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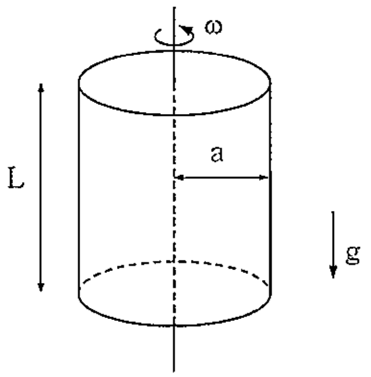
과목명 : 통계역학 2011 . 12 . 22 시행

1. An ideal non-interacting gas of N atoms is enclosed in a cylinder of radius a and length L. The cylinder rotates with angular velocity ω about its symmetry axis and the gravity acts along the axis. The ideal gas is in thermal equilibrium at temperature T in the referfence frame rotating with the cylinder. Assume that the atoms have mass m, have no internal degrees of freedom, and obey classical statistics.



(a) What is the partition function of the system? (10 pts)
(b) Find the average particle number density as a function of r and z. (10 pts)
(c) When ω=0, compute the constant volume specific heat C_v of this system as a function of T. What are the values of C_v for the limiting cases, $T\to 0$ and $T\to\infty$. (10 pts)

2. Consider a solid surface consisting of a two-dimensional lattice with M adsorption sites, which is in equilibrium with a mono-atomic ideal gas. Each lattice site can be either empty or occupied with a single adsorbed atom. An adsorbed atom has a binding energy $-\Delta$ and its kinetic energy and other interactions between the atoms can be neglected.

(a) Calculate the grand canonical partition function for the lattice with adsorbed atoms as a function of temperature T, lattice size M, and chemical potential μ_a(related to the number of adsorbed atoms). (Note that the grand canonical partition function is given by $Z=\sum_{N=0}^{\infty} e^{\beta\mu N}\mathrm{tr}(e^{-\beta \mathrm{H}_N})$, where $\beta=1/k_BT$).

(b) Calculate the fraction of occupied adsorption sites N_a, $f=N_a/M$, as a function of T, M, and μ_a.

(c) The surface is exposed to an ideal gas of the atoms at pressure p and at the same temperature T as the surface. For a mono-atomic gas, the single particle partition function is given as $\zeta = V\lambda_T^{-3}$ where λ_T is the thermal wavelength, i.e., $\lambda_T=\sqrt{2\pi\hbar^2/mk_BT}$. Show that the grand partition function for the indistinguishable particles (without considering the symmetry restriction of the wavefunction) is

$$Z_g = \exp(V\lambda_T^{-3}e^{\beta\mu_g}).$$

Also express the gas density $n_g = N_g/V$ and the pressure p in terms of λ_T and μ_g. (8 pts)

(d) Assuming $\mu_a=\mu_g$, express the fraction $f(=N_a/M)$ in terms of p, Δ, T, and other physical constants. (5 pts)

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과목명 : 양자역학 2011 . 12 . 22 시행

1. A particle of mass m is placed in a one-dimensional infinite potential well of width L,

$$V(x)=\begin{cases}0, & 0 \le x \le L \\ \infty, & x<0, x>L\end{cases}$$

with an additional perturbation

$$H_1(x)=\frac{\hbar^2}{2m}\alpha\,\delta\left(x-\frac{L}{2}\right),$$

where $0<\alpha<1$ is a real positive constant with the dimension of inverse length.

