| 소속대학원 | 물리학부 | 학번 | | 성 명 | | 감독교수 확 인 | (인) |
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# 물리학부 석박사학위 자격시험

**과목명 : 양자역학** **2013 . 06. 28 시행**

1. **(35 pts)** Consider an electron in three dimensional (3D) space whose Hamiltonian is given by $H = \frac{\hat{p}_x^2 + \hat{p}_y^2 + \hat{p}_z^2}{2m_e} + V(x, y, z)$ where $m_e$ is the mass of an electron and

$$V(x,y,z) = \begin{cases} 0 & \text{if } 0 \le x \le L,\ 0 \le y \le L,\ 0 \le z \le L \\ \infty & \text{otherwise} \end{cases}.$$

(a) What is the most general form of the wavefunction of the eigenstates (including an appropriate normalization constant) and the corresponding energy eigenvalues? (7 pts)

(For problems (b) and (c)) A photon can induce a transition of an electron from an occupied state to an empty (higher-energy) state.

(b) If the system (3D infinite potential well) has 8 electrons, what is the minimum photon energy that can induce such an electronic transition? Remember that electrons are fermions. (Neglect electron-electron interactions.) (8 pts)

(c) Assuming that $L$ = 1 nm, estimate the numerical value for the photon energy you found in (b) in eV. What is the corresponding wavelength? Is it infrared, visible, ultraviolet or x-ray light? Retain one significant digit in the final numerical answers. (10 pts)

(d) Now suppose that the system has one electron and it is in the ground state. The potential term $V(x, y, z)$ is suddenly replaced at $t = 0$ by

$$\widetilde{V}(x,y,z) = \begin{cases} 0 & \text{if } -L \le x \le L,\ 0 \le y \le L,\ 0 \le z \le L \\ \infty & \text{otherwise} \end{cases}$$

Suppose that we measure the energy of the electron either at $t < 0$ or at $t > 0$, what are the possible observable energy values in each of the two cases? (10 pts)

2. **(45 pts)** Consider that one electron $\left(s = \frac{1}{2}\right)$ is bound to a radial potential $U(r)$ which respects the rotational symmetry, and that the orbital angular momentum quantum number of the electron is $l = 1$, (i.e., it is a *p-wave* state).

(a) Find out all the possible total (orbital plus spin) angular momentum quantum number $j$'s. For each $j$, write down all the possible magnetic quantum number $m_j$'s. (8 pts)

(b) Now we consider the spin-orbit coupling $H_{\rm SOC} = \lambda_{\rm SOC} \frac{\mathbf{L}}{\hbar} \cdot \frac{\mathbf{S}}{\hbar}$ in this system. Show that a total angular momentum eigenstate $|j, m_j\rangle$ is an eigenstate of $H_{\rm SOC}$. Also, find the eigenvalue of $H_{\rm SOC}$ for each $(j, m_j)$ pair you found in (a).
(Note that these states satisfy $\mathbf{J}^2|j,m_j\rangle = j(j+1)\hbar^2|j,m_j\rangle$, $J_z|j,m_j\rangle = m_j\hbar|j,m_j\rangle$ and $J_\pm|j,m_j\rangle = \sqrt{j(j+1) - m_j(m_j \pm 1)}\,\hbar|j, m_j \pm 1\rangle$ where $\mathbf{J} = \mathbf{L} + \mathbf{S}$ is the total angular momentum operator and $J_\pm = J_x \pm iJ_y$.) (12 pts)

(c) Let us confine our interest to a hydrogen atom where we have a radially symmetric potential and a single electron. Estimate the numerical value of $\lambda_{\rm SOC}$ in eV. Retain one significant digit in the final answer.

[Hint: Spin-orbit coupling is a relativistic effect. An electron moving at a velocity $v$ in a direction perpendicular to an electric field with magnitude $E$ (without a magnetic field in the lab frame) feels a magnetic field of magnitude $B \sim \frac{v}{c^2}E$ (SI unit) where $c$ is the speed of light. For an electron in a hydrogen atom, the electric field is provided by nucleus (Coulomb field). An electron with spin expectation value

| 소속대학원 | 물리학부 | 학번 | | 성 명 | | 감독교수 확 인 | (인) |
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**과목명 : 양자역학** **2013 . 06. 28 시행**

