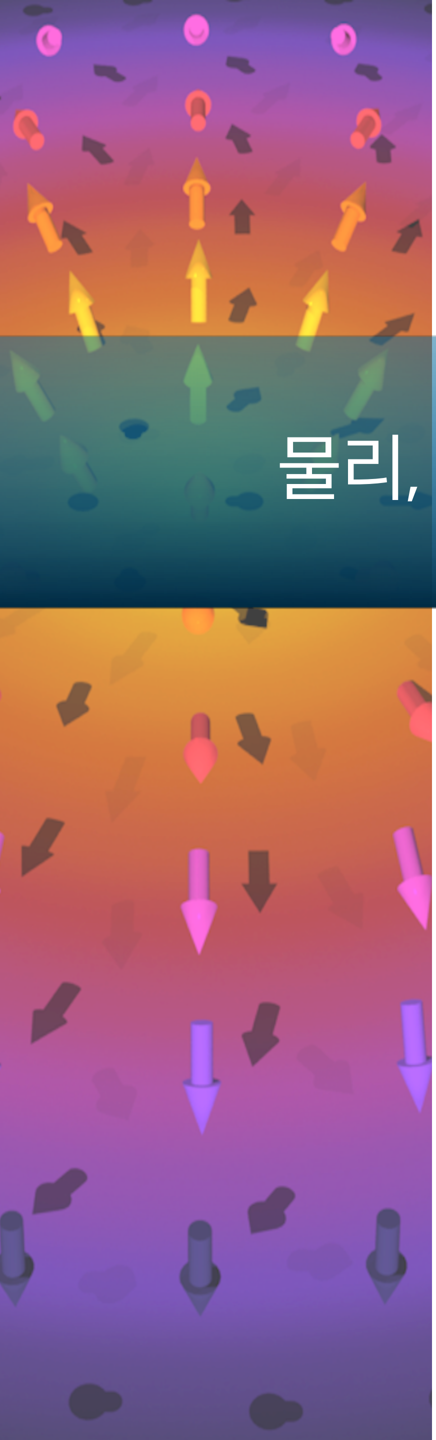


KOREA INSTITUTE FOR ADVANCED STUDY

물리, 위상수학을 만나다 Physics meets topology

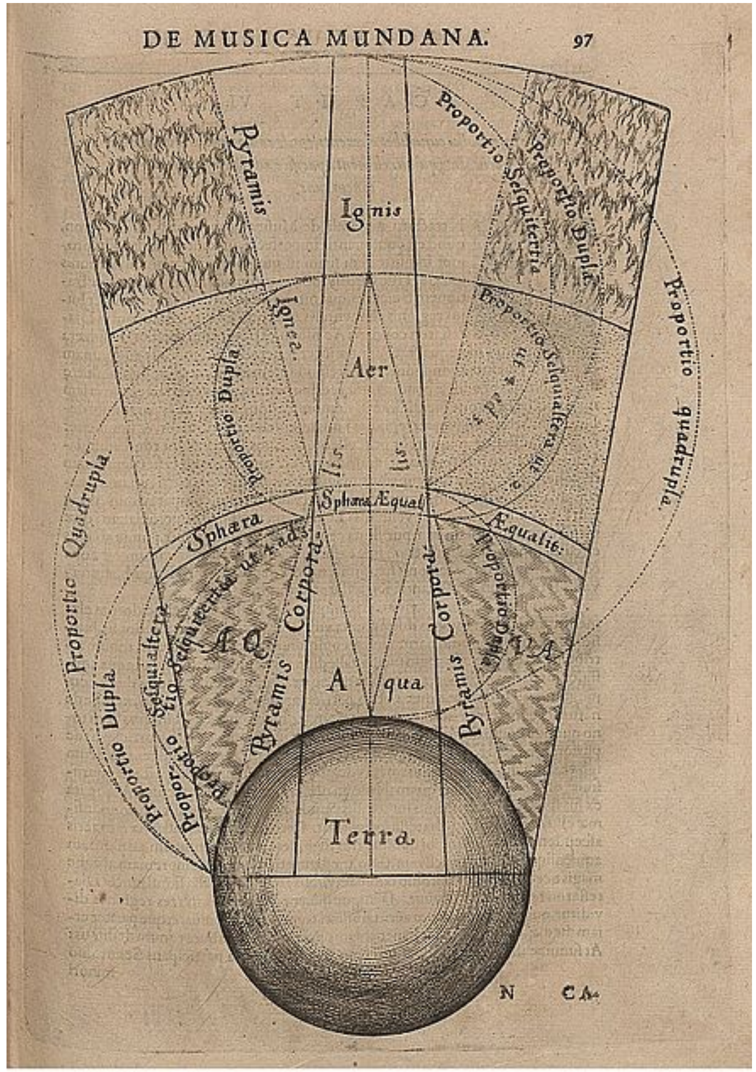
박권
Kwon Park

Colloquium
Seoul National University
2017년 4월 26일

Together physics and topology open up the new world of quantum matter.

물리와 위상수학이 만나서 양자물질이라는 새로운 세계가 열리다.

4원소 Four Elements



Quelle: Deutsche Fotothek

Robert Fludd (1617)

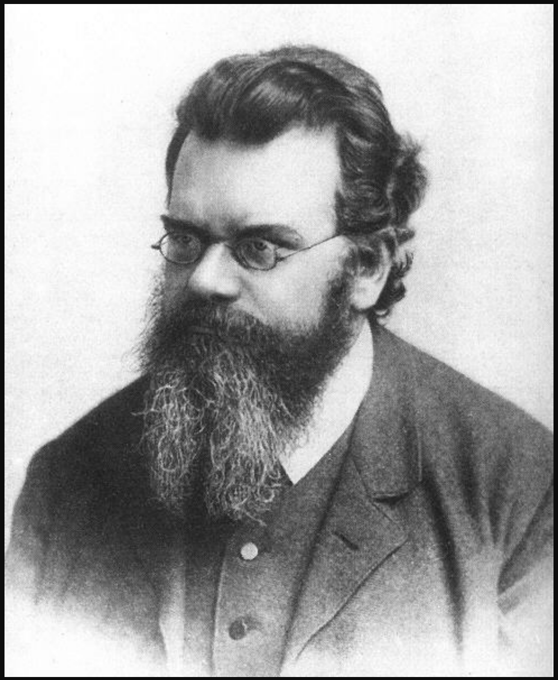
Ignis (fire)

Aer (air)

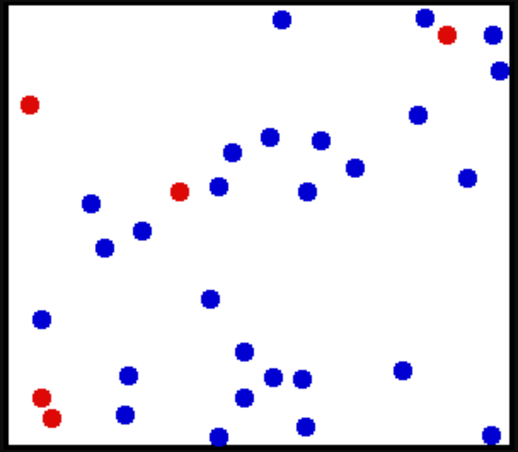
Aqua (water)

Terra (earth)

볼츠만의 기체 동역학 Boltzmann's Kinetic Theory of Gases



Ludwig Boltzmann (1890's)

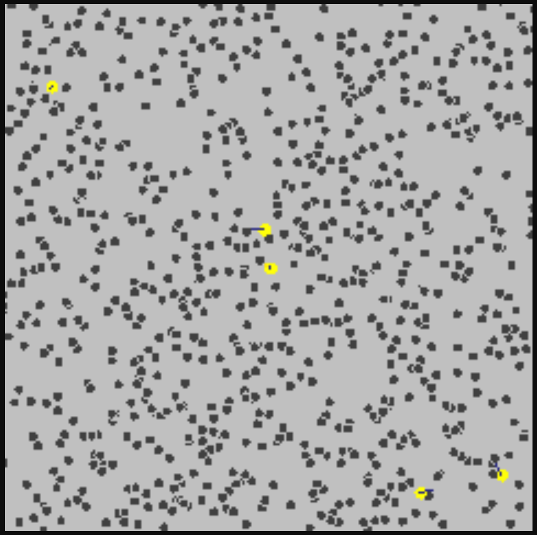


$$S = k_B \ln W$$

Based on the existence of atoms and molecules, Boltzmann's kinetic theory of gases laid the foundation of statistical mechanics.