(a) Find the energy eigenvalues and corresponding eigenfunctions for $\alpha=0$, i.e., without the perturbation. Label the states by integer n (take n=1 as the ground state, n=2 as the first excited state, and so on.) (5 pts)
(b) Find the energy shift due to $H_1(x)$ for all the energy eigenvalues to the first order in the perturbation theory. (10 pts)
(c) Evaluate the energy shift to the second order in α for all the energy eigenvalues. (15 pts)
(d) Suppose that $\psi_n(x)$ denotes the exact eigenstate corresponding to the state "n" of (a). For $\alpha \neq 0$, $\frac{d\psi_n(x)}{dx}$ has a discontinuity at $x=\frac{L}{2}$. Find the expression for discontinuity

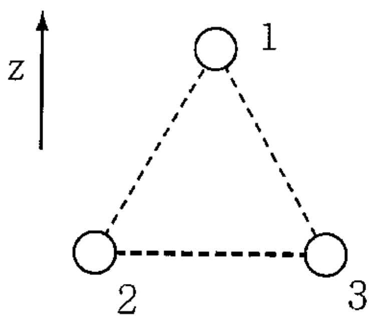
$$A_n=\frac{d\psi_n}{dx}\Big|_{x=\frac{L}{2}+0}-\frac{d\psi_n}{dx}\Big|_{x=\frac{L}{2}-0}$$

Sketch the wavefunction $\psi_{n=1}(x)$ and $\psi_{n=2}(x)$. (10 pts)

2. A spinless particle of charge q and mass m is moving in a two-dimensional xy-plane in the presence of a uniform magnetic field $\mathbf{B}=B\hat{z}$. Introducing a vector potential $\mathbf{A}$ such that $\mathbf{B}=\nabla\times\mathbf{A}$, we can write down the Hamiltonian as $H=\frac{1}{2m}\vec{\Pi}^2$ where $\vec{\Pi}=(\mathrm{p}-q\mathrm{A})$ with the vector potential $\mathrm{A}=\frac{1}{2}B\hat{z}\times\mathrm{r}=-(By/2)\hat{x}+(Bx/2)\hat{y}$.

(a) By considering the classical equation of motion and the Bohr-Sommerfeld quantization rule, i.e., $\oint \mathrm{p}\cdot d\mathrm{x}=h(n+1/2)$, find the radius l_B and the angular frequency ω_B of the smallest cyclotron orbit. (10 pts)
(b) Consider the quantum mechanical operators Π_x and Π_y. Show that the operators Π_x and Π_y satisfy the commutator relation $[\Pi_x,\Pi_y]=i\gamma^2$, where γ is some constant. Express γ in terms of $\hbar$, q, m, and B, and find the relation between γ and l_B. (10 pts)
(c) Solve the eigenvalue equation: $H|n\rangle=\epsilon_n|n\rangle$, and obtain the eigenvalues. (Note that the relation $[\Pi_x,\Pi_y]=i\gamma^2$ is analogous to $[X,P]=i\hbar$.) (10 pts)
(d) Obtain the average radius of the n-th eigenstate $|n\rangle$, i.e., $\bar{r}=(\langle n|(X^2+Y^2)|n\rangle)^{1/2}$. (Hint: In the classical motion, the radius r of a cyclotron orbit is related to $r=v/w$ and $\vec{\Pi}=m\vec{v}$.) (10 pts)

3. A molecule is made up of three identical atoms at the corners of an equilateral triangle as shown in the figure below. We want to consider its ion which is made by adding one electron to this configuration. Let $|i\rangle$ denote the localized state of the electron at the ith atom site. The matrix elements of the Hamiltonian for the system are $\langle i|H|j\rangle=-a$ (for $i\neq j$) and $\langle i|H|i\rangle=E_0$ $(i,j=1,2,3)$.



(a) Calculate the energy levels for the system. (10pts)
(b) Suppose that an electric field in the z direction is applied, so that the potential energy for the electron at the top is lowered by b with $|b|\ll|a|$. Now calculate the changed energy levels. (10 pts)
(c) Find the normalized ground state in the presence of the electric field. (10 pts)
(d) Suppose the electron is in the above ground state. Suddenly the field is rotated by 120° and points toward the site 2. Calculate the probability for the electron to remain in the new ground state. (10 pts)