$S$ has a magnetic moment $\mu = \frac{e}{m_e} S$ where $e$ and $m_e$ are the charge and mass of an electron, respectively. The magnetic dipole energy then is roughly given by $H_{\mathrm{SOC}} \sim \mu B \sim \frac{ev}{m_e c^2} ES$.] (12 pts)

(d) Consider the situation where an external electro-magnetic field induces a transition between the total angular momentum eigenstates $|j, m_j\rangle$'s (which are the energy eigenstates in an atom with strong spin-orbit coupling). If the light is polarized along the $z$ direction, the dominant perturbing (electric-dipole radiation) Hamiltonian is of the form $H_{\mathrm{rad}} = b\,\hat{p}_z$ where $b$ is a constant and $\hat{p}_z$ is a momentum operator in the $z$ direction. Assume that the initial state is $\left| j = \frac{3}{2}, m_j = -\frac{3}{2} \right\rangle$ and it is made from electron orbital states with $l = 1$. Enlist possible $(l, j, m_j)$ pairs for the final state. You might want to consider selection rules regarding total angular momentum and parity. (13 pts)

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# 물리학부 석박사학위 자격시험

**과목명 : 통계역학** **2013 . 06. 28 시행**

1. **(40 pts)** Consider a crystal of $N$ atoms ($N$~$10^{23}$) with spin quantum numbers $s=1/2$ and $m_s=\pm 1/2$. The magnetic moment of the $i$-th atom is $\vec{\mu_i}=g\mu_B\vec{s}_i$, where $g$ is the Landé $g$-factor, and $\mu_B=e\hbar/2mc$ is the Bohr magneton. Assume that the atoms do not interact, but are in equilibrium at temperature $T$ and are placed in an external magnetic field $\vec{H}=H\hat{z}$.

(a) Show that the partition function is given as $Z=(2\cosh\eta)^N$ where $\eta=g\mu_B H/2kT$. (8 pts)

(b) Find an expression for the entropy $S$ of the crystal, only considering the contribution from the spin states. Evaluate $S$ in the strong field ($\eta\gg 1$) and weak field ($\eta\ll 1$) limits. (8 pts)

(c) The magnetization $M$ of the crystal is given by $M=\left\langle \sum_{i=1}^{N}(\mu_i)_z \right\rangle$. Find the expression for $M$ and sketch its behavior as a function of $\eta$. In the weak field limit, evaluate the magnetic susceptibility $\chi=M/H$. (12 pts)

Now suppose each atom interacts with each of its nearest $n$ neighbors. To include these interactions approximately, we assume that the nearest $n$ neighbors generate a 'mean field' $\overline{H}$ at the site of each atom, where $g\mu_B\overline{H}=2\alpha\left\langle \sum_{j=1}^{n}(s_j)_z \right\rangle$, $\alpha$ is a parameter which characterizes the strength of the interactions.

(d) Using this mean field approximation together with the results of part (c), calculate the susceptibility $\chi$ in the weak field (i.e. the high temperature) limit. At what temperature, $T_c$, does $\chi$ become infinite? (12 pts)

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# 물리학부 석박사학위 자격시험

**과목명 : 전자기학** **2013 . 06. 28 시행**

**1. (40 pts)** We want to study physics of a high-mobility electron inside two-dimensional MOS (metal oxide semiconductor) in magnetic field and planar confinement.

(a) Consider a circular ring of radius R, carrying an electric current I counterclockwise. In Cartesian coordinates, the ring lies on x-y plane at z=0 and centered at (0,0,0). Draw qualitatively how the magnetic field lines in (x,0,z) slice and (x,y,0) slices look like. (3 pts)

(b) By integrating the Biot-Savart's law, show that the z-component of the magnetic field is given by

$$\vec{B}(0,0,z)\cdot\hat{z} = \frac{\mu_o I}{2}\cdot\frac{R^2}{(R^2+z^2)^{3/2}}. \qquad (1)$$

On symmetry ground, explain why the magnetic field $\vec{B}$ at (0,0,z) must have z-component only. (8 pts)

(c) Consider next a solenoid of radius R, of infinite height and of $n$ turns per meter along z-direction, carrying an electric current I. Draw qualitatively how magnetic field lines around the solenoid look like. Using the result (1) or otherwise, show that the magnetic field B is constant and is given by (7 pts)

$$\vec{B}(r,\theta,z) = \begin{cases}\mu_0 n I\ \hat{z} & \text{for} \quad r \le R \\ 0 & \text{for} \quad r \ge R\end{cases} \qquad (2)$$