브라운 운동 Brownian motion

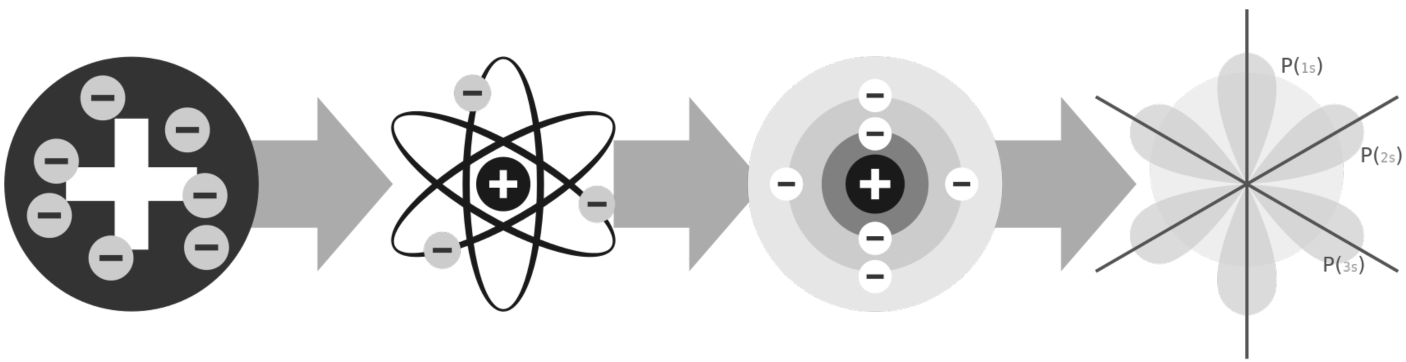
Jean Baptiste Perrin (1908)



Perrin verified Einstein's theory of Brownian motion and thereby confirmed the atomic nature of matter.

Nobel prize awarded in 1926 "for his work on the discontinuous structure of matter, and especially for his discovery of sedimentation equilibrium"

원자 모형의 진화 Evolution of the Atomic Model



J. J. Thomson's
Plum Pudding Model
(1904)

Rutherford's
Model
(1911)

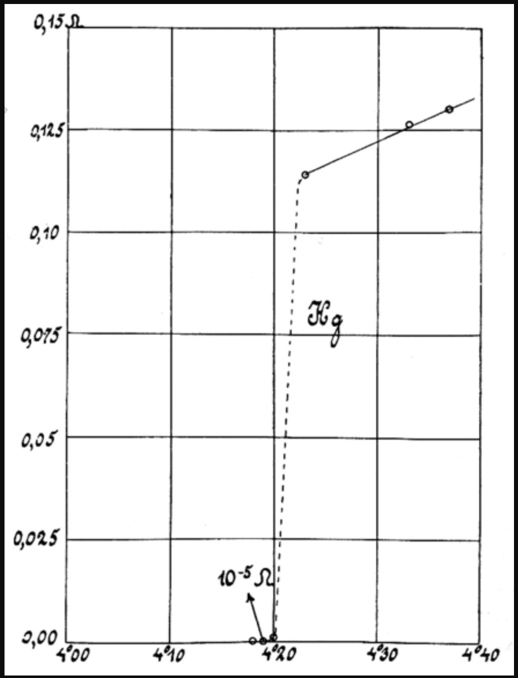
Bohr's
Model
(1913)

Quantum
Mechanics
(1926)

통계물리학 + 양자역학 = 응집물질물리학

“새로운 세계, 양자물질의 서막”
과학동아 **376**, 52-63 (2017)

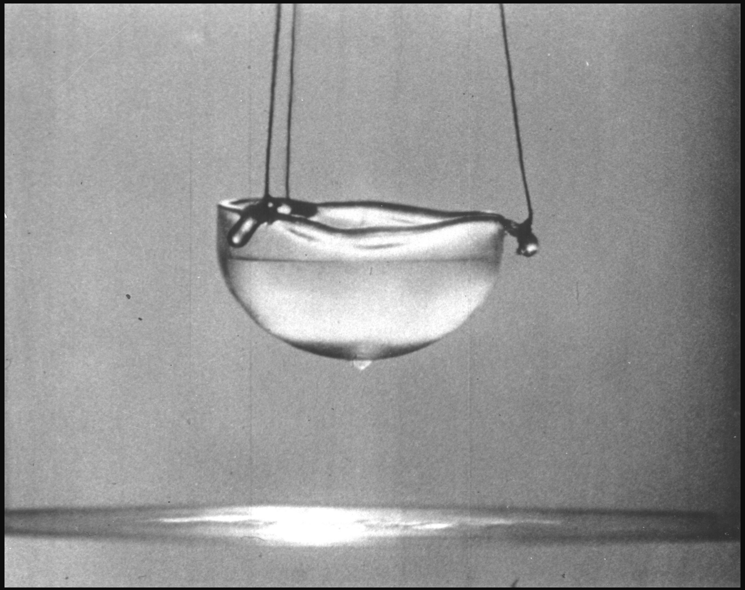
초전도체 Superconductor



Heike Kamerlingh Onnes (1911) discovered that the resistance of mercury suddenly vanishes below a critical temperature.

Nobel prize awarded in 1913 “for his investigations on the properties of matter at low temperatures which led, inter alia, to the production of liquid helium”

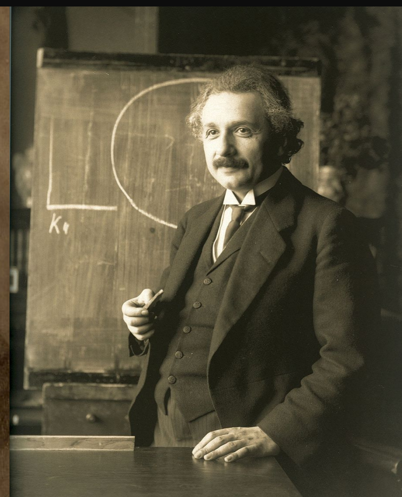
초유체 Superfluid



Pyotr Kapitsa (1937) discovered the superfluidity effect of liquid helium, i.e., fluidity with zero viscosity and entropy.

Nobel prize awarded in 1978 “for his basic inventions and discoveries in the area of low-temperature physics”

보즈-아인슈타인 응축   Bose-Einstein Condensate



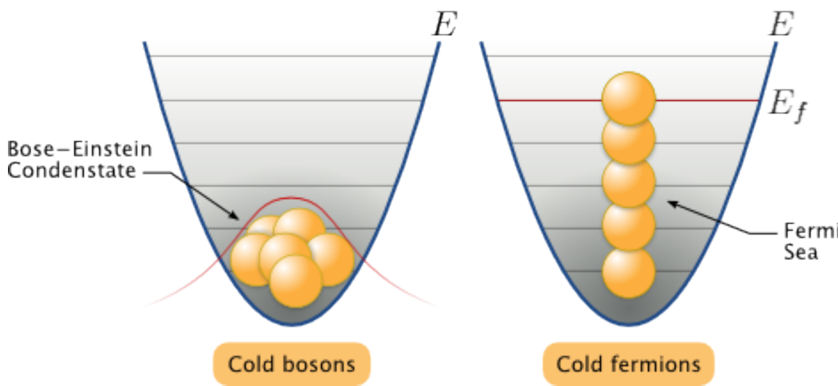
Satyendra Bose (1924)   Albert Einstein (1924)

Bose first sent a paper to Einstein on the quantum statistics of photons. Einstein was so impressed that he translated the paper himself from English to German and submitted it for Bose to Z. Physik. Einstein then extended Bose's idea to matter.

Fritz London (1938)

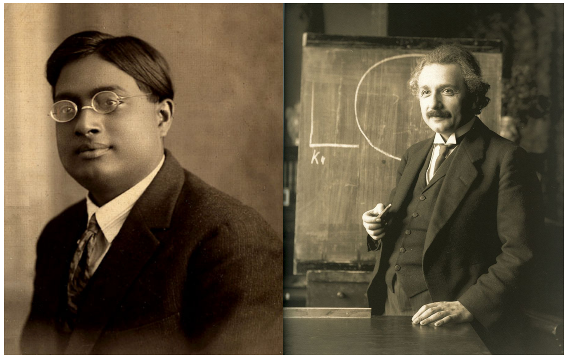
London proposed BEC as a mechanism for superfluidity in ^4He.

