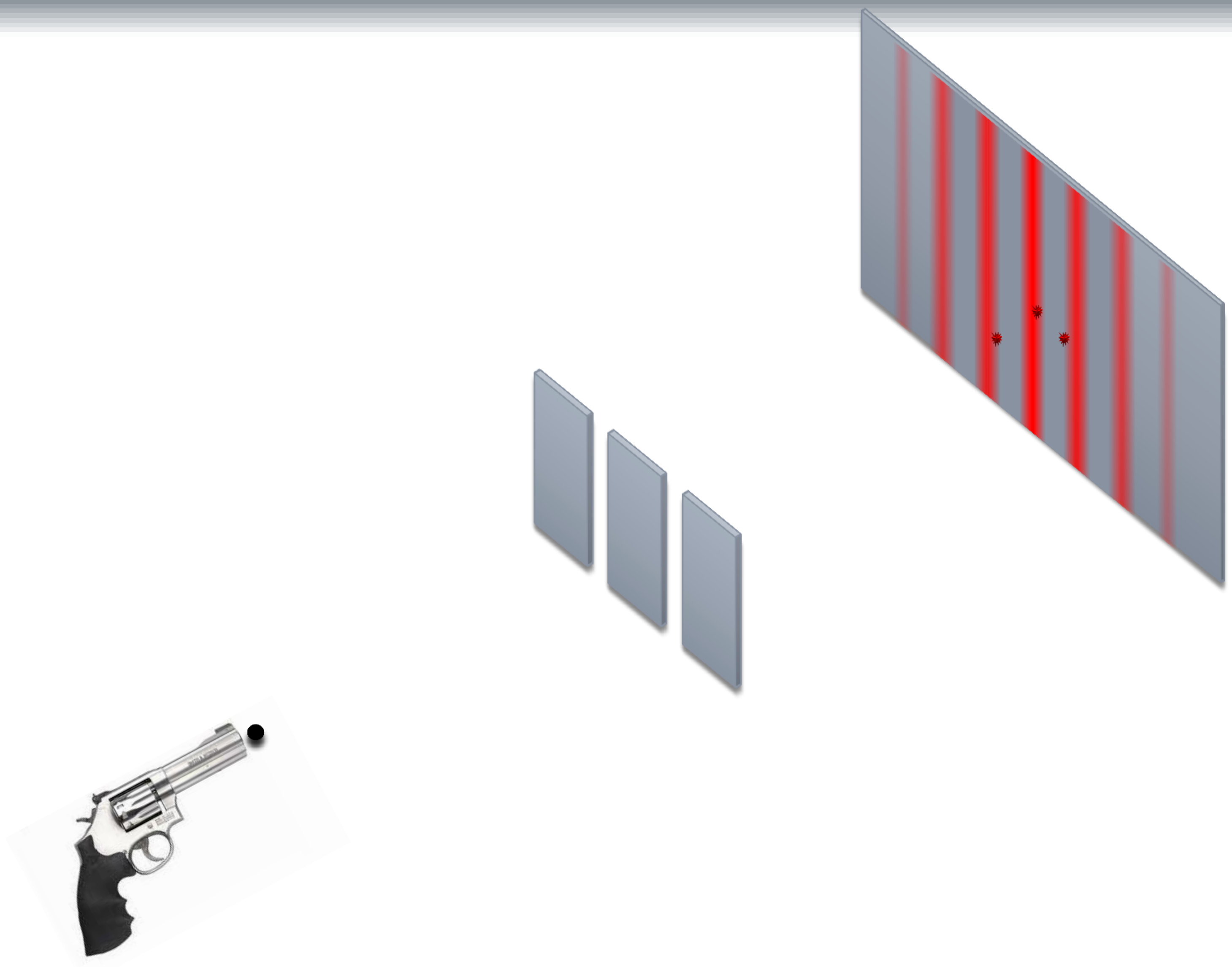
Framework





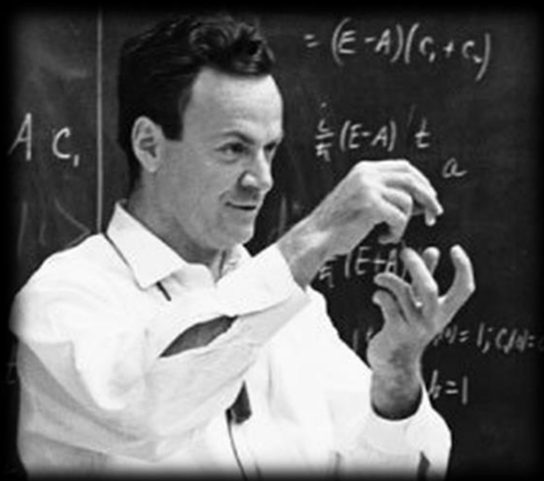
Controlled dephasing

Double slit experiment

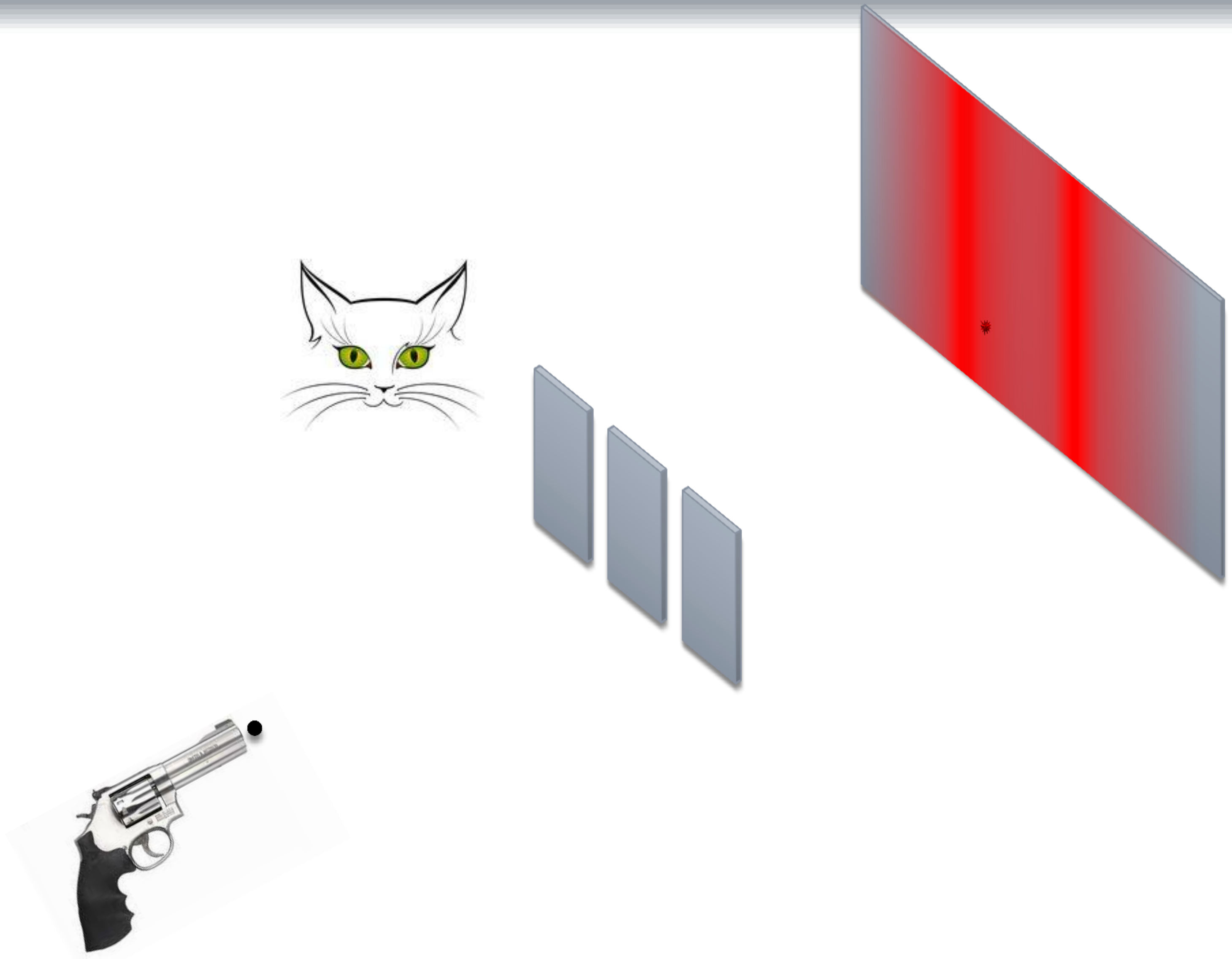


“It is impossible to design an apparatus to determine which hole the electron passes through, that will not at the same time disturb the electron enough to destroy the interference pattern”