(d) A real solenoid you deal with in laboratory has only a finite height, and has lead wires at each ends through which the current flows in and out. You may view this solenoid as a combination of a stack of large but finite number of the ring studied in (a) and of a straight, infinite wire along the z-axis. Analyzing magnetic fields generated by each, draw qualitatively how the magnetic field lines around the solenoid look like, with particular emphasis any changes that would occur compared to the idealized situation of (c). (Hint: use superposition principle of magnetic fields) (10 pts)

(e) A two-dimensional MOS is threaded to the infinite solenoid (2) on the (x,y)-plane at z=0. The center of solenoid on MOS is at (0,0,0). We want to study motion of a free electron inside MOS under the magnetic field. To confine the electron, we also exert mechanical potential

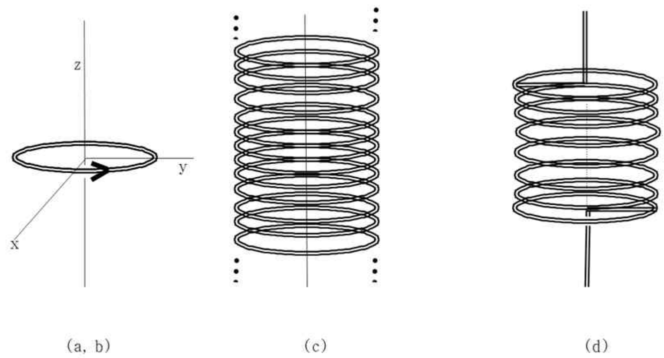
$$V(x,y) = \frac{1}{2}K(x^2+y^2) \qquad (3)$$

Write down classical equations of motion of the electron confined to MOS FET and discuss what kind of motions the electron will execute for both inside and outside of the solenoid. You may answer for the following three possible initial conditions: (i) $\vec{x_o} = (r_0 < R, 0), \vec{v_o} = (0,0)$,

(ii) $\vec{x_o} = (r_o > R, 0), \vec{v_o} = (0,0)$,

(iii) $\vec{x_o} = (r_o, 0), \vec{v_o} = (0, v_o)$.

(12 pts)




Figures for problem 1.

| 소속대학원 | 물리학부 | 학번 | | 성 명 | | 감독교수<br>확 인 | (인) |
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# 물리학부 석박사학위 자격시험

**과목명 : 전자기학 2013 . 06. 28 시행**

**2. (40 pts)**

(a) Consider an electron moving in coherent radio wave

$$\overrightarrow{E}(t) = \hat{x}E_0 \sin(\omega t) \qquad (1)$$

Initially, it was at $x=0, v_x=0$. Find the equation of motion and show that the solution is given by

$$x(t) = -\frac{a_0}{\omega}t + \frac{a_0}{\omega^2}\sin(\omega t) \quad (a_0 = eE_0/m) \qquad (2)$$

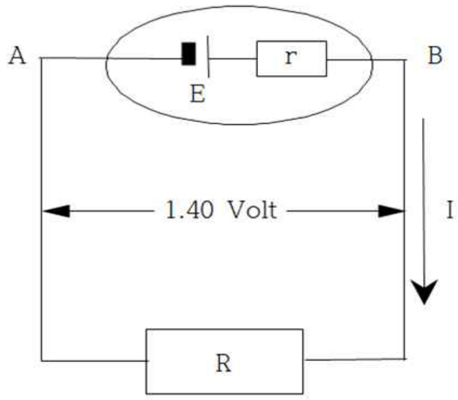
The second term in (2) is easily expected from (1), and describes the jiggling motion of the electron. The first term in (2), however, indicates that the electron starts to drift away. Explain why this is happening (6 pts).

(b) In the Bohr model of the hydrogen atom, the electron follows a circular path around the proton. Its speed is $2.2\times10^6 m/s$, orbit radius is $5.3\times10^{-11}m$. Show that the effective current in the orbit is $ev/2\pi r$ and the magnetic dipole moment $\overrightarrow{\mu} = -(\frac{e}{2m})\overrightarrow{L}$, where $\overrightarrow{L} = m\overrightarrow{r}\times\overrightarrow{v}$ is orbital angular momentum. Calculate up to first significant digit (4 pts).