보존 대 페르미온 Boson versus Fermion

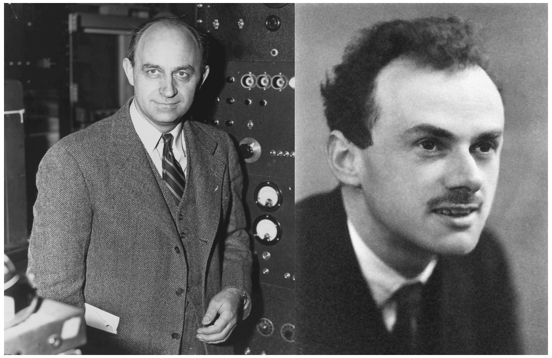


$$f_{\mathrm{BE}}(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/k_{\mathrm{B}}T} - 1}$$

$$f_{\mathrm{FD}}(\epsilon) = \frac{1}{e^{(\epsilon-\mu)/k_{\mathrm{B}}T} + 1}$$



Bose-Einstein Statistics



Fermi-Dirac Statistics

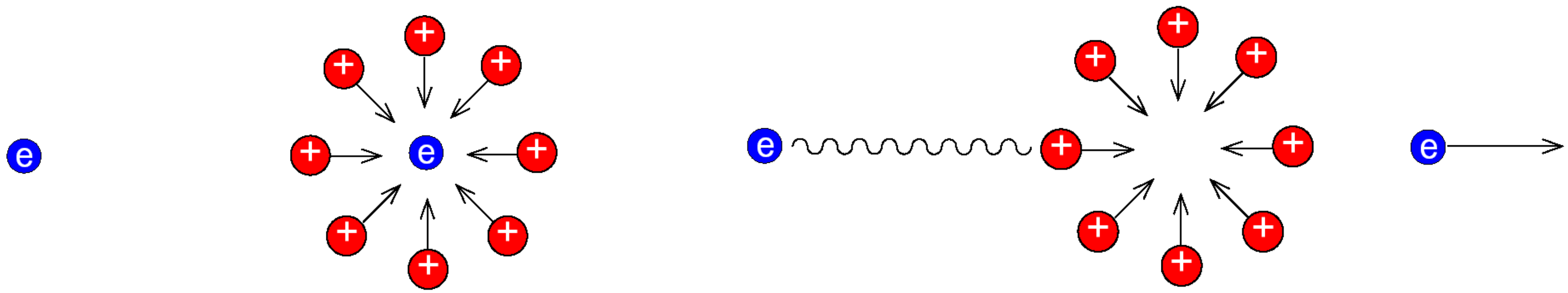
Bardeen-Cooper-Schrieffer (BCS) Theory of Superconductivity

John Bardeen, Leon Cooper, and John Schrieffer (1957)

Nobel prize awarded in 1972 "for their jointly developed theory of superconductivity, usually called the BCS theory"

BCS 이론을 요약하면 BCS Theory in a Nutshell

- Electrons are paired via the “exchange of phonon (포논의 교환).”

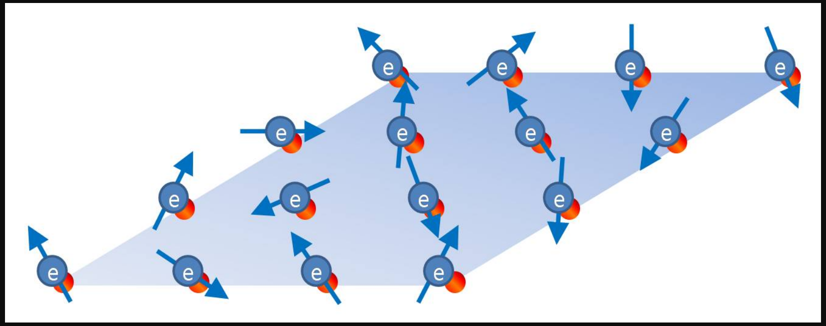


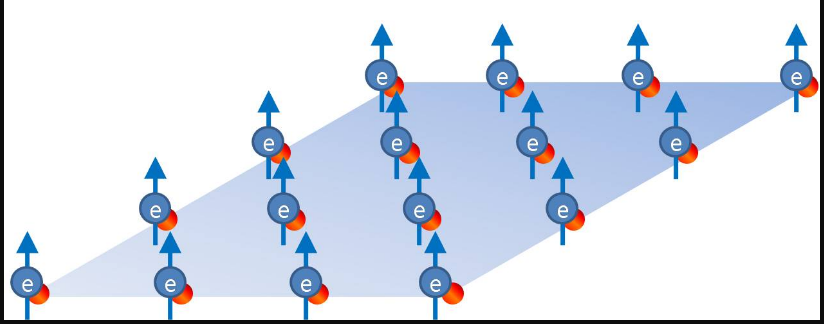
- Electrons pairs, known as Cooper pairs (쿠퍼쌍), behave more or less as if they are usual bosons.
- Similar to bosons in superfluidity, Cooper pairs can move freely once the phases of all Cooper pairs become coherent, i.e., aligned (쿠퍼쌍의 위상값이 정렬하다).
- The gauge symmetry is spontaneously broken (게이지 대칭성이 자발적으로 붕괴하다).

자발적 대칭성 붕괴 Spontaneous Symmetry Breaking

질서가 없는 Disordered

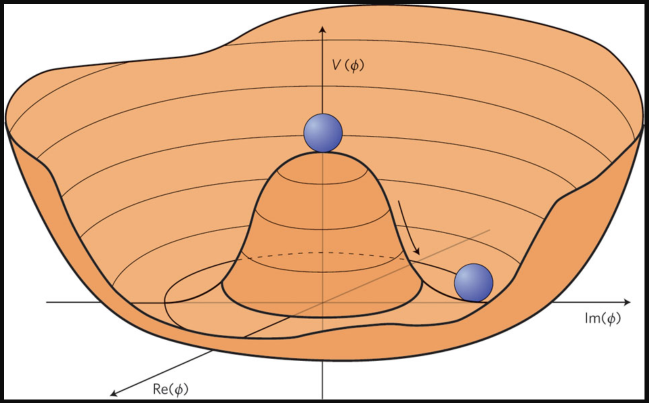
질서가 있는 Ordered





Spontaneous Symmetry Breaking:
Anderson-Higgs Mechanism

Philip W. Anderson (1962)