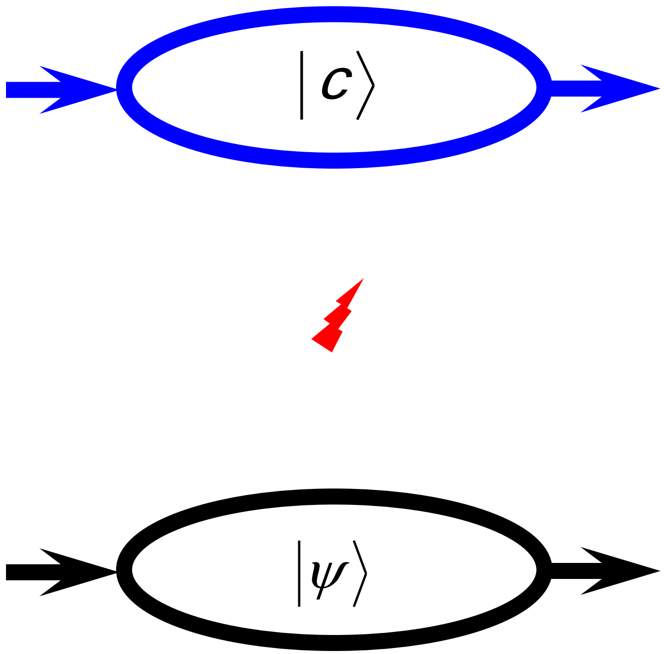
Richard Feynman



Double slit experiment



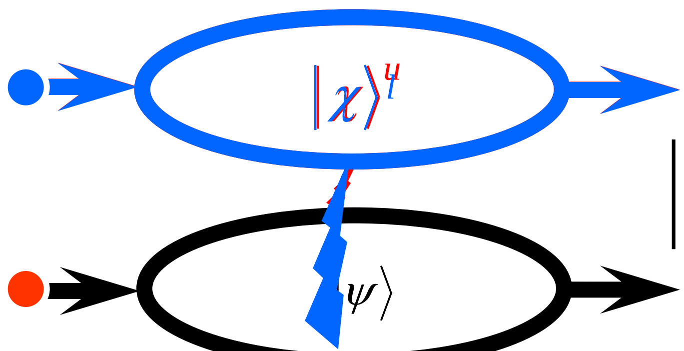
Pure state (separable)



composite system

$$|\Psi_{total}\rangle = |\psi\rangle \otimes |c\rangle$$

Entangling systems *via* Coulomb interaction

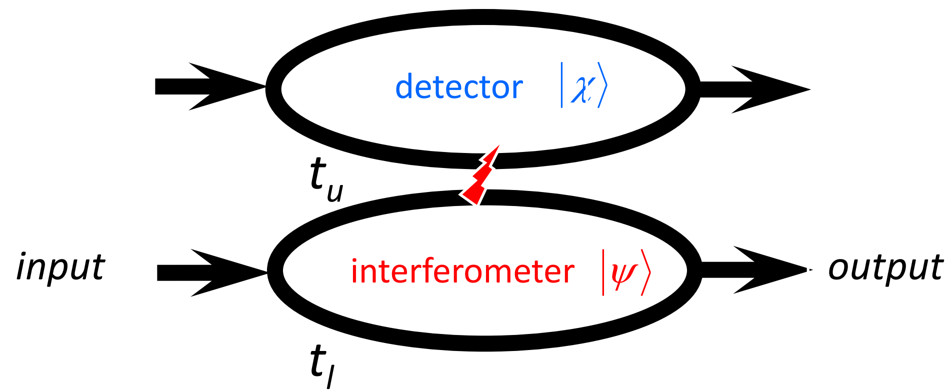


$$|\psi\rangle = |\psi_u\rangle + |\psi_l\rangle$$

$$|\Psi_{total}\rangle = |y_u\rangle \text{Ä} |c^u\rangle + |y_l\rangle \text{Ä} |c^l\rangle$$

entangled state

Entangling detector - interferometer



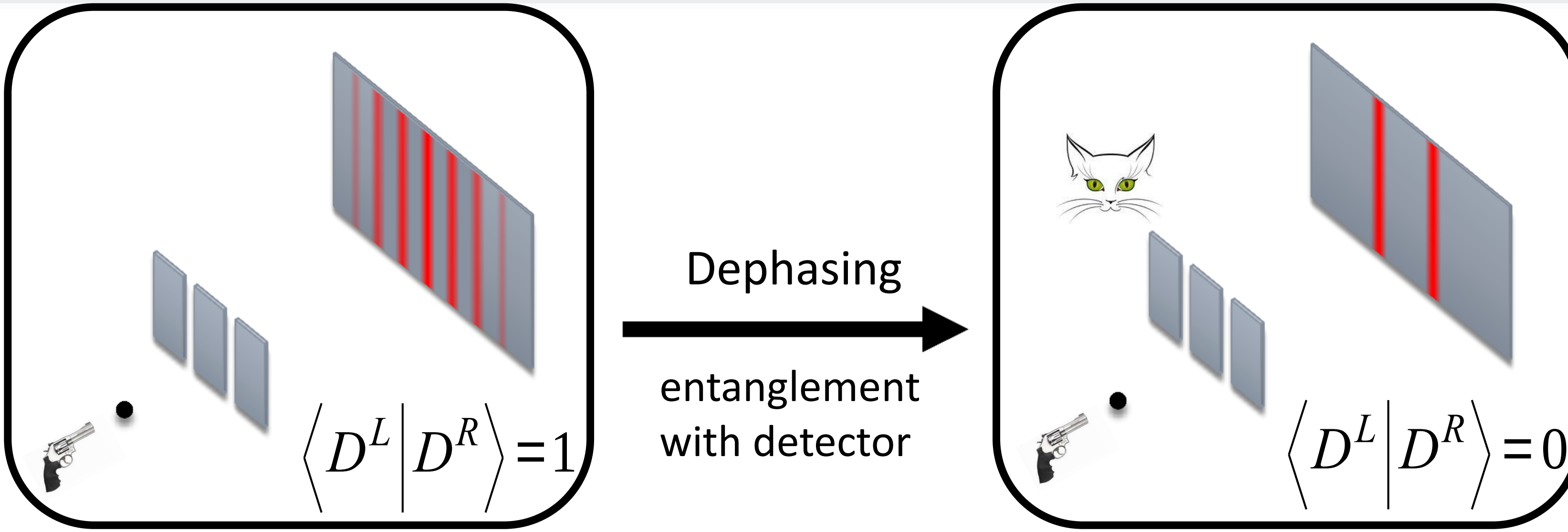
$$|\Psi_{total}\rangle = |\psi_l\rangle \otimes |\chi^l\rangle + e^{i\varphi_{AB}} |\psi_u\rangle \otimes |\chi^u\rangle$$

interference term

$$P_{output} = |\langle \Psi_{total} | output \rangle|^2 = |t_l|^2 + |t_u|^2 + 2 t_l t_u \cos \Delta\varphi_{AB} \langle \chi^l | \chi^u \rangle$$

'which path' information

What is dephasing ?



entangled state interferometer - detector

$$|\Psi_{total}\rangle = |\psi_L\rangle \otimes |D^L\rangle + |\psi_R\rangle \otimes |D^R\rangle$$

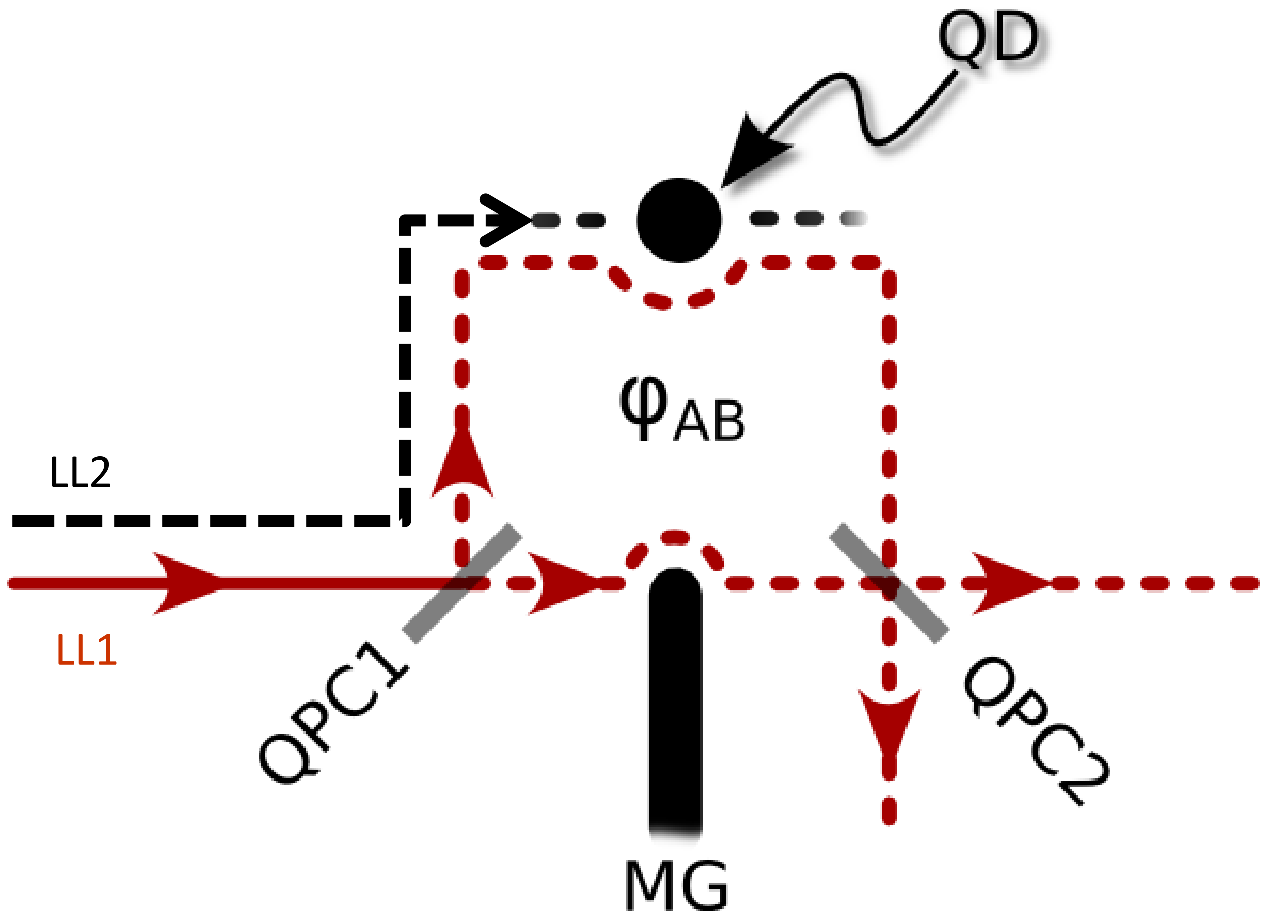
interference ‘*which path*’ information

$$P_{output} = |\langle \Psi_{total} | output \rangle|^2 = |t_L|^2 + |t_R|^2 + 2 t_L t_R \cos \Delta\varphi \, \langle D^L | D^R \rangle$$