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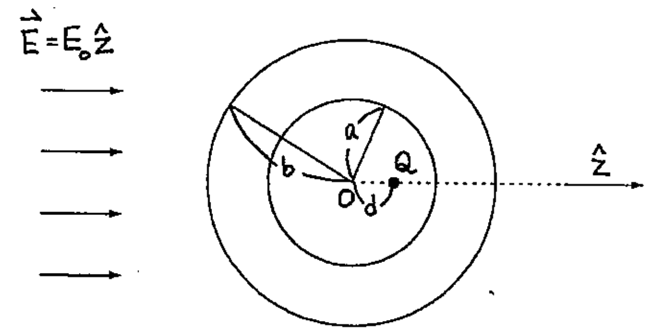
과목명 : 전기역학 2011 . 12 . 22 시행

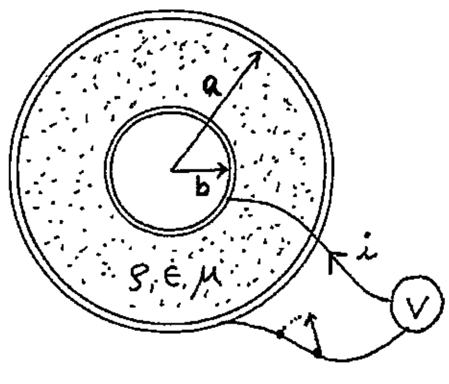
1. Let us consider the case of a conducting spherical shell (having zero net charge) of inner radius a and outer radius b in the presence of a uniform electric field $\vec{E}=E_0\hat{z}$. Inside the conducting shell we have a point charge Q, located at the position (0, 0, d) $\equiv d\hat{z}$ (here $d<a$) when the spatial origin is taken at the center of the shell. (The configuration is shown in the figure below.)



(a) Determine the electrostatic potential in the outside of the conducting shell (i.e., in the region $r>b$). (10 pts)
(b) Find the induced surface charge density on the outer surface of the conductor. (10 pts)
(c) Obtain also the electrostatic potential in the inside of the conducting shell (i.e., in the region $r<a$). (10 pts)
(d) Determine the force that the charge Q experiences. (10 pts)

2. Two concentric spheres of radii a and b ($a< b$), consisting of perfect metal, are maintained at a constant voltage difference of V_0. The space between the spheres are filled with a material with dielectric constant ε, magnetic permeability μ, and resistivity ρ.

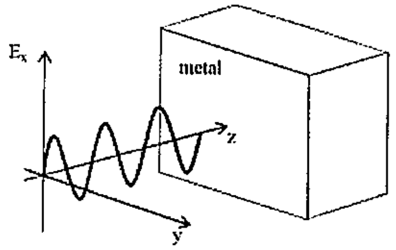
(a) Find the resistance R and the total current i_0 flowing between the spheres in terms of ρ, a, b, and V_0. (Hint: R is proportional to the length and inversely proportional to the area of the material.) (10 pts)



Now assume that the potential difference V suddenly drops to zero after $t=0$, *i.e.*, $V(t)=V_0\theta(-t)$.

(b) Obtain the amount of charge Q_0 on the inner sphere at $t=0$ and find the charge $Q(t)$. (10 pts)
(c) Calculate the electric field $E(r,t)$ for $a<r<b$. And show that no polarization charge exists, i.e., $\nabla\cdot\mathbf{P}=0$ between the spheres. (10 pts)
(d) Prove that, despite the current flow, no magnetic field **B** is generated for $t>0$ in the region between the spheres. (10 pts)

3. Consider a linearly polarized electromagnetic plane wave which is incident on a non-magnetic metal with an electric conductivity σ, dielectric constant ϵ, and magnetic permeability μ, as depicted in the following figure.