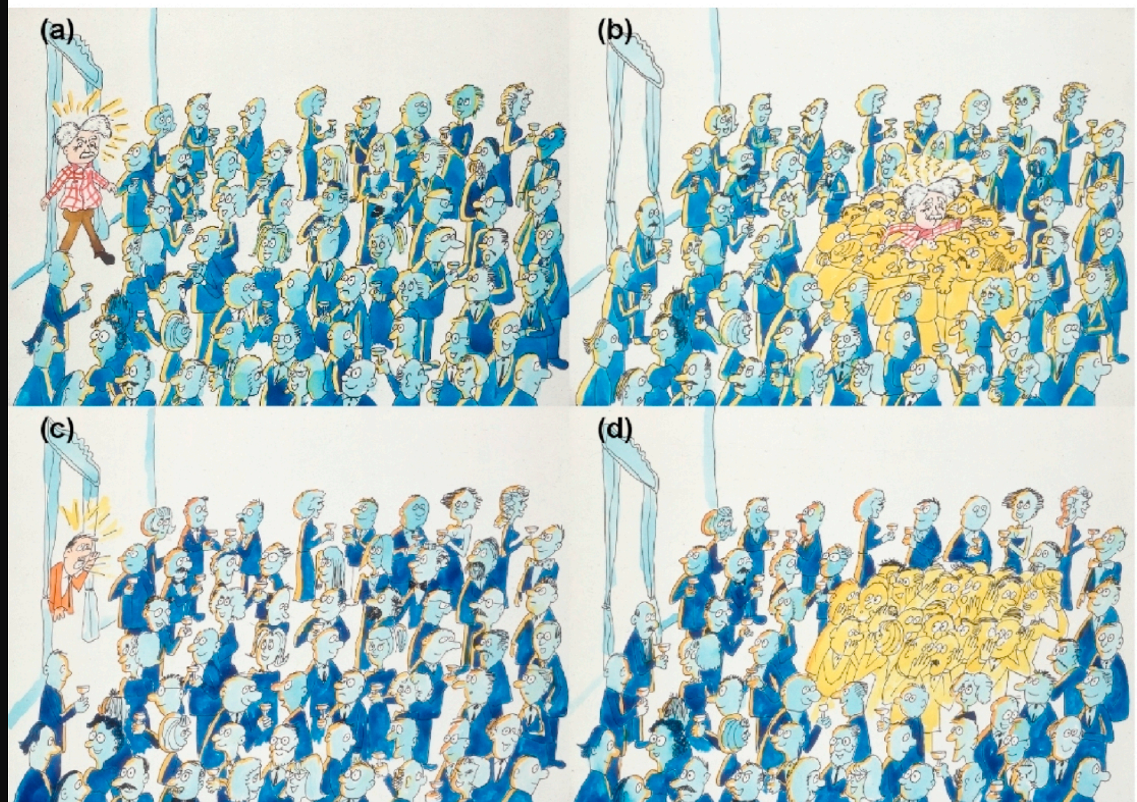
Spontaneous Symmetry Breaking

Meissner Effect

뫼비우스 띠에서의 양자부양
Quantum Levitation on a Mobius Strip

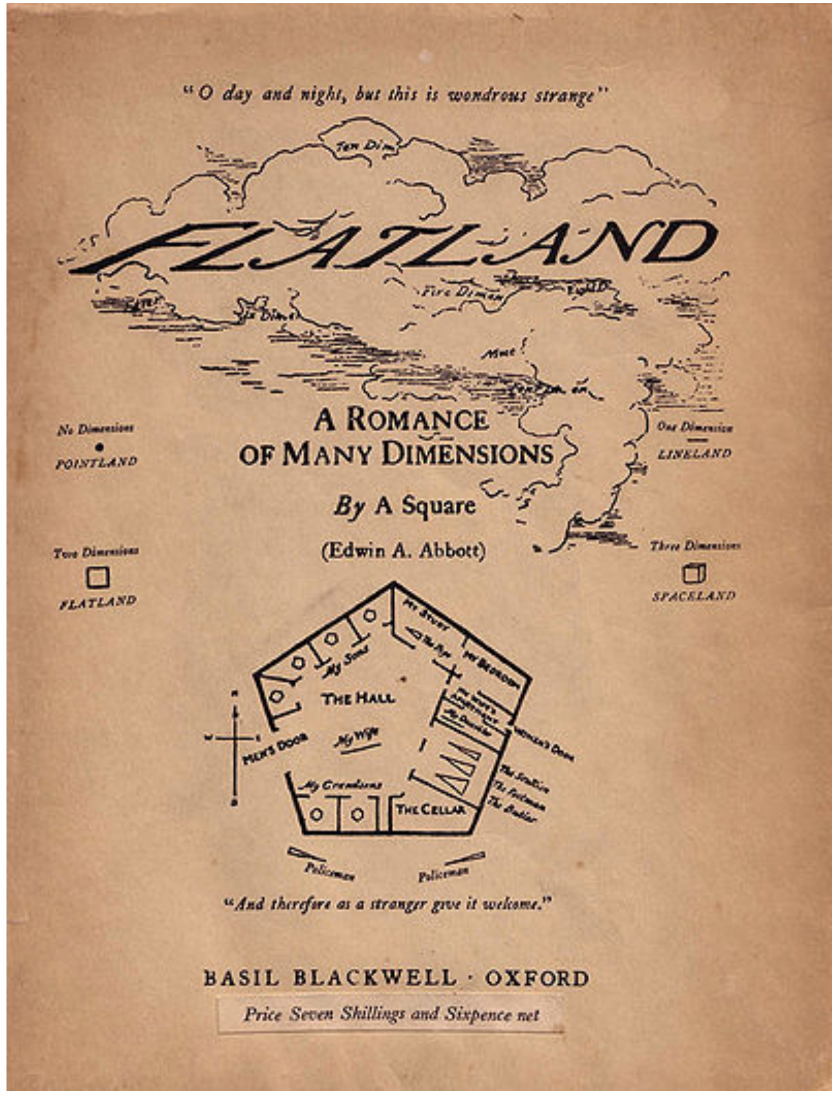
Higgs Mechanism (힉스 메커니즘):
How to Give Mass to Particles

Figure 1 from S Alsop and S Beale 2013 Phys. Educ. 48 670



Crowd analogy originally proposed by David Miller in "Politics, Solid State, and the Higgs," Physics World (1993)

플랫랜드 Flatland



Written by Edwin A. Abbott in 1884

2차원에는 질서가 없다? No Order in the Two-dimensional World?

- **Mermin-Wagner theorem (머민-와그너 정리):**

Thermal fluctuations destroy all orders (obtained by breaking the continuous symmetry) in a flat, two-dimensional world even in the limit of absolute zero temperature.

- Superconductivity and superfluidity cannot exist in a thin film? 초전도체와 초유체가 얇은 필름 형태로는 존재할 수 없다?

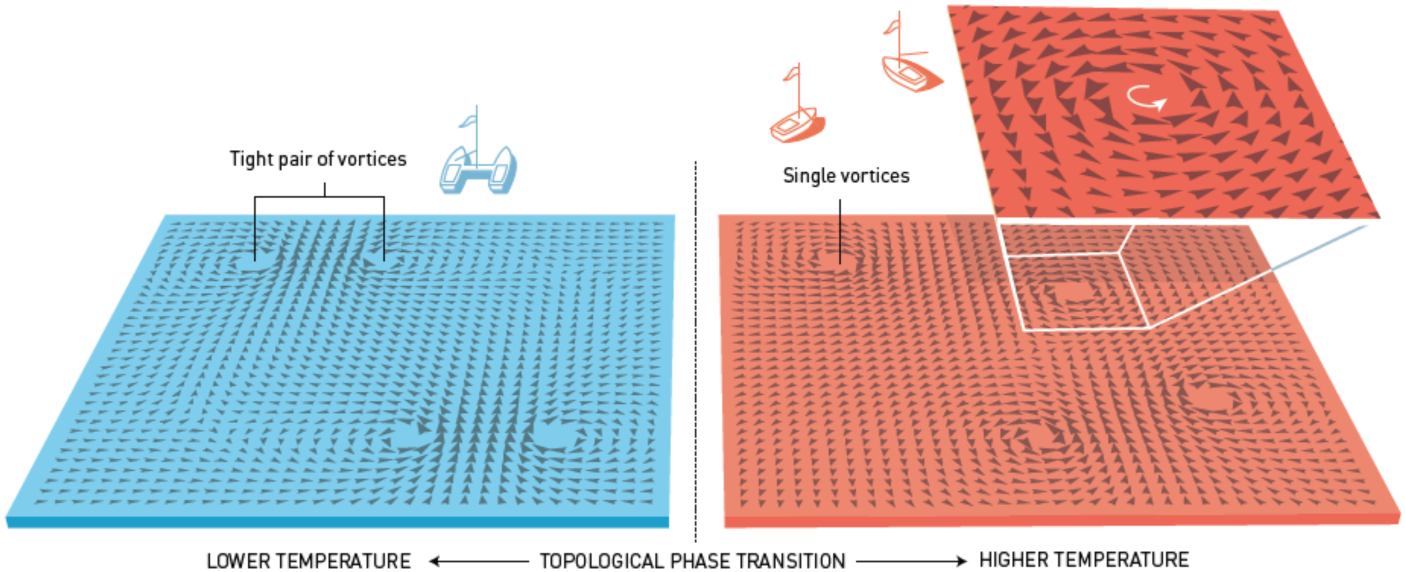
Both are long-range ordered states, where the phase of Cooper pairs and superfluid bosons, respectively, should be all aligned like little arrows in a ordered state.

이 두 가지 물질 상태에서는 작은 화살표로 표현되는 질서 변수가 모두 한 방향으로 정렬되어야 한다.

코스털리츠-싸울리스 상전이 Kosterlitz-Thouless Transition

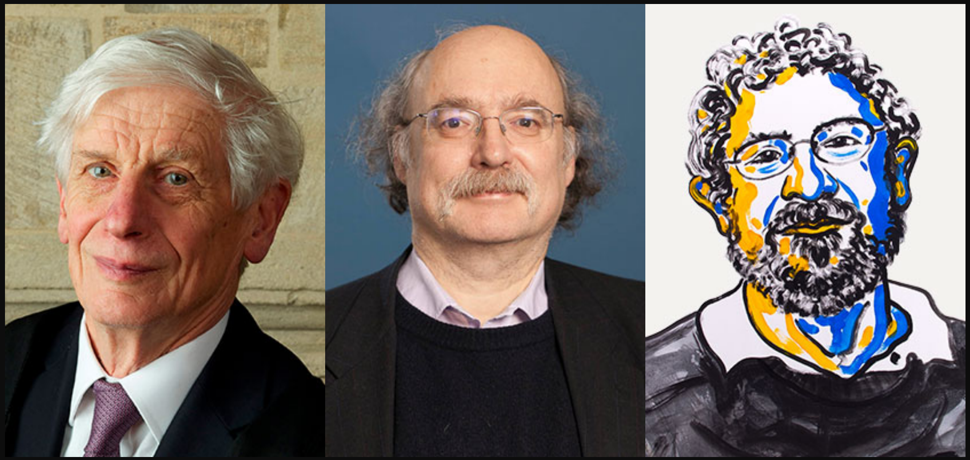
Kosterlitz, Thouless, J. Phys. C: Solid State Phys. **5**, L124 (1972); *ibid.* **6**, 1181 (1973)

- **The Kosterlitz-Thouless theory** tells us that there can be a quasi-ordered state, which undergoes a topological phase transition with proliferation of vortices (소용돌이의 마구잡이 확산을 통한 위상학적 상전이) at a critical temperature.