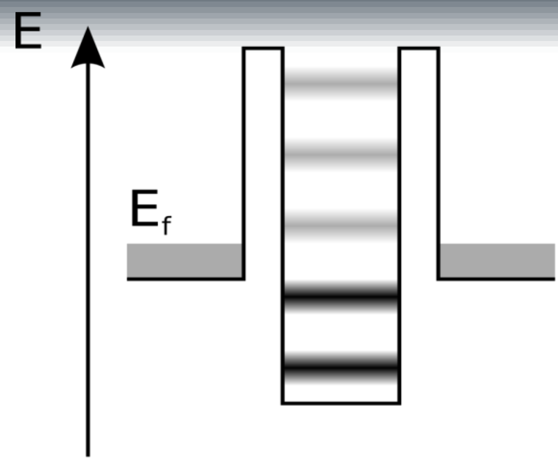
How to build a good detector ?

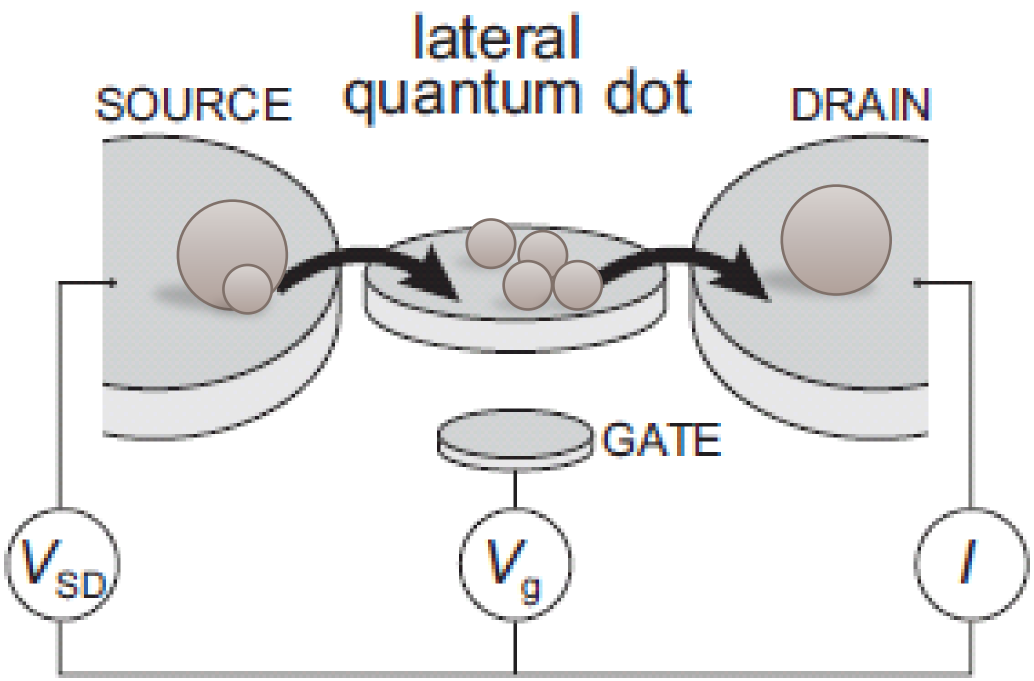
$$\left\langle D^L \middle| D^R \right\rangle = 0$$

