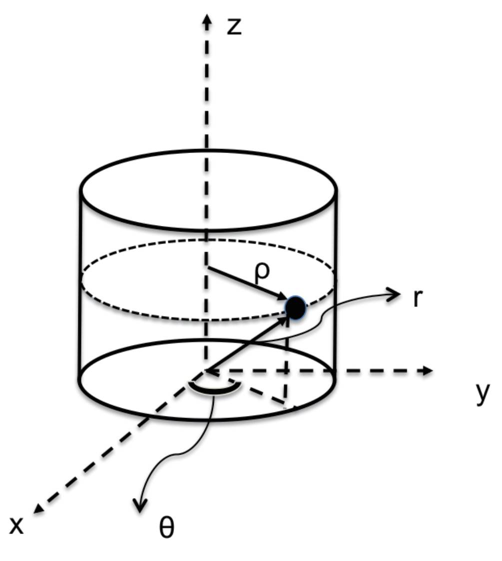
소속대학원	물리학부	학번		성명		감독교수 확인	(인)

과목명 : 고전역학 2016. 4. 29. 시행

1. (20 pts) Consider a bead of mass m which oscillates as a simple harmonic oscillator (SHO): $\vec{F}_{SHO} = -k\vec{r}$ where $\vec{r} = (x, y, z)$ and k is constant.

   a) (5 pts) Write down the equations of motion in each dimension and find a solution of the equations. If there is a damping force, $\vec{F}_{\text{damp}} = -b\dfrac{d\vec{r}}{dt}$ where b is constant, find solutions changed by the damping force with underdamping condition.

   Now the bead is constrained to move on a frictionless cylinder (on the surface) of radius R as shown in the figure. The force of constraint (the normal force of the cylinder) and the gravity $\vec{F}_g = -mg\hat{\mathbf{z}}$ act on the bead with the $\vec{F}_{SHO} = -k\vec{r}$ (ignore the damping force).



   b) (5 pts) Using cylindrical coordinates (ρ, θ, z), find the Lagrangian L and write down the Lagrange's equations of motion.

   c) (5 pts) Find a solution of the Lagrange's equations of motion. Is the angular momtnum conserved about the axis of symmetry of the system?

   d) (5 pts) The cylinder surface is now changed by the shape of a parabola, $cz = \rho^2$. Show how the Lagrange's equations of motion are changed.

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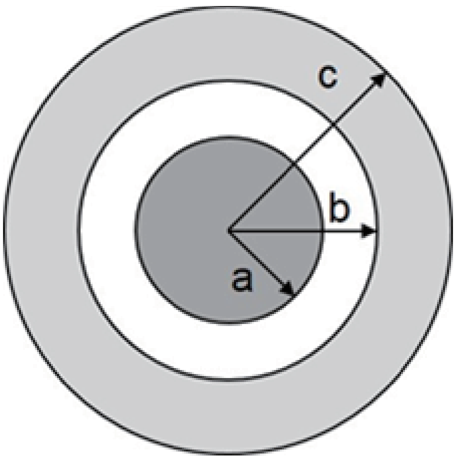
과목명 : 전기역학 2016. 4. 29. 시행

2. (20 pts) Consider a coaxial cable consisting of a copper wire of radius a, surrounded by a concentric copper tube of inner radius c. The space between the wire and the tube is partially filled (from b to c) with material of dielectric constant ϵ as shown in the figure. Q is the charge on a length ℓ of the inner conductor. (The space from a to b is vacuum.)



a) (5 pts) Find the electric displacement $\mathbf{D}$ at distance r $(a < r < c)$.

b) (5 pts) Find the electric field $\mathbf{E}$ at distance r $(a < r < c)$.

c) (5 pts) Find the potential difference between $r = c$ and $r = a$.

d) (5 pts) Find the capacitance per unit length of this cable.