2016 Nobel prize press release

2016년도 노벨 물리학상 2016 Nobel Prize in Physics



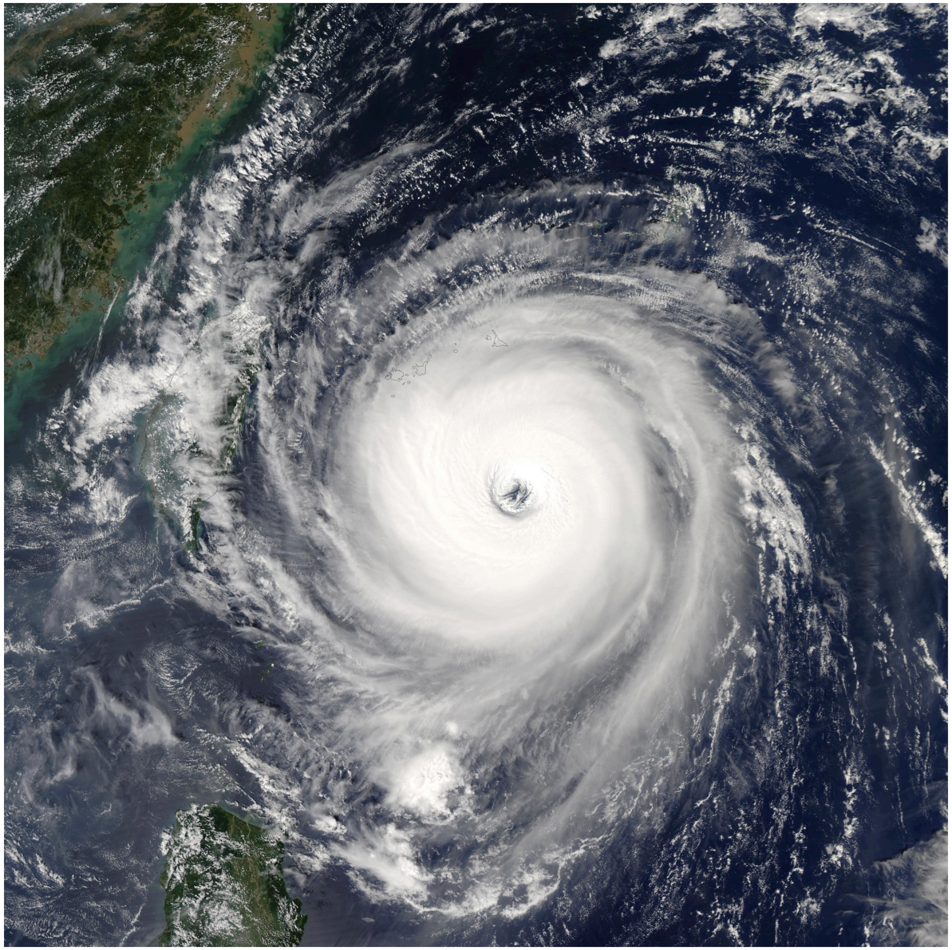
David J. Thouless (1/2), F. Duncan M. Haldane (1/4), and J. Michael Kosterlitz (1/4)
“for theoretical discoveries of topological phase transitions and topological phases of matter”

“위상학적 상전이와 위상학적 물질상태를 이론적으로 발견한 공로로” 노벨 물리학상을 수여하다.

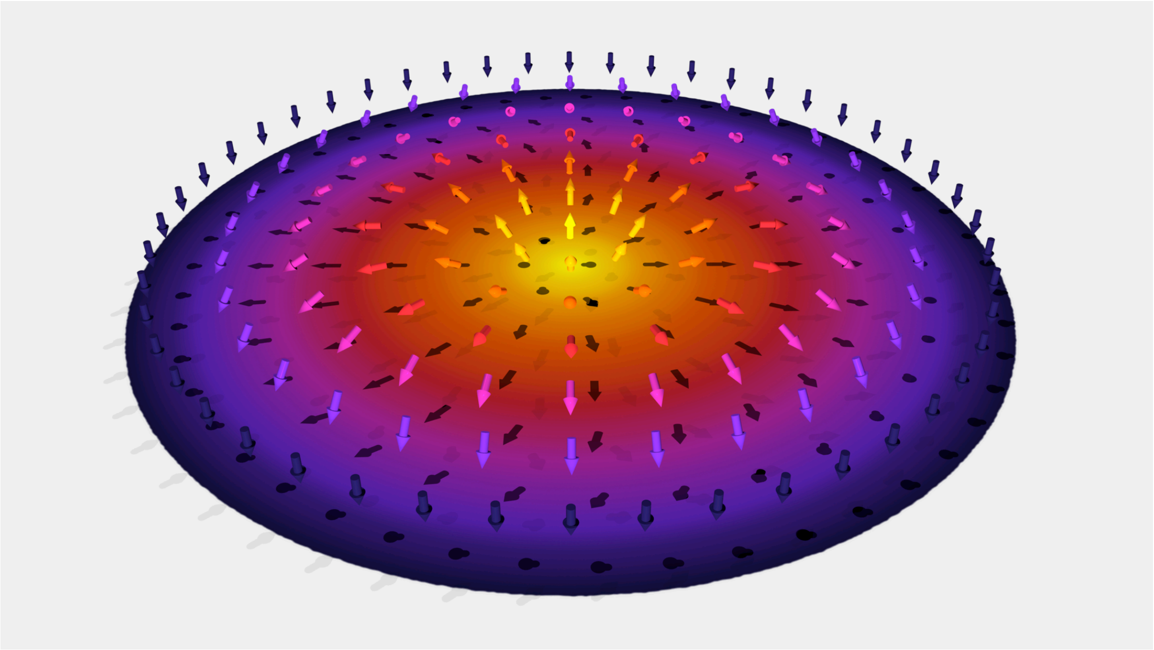
What is topology?

위상수학이란?

위상학적 물체로서의 소용돌이 Vortex as a Topological Object



3차원 소용돌이, 스커미온 3D Vortex, Skyrmion

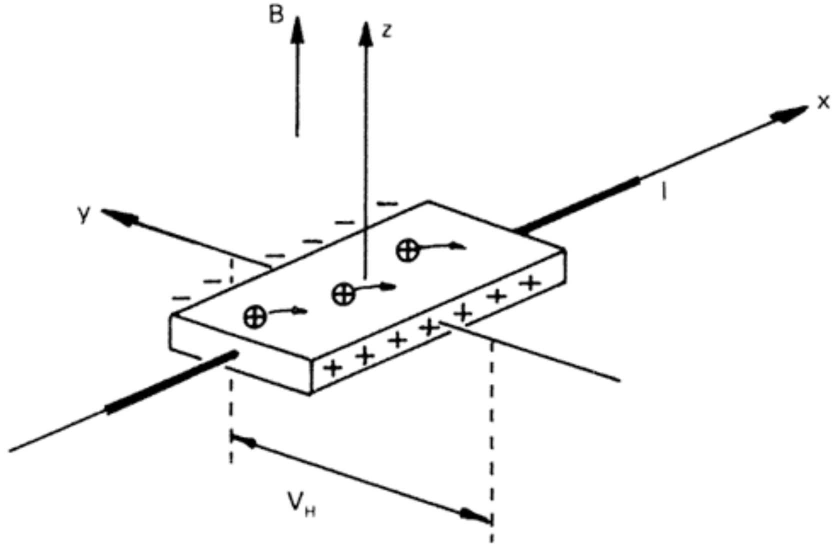


Chern number, or winding number: $\mathcal{C} = \frac{1}{4\pi} \int d^2\mathbf{r}\ \mathbf{n} \cdot (\partial_x \mathbf{n} \times \partial_y \mathbf{n})$

Can a topological phase of matter exist as a stable ground state?

안정된 바닥상태로서 위상학적 물질상태가 존재할 수 있을까?