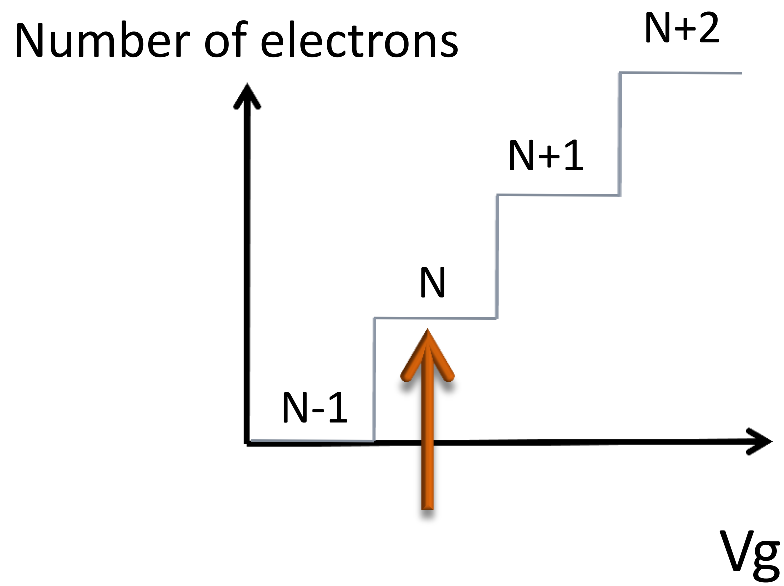
Quantum dot detector

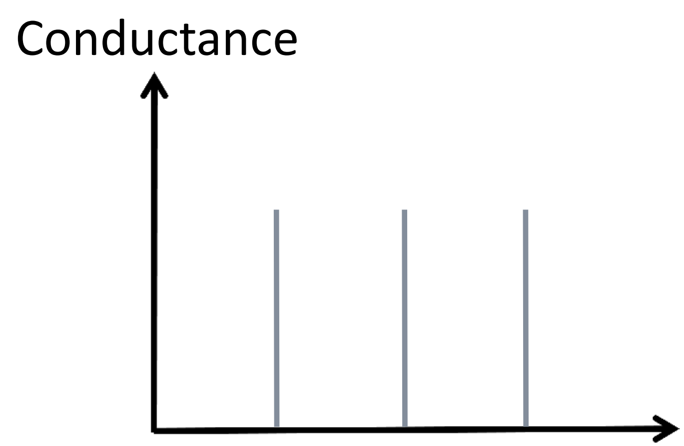


Quantum dot

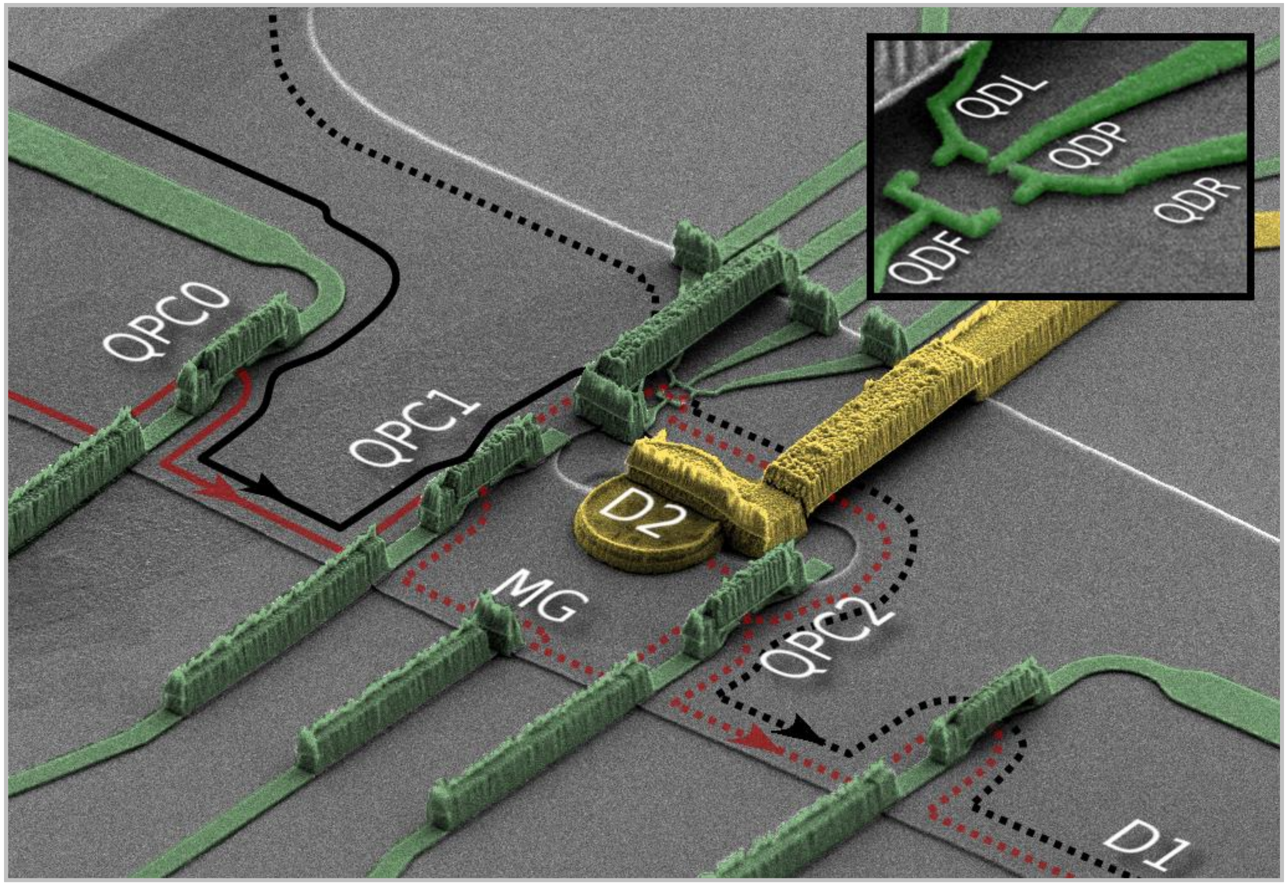

E
E_f


lateral
quantum dot
SOURCE
DRAIN
GATE
V_{SD}
V_g
I


Number of electrons
N+2
N+1
N
N-1
Vg


Conductance

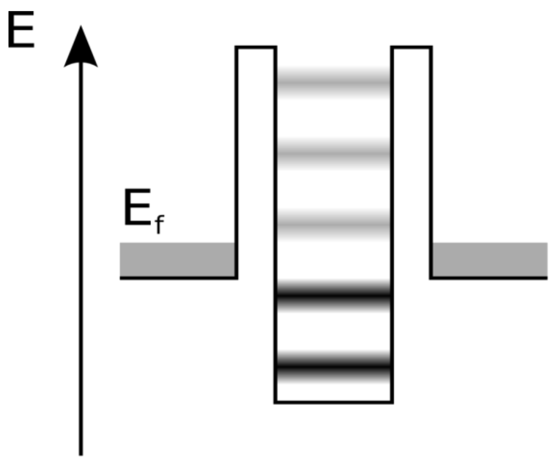
Quantum dot detector

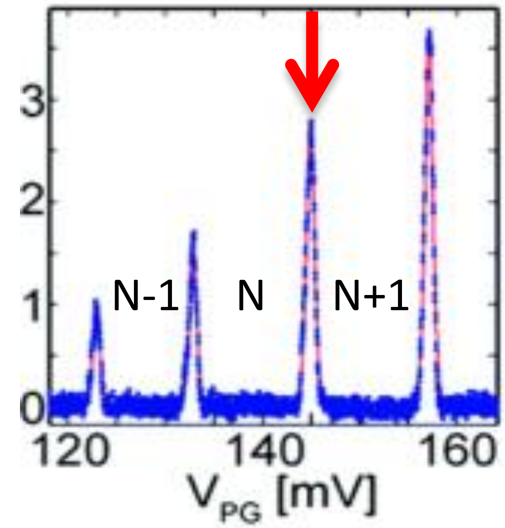


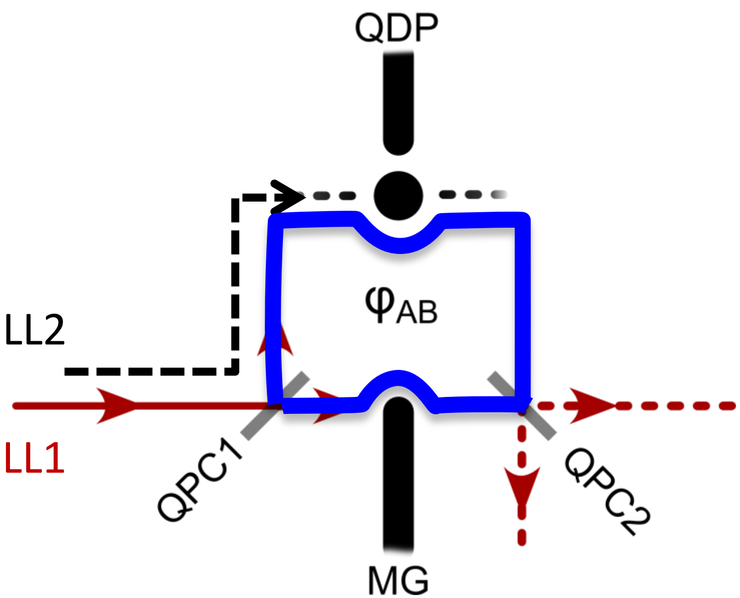
Experiment: E. Weisz, H. K. Choi *et al.*, PRL **109**, 250401 (2012)

Theory: SC. Youn et al, PRB (2009); B. Rosenow & Y. Gefen PRL (2012)

Quantum dot detector





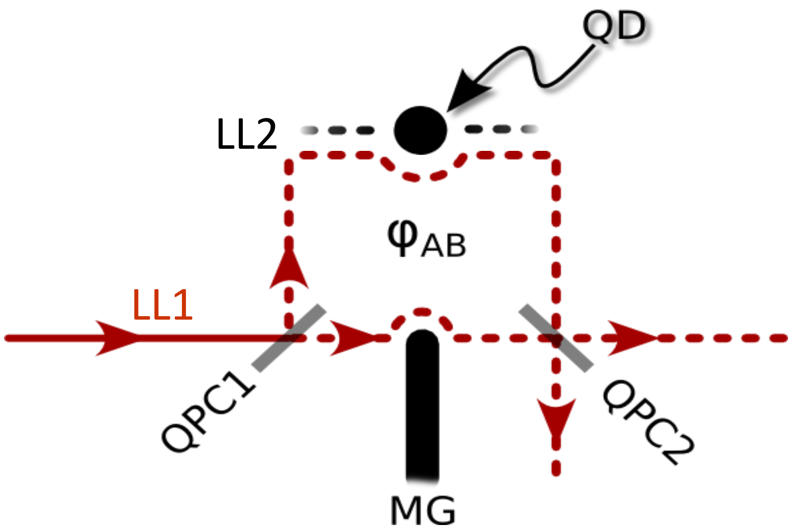


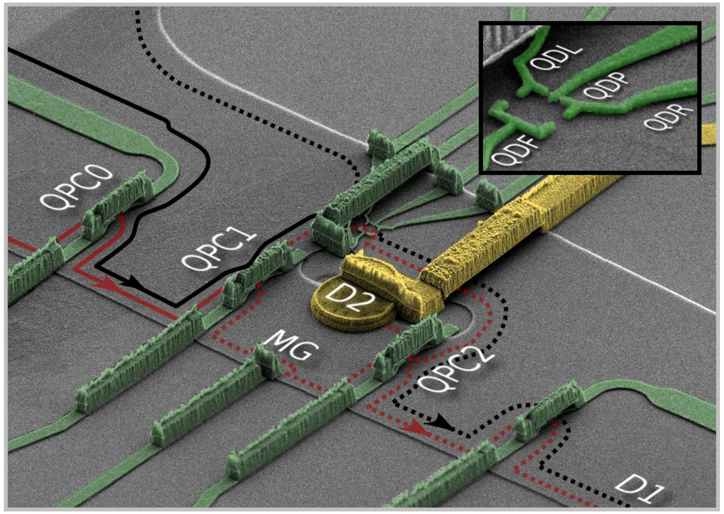
At resonance, E(N)=E(N+1)

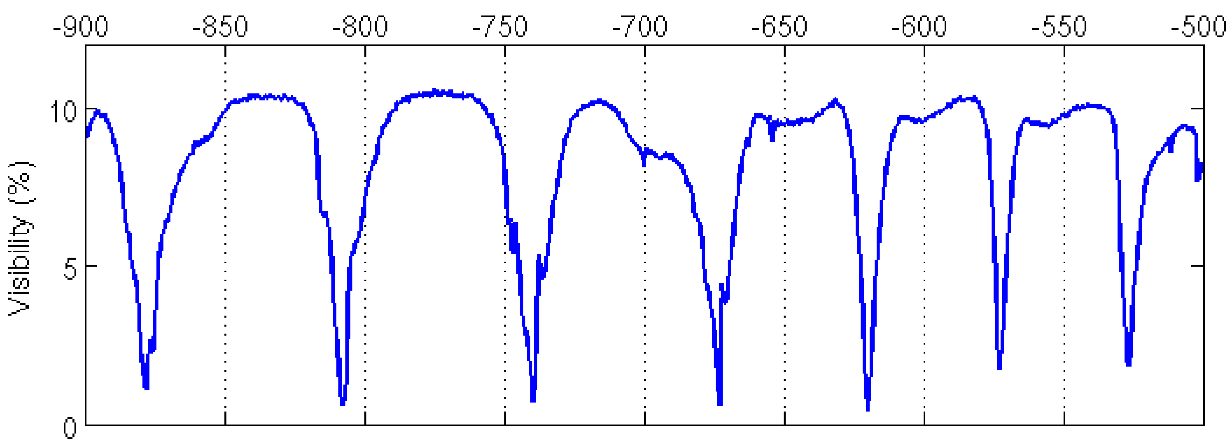
$$\left|\mathrm{Y}_{total}\right\rangle = \left|y_U\right\rangle \text{Ä} \left|N\right\rangle + \left|y_L\right\rangle \text{Ä} \left|N+1\right\rangle$$

