

1. For the following symmetrical linear triatomic molecules with the length of $l$ between the $A$ and $B$ molecules and masses $m_{\mathrm{A}}$ and $m_{\mathrm{B}}$,

(a) Write down the Lagrangian of the molecular system for the longitudinal motions when it is assumed that the potential energy of the molecule depends only on the distance between $A$ and $B$ molecules. Use longitudinal displacements $x_{1}, x_{2}$, and $x_{3}$ of the atoms and the force constant $k_{1}$.
(b) Rewrite down the Lagrangian of the molecular system for the longitudinal motions using the new symmetric and antisymmetric coordinates.
(c) Determine the frequencies of molecular vibration and discuss the vibrational motions.
2. A large parallel-plate capacitor with uniform surface charge density $+\sigma$ on the upper plate and $-\sigma$ on the lower is moving with a constant speed $v$ as shown in the figure. Assume that the distance between the plates is $d$. (All the quantities are measured in the laboratory frame, and therefore there is no need to consider the theory of special relativity.)
(a) What is the electrical force per unit area on the upper plate?
(b) Find the magnetic field between the plates and also above and below them.
(c) Find the magnetic force per unit area on the upper plate.
(d) At what speed would the magnetic force balance the electrical force? Express the result as a fraction of the speed of light c.

3. A Stern-Gerlach experiment is performed along $x$-axis and "spin down" is shielded (i.e. only "spin up" is selected for the following experiment.) The $y$-axis corresponds to the direction of the emitted spin $-\frac{1}{2}$ particles. Another Stern-Gerlach experiment is then performed, being tilted by 45 degree toward $z$-axis (from the x-axis) using "selected" particles.
(a) What is the expectation value of the final spin measurement?
(b) What is the probability of getting "spin down" for the outcome of the final measurement?

* Spin observables:
$S_{x}=\frac{\hbar}{2}\left(\begin{array}{ll}0 & 1 \\ 1 & 0\end{array}\right), \quad S_{y}=\frac{\hbar}{2}\left(\begin{array}{cc}0 & -i \\ i & 0\end{array}\right), \quad S_{z}=\frac{\hbar}{2}\left(\begin{array}{cc}1 & 0 \\ 0 & -1\end{array}\right)$
*Eigenvectors of the spin operator $S_{r}$ for arbitrary direction $\quad \vec{r} \quad \chi_{+}^{(r)}=\binom{\cos (\theta / 2)}{e^{i \phi} \sin (\theta / 2)}, \quad \chi_{-}^{(r)}=\binom{e^{-i \phi} \sin (\theta / 2)}{-\cos (\theta / 2)}$

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\widehat{r}=\sin \theta \cos \phi \widehat{i}+\sin \theta \sin \phi \widehat{j}+\cos \theta \widehat{k}
$$


4. Consider a lattice with $N \gg 1$ sites, each occupied normally by one atom. There are the same number of interstitial locations where atoms can be misplaced, and it costs an energy $\varepsilon$ to misplace an atom. Denote by $n_{0}$ and $n_{1}$ the occupation numbers of the energy levels 0 and $\varepsilon$, respectively ( $N=n_{0}+n_{1}$ ). The fixed total energy of the system is $U\left(U=n_{1} \varepsilon\right)$.
(a) Find the entropy of the system. Use the Stirling's approximation $(\ln N!\simeq N \ln N-N)$.
(b) Find the temperature of the system, $T$, as a function of $n_{0}$ and $n_{1}$. For what range of values of $n_{0}$ is $T<0$ ?
(c) In which direction does heat flow when a system of negative temperature is brought into thermal contact with a system of positive temperature? Why?