고전 홀 효과 Classical Hall Effect



Lorentz force due to magnetic field
로렌츠 힘

$$\mathbf{F} = \frac{e}{c}\mathbf{v} \times \mathbf{B} = \frac{\mathbf{j}}{\rho c} \times \mathbf{B}$$

Electric force due to charge accumulation
전기력

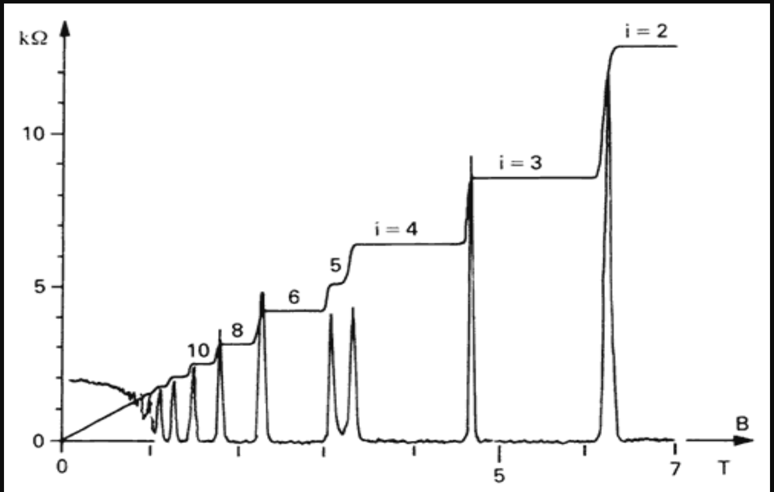
$$\mathbf{F} = e\mathbf{E}$$

- The Hall resistivity is given by the steady-state condition balancing the two forces.

$$R_{xy} = \frac{E_y}{j_x} = \frac{B}{\rho e c}$$

양자 홀 효과 Quantum Hall Effect

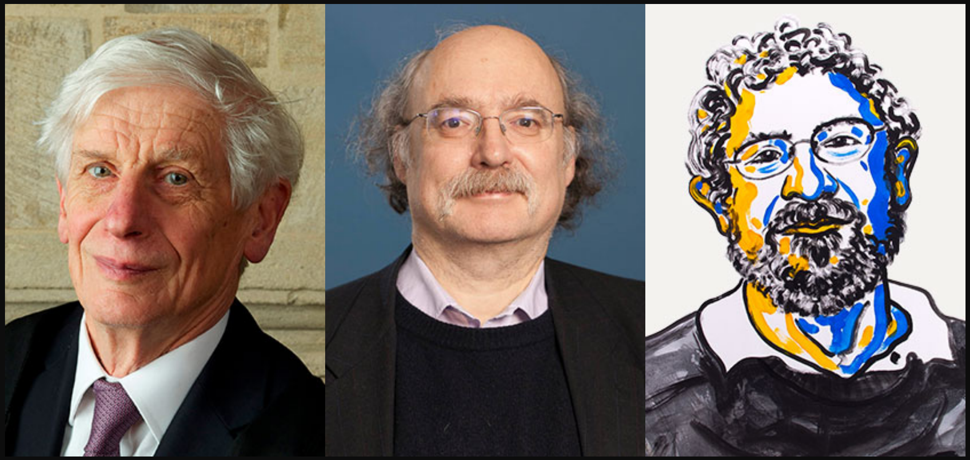
Klaus von Klitzing (1980)



$$R_{xy} = \frac{h}{ne^2}$$

Nobel prize awarded in 1985 "for the discovery of the quantized Hall effect"

2016년도 노벨 물리학상 2016 Nobel Prize in Physics



David J. Thouless (1/2), F. Duncan M. Haldane (1/4), and J. Michael Kosterlitz (1/4)
“for theoretical discoveries of topological phase transitions and topological phases of matter”

“위상학적 상전이와 위상학적 물질상태를 이론적으로 발견한 공로로” 노벨 물리학상을 수여하다.

TKNN 공식 TKNN Formula

Thouless, Kohmoto, Nightingale, den Nijs, PRL **49**, 405 (1982)

- **The Thouless-Kohmoto-Nightingale-den Nijs (TKNN) formula** relates the topological invariant called the Chern number (천 숫자) with the Hall conductivity (홀 전도도).

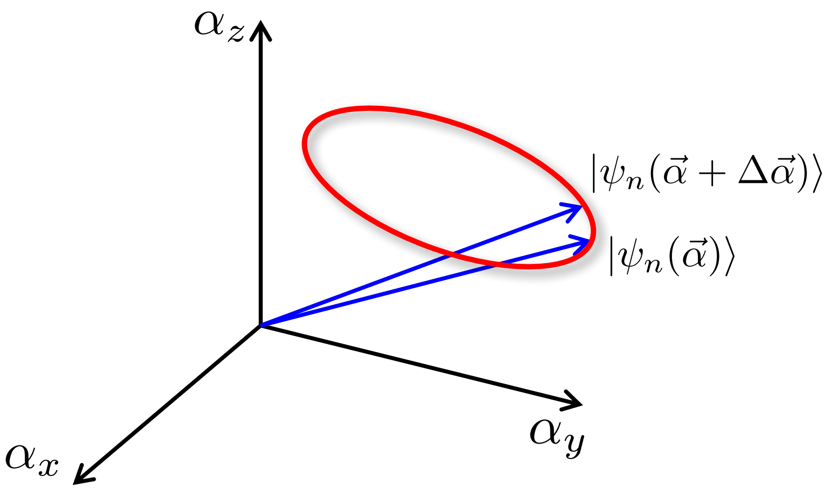
$$\sigma_{xy} = n\frac{e^2}{h}$$

$$n = \sum_{\epsilon_\mu(\mathbf{k})<\epsilon_{\mathrm{F}}} \mathcal{C}_\mu$$

$$\mathcal{C}_\mu = \frac{i}{2\pi}\int_{\mathbf{k}\in\mathrm{BZ}} d^2\mathbf{k}\langle\nabla_{\mathbf{k}}u_\mu(\mathbf{k})| \times |\nabla_{\mathbf{k}}u_\mu(\mathbf{k})\rangle\cdot\hat{z}$$

Berry flux in the momentum space (운동량 공간에서의 **베리 플럭스**)

베리 위상 Berry phase



$$\langle\psi_n(\vec{\alpha})|\psi_n(\vec{\alpha}+\Delta\vec{\alpha})\rangle$$
$$= 1 + \Delta\vec{\alpha}\,\langle\psi_n(\vec{\alpha})|\,\nabla_{\vec{\alpha}}\,|\psi_n(\vec{\alpha})\rangle$$
$$= e^{-i\Delta\vec{\alpha}\cdot\vec{\mathcal{A}}_n(\vec{\alpha})}$$

- Berry connection: *vector potential*

$$\vec{\mathcal{A}}_n(\vec{\alpha}) = i\,\langle\psi_n(\vec{\alpha})|\,\nabla_{\vec{\alpha}}\,|\psi_n(\vec{\alpha})\rangle$$

- Berry curvature: *magnetic field*

$$\vec{\mathcal{B}}_n(\vec{\alpha}) = \nabla_{\vec{\alpha}} \times \vec{\mathcal{A}}_n(\vec{\alpha}) = i\,\langle\nabla_{\vec{\alpha}}\psi_n(\vec{\alpha})| \times |\nabla_{\vec{\alpha}}\psi_n(\vec{\alpha})\rangle$$

- Berry phase: *Aharonov-Bohm phase*

$$\Gamma_n = \oint_C d\vec{\alpha}\cdot\vec{\mathcal{A}}_n(\vec{\alpha}) = \int_A d\vec{S}\cdot\vec{\mathcal{B}}_n(\vec{\alpha}) = i\int_A d\vec{S}\cdot\langle\nabla_{\vec{\alpha}}\psi_n(\vec{\alpha})| \times |\nabla_{\vec{\alpha}}\psi_n(\vec{\alpha})\rangle$$

Interpretation of the TKNN formula via the Berry phase

- The TKNN formula tells us that the Hall conductivity is proportional to the Berry phase of a closed path encompassing the entire Brillouin zone.