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과목명 : 양자역학 1 　　　　　　　　　　 2016. 4. 29. 시행

1. (15 pts) Consider a one dimensional periodic potential of N potential cores with a period a, such that $U(r+a) = U(r)$. Hamiltonian of this system is given by

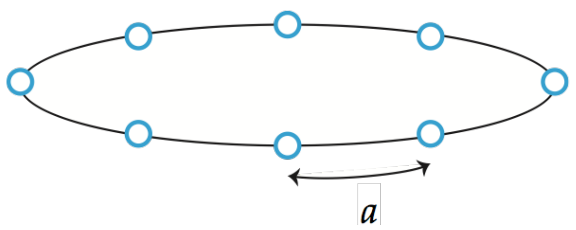
$$\widehat{H} = \frac{\widehat{P}^2}{2m} + U(\widehat{R}),$$

where $\widehat{P}$ is a momentum operator and $\widehat{R}$ is a position operator. The eigenfunctions of this Hamiltonian $\widehat{H}$ satisfy a periodic boundary condition

$$\psi(r) = \langle r|\psi\rangle = \langle r+Na|\psi\rangle = \psi(r+Na).$$

We can define a translation operator which translates r by a as $\widehat{T}_a$. That is, if one applies $\widehat{T}_a$ on a spatial position vector $|r\rangle$,

$$\widehat{T}_a|r\rangle = |r+a\rangle, \quad \langle r|\widehat{T}_a^\dagger = \langle r+a|.$$



a) (3 pts) Show that $\widehat{T}_a$ can be expressed as $\widehat{T}_a = e^{-i\widehat{P}a/\hbar}$.

b) (3 pts) Show that one can find $|\psi\rangle$ which is a simultaneous eigenfunction of both $\widehat{T}_a$ and $\widehat{H}$ of this system.

c) (3 pts) Let $|\psi\rangle$ be a simultaneous eigenfuction of $\widehat{T}_a$ and $\widehat{H}$. Say $\widehat{T}_a^\dagger|\psi\rangle = C_a|\psi\rangle$, $\widehat{H}|\psi\rangle = E|\psi\rangle$. Let $|k\rangle$ be an eigenfunction of $\widehat{P}$, $\widehat{P}|k\rangle = \hbar k|k\rangle$. By calculating $\langle k|\widehat{T}_a^\dagger|\psi\rangle$ find the eigenvalue C_a when $\langle k|\psi\rangle \neq 0$. Also express $\psi(r+ma)$ in terms of $\psi(r)$ for such k using the C_a value you found (m : an integer).

d) (3 pts) Express the eigenfunction $\psi(r) = \langle r|\psi\rangle$ of this system's Hamiltonian for a k ($\langle k|\psi\rangle \neq 0$) in terms of a periodic function $u(r) = u(r+a)$. In other words, express $\psi(r)$ as a product of periodic function of r and the rest.

e) (3 pts) Can the C_a value be continuous? Explain. How many distinctive C_a values are possible? Explain.

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과목명 : 양자역학 2 2016. 4. 29. 시행

2. (12 pts) Let us consider an electron with spin 1/2. Its spin operator $\mathbf{S} = (S_x, S_y, S_z)$ can be written in terms of three Pauli matrices, $\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$,

$$\mathbf{S} = \frac{\hbar}{2}\boldsymbol{\sigma},$$

where

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix},$$
$$\sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix},$$
$$\sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}.$$

Now we can define the two eigenstates of σ_z, such that $\sigma_z |+\rangle = +|+\rangle$ and $\sigma_z |-\rangle = -|-\rangle$

a) (3 pts) Find the eigenvalues and eigenstates for S_x. Express these eigenstates in terms of $|+\rangle$ and $|-\rangle$, i.e. the eigenstates of σ_z defined earlier.

b) (3 pts) One of the eigenstates obtained in (a) should satisfy $\sigma_x |\psi_x\rangle = +|\psi_x\rangle$. Find expectation values for three spin operators S_x, S_y and S_z for this eigenstate $|\psi_x\rangle$.

After preparing an electron in $|\psi_x\rangle$ state defined in (b), a uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}(B_0 > 0)$ has been applied to the electron at time $t = 0$. We would like to study the spin wave function of this electron under the uniform magnetic field as a function of time. The electron has the magnetic moment and its operator can be written as

$$\boldsymbol{\mu} = \frac{gq}{2m_e}\mathbf{S},$$

where q and m_e are the charge and the mass of the electron, while g is called g-factor and is equal to 2. Using the potential energy of a magnetic moment under the magnetic field, the Hamiltonian of this electron under $\mathbf{B} = B_0\hat{\mathbf{z}}(B_0 > 0)$ can be written as

$$\widehat{H} = -\boldsymbol{\mu} \cdot \mathbf{B}.$$

c) (3 pts) Show that the two eigenstates $|+\rangle$ and $|-\rangle$ are also eigenstates for this Hamiltonian $\widehat{H}$. What are the energy eigenvalues for $|+\rangle$ and $|-\rangle$ for this electron in $|\psi_x\rangle$ state under the uniform magnetic field $\mathbf{B} = B_0\hat{\mathbf{z}}(B_0 > 0)$?

d) (3 pts) Find the expectation value of S_x for this electron as a function of time t.