$$\left\langle N \middle| N+1 \right\rangle = 0$$ Full detectability

Quantum dot detector



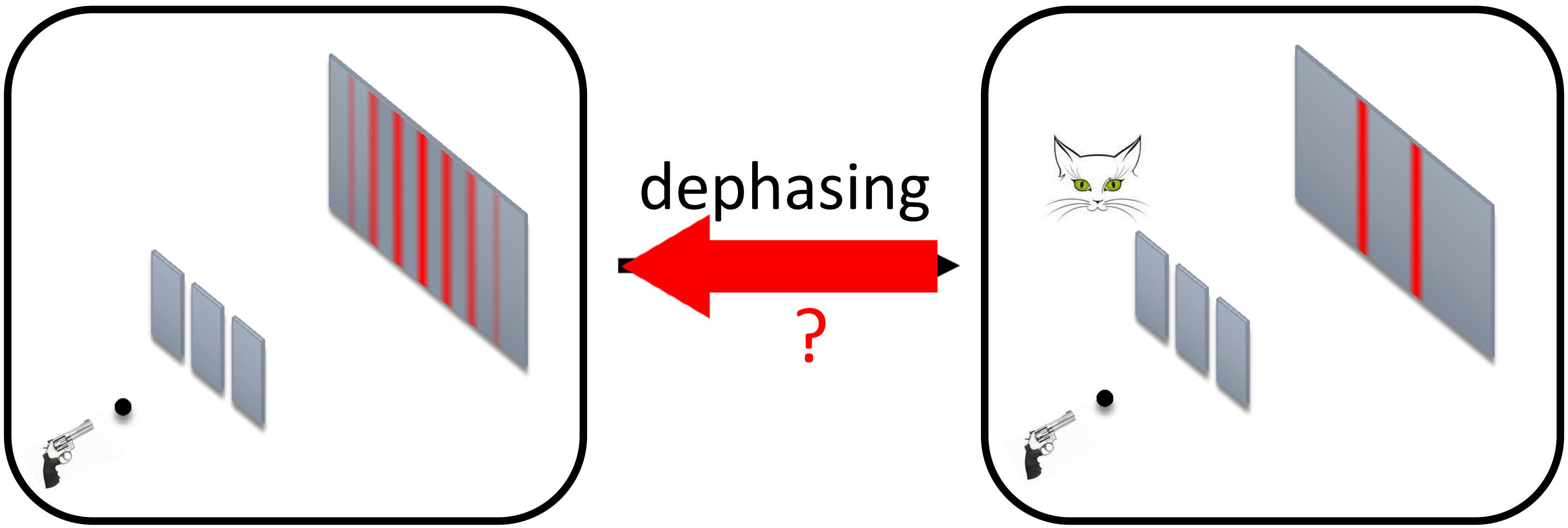




Full dephasing without current in Detector

E. Weisz, H. K. Choi *et al.*, PRL **109**, 250401 (2012)

Can lost interference be recovered ?

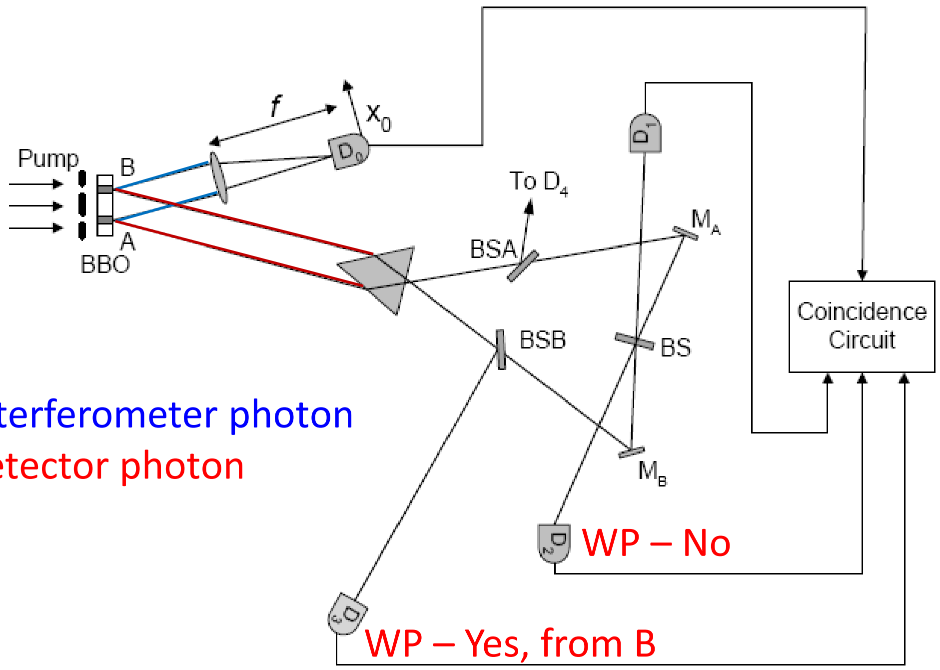


Yes, by erasing which-path information

Asking to  " Which path did electron pass through ?"