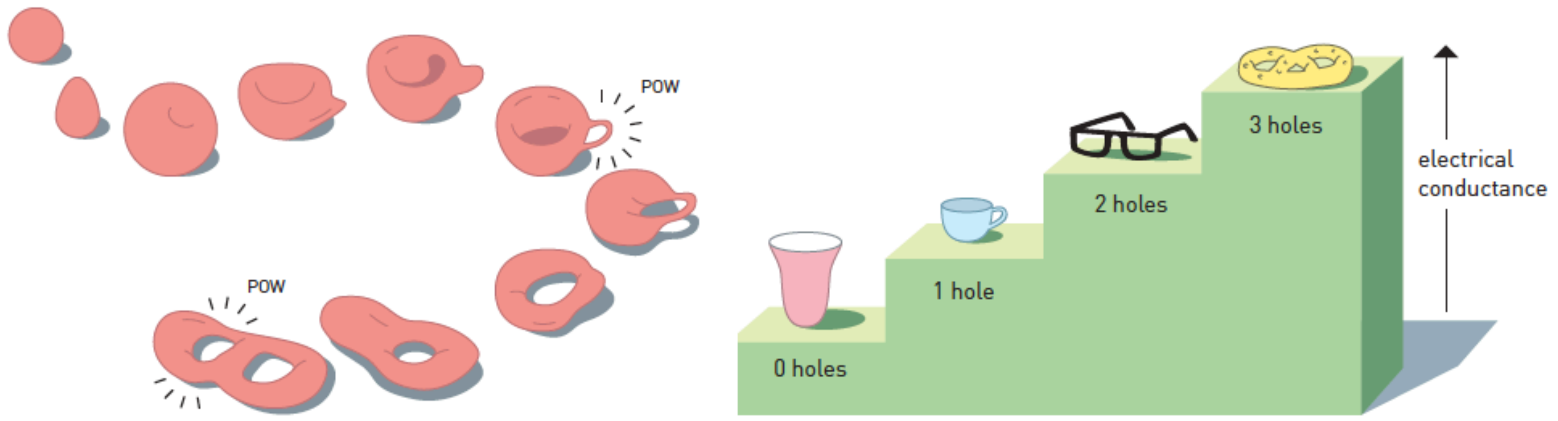
$$\Gamma_n = i \int_A d\vec{S} \cdot \langle \nabla_{\vec{\alpha}} \psi_n(\vec{\alpha}) | \times | \nabla_{\vec{\alpha}} \psi_n(\vec{\alpha}) \rangle$$

$$\Updownarrow$$

$$\vec{\alpha} \leftrightarrow \mathbf{k}$$

$$\sigma_{xy} \propto \int_{\mathbf{k} \in \mathrm{BZ}} d^2\mathbf{k} \sum_{\epsilon_\mu(\mathbf{k}) < \epsilon_\mathrm{F}} \langle \nabla_\mathbf{k} u_\mu(\mathbf{k}) | \times | \nabla_\mathbf{k} u_\mu(\mathbf{k}) \rangle \cdot \hat{z}$$

TKNN 공식을 만화로 표현하면 Cartoon for the TKNN Formula



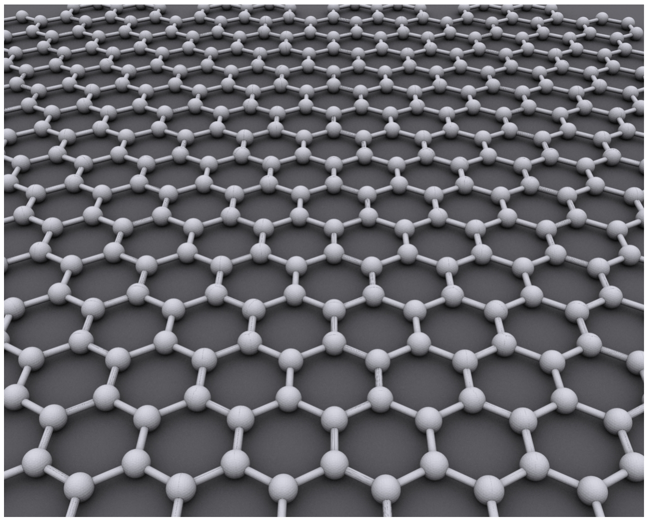
2016 Nobel prize press release

Can a topological phase of matter exist without the magnetic field?

자기장 없이도 위상학적 물질상태가 존재할 수 있을까?

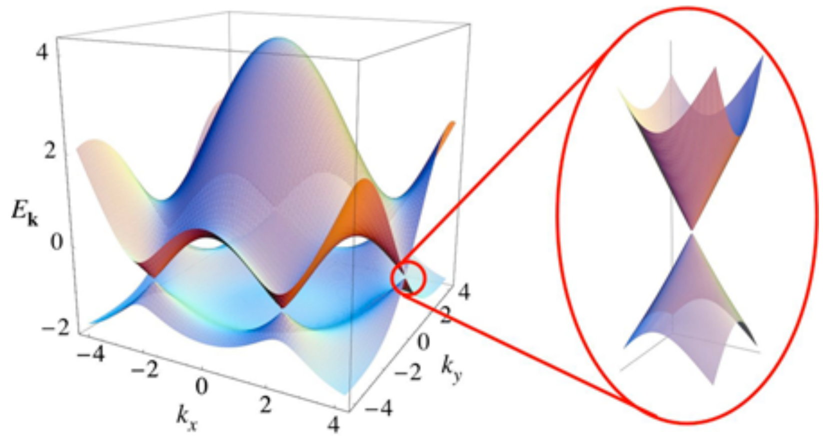
그래핀 Graphene

Novoselov, Geim, *et al.*, Science **306**, 666 (2004); Zhang, Tan, Stormer, Kim, Nature **438**, 201 (2005)



Two-dimensional carbon matter
in honeycomb lattice structure

벌집구조의 2차원 탄소 물질



Relativity manifests itself in condensed matter!

응집물질에서 상대성 이론이 나타나다!

할데인 모형 Haldane model

VOLUME 61, NUMBER 18 PHYSICAL REVIEW LETTERS 31 OCTOBER 1988

Model for a Quantum Hall Effect without Landau Levels: Condensed-Matter Realization of the "Parity Anomaly"

F. D. M. Haldane

Department of Physics, University of California, San Diego, La Jolla, California 92093

(Received 16 September 1987)

A two-dimensional condensed-matter lattice model is presented which exhibits a nonzero quantization of the Hall conductance σ^{xy} in the *absence* of an external magnetic field. Massless fermions *without spectral doubling* occur at critical values of the model parameters, and exhibit the so-called "parity anomaly" of (2+1)-dimensional field theories.

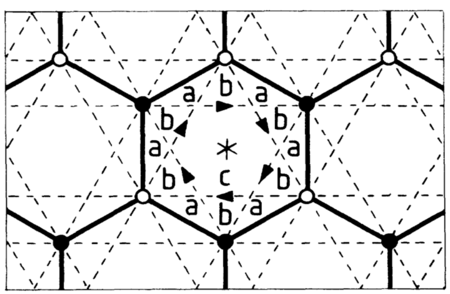


FIG. 1. The honeycomb-net model ("2D graphite") showing nearest-neighbor bonds (solid lines) and second-neighbor bonds (dashed lines). Open and solid points, respectively, mark the *A* and *B* sublattice sites. The Wigner-Seitz unit cell is conveniently centered on the point of sixfold rotation symmetry (marked "*") and is then bounded by the hexagon of nearest-neighbor bonds. Arrows on second-neighbor bonds mark the directions of positive phase hopping in the state with broken time-reversal invariance.

$$H(\mathbf{k}) = \begin{pmatrix} g_{\mathbf{k}} & f_{\mathbf{k}} \\ f^*_{\mathbf{k}} & -g_{\mathbf{k}} \end{pmatrix} = \mathbf{d}_{\mathbf{k}} \cdot \boldsymbol{\sigma}$$

$$\mathbf{d}_{\mathbf{k}} = (\mathrm{Re} f_{\mathbf{k}}, -\mathrm{Im} f_{\mathbf{k}}, g_{\mathbf{k}})$$

$$f_{\mathbf{k}} = t_1 \sum_i e^{i\mathbf{k}\cdot\mathbf{a}_i}$$

$$g_{\mathbf{k}} = M + 2t_2 \sum_i \cos(\mathbf{k}\cdot\mathbf{b}_i + \phi)$$

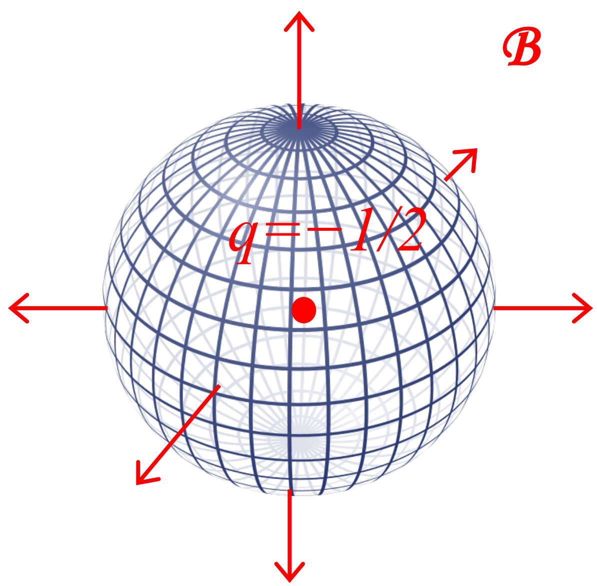
Magnetic monopole in the pseudospin space

$$H(\vec{\alpha}) = \vec{\sigma} \cdot \vec{\alpha}$$

$$\vec{\mathcal{B}}_n(\vec{\alpha}) = \nabla_{\vec{\alpha}} \times \vec{\mathcal{A}}_n(\vec{\alpha}) = i \langle \nabla_{\vec{\alpha}} \psi_n(\vec{\alpha}) | \times | \nabla_{\vec{\alpha}} \psi_n(\vec{\alpha}) \rangle$$