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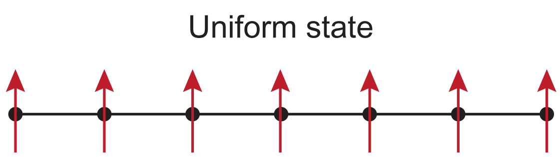
과목명 : 열통계 2016. 4. 29. 시행

3. (13 pts)Consider a one-dimensional finite-size lattice system composed of Ising magnetic dipoles S_i, which can either point up ($S_i = 1$) or point down ($S_i = -1$), with open boundaries at both ends. Here the subscript $i = 1, 2, ..., N$ denotes the lattice sites where an Ising dipole is located. Assuming that it is energetically favorable to have nearest-neighbor spins aligned in parallel, the sytem can be described by the following Hamiltonian,

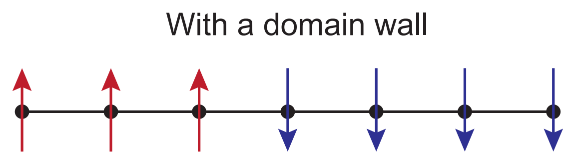
$$H = -J \sum_{i=1}^{N-1} S_i S_{i+1}, \quad J > 0.$$

   a) (4 pts) Show that the energy of a uniform state (as in the figure) with all spins pointing up is $E = -J(N-1)$.



   What is the free energy $F_{\text{uniform}} = E - TS$ of such a uniform spin configuration at a finite temperature T? (Hint: use the definition of the entropy $S = k_B \ln N_{\text{config}}$ where N_{config} is the number of possible configurations with the same energy.)

   b) (4 pts) Suppose that the system is composed of a spin-up domain and a spin-down domain with a domain wall in-between as shown in the following figure.



   What is the free energy F_{domain} of such a system? (Hint: the location of the domain wall can be anywhere in the lattice.)

   c) (5 pts) Comparing F_{uniform} and F_{domain}, argue that the average magnetization $m = \frac{1}{N} \sum_{i=1}^{N} \langle S_i \rangle$ is always zero at a finite temperature when $N \to \infty$.

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과목명 : 실험 2016. 4. 29. 시행

4. (20 pts) In research laboratories, we often have to measure the energy of particles such as photons and electrons. In case of light, one can use gratings to disperse the light (making different wavelengths go into different directions) and find the energy of the light from the direction. Such an instrument to disperse particles with different energies is, in general, called a spectrometer. From a practical point of view in physics research labs, one should design a spectrometer that is capable of measuring the energy over quite a large range. Let's imagine that we need to build a spectrometer to measure the energies of electrons in an electron beam in vacuum (that is, an electron analyzer). You can assume that the electrons have kinetic energies in the range of 100 eV – 10 keV which are typical energy values for electron beams for common physics instruments such as electron microscopes and electron analyzers.

   a) (8 pts) In case of electron beams, electrons with different kinetic energies travel along different paths in an electric or magnetic field. Please design an experimental set-up that uses electric or magnetic fields to disperse electrons and measure their energies. You can use one or more of the following parts: conducting plates, conducting spheres, conducting cylinders, solenoid, toroid, conducting wires, AC/DC power supplies, voltmeters, ammeters, and electron detectors. You can also use other instruments if necessary. Please be specific on experimental details such as generation of electric (or magnetic) fields and arrangements of spectrometer parts.

   b) (7 pts) Please explain the equation to extract electron energies from the measurements. In addition, discuss possible measurement errors.

   c) (5 pts) On the other hand, one can also imagine using a grating to disperse electron beams with different energies via the wave nature of electrons. Considering previous research in physics history, please discuss what can be used as a grating for electron beams. Also, from a practical point of view, discuss possible problems using such a grating for an electron analyzer.