Quantum eraser

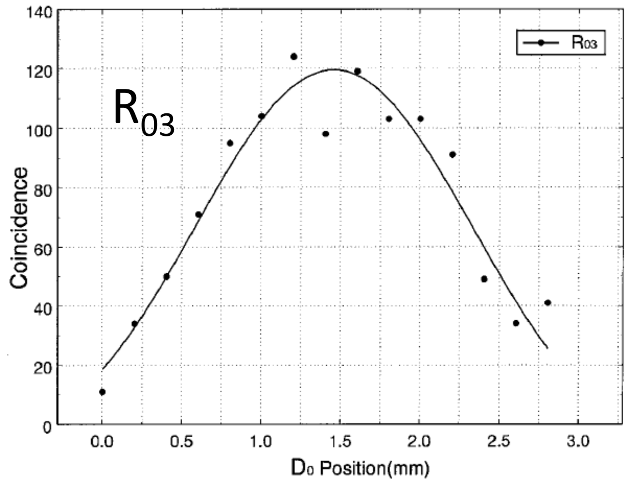
Quantum eraser with photon



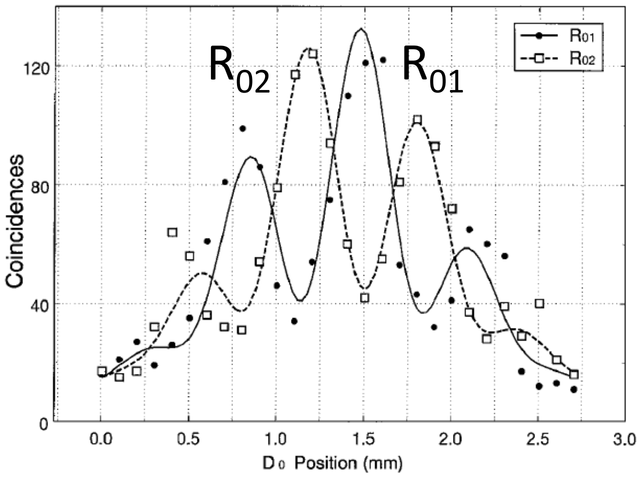
interferometer photon

detector photon

WP : Yes→ No interference

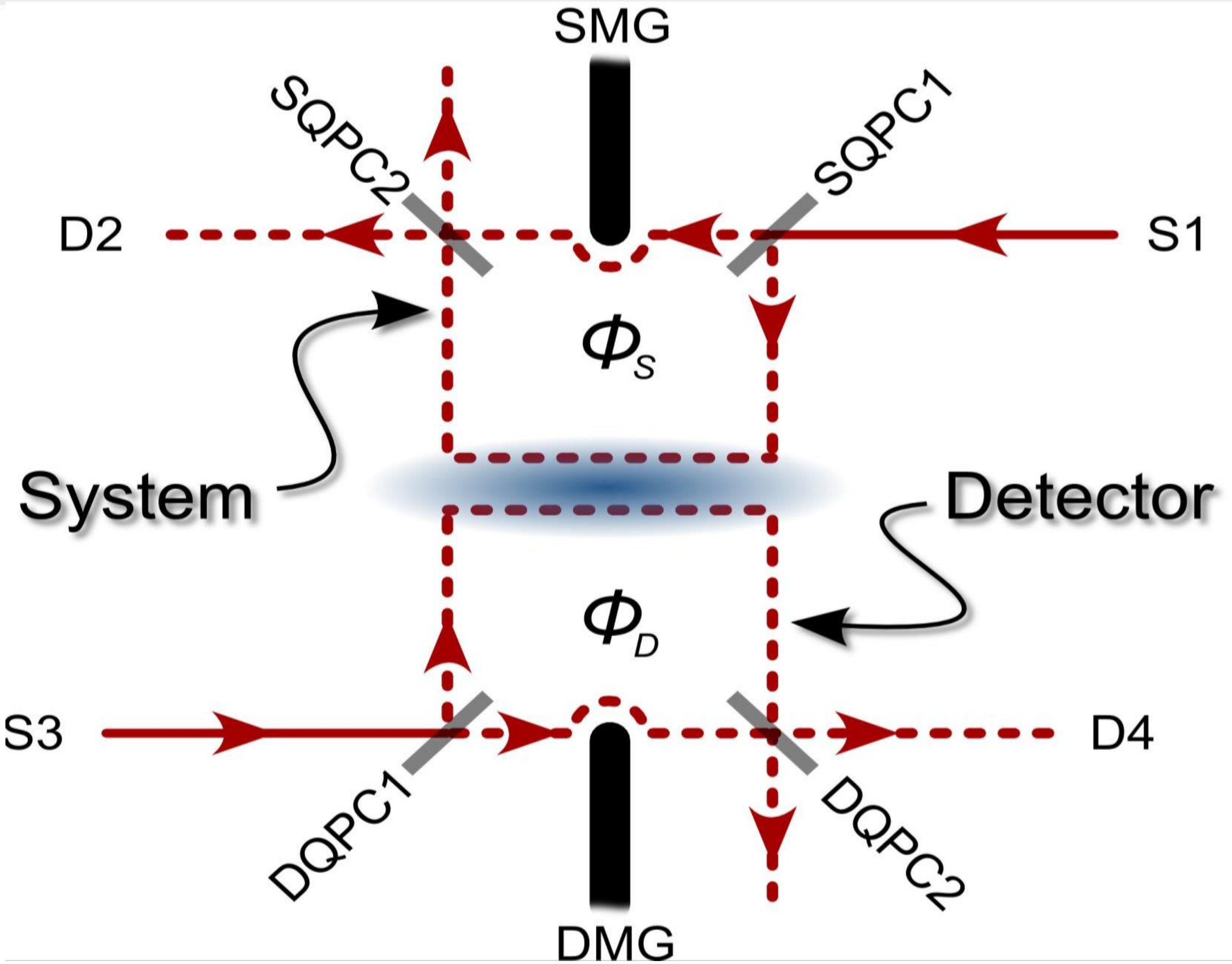


WP : No → interference



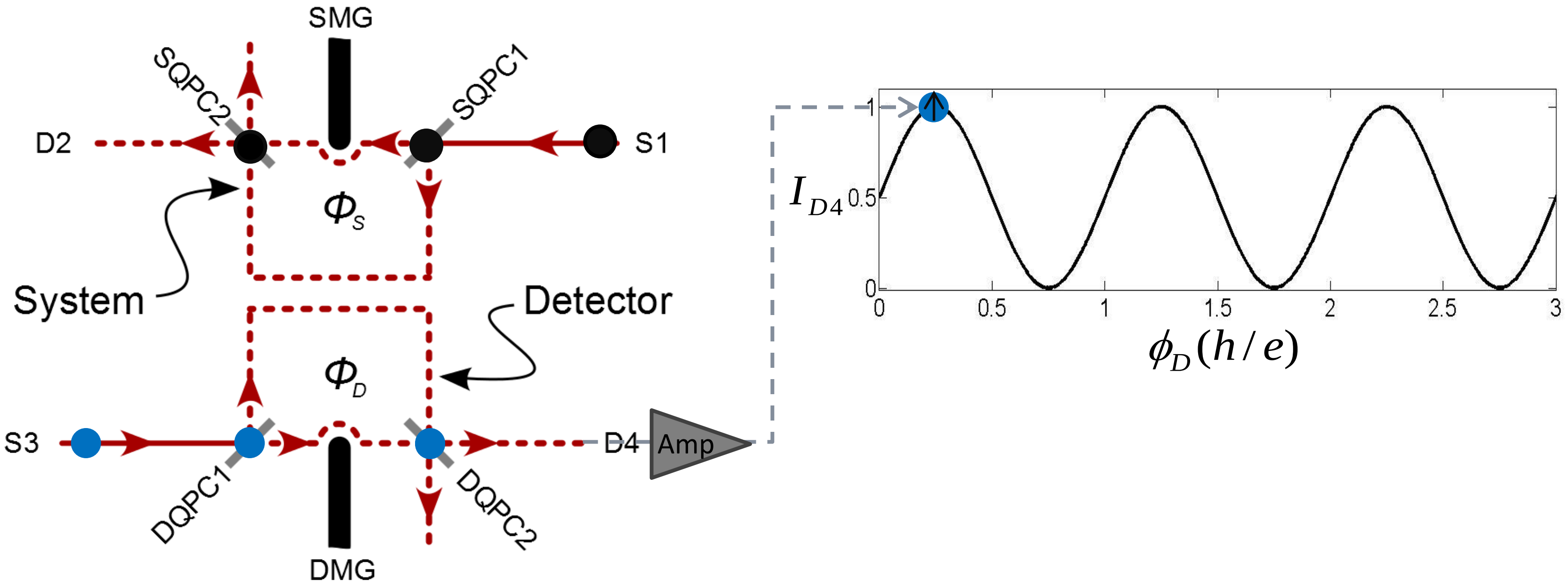
Y.H. Kim *et al.*, PRL **84**,1 (2000)

Electronic Double MZI for QE



$$|\Psi_{total}\rangle = |\uparrow_S\rangle \otimes |D^{\uparrow}\rangle + |\downarrow_S\rangle \otimes |D^{\downarrow}\rangle$$

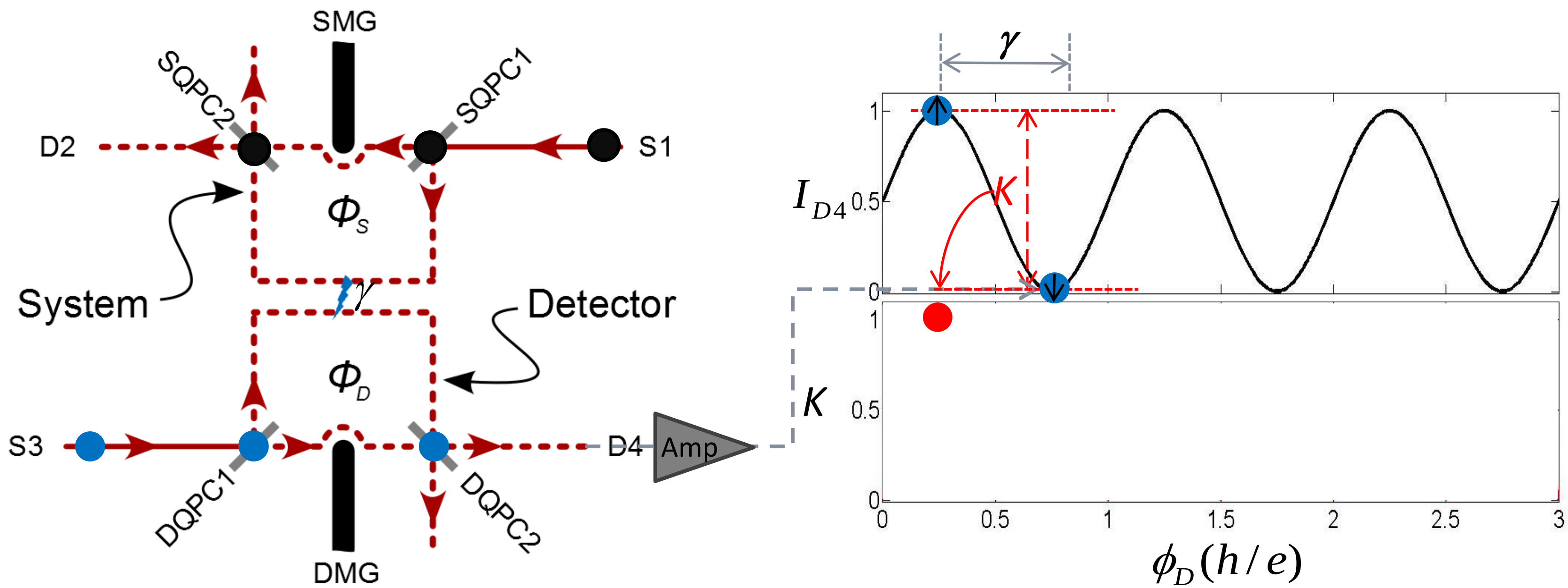
Double MZI



When System electron pass through upper path

Detector current : $P(\mathrm{D4} \mid \uparrow_S)$

Double MZI



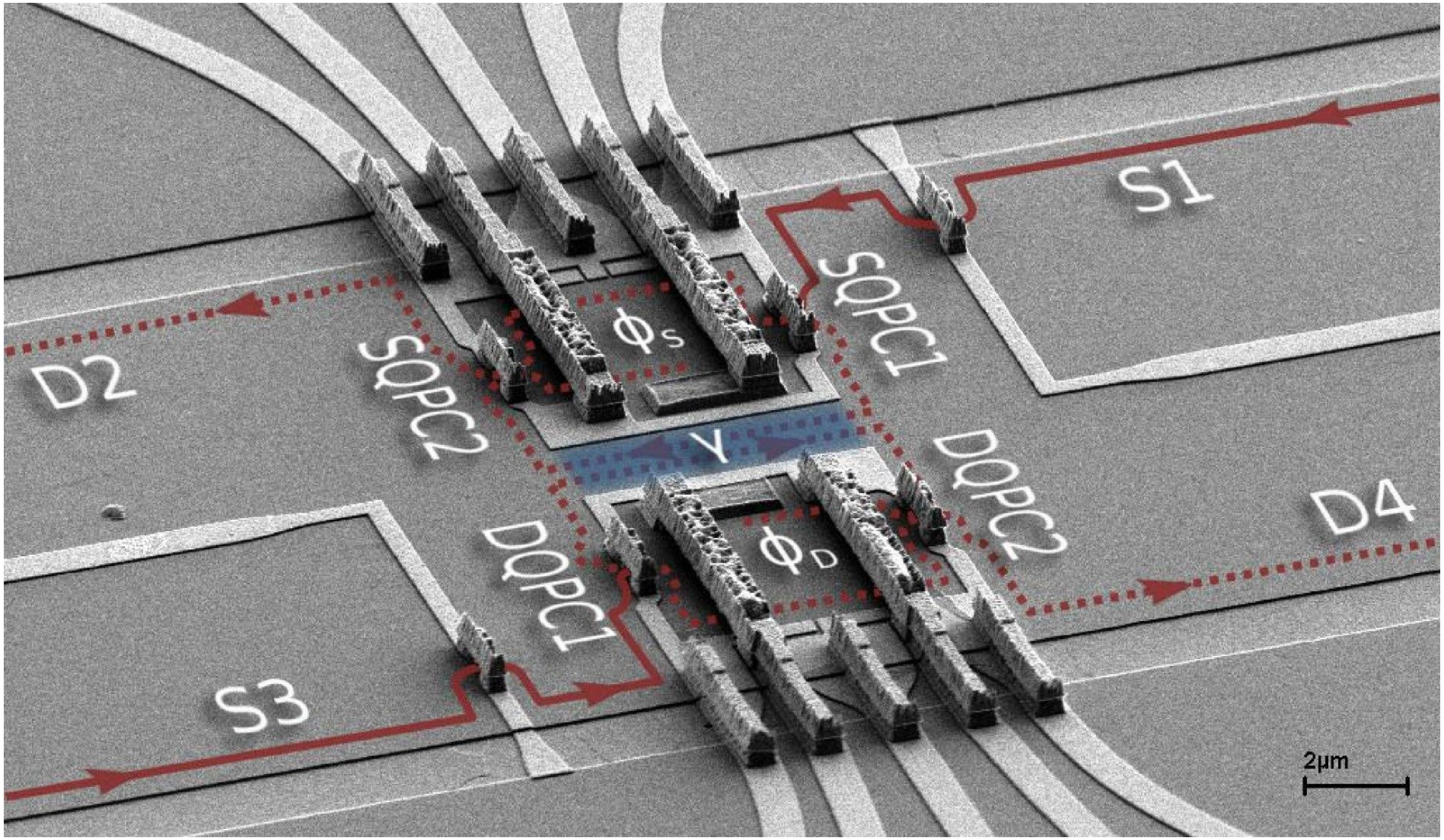
When System electron pass through lower path

Measured which-path information *K*:

Detector current : $P(\mathrm{D4}|\downarrow_S)$ here, $\gamma=\pi$

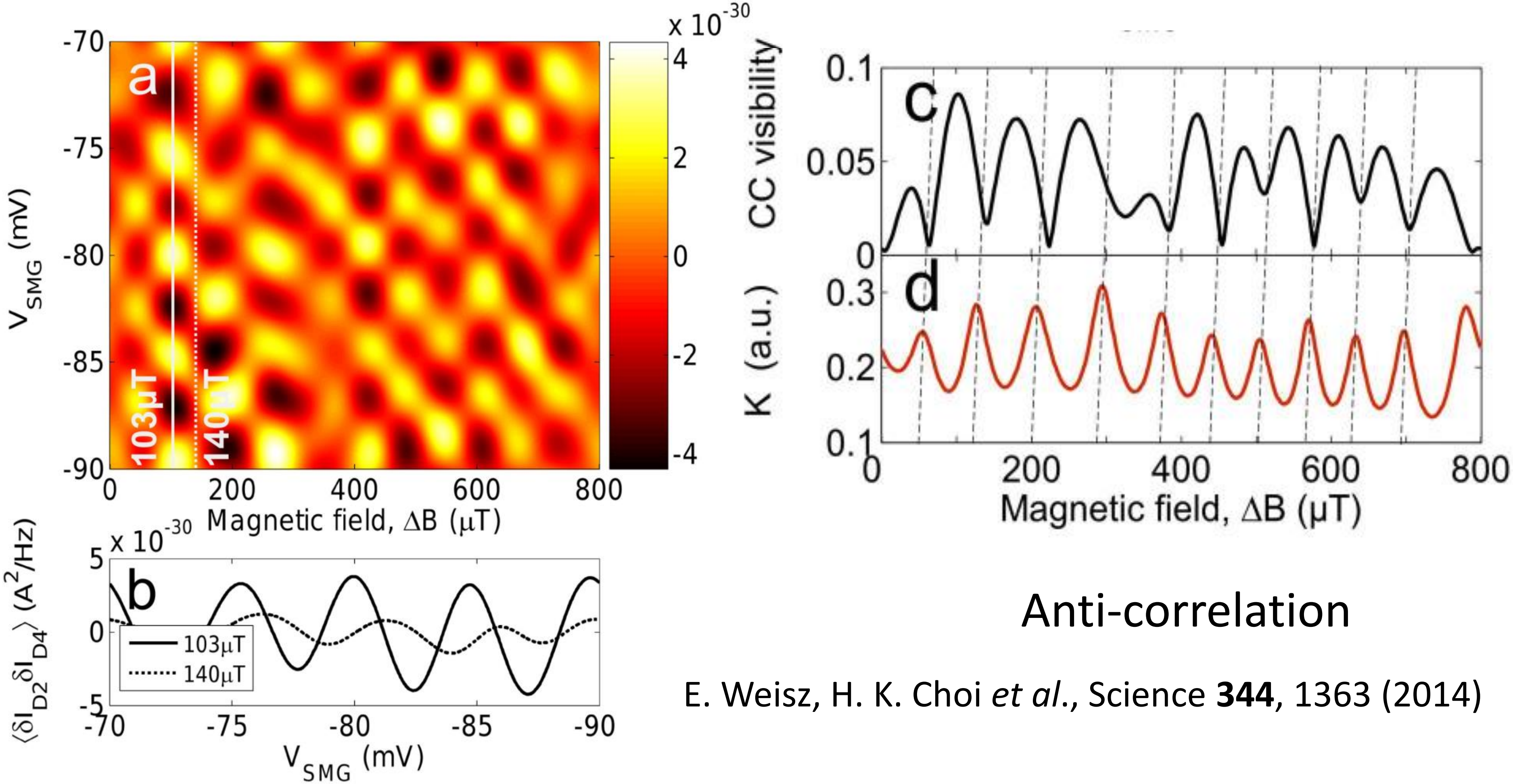
$$K(\phi_D) \equiv \left|P(\mathrm{D4}|\uparrow_S) - P(\mathrm{D4}|\downarrow_S)\right| \propto \left|\sin(\phi_D)\right|$$

Realization of coupled MZIs



Experiment: E. Weisz, H. K. Choi *et al.*, Science (2014)
Theory: K. Kang PRB (2007)

Electronic Quantum eraser



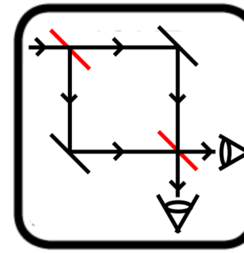
Anti-correlation

E. Weisz, H. K. Choi *et al.*, Science **344**, 1363 (2014)

"manifestation of complementarity in electronic system"

Summary

Framework

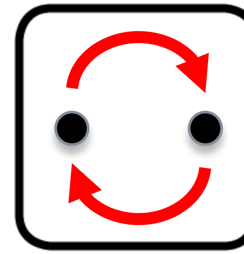


Motivations

Fundamental
Q-mechanics

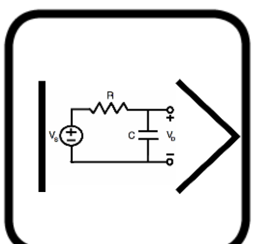
OK

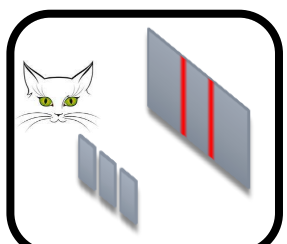
Anyonic
statistics

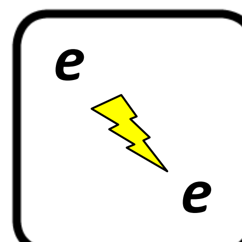


Tools

In order to solve the mystery, we must understand the systems

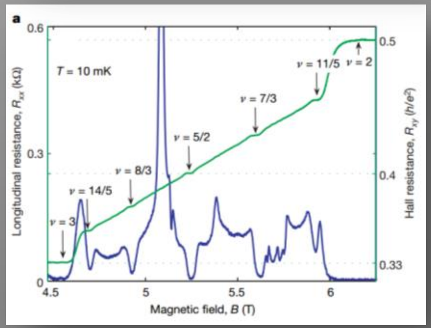


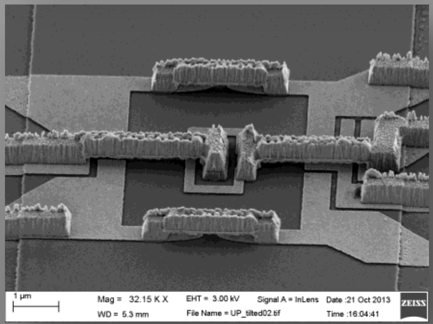


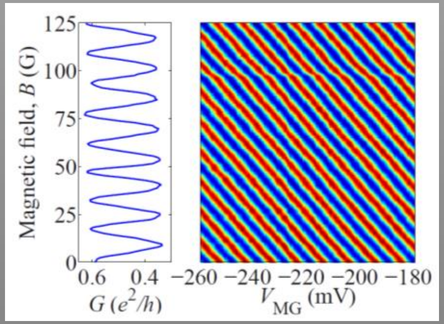


Experimental physics

Experimental physics



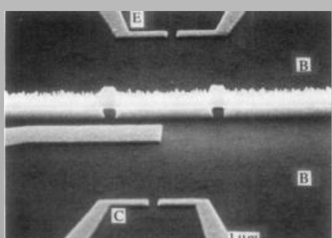




Electronic interferometers

Electronic interferometers show unexpected behavior

Double-slit interferometer

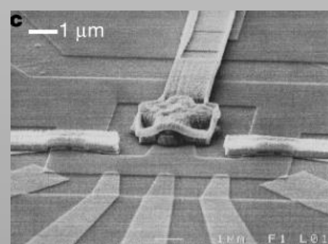


"Unexpected double periodicity…"

(Time-reversal symmetry)

A. Yacoby et al
PRL (1994)

Mach-Zehnder interferometer

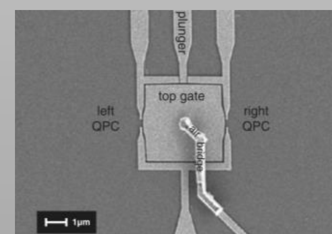


"Unexpected non-linear behavior…"

(Lobe-structure)

I. Neder et al
PRL (2006)

Fabry-Perot interferometer



"Role of interactions…"

(CD regime)

N. Ofek et al
PNAS (2010)

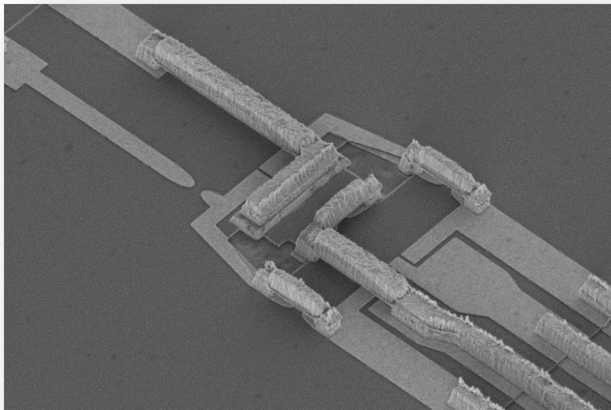
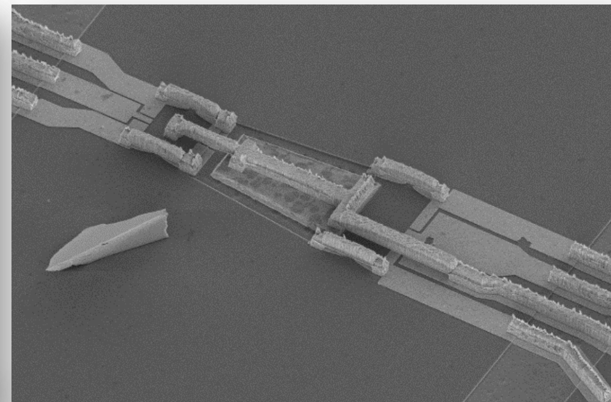
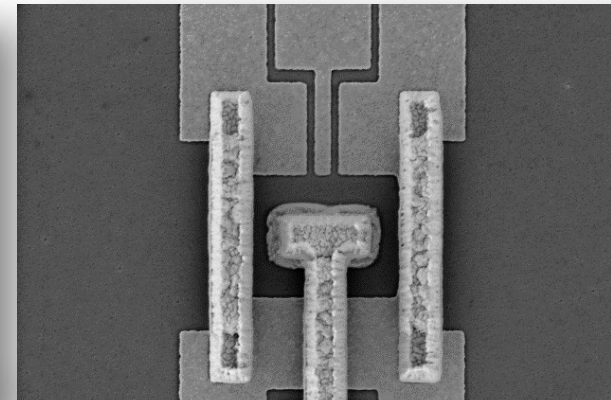
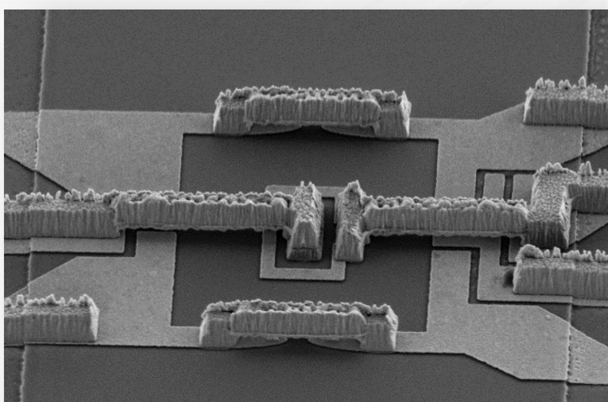
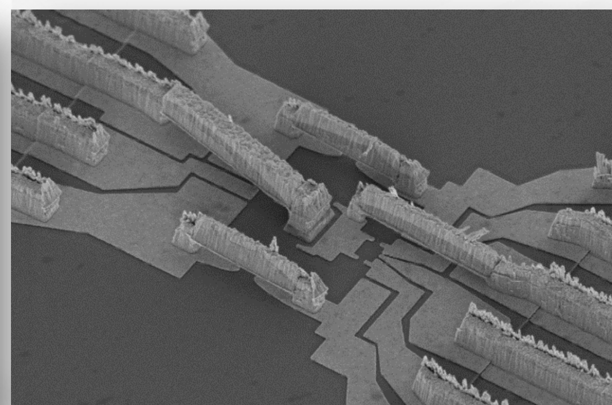
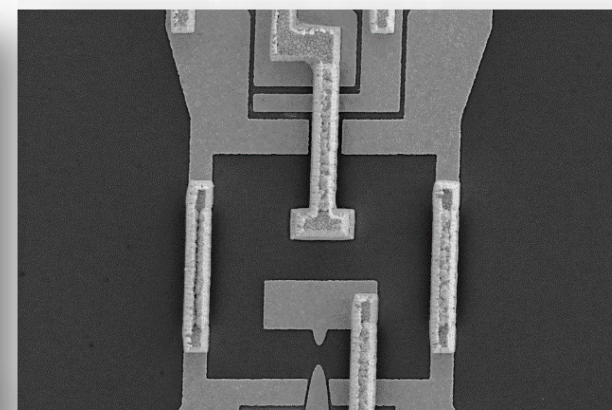
Fabry-Perot interferometer

"Unexpected pairing…"

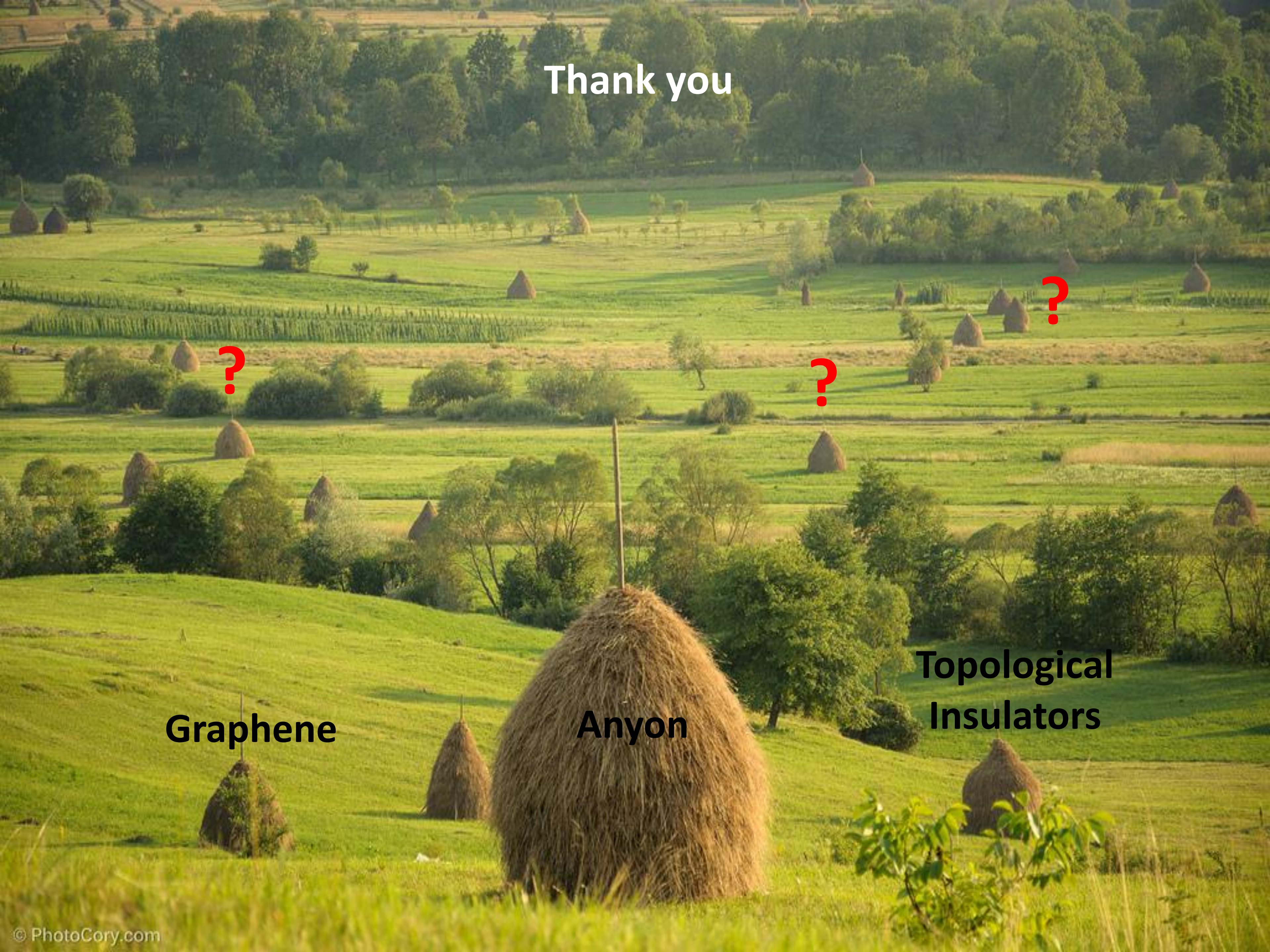
(To be given a name)

HK Choi et al
Nat. Com. (2015)

Waiting for your ideas…

How to find needles


Thank you
?
?
?
Graphene
Anyon
Topological
Insulators
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