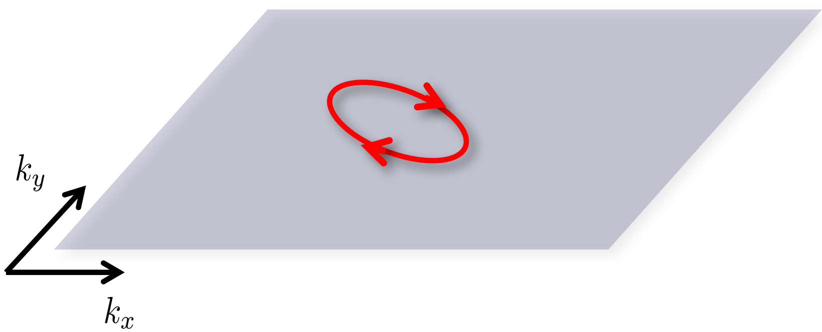
$$\vec{\mathcal{B}}_+(\vec{\alpha}) = -\frac{\hat{\alpha}}{2\alpha^2}$$

- This is precisely equal to the "magnetic field" exerted by a Dirac monopole at the center with monopole strength −1/2

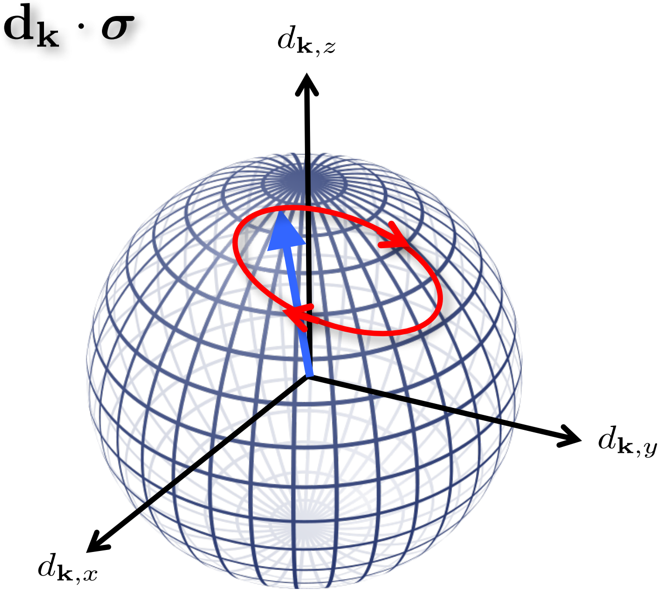


Equivalence between the Berry flux and the Chern number

$$H(\mathbf{k}) = \begin{pmatrix} g_{\mathbf{k}} & f_{\mathbf{k}} \\ f^*_{\mathbf{k}} & -g_{\mathbf{k}} \end{pmatrix} = \mathbf{d}_{\mathbf{k}} \cdot \boldsymbol{\sigma}$$



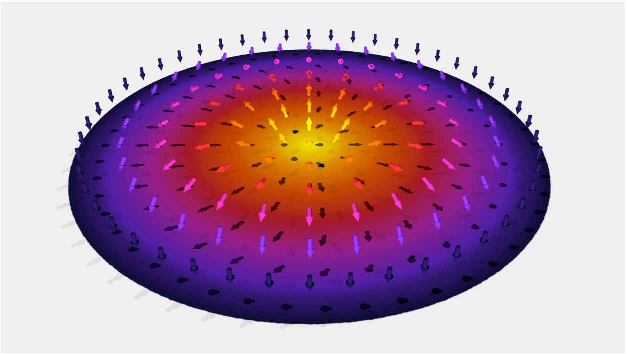
Momentum space



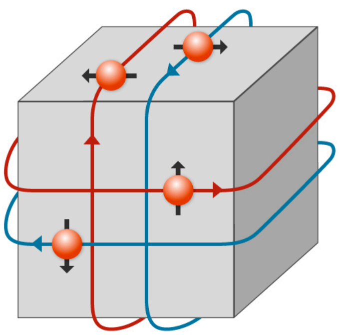
Bloch sphere

Berry flux = Chern number

$$\mathcal{C} = \frac{1}{4\pi} \int d^2\mathbf{k} \;\; \hat{\mathbf{d}}_{\mathbf{k}} \cdot (\partial_{k_x} \hat{\mathbf{d}}_{\mathbf{k}} \times \partial_{k_y} \hat{\mathbf{d}}_{\mathbf{k}})$$

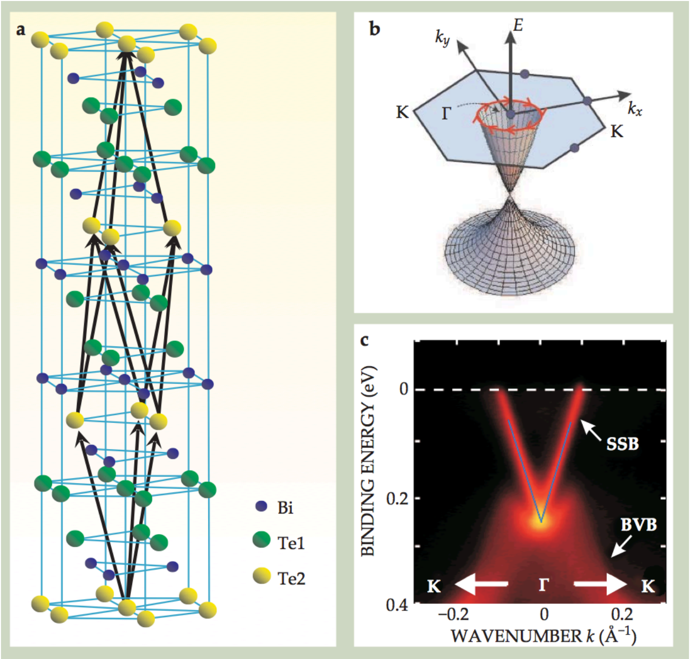


위상 절연체 Topological Insulator



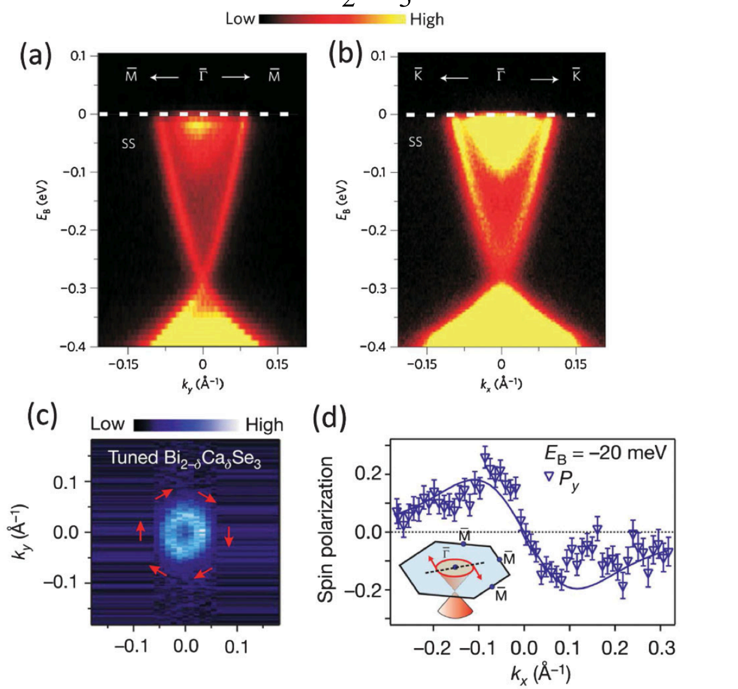
- Electrons can move freely on the surface of 3D topological insulators, behaving as if they are relativistic particles. These states are known as helical surface states (나선형 표면 상태).

Bi_2Te_3



Chen *et al.*, Science **325**, 178 (2009), adapted by Qi, Zhang, Phys. Today **63**, 33 (2010)

Bi_2Se_3



Xia *et al.*, Nature Phys. **5**, 398 (2009), Hsieh *et al.*, Nature **460**, 1101 (2009), adapted by Qi, Zhang, RMP **83**, 1057 (2011)

Quantum matter has become one of the most important subjects in modern physics.

양자물질은 현대 물리에서 가장 중요한 주제 중의 하나가 되었다.