(a) From the Maxwell equations, show that the electric field inside the metal satisfies the following wave equation. (10 pts)

$$c^2\frac{\partial^2 E_x}{\partial z^2}=\epsilon\frac{\partial^2 E_x}{\partial t^2}+\frac{\sigma}{\epsilon_0}\frac{\partial E_x}{\partial t}$$

Here, c = speed of light, ϵ_0 = permittivity of vacuum, and Maxwell equations are given as

$$\vec{\nabla}\cdot\vec{D}=\rho \qquad \vec{\nabla}\times\vec{E}=-\frac{\partial\vec{B}}{\partial t}$$

$$\vec{\nabla}\cdot\vec{B}=0 \qquad \vec{\nabla}\times\vec{H}=\vec{J}+\frac{\partial\vec{D}}{\partial t}$$

(b) Show that we can write the solution to the wave equation in (a) in the form of

$$E_x=E_0\exp[i\omega(t-\frac{zn}{c})]$$

with a complex index of refraction n, where ω is the angular frequency of the electromagnetic wave. Write down the corresponding magnetic field inside the metal. (10 pts)

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(c) If we write the complex index of refraction as $n = n_r - ik$ (n_r and k are real quantities), we can parameterized n_r and k as

$$n_r^2 = \frac{1}{2}\left(\sqrt{\alpha^2+\beta^2}+\alpha\right) \quad k^2 = \frac{1}{2}\left(\sqrt{\alpha^2+\beta^2}-\alpha\right)$$

Find the expressions for α and β in terms of $\epsilon, \epsilon_0, \sigma$, and ω. (10 pts)

(d) Show that the electric field decays exponentially when it propagates in the metal. From this exponential decay, the skin depth is defined as the penetration depth at which the intensity of the light decreases to 1/e of its original value. What is the skin depth of the metal for low frequencies $\omega \ll \sigma/\epsilon_0$? (10 pts)

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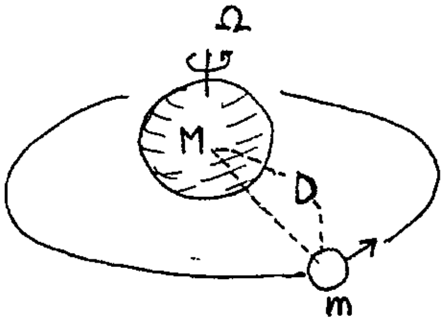
과목명 : 고전역학 2011 . 12 . 22 시행

1. A moon of mass m orbits with angular velocity ω around a planet of mass M. Assume a circular orbit and $m \ll M$. The rotation of the moon can be neglected, but the planet rotates about its axis with angular velocity Ω. The axis of rotation of the planet is perpendicular to the plane of the orbit. The moment of the inertia of the planet about its axis is I



(a) Find the distance from the moon to the center of the planet, D, in terms of ω. (5 pts)

(b) Find the expressions for the total angular momentum L and the total energy E of the system. (10 pts)

(c) Generally the two angular velocities ω and Ω are unequal. Suppose there is a mechanism such as tidal friction which can reduce E if $\omega \neq \Omega$, but conserves angular momentum. By examining the behavior of E as a function of ω, show that there is a range of initial conditions such that eventually $\omega = \Omega$ and a stable final configuration obtains. (10 pts)

(d) Show that the moon drifts away from the planet when $\Omega \gg \omega$. Can the moon completely escape from the planet? (5 pts)

2. Consider a particle of mass m which is constrained to move in a plane. It is subject to a force (directed exactly at the origin O)

$$F(r) = -\frac{k}{r^2} + \frac{k'}{r^3}\ , \ (k>0)$$

where r is the distance from O.

(a) What is the Lagrangian for the system in terms of the polar coordinates r, θ and their velocities? (7 pts)

(b) Write down the equation of motion for r and θ, and show that the orbital angular momentum l is a constant of motion. (7 pts)

(c) Assuming that $l^2 + mk' > 0$, find the equation for the orbit, i.e., r as a function of θ explicitly. (12 pts)

(d) For relatively small, but nonzero, k', what is the most characteristic feature of the orbit (consider a bounded orbit) compared to that with a strictly vanishing value for k'? (4 pts)