(c) A very old battery may have a rather large internal resistance $r$, as shown in next figure. Suppose that a voltmeter of resistance $R$ = 280 Ohm reads the voltage of such a cell to be 1.40V, while a potentiometer (ellipse in the figure) reads its voltage to be 1.55V. What are (1) the EMF (electromotive force) of the battery, (2) its internal resistance $r$? (3) Now, instead of voltmeter, attach a resistor $R$. As you choose different values of $R$, find the maximum power this resistor can draw from the battery. Calculate all numerical values to the first significant digit. (12 pts)

(d) Consider the three-dimensional wave equation in spherical coordinate system

potentiometer (r= large internal resistance)




Voltmeter

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$$\nabla^2\phi - \frac{1}{c^2}\frac{\partial^2\phi}{\partial t^2} = 0$$

$$\nabla^2 = \frac{1}{r^2}\frac{\partial}{\partial r}(r^2\frac{\partial}{\partial r}) + \frac{1}{r^2}\frac{\partial^2}{\partial\theta^2} + \frac{1}{r^2 sin^2\theta}\frac{\partial^2}{\partial\varphi^2}$$

By solving the partial differential equation, show that spherical waves emanating from a regular point source takes the form (6 pt)

$$\phi(r,t) = C_+\frac{1}{r}\sin(kr-kct) + C_-\frac{1}{r}\sin(kr+kct).$$

(e) A beam of protons moves at $v = 5\times10^7 m/s$ and carries line charge density $\sigma = 2\times10^{-12}C/m$ as measured by an observer in the laboratory frame. An isolated proton moves at the same velocity, parallel to and at a distance 10mm from the beam.

(1) Compute the electric and magnetic forces on the protons as measured by an observer in the laboratory frame to the first significance digit.

(2) Compute the electric and magnetic forces on the proton as measured by an observer moving with the proton to the first significance digit. In other words, the observer is in the rest frame of the proton.

(12 pts)

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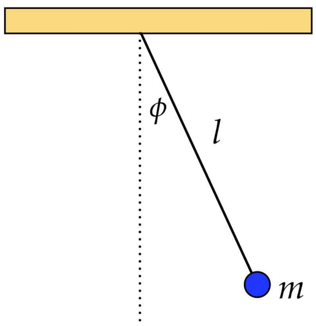
# 물리학부 석박사학위 자격시험

**과목명 : 역학** **2013 . 06. 28 시행**

**1. (40 pts)**

Consider a simple pendulum under the gravity as in the figure.

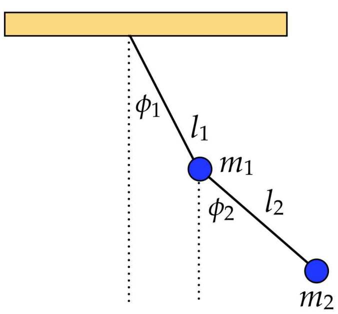



(a) Suppose that the pendulum has been released from an angle $\phi_0$. Find the speed of the pendulum as a function of $\phi$. (5 pts)

(b) When $\phi$ is limited to small angles, find the period of the pendulum. (4 pts)

(c) Discuss qualitatively (or quantitatively, if you want) the change in the period as the amplitude of the pendulum increases. (6 pts)

(d) Now the previous pendulum has been modified as follows. (10 pts)




Find the equations of motion for each mass, $m_1$ and $m_2$. You can assume that $\phi_1$ and $\phi_2$ are small.

(e) When $l_1 = l_2 = l$ and $m_1 = m_2 = m$, the equation of motion can be simplified as follows.

$$l\ddot{\phi}_1 = -2g\phi_1 + g\phi_2$$

$$l\ddot{\phi}_2 = 2g\phi_1 - 2g\phi_2$$

Find the two angular frequencies $\omega_1$ and $\omega_2$ for the normal modes of this two pendulum system. You can assume that $\phi_1$ and $\phi_2$ are small. (8 pts)

(f) Continuing the configuration (e) to $N$ identical particles and taking continuum limit $N \to \infty, \ell \to 0$ with $N\ell = L =$finite, $m/l = \mu =$finite, discuss "qualitatively" dynamics of the system when the lowest endpoint is shaken up around equilibrium point. (8 pts) [Hint: Try to estimate the angular frequency of the lowest normal mode in comparison with a horizontal string with a constant